SECTION 1

Standards and Activities for Grade 6

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Ratios and Proportional Relationships: 6.RP.1

"Understand ratio concepts and use ratio reasoning to solve problems."

1. "Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities."

BACKGROUND

A ratio is a comparison of two numbers. A ratio can compare a part to a whole, a whole to a part, a part of a whole to another part, or a rate (a comparison of two different quantities).

Ratios may be expressed in three ways: with "to," by a colon, or by a fraction bar. For example, a ratio that compares the value of a quarter to a dollar can be expressed as 1 to 4, 1:4, or $\frac{1}{4}$.

ACTIVITY: RATIOS ALL AROUND US

Working in groups of three or four, students will select a topic and write ratios that compare numbers associated with their topic. They will then create a poster, illustrating the meaning of select ratios.

MATERIALS

Math, science, and social studies texts; reference books, particularly almanacs and atlases; poster paper; markers; rulers; scissors; glue sticks. Optional: computers with Internet access.

PROCEDURE

1. Explain that numbers are constantly compared. Provide examples such as 1 inch on a map equals 50 miles, 1 pound of chopped meat makes 4 hamburgers, and a team's record of wins to losses is 2 to 1. These are all examples of ratios.
2. Explain that each group is to select a topic and write at least ten ratios associated with their topic. They are to then choose five of their ratios and create a poster that illustrates the meanings of these ratios.

3. To help your students get started, offer a broad list of topics and examples of possible ratios they may consider, such as:
    - Transportation: distance between cities, gas mileage, amount of luggage per person, costs per trip.
    - Cooking and baking: servings per person, cooking times per pound of food, ratios of ingredients.
    - Sports: per-game averages for individual players and teams, won-loss records, attendance.
    - Amusement parks: admission prices, types of attractions, roller-coaster statistics.
    - Information about their state, town or city, or school.
4. Encourage your students to brainstorm other possible topics.
5. After students have selected their topics, they should research the topics. Along with using books, they may also find the Internet helpful, especially for finding statistics on various topics. Remind them that they are to write at least ten ratios associated with their topics. From these they should select five that they will illustrate on a poster.
6. Encourage your students to be creative, neat, and accurate with their posters.

CLOSURE

Have each group of students present their poster to the class, explaining their selections of ratios. Display the posters in the room.

Ratios and Proportional Relationships: 6.RP.2

"Understand ratio concepts and use ratio reasoning to solve problems."

> 2. "Understand the concept of a unit rate a/b associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship."

BACKGROUND

A ratio is a rate that compares two quantities. A unit rate compares a quantity to 1.

To find a unit rate, divide the numerator and denominator of a ratio expressed as $\frac{a}{b}$ by b where $b \neq 0$. For example, if Milo ran 1 lap in $1\frac{1}{4}$ minutes, the ratio of laps to minutes is $\frac{1}{1\frac{1}{4}}$. The unit rate is found by dividing the numerator and denominator by $1\frac{1}{4}$ so the denominator is equal to 1. Milo ran $\frac{4}{5}$ of a lap in 1 minute.

ACTIVITY: UNIT RATE TIC-TAC-TOE

This activity is best implemented in two days. Students will first work individually and then in groups of four or five, with each group divided into two teams. Each team will create problems about finding unit rates that the other team will solve. As they solve, or fail to solve, the problems, the teams will complete a tic-tac-toe board.

PROCEDURE

1. On the first day of the activity, explain that each student is to create five unit rate problems. They should make an answer key for their problems on the back of the sheet. Caution them not to show their problems to other students. After students have finished their problems, collect them and check that the answers to the problems are correct.
2. The next day, return the problems to their owners and divide students into groups.
3. Within each group, students should form two teams: One team will be the "X" team and the other will be the "O" team. One student should draw a tic-tac-toe board.

4. Explain the rules of the game:
    - A member of the X team will read a unit rate problem, which the members of the O team must solve. If the O team solves the problem correctly, they may place an O on any square of the tic-tac-toe board. If their answer is incorrect, they must place an X on any square of the board.
    - A member of the O team now reads a unit rate problem, which the members of the X team must solve. If the X team solves the problem correctly, they may place an X on any square of the tic-tac-toe board. If their answer is incorrect, they must place an O on any square of the board.
    - The object of the game, of course, is to get three Xs or three Os in a row. The game continues until there is a winner or there is a draw. If a second game is played, a member of the O team reads the first problem. To play more games, students may need to create more unit rate problems.
5. As students play, you may find it necessary to assume the role of referee and provide an explanation for answers that students challenge.

CLOSURE

Announce the winners of the games. Review any problems that proved to be particularly troublesome. Ask students to summarize how to find a unit rate

Ratios and Proportional Relationships: 6.RP.3

"Understand ratio concepts and use ratio reasoning to solve problems."

3. "Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- **a.** "Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- **b.** "Solve unit rate problems including those involving unit pricing and constant speed.
- **c.** "Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.
- **d.** "Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities."

BACKGROUND

Problems involving ratios can be solved using various methods, such as creating tables, tape diagrams, double number line diagrams, graphs, or writing equations. Most students are familiar with using tables, graphs, and equations.

To construct a table, use the ratio in the problem and create equivalent ratios. For example, if Sarah earns $200 during a typical 8-hour workday, create a table to show the amount of money she earns each hour. First, scale the ratio $\frac{\$200}{8\text{hr}}$ down to its unit rate by dividing the numerator and denominator by 8. The unit rate is $25 per hour. Using this information, you can create a table showing the money Sarah earns each hour. The data display in a table can also be used to find the missing values. For example, if Sarah worked 5 hours, write a proportion $\frac{4}{100} = \frac{5}{x}$ and solve for x. Sarah earned $125. If she earned $175, set up a proportion $\frac{6}{150} = \frac{x}{175}$ and solve for x. Sarah worked 7 hours. (Note that any ratio of time to money may be used in writing a proportion.)

Time (hours)	Money Earned
1	$25
2	$50
3	$75
4	$100
5	
6	$150
	$175
8	$200

The data from this table can also be used to create a graph. Time in this situation is the independent variable (graphed on the *x*-axis) and money earned is the dependent variable (graphed on the *y*-axis). The time and money earned represent an ordered pair on the graph. On this graph, the first ordered pair is (1, 25). An equation to represent this situation can be written from the table, graph, or unit rate. The equation $y = 25x$, where y represents the money earned and x represents the amount of time, shows the relationship between time and money earned. This equation can be used to find the amount of money earned or hours worked.

ACTIVITY: THE FASTER RATE

Working in groups of three or four, students will walk or run 20 meters and record their times. They will use their data to construct a table and a graph and write an equation. They will then use each representation to compare their rate to the rates of other students in the class.

MATERIALS

One stopwatch for each group; graph paper; at least two cones.

PROCEDURE

1. Take your students outside or to the gym and place two cones 20 meters apart. (If you have a large class, you may want to set up additional pairs of cones.)
2. Explain that students will walk or run 20 meters and record their data. As one student walks or runs, another student will use a stopwatch to track the time it takes the student to travel from one cone to the other. A third student will write this time down on a sheet of paper. The students will then switch tasks until each person has walked or run from one cone to the next.
3. After all students have finished walking or running, return to the classroom and instruct groups to write their rates as ratios of distance to time. Explain that they must create a table showing the distance they were at during each second of their walk or run, assuming they were walking or running at a constant speed. In order to do this, students must find the unit rate by dividing both the distance and time by the time. Instruct your students to round their answers to the nearest tenth. After they create a table, they must use the information on the table to create a graph of their data. Finally, they will write an equation, expressing their rate as a function of time.
4. After all of the students have represented their rates in a table, graph, and equation, instruct them to compare their rates with the rates of another group. In these larger groups, students should examine the tables, graphs, and equations to determine who was walking or running at the fastest rate. For each representation, they should discuss how the fastest rate is shown. For example, by looking at a table, the fastest rate is shown by the greatest increase in distance per second. In a graph, the fastest rate is

shown by the steepest line. In an equation, the fastest rate is shown by the slope. (The greatest coefficient of x indicates the fastest rate.)

5. Working as a class, list the results of each group on the board. Ask for volunteers to explain how they could use this data to find a percent of a quantity as a rate per hundred. For example, if a student walked 3 meters in the first second, find the percent of the distance he traveled. This can be expressed as $\frac{3\text{ m}}{20\text{ m}} = \frac{x}{100}$ or 15%. The student finished 15% of his total walk in one second. Note that students can verify this by finding 15% of 20 meters to get 3 meters.

6. Instruct your students to work in their original groups to generate a problem involving the whole, given a part and a percent. For example, suppose a student walked 15 meters, which was 20% of the total distance. To find the total distance, write and solve a proportion $\frac{20}{100} = \frac{15}{x}$. Share the problems the groups generate with the class. Students should solve the problems of other groups.

7. Working as a class again, instruct your students to use ratios to convert measurement units; for example, meters to centimeters, meters to kilometers, or meters to feet. For example, suppose a student walked 3 meters in one second. Meters per second can be converted to centimeters per second by using ratios in a proportion: $\frac{3\text{ m}}{1\text{ sec}} \times \frac{100\text{ cm}}{1\text{ m}} = \frac{300\text{ cm}}{1\text{ sec}} = 300$ cm per second. Note that setting up the ratios and proportions correctly ensures that meters cancel out and the answer will be centimeters per second. Also explain that because 100 cm = 1 m, multiplying by $\frac{100\text{ cm}}{1\text{ m}}$ is the same as multiplying by 1.

8. Instruct your students to work in their original groups to generate a problem involving the use of ratios to convert measurement units. Share the problems the groups generate with the class. Students should solve the problems of other groups.

CLOSURE

Discuss the overall results of your students as a class. Review the process of taking a rate and representing it in a table, graph, and equation. Also discuss how to determine faster rates in tables, graphs, and equations, and how students may use ratio reasoning to solve problems.

The Number System: 6.NS.1

"Apply and extend previous understandings of multiplication and division to divide fractions by fractions."

1. "Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem."

BACKGROUND

A division problems has three components—a dividend, a divisor, and a quotient. For example, in $\frac{1}{2} \div \frac{1}{6} = 3$, $\frac{1}{2}$ is the dividend, $\frac{1}{6}$ is the divisor, and 3 is the quotient.

To write a related multiplication problem, multiply the divisor by the quotient, which will equal the dividend. $\frac{1}{6} \times 3 = \frac{1}{2}$. This may also be written as $\frac{1}{6}$ of $3 = \frac{1}{2}$.

ACTIVITY: MODELING DIVISION OF FRACTIONS

This two-day activity requires students to work in pairs or groups of three. On the first day, each group will write a word problem, create a model, and write an equation to solve the problem. On the second day, groups will present their work to the class.

MATERIALS

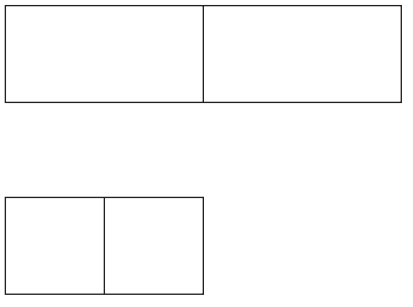
Fraction bars and fraction circles to make models (rulers and compasses may be used instead).

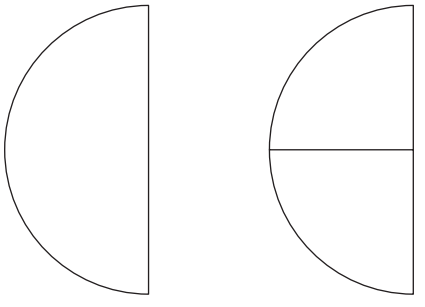
PREPARATION

At the end of the first day, collect each group's work. Make a copy of the work that contains each group's problem, model, and equation. (You will return the copy of their work to the group to use as an answer key.) Keep the original and cross out the names of the group's members. Make enough copies of each original for each group to have one copy of every group's work. Use a paper cutter to cut each copy you made into thirds—each third containing a problem, model, and equation—which you will distribute on the second day. Each pair or group of students will then have a separate copy of every other group's problem, model, and equation.

PROCEDURE

1. On day one, explain that division involves finding the number of parts in the number to be divided. Be sure your students understand these terms: dividend, divisor, and quotient.
2. Offer this problem: How many quarters are in half of a dollar?
   - Present the following models to your students. The first one uses fraction bars to show the number of quarters in a half dollar, and the second uses fraction circles.





   - Ask your students how many fourths can be placed on the model of half of the fraction bar. Or, using the fraction circles, ask how many fourths of a circle can be placed in half of the circle. In both cases, the answer is 2.
   - Explain that this problem can also be modeled by the equation $\frac{1}{2} \div \frac{1}{4} = 2$. The answer can be checked by multiplying the divisor by the quotient, which should equal the dividend. $\frac{1}{4} \times 2 = \frac{1}{2}$, which is the same as $\frac{1}{4}$ of $2 = \frac{1}{2}$.
3. Explain that each pair or group of students is to create a word problem that can be solved by dividing fractions. They are to create a model using fraction bars or fraction circles that represent their problem and write an equation that can be used to solve the problem. Remind students that their work must be accurate and neat, because it will be read by other students. After everyone in the group agrees that their work is correct, they should recopy it in the following manner:
   - Start by holding a sheet of lined paper so that the lines are horizontal.
   - Divide the paper into thirds by folding the bottom of the paper up and the top of the paper down.
   - Write the problem in the upper third of the paper, sketch the model in the middle third, and write the equation in the lower third.

   After they have finished, each group should hand their paper to you, which you will then copy and cut into thirds. Organize the thirds into three sets, one set for the problems, the second for the models, and the third for the equations.

4. On the second day, randomly distribute the copies of the problems, models, and equations (all of which were previously cut into thirds) to each group. Each group should receive a problem, model, and equation that was created by every other group.
5. Instruct your students to match each problem with the model and its equation.

CLOSURE

Share students' results by having one member of a group read the group's original problem, model, and equation. The other groups can then check their work.

The Number System: 6.NS.2

"Compute fluently with multi-digit numbers and find common factors and multiples."

2. "Fluently divide multi-digit numbers using the standard algorithm."

BACKGROUND

The standard algorithm for long division involves the following steps:

1. Divide.
2. Multiply.
3. Subtract.
4. Check that the remainder is smaller than the divisor. If the remainder is larger than the divisor, increase the quotient by 1 and redo the multiplication and subtraction.
5. Bring down the next number and repeat the process until there are no more digits to bring down.

ACTIVITY 1: SNORK'S LONG DIVISION

Students will practice long division on a Web site.

MATERIALS

Computers with Internet access.

PROCEDURE

1. Review the process of long division. Present examples such as $41\overline{)515}$ (Answer = 12 R23) and $58\overline{)3{,}604}$ (Answer = 62 R8) by explaining each step. Remind students that they can check the answer of any division problem by multiplying the quotient by the divisor and adding any remainder. If their quotient is correct, their answer should equal the dividend.
2. Instruct your students to go to the Web site http://www.kidsnumbers.com/long-division.php. Direct them to the box of Snork's Long Division and tell them to type in the highest number you want them to work with. They should click on "Play," and they will

be guided through the process of long division. After completing a problem, they will be given the opportunity to try another one by clicking on ''Again.''

3. Allow students about ten to fifteen minutes to practice problems on the Web site.

CLOSURE

Ask students to summarize the steps for long division.

ACTIVITY 2: LONG DIVISION RELAY RACE

Sitting at their desks in rows, students will complete a long division relay race. The first row to complete the most problems correctly is the winner.

MATERIALS

Blank sheets of paper.

PREPARATION

On a blank sheet of paper, write a long division problem on the upper left side. Write the quotient on an answer sheet for your reference. With a ruler, draw a line down the middle of the paper. At the top of the right side, write ''Work Space.'' Do this four more times, writing a new problem on a new sheet of paper each time. Make photocopies of each sheet so that there is one set of sheets (five separate problems) per row. Label each sheet in each set by row, for example, ''Row 1,'' ''Row 2,'' and so on. (This will make it easy to keep track of correct answers for each row as the race goes on.) Place each set of sheets for each row in a separate pile. These sheets will serve for one relay race.

PROCEDURE

1. Arrange students' desks into rows, designating the rows by numbers.
2. Explain that students will take part in a long division relay race. Following are the rules of the relay:
   - You will hand a sheet of paper, face down, with the same division problem to the first student in each row. Note that students should work the problem out on the left-hand side of the paper, but may use the right-hand side for additional computation, if necessary.

- Once the race begins — on your signal — the first person in each row will do the first step of the problem, which is to divide. He will then pass the paper to the student behind him who will do the next step, multiplication. This student then passes the sheet to the student behind him, who will subtract. This student then passes the paper to the student behind him, who will compare the difference with the divisor, making any corrections, if necessary. He then passes the paper to the student behind him who brings the next number down and divides. This procedure continues until the problem is completed, at which point the student who completes the problem brings it to you. If a row has fewer students than other rows, the last person must bring the problem back to the first person in the row to complete the next step.
- Emphasize that at any step along the way, if a student finds a mistake made by someone earlier in the row, she should correct it. Otherwise, her row will not find a correct answer.
- After a row finishes the problem and the last person who worked on the problem brings it to you, you will give this student another sheet and the relay race continues by this student starting the division process on the new problem. In the meantime, you will assign one point to the row if their answer was correct, but no point if it was incorrect.
- The race ends when all of the rows have completed all of the problems. The row that completes the most problems correctly is the winner of the race. In case of a tie, the row that was first to finish is the winner.

3. Start the race. May the fastest (and most correct!) row win.

CLOSURE

Discuss any problems that students had difficulty solving.

The Number System: 6.NS.3

"Compute fluently with multi-digit numbers and find common factors and multiples."

3. "Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation."

BACKGROUND

Even in this age of lightning-fast calculators, understanding basic operations is an essential skill. Not only does a solid understanding of operations with decimals provide a foundation for mastering higher mathematical skills, but there will be times students will need to add, subtract, multiply, and divide decimals without the aid of a calculator.

ACTIVITY: DECIMAL OPERATION TOURNAMENT

Students will compete in a decimal operation tournament in which they will have to add, subtract, multiply, and divide decimals on the board. The fastest and most accurate problem-solver wins.

MATERIALS

Depending on whether you have a traditional chalkboard or whiteboard, you will need enough chalk or markers for three to five students to work at the board at a time.

PREPARATION

Make a list of problems—addition, subtraction, multiplication, and division—and their answers for the tournament. You might simply use problems from your text. Your list should contain about 30 problems. You should also have a list of students' names to use as a score sheet.

PROCEDURE

1. Explain the rules of the tournament:
    - You will choose students (the number will be determined by the amount of board space you have) to go to the board. Starting with one row and then working through the others is a good plan, making it easy to keep track of students.

- You will present a problem; for example, $0.4 + 1.23$. Students will write the problem on the board.
- On your word, "Go," students will begin solving the problem. The first student to solve the problem correctly receives a point and remains at the board. The others return to their seats and are replaced by new contestants. Be sure to mark a point by the winner's name on your score sheet.
- You present another problem and the tournament continues.
- Students at their seats should also try to solve the problem. As they finish the problem, the first student to finish should raise her hand and call out—but not too loudly—"One," the second student "Two," and the third "Three." Only the first three should call out. Should no one at the board solve the problem correctly, the first of the students at their seats to raise her hand and solve the problem correctly wins the point. This student then automatically goes to the board with the next set of students. (She does not have to wait for her turn to come later.) If the first student at her seat does not have the correct answer, the student who called "Two" gets a chance to provide the answer, and if this student's answer is also incorrect, the student who called "Three" gets a chance. If no one gets the right answer, a new set of students replaces those who were at the board and the tournament continues. Students at their seats will quickly realize the advantage of trying to solve the problem, which is that they will get to the board more often where they will have a chance of accumulating more points.
- The game should continue until all students have a chance to get to the board a few times.

2. Start the tournament. Beware: your students will likely participate with enthusiasm and energy.

CLOSURE

Discuss any problems that the class found difficult to solve.

The Number System: 6.NS.4

"Compute fluently with multi-digit numbers and find common factors and multiples."

> 4. "Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor."

BACKGROUND

A factor is a number that evenly divides into a larger number. The greatest common factor (GCF) is the largest number that divides into two or more numbers evenly. Students can find the GCF by listing the factors of two or more numbers and selecting the largest number that appears in both lists.

A multiple of a number is the product of the number and a counting number. A common multiple is a number that two or more numbers evenly divide into. The least common multiple (LCM) is the smallest multiple of the numbers. Students can find the LCM by listing the multiples of the numbers and selecting the smallest number that is on both lists, or they can find the product of the two numbers and divide it by the GCF.

The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$. It relates multiplication to addition because students can distribute a factor to each term enclosed in the grouping symbol.

ACTIVITY: THE NUMBERS GAME

Students will work in groups of three or four for this activity. They will play a game in which they will find the greatest common factor, the least common multiple, and use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factors other than 1.

MATERIALS

3″ × 5″ index cards; dark marker; small whiteboards. Optional: graphing calculator.

PREPARATION

To play the game, you will need to generate sets of random numbers. Using a dark marker, write the numbers 1–100 on index cards, one number per card. Write the numbers 1–12 on a second set of index cards, one number per card. On ten index cards, write the sum of two

whole numbers that have a common factor, one sum per card; for example, $12 + 15$. Base the sums on the abilities of your students. The first set of cards will be used to find the GCF, the second set will be used to find the LCM, and the third will be used with the distributive property. (Note: A graphing calculator may instead be used for generating random numbers, using the random integer menu.)

PROCEDURE

1. Write each group's number on the board or screen for keeping score; for example, Group 1, Group 2, and so on.
2. Explain how the game is played. Groups play against each other. The game has three rounds. In Round 1, you will present two random numbers. The students in each group are to find the GCF of the numbers. In Round 2, you will present two random numbers. Groups are to find the LCM of the numbers. In Round 3, you will present a sum of two numbers. Groups are to use the distributive property to rewrite the sum as a multiple of a sum of two numbers that has no common factor other than 1. Each correct answer is worth one point. The group with the most points at the end of the game wins. Play a sample of each round of the game so that everyone understands what to do. If necessary, review finding the GCF, finding the LCM, and using the distributive property.
3. Play the game as described below:
   - Start Round 1 by shuffling your first set of cards and randomly drawing two cards. Write the numbers of the cards on the board. Groups are to find the GCF of the two numbers. As students are working to find the answer, you should find the answer, too. Placing a time limit of 30 to 45 seconds for groups to work out their answers keeps the game moving. Each group should write their answer on their whiteboard and place the board face down. (Instead of the whiteboards, you may substitute clear page protectors. Place a white sheet of paper behind the page protector for a background.) Ask that a member of each group hold up the group's whiteboard. Check the answers and place a checkmark by the names of the groups that have the correct answer. Repeat the procedure. Choosing five to ten pairs of numbers makes for a reasonable round.
   - For Round 2, shuffle your second set of cards and choose two cards. Students are to find the LCM. Follow the procedure established in Round 1.
   - For Round 3, select a card from your third set. Students are to use the distributive property to rewrite the sum of the numbers as a multiple of the sum of two whole numbers that have no common factor other than 1. Follow the established procedure.

CLOSURE

Tally the scores. The group with the most points wins. For a tie-breaker, present another problem. The first group to find the correct answer wins.

The Number System: 6.NS.5

"Apply and extend previous understandings of numbers to the system of rational numbers."

> 5. "Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation."

BACKGROUND

Numbers are used to measure or describe real-world situations such as those noted above. Every number except zero has an opposite. 5 is the opposite of -5, $-\frac{2}{3}$ is the opposite of $\frac{2}{3}$, a gain of 1 pound, which can be represented by $+1$ or 1, is the opposite of the loss of a pound, which can be represented by -1. The meaning of zero depends on the situation.

In the set of real numbers, zero is the reference point that separates positive numbers from negative numbers. In terms of losing or gaining weight, for example, zero represents the original weight before any losses or gains.

ACTIVITY: FINDING THE OPPOSITE

Working individually or in pairs, students will create an origami fortune teller whose flaps will reveal the opposite of a number or quantity, instead of a fortune.

MATERIALS

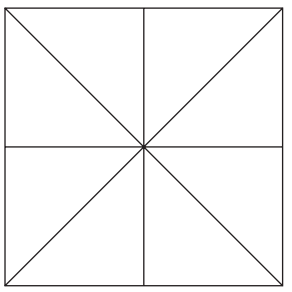
$8\frac{1}{2}'' \times 11''$ sheets of unlined paper, one per student.

PREPARATION

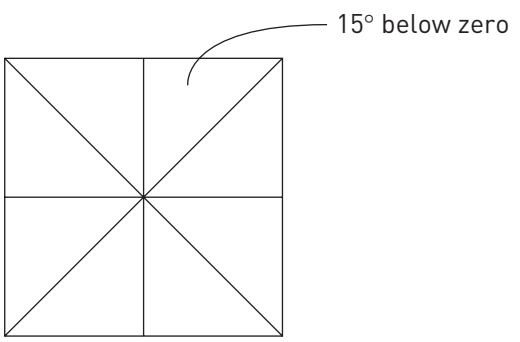
For each student, make an $8\frac{1}{2}'' \times 8\frac{1}{2}''$ square by cutting a $2\frac{1}{2}'' \times 8\frac{1}{2}''$ strip from a sheet of unlined paper. Go to the Web site http://www.dltk-kids.com/world/japan/mfortune-teller.htm where you will find directions for making an origami fortune teller. (You may also search for other directions with the term "directions for making an origami fortune teller.") Create an origami fortune teller for yourself. Not only will you have an example to show your students, but you will familiarize yourself with the procedure so that you can assist students in making their own origami fortune tellers.

PROCEDURE

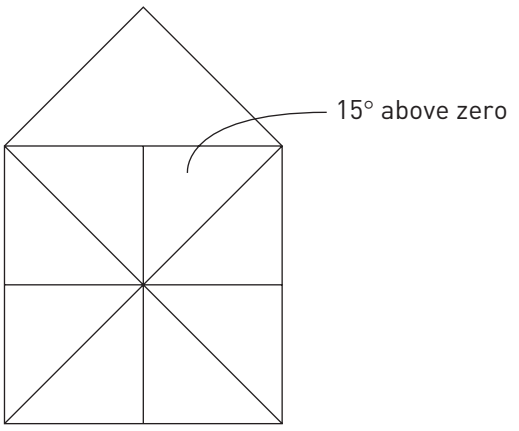
1. Explain to your students that they will make an origami fortune teller, but instead of the flaps revealing a fortune, they will reveal the opposite of the situation described on the flap.
2. Direct your students to the Web site given under Preparation for directions. (You may instead prefer to project the Web site onto a screen and guide your students through the steps.)
3. As students are working on their origami fortune tellers, circulate around the room, providing help as necessary. There will be questions.
4. After students have completed folding their fortune tellers, instruct them to write on the flaps in the following manner:
   - Fold the paper in half and write "Left" on one square and "Up" on the other.
   - Turn the paper over and write "Right," then "Down."
   - Open the paper and turn it over to reveal eight triangles, as shown below.



   - Choose one triangle and write a situation that can be modeled by a positive or negative integer; for example, 15° below zero. Do this for each of the triangles so that eight situations are described.



- Lift up a flap. Write the opposite of the situation described on the flap above it. Repeat this process so that the opposite of each of the eight situations is described.



- Refold the paper.

5. Instruct your students to use their fortune tellers to ask a partner (or a classmate if they are working individually) to choose "Left," "Up," "Right," or "Down." Explain that they should do the following:
   - Flap the fortune teller while spelling out the word their partner chooses.
   - Ask their partner to choose one of the situations that is revealed and ask him to identify the opposite of the situation.
   - Lift the flap to see if he is correct.
6. Students should switch roles and continue.

CLOSURE

Ask your students to share some situations and opposites. Ask them to explain the meaning of zero in each of the situations.

The Number System: 6.NS.6

"Apply and extend previous understandings of numbers to the system of rational numbers."

6. "Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- **a.** "Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- **b.** "Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the location of the points are related by reflections across one or both axes.
- **c.** "Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane."

BACKGROUND

Every rational number can be graphed on a number line. If the number line is horizontal, positive numbers are located to the right of zero and negative numbers are located to the left of zero. If the number line is vertical, positive numbers are above zero and negative numbers are below zero. Numbers that have opposite signs are graphed the same distance from zero, but in opposite directions.

A coordinate plane is formed when a horizontal number line and a vertical number line intersect at a right angle. The point where the two number lines intersect is the origin. It is represented by (0, 0). Every ordered pair can be graphed in a coordinate plane.

ACTIVITY 1: GRAPHING ON A NUMBER LINE

Students will find, record, and graph on a number line the record high temperatures and record low temperatures of two states in the United States. This activity may be done in class or at home.

MATERIALS

Rulers; blank paper; markers or colored pencils; current almanacs and other reference books; computers with Internet access.

PREPARATION

Make a ''Student Sign-Up Sheet for States'' listing each state to serve as a record of the states students will research.

PROCEDURE

1. Explain that each student will sign up to research the record high temperature and record low temperature of two states of the United States. (If you have fewer than 25 students in your class, perhaps some students can research an additional state. If you have more than 25 students, some students may research the same states.)
2. Assign, or allow students to select, the states they will research. Having them write their names next to their states on the ''Student Sign-Up Sheet for States'' is a simple way to record which students are doing which states.
3. Suggest that students use reference materials or the Internet to find their information. They may search the Internet with terms such as ''highest and lowest temperature in U.S. states.'' They may search specific states as well. They should record the dates of the highest and lowest temperatures, as well as the source of their data.
4. Instruct your students to record all temperatures in degrees Fahrenheit. If they find a temperature in degrees Celsius, they should use the formula $F = 1.8C + 32$ to convert the temperature to the Fahrenheit scale.
5. Students should construct either a horizontal or vertical number line by finding the range of values, choosing a scale for each interval, and then graphing each temperature on the number line. They should label the corresponding values for the highest and lowest temperatures of their state and the name of the state. Students should also record the date of the temperatures beneath the temperature on their number line.

CLOSURE

Display students' number lines in your classroom. Ask your students to identify both the highest and lowest temperatures in the United States. You might also ask them to look for patterns. For example, how does elevation affect temperature? Which states, in general, have the highest ''high'' temperatures, and which have the lowest ''low'' temperatures. What are some of the biggest ranges of low and high temperatures?

ACTIVITY 2: BONK THE MOLE

Students will complete this activity on a Web site. They will find and bonk a mole by identifying the mole's coordinates on the coordinate plane. Most students will enjoy bonking the mole as they are learning about the coordinate plane.

MATERIALS

Computers with Internet access.

PROCEDURE

1. Instruct your students to go to the Web site http://funbasedlearning.com/algebra/graphing/default.htm.
2. Explain that they will play a game where they must "bonk" a mole before he eats vegetables. To bonk him, they must identify his coordinates.
3. Review the coordinate plane by instructing your students to click on "Easy Version of Graph Mole." Instruct them to click on "Introduction," read the introduction, then play the game.
4. Once your students have mastered the easy version of the game, encourage them to try the medium and harder versions.

CLOSURE

Ask your students questions such as the following: In what quadrant are both coordinates positive? (Quadrant I) In what quadrant are both coordinates negative? (Quadrant III) In what quadrant is x positive and y negative? (Quadrant IV) In what quadrant is x negative and y positive? (Quadrant II) If two ordered pairs differ by the sign of the x-coordinates, the points are related by a reflection across an axis. Which axis are the points reflected across? (the y-axis)

The Number System: 6.NS.7

"Apply and extend previous understandings of numbers to the system of rational numbers."

7. "Understand ordering and absolute value of rational numbers.
   - **a.** "Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
   - **b.** "Write, interpret, and explain statements of order for rational numbers in real-world contexts.
   - **c.** "Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
   - **d.** "Distinguish comparisons of absolute value from statements about order."

BACKGROUND

Every real number can be represented as a point on a number line. A point labeled 0 is called the origin of the number line. Points to the right of 0 represent positive numbers; points to the left of 0 represent negative numbers. Integers, fractions, and decimals can be graphed on a number line. The absolute value of a number is the distance a number is from 0 on the number line. Because a distance is always a positive number, the absolute value of any number, except 0, is always positive.

A number line is divided into equally spaced intervals. For most number lines, each interval represents an integer. To graph a number on a number line, mark the point on the number line that is paired with the number.

ACTIVITY: AN OLD-FASHIONED NUMBER LINE

Working in groups of three or four, students will create number lines and display points that relate the ordering and absolute value of rational numbers. After completing their number lines, they will share their work with the class.

MATERIALS

Poster paper; rulers; different-colored markers.

PROCEDURE

1. Explain to your students that the ordering of numbers and absolute values can be represented on a number line. ''Seeing'' the position of numbers and values on a number line can help in understanding numerical relationships.
2. Offer this example to your students: $-5 < 8$. This can be shown on the number line by graphing -5 and then graphing 8. This can be stated as -5 is to the left of 8 or 8 is to the right of -5.
3. Write the following statements on the board:
   - $-2 > -3$
   - $3 < 4$
   - $-\frac{1}{2} < 0$
   - $-8 > -8\frac{1}{2}$
   - $5^{\circ}\text{F} > 0^{\circ}\text{F}$
   - 25 mph $>$ 15 mph
   - 10 feet below sea level $<$ 2 feet below sea level
   - $|-12| > 10$
   - A \$20 debt $<$ a \$14 credit
   - 3 seconds before liftoff $<$ 3 seconds after liftoff
4. Explain that students are to draw a number line on poster paper. They are then to interpret the statements above and graph the numbers on their number line. They should also create two inequalities of their own and graph these as well. On a separate sheet of paper, they are to include brief descriptions as to why they placed these points as they did, mentioning the location of the numbers on the number line and how the location relates to the inequality, using the example you provided earlier as a guide.
5. Offer these suggestions for creating number lines:
   - Create a number line with a range of -25 to 25. The origin, 0, should be located in the middle of the number line.
   - Use rulers to draw an accurate number line. Make sure the units on the number line are equidistant. Students may include a tick mark for every unit, but instead may prefer to label numbers in multiples of 5.
   - Print neatly. Use markers with bold colors. Make the number line appealing and clear.

CLOSURE

Have students share their posters with the class. Discuss the posters and review the statements the groups generated. Display the posters.

The Number System: 6.NS.8

"Apply and extend previous understandings of numbers to the system of rational numbers."

8. "Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate."

BACKGROUND

The coordinate plane is formed by the intersection of a horizontal number line, the *x*-axis, and a vertical number line, the *y*-axis. The two lines intersect at their zero points, which is called the origin and is written as the ordered pair (0, 0). Every ordered pair consists of an *x*-coordinate and *y*-coordinate and can be graphed in the coordinate plane. The *x*-coordinate tells how many places horizontally from zero, or the origin, a point is located. The *y*-coordinate tells how many places vertically from zero a point is located. The coordinate plane is divided into four quadrants, or sections, that are formed by the intersection of the *x*-axis and *y*-axis. The quadrants are labeled counterclockwise beginning with Quadrant I in the upper right and ending with Quadrant IV in the lower right. The coordinate plane is used to locate points and calculate distances between them.

ACTIVITY: THE MAZE GAME

Students will work at a Web site where they will use an interactive game to move a robot through a mine field to its target. Students must correctly input ordered pairs to direct the robot's movements.

MATERIALS

Graph paper; computers with Internet access.

PROCEDURE

1. Review concepts of the coordinate plane with your students. They should be familiar with locating ordered pairs before beginning this activity.
2. Instruct your students to go to the Web site at http://www.shodor.org/interactivate/activities/MazeGame/.

3. Explain that they will play The Maze Game, the goal of which is to move a robot from its starting location by entering the *x*-coordinate and *y*-coordinate of the robot's next step to the target without hitting any mines. Note that students can only move the robot vertically or horizontally. As students work through this activity, they should record the robot's movements on graph paper. For each move horizontally and vertically, students should record the distance on their graph paper. For example, if the robot starts at (−5, 3) and a student moves it to (−5, −5), the student would graph these points on her graph paper. Then she would record a distance of 8 units (found by subtracting the *y*-coordinates and the absolute value) along that line to show that the robot moved 8 units. She would continue to do this for each move until the robot reached its target.
4. Explain that when students have passed one mine field, they may continue this procedure advancing through the next levels of mines. (They can select 5, 10, 15, 20, 25, or 30 mines to be placed throughout the coordinate plane.)
5. Allow time for students to complete several levels of the game.

CLOSURE

Discuss this activity as a class. Select a few students to show their graphs to the class and explain their strategies for winning the game. Also, discuss how students found the distances the robot moved. Finally, ask your students how graphing on the coordinate plane can be used to solve real-world problems.

Expressions and Equations: 6.EE.1

"Apply and extend previous understandings of arithmetic to algebraic expressions."

1. "Write and evaluate numerical expressions involving whole-number exponents."

BACKGROUND

An exponent shows the number of times a base is used as a factor. Following are some examples:

- 3^2 is read "3 squared" or "3 to the second power." $3^2 = 3 \times 3 = 9$
- 4^3 is read "4 cubed" or "4 to the third power." $4^3 = 4 \times 4 \times 4 = 64$
- 5^1 is read "5 to the first power" and is equal to 5. Any base raised to the first power is equal to the base.
- 2^0 is read "2 to the zero power" and is equal to 1. Any base (except 0) raised to the zero power is equal to 1.

If negative numbers are used, students may have trouble recognizing whether the base is positive or negative. Following are some examples:

- $(-3)^4$ is read "-3 to the fourth power." $(-3)^4 = -3 \times (-3) \times (-3) \times (-3) = 81$
- -3^4 is read "the opposite of 3^4." $-3^4 = -(3 \times 3 \times 3 \times 3) = -81$

ACTIVITY: FIND WHICH DOES NOT BELONG

Working individually, then in pairs or groups of three, students will decide which expression on the "Expressions Grid" differs from the other two expressions in the same row.

MATERIALS

Reproducible, "Expressions Grid."

PROCEDURE

1. Discuss the meanings of bases and exponents, using the information in the background as a guide. If necessary, provide additional examples.
2. Explain that expressions with exponents may be described numerically or verbally, but that each expression has only one value. For example, "three squared," "three to the second power," and 3^2 all equal 9.

3. Distribute copies of the reproducible, one to each student. Explain that two of the three expressions in each row have the same value. Working alone, students must choose the expression in each row that is different in value from the other two. They are to provide an explanation for their choice.
4. After students have completed the reproducible, instruct them to work with their partner or group and discuss their answers, correcting any wrong answers.

CLOSURE

Provide the answers to the reproducible and discuss any expressions that students found difficult to understand.

ANSWERS

The expression that does not belong is shown, followed by the reason. Row 1: 6; the other expressions equal 8. Row 2: -7^2 equals -49; the other expressions equal 49. Row 3: 9; the other expressions equal 27. Row 4: 6; the other expressions equal 1. Row 5: -1; the other expressions equal 1. Row 6: 8^2 equals 64; the other expressions equal 16. Row 7: -25; the other expressions equal 25. Row 8: 4^0 equals 1; the other expressions equal 4. Row 9: -100; the other expressions equal 100. Row 10: 100; the other expressions equal -100. Row 11: 5^2 equals 25; the other expressions equal 32. Row 12: 2^3 equals 8; the other expressions equal 1.

EXPRESSIONS GRID

Row #			
1	2^3	2 to the third power	6
2	(–7) squared	-7^2	49
3	3^3	9	27
4	6	1^6	6^0
5	$(-1)^2$	$(-1)^0$	–1
6	8^2	4^2	16
7	The opposite of –5, squared	25	–25
8	4^1	4^0	4
9	10^2	–100	10 squared
10	-10^2	–100	100
11	5^2	2^5	32
12	8^0	2^3	1

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Expressions and Equations: 6.EE.2

"Apply and extend previous understandings of arithmetic to algebraic expressions."

2. "Write, read, and evaluate expressions in which letters stand for numbers.
   - **a.** "Write expressions that record operations with numbers and with letters standing for numbers.
   - **b.** "Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
   - **c.** "Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations)."

BACKGROUND

There are two types of expressions: numerical expressions and algebraic expressions. A numerical expression names a number. Examples include $3 + 4$, $2(3 + 7)$, $18 \div 6$, and 104×5. An algebraic expression is an expression that contains a variable; for example, $2x + 3$, $3(4x - 5)$, and $\frac{1}{3}x$.

To evaluate a numerical expression, students must follow the order of operations. To evaluate an algebraic expression, they must substitute a value for the variable, then use the order of operations.

An equation is a statement that two expressions are equal. Students may solve equations by first evaluating the expression or expressions on one or both sides of the equation.

ACTIVITY: AND IT EQUALS...

Students will be given a slip of paper that contains an expression or formula and a number or expression. Working individually or in pairs, students will identify the value or expression that matches an expression or formula.

MATERIALS

Reproducible, "Expressions, Equations, and Values."

PREPARATION

Make one copy of the reproducible. (You may prefer to enlarge the reproducible before photocopying.) Cut out each box so that you have a total of 21 slips of paper. The original will serve as your answer key. The slips are arranged in order on the reproducible, each providing the answer that correctly matches an expression or formula written on the previous slip, except the match to the last formula which is written on the first slip.

PROCEDURE

1. Mix the slips up, then distribute one slip of paper to each student (or a slip to pairs of students). For a small class, you may give some students two slips. You must distribute all 21 slips.
2. To start, choose a student to read the expression or formula that is written on the right side of his slip. You may find it helpful to write the expression or formula on the board. All students should try to find the value or expression on the left side of their slips that matches the expression or formula that was presented. Because of the way the slips are designed, only one will contain a correct match. The student who has the slip with the correct answer should say "I have . . . " and then provide the answer. If the student is correct, he then reads the expression or formula written on the right side of his slip. If he is incorrect, point out his error. Another student should then provide the correct answer printed on the left side of her slip.
3. Continue the process until the student who read the first formula has the match to the last formula.

CLOSURE

Discuss the activity. Did students find other answers in addition to those on the cards? Explain that an expression may be written in several ways.

EXPRESSIONS, EQUATIONS, AND VALUES

I have	Find P ; $s = 1.5$
26	$P = 4s$

I have	Find A ; $s = 1.5$
6	$A = s^2$

I have	Three less
2.25	than a number

I have	The quotient of
$n - 3$	a number and 4

I have	The product
$\frac{n}{4}$	of 3 and a number

I have	The
$3n$	coefficient of $4n$

I have	Two factors
4	of $2(n + 3)$

I have	A number
2 and $(n + 3)$	that represents $(3 - 12)$

I have	The sum of a
–9	number and 3

I have	Find A ; $s = 1.5$
$n + 3$	$A = 6s^2$

I have	Find V ; $s = 1.5$
13.5	$V = s^3$

I have	$x = 2$
3.375	Find $8x^2$

I have	The terms of
32	$(-1 + \frac{3}{4})$

I have	Find the value
-1 and $\frac{3}{4}$	of $19 - 5 - 2$

I have	Six times
12	a number

I have	A number
$6n$	divided by $\frac{1}{4}$

I have	Find the value
$4n$	$3^2 - 1$

I have	Find the value
8	of $3 + 8 \times 2$

I have	Find the value
19	of $3(-6 + 2)$

I have	$x = \frac{1}{2}$
–12	Find $-2x^2$

I have	Find C ; $t = 4$
$-\frac{1}{2}$	$C = 6.5t$

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Expressions and Equations: 6.EE.3

"Apply and extend previous understandings of arithmetic to algebraic expressions."

3. "Apply the properties of operations to generate equivalent expressions."

BACKGROUND

Several properties are true for all rational numbers. Some of the most important include:

- Commutative property of addition: $a + b = b + a$
- Commutative property of multiplication: $ab = ba$
- Associative property of addition: $(a + b) + c = a + (b + c)$
- Associative property of multiplication: $a \times (b \times c) = (a \times b) \times c$
- Distributive property: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$
- Addition property of zero: $a + 0 = 0 + a = a$
- Sum of opposites property: $a + (-a) = -a + a = 0$
- Multiplication property of one: $a \times 1 = 1 \times a = a$
- Multiplication of reciprocals property: $a \times \frac{1}{a} = \frac{1}{a} \times a = 1, a \neq 0$

Understanding properties helps students not only to compute accurately but also to clarify their mathematical reasoning and recognize mathematical relationships.

ACTIVITY: PRESENTING PROPERTIES

For this activity, students will work in groups of four or five. They will create a presentation of a property of mathematics, explain the property and include various examples of how the property can be applied to generate equivalent expressions. This activity will require two class periods—the first for students to create and practice their presentations; the second for students to make their presentations to the class.

MATERIALS

Depending on the presentations, computers; digital projector; whiteboard or screen; overhead projector; transparencies; markers; math texts; math reference books. Optional: computers with Internet access.

PROCEDURE

1. Explain that each group is to create a presentation in which they will explain a mathematical property to the class. (Depending on the size of your class, you may prefer that each group of students presents only one property, for example, the commutative property of addition, or two properties, for example, the commutative properties of addition and multiplication.) Note that students will also include examples that show how the property can be applied to generate equivalent expressions. Offer the following examples of equivalent expressions of the associative property of addition:

$$(3 + 6) + 7 = 3 + (6 + 7)$$

$$9 + 7 = 3 + 13$$

$$16 = 16$$

$$\left(\frac{3}{10} + \frac{1}{10}\right) + \frac{7}{10} = \frac{3}{10} + \left(\frac{1}{10} + \frac{7}{10}\right)$$

$$\frac{4}{10} + \frac{7}{10} = \frac{3}{10} + \frac{8}{10}$$

$$\frac{11}{10} = \frac{11}{10}$$

$$1\frac{1}{10} = 1\frac{1}{10}$$

2. Explain that the presentations may take various forms, for example: a mini-lesson that the members of the group teach to the class; a skit (perhaps a group member assuming the role of a teacher and the other members assuming roles of students); or a demonstration (perhaps in which group members explain their property and provide examples in the form of charts or posters). Encourage students to use materials that will help them to make clear and informative presentations. Caution them that any props they use should be simple, safe, and easily obtainable.
3. Encourage your students to research their properties using their math texts and math reference books. Researching properties on the Internet (using the property's name as a search term) is an option. Emphasize that the information in their presentations must be accurate.
4. During their presentations, students should explain their property and provide examples. Encourage all group members to participate in their group's presentation.

CLOSURE

Have a member of each group briefly summarize the group's property for the class.

Expressions and Equations: 6.EE.4

"Apply and extend previous understandings of arithmetic to algebraic expressions."

4. "Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them)."

BACKGROUND

Expressions are mathematical symbols that represent a number. An expression can be numerical or algebraic:

- A numerical expression contains only numbers and operations, and/or grouping symbols. A numerical expression names a particular number. Examples: $15 - 8$; 6×5; $4(3 + 2)$; $1\frac{1}{2} \div \frac{3}{4}$
- An algebraic expression includes a variable or variables. It may also include numbers, operations, and/or grouping symbols. Two algebraic expressions are equivalent if they name the same number, no matter what value is substituted for the variable. Examples: $8n$; $4n + 7$; $4(x - y)$; $\frac{y}{18}$

ACTIVITY: PARTNER QUIZ

Students will first work individually, then work with a partner or in a group of three. They will create quizzes of equivalent expressions that their partner must solve.

PROCEDURE

1. Explain that expressions are mathematical symbols that represent a number, or can represent a number if values are provided for the variables. Also explain the difference between numerical expressions and algebraic expressions, offering the examples that are included in the Background for this activity. Note that for this activity, students will focus on algebraic expressions.
2. Provide examples of equivalent algebraic expressions such as: $x + x + x + x = 4x$; $\frac{1}{2}y = \frac{y}{2}$; and $3(x + 1) = 3x + 3$.

3. Explain to your students that they are to create a quiz that focuses on algebraic expressions. Suggest that they consider a ''matching'' quiz set up in two columns. (They can create other forms of quizzes, but a simple quiz in which expressions are matched is easy to do, freeing students to concentrate on the math.) In the first column, they may write five algebraic expressions, numbered 1 through 5. In the second column, they should write an equivalent algebraic expression (not in the same order) for each expression in the first column. Instead of numbers, they should designate each expression in the right column by a letter. On the back of their paper, they should write the correct answers.
4. After students have created their quizzes, they are to exchange their quizzes with another student. They then solve each other's quizzes by matching the correct algebraic expressions.
5. After students have completed each other's quizzes, they should return and correct them.

CLOSURE

Ask for volunteers to share what they felt were particularly challenging problems. Write these problems on the board and discuss them as a class. Also correct any problems whose answers are in dispute.

Expressions and Equations: 6.EE.5

"Reason about and solve one-variable equations and inequalities."

> 5. "Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true."

BACKGROUND

Equations and inequalities may be true or false, depending on the values that are substituted for variables. If a value makes an equation or inequality true, then the value is a solution to the equation or inequality.

ACTIVITY: THREE IN A ROW

Working in pairs or groups of three, students will rearrange equations and inequalities so that each equation or inequality in the same row matches a specific solution.

MATERIALS

Scissors; glue sticks; reproducible, "Equations and Inequalities."

PREPARATION

Make two copies of the reproducible for each pair or group of students.

PROCEDURE

1. Explain that a solution to an equation or inequality is a value that makes the statement true. Students can determine if a value is a solution to an equation or inequality by substituting the values of the variable into the equation or inequality.
2. Distribute two copies of the reproducible to each pair or group of students.

3. Explain that they are to cut out each of the equations and inequalities on one of the reproducibles. They will have 24 separate cards containing either equations or inequalities. Using the second copy of the reproducible, they are to work together to match three cards with the solution provided in each row. (On the original, the equations and inequalities do not match the solutions in their rows.) Students should place a matching equation or inequality over an original equation or inequality on the reproducible. Once students have agreed they have placed each card in the correct row, they should glue the cards. When they are finished, the solution of each row should be a solution to the equations and inequalities that follow it. Note that there is only one way to arrange the cards.

CLOSURE

Discuss the answers as a class.

ANSWERS

Answers for each row may be in any order: **Row 1:** $-2n = -16$; $9 + n = 17$; $-3n = -24$. **Row 2:** $25 - n \geq 5$; $2n = 40$; $3 + n < 25$. **Row 3:** $36 = n + 10$; $2n \geq 52$; $24 = n - 2$. **Row 4:** $2n = 0$; $15 - n = 15$; $0 \div 10 = n$. **Row 5:** $-16 \div -4 = n$; $5n < 24$; $3n = 12$. **Row 6:** $6 \div n = 3$; $-2n = -4$; $8n = 16$. **Row 7:** $48 \div 4 = n$; $2n = 24$; $12 \div n = 1$. **Row 8:** $7 = 8 + n$; $9 - 10 = n$; $3n \leq -3$.

Row #	Solution	Equations and Inequalities		
1	$n = 8$	$3n \leq -3$	$7 = 8 + n$	$-16 \div -4 = n$
2	$n = 20$	$5n < 24$	$9 - 10 = n$	$48 \div 4 = n$
3	$n = 26$	$12 \div n = 1$	$2n \geq 52$	$-2n = -4$
4	$n = 0$	$2n = 24$	$8n = 16$	$-2n = -16$
5	$n = 4$	$2n = 40$	$3n = 12$	$3 + n < 25$
6	$n = 2$	$-3n = -24$	$25 - n \geq 5$	$36 = n + 10$
7	$n = 12$	$2n = 0$	$9 + n = 17$	$24 = n - 2$
8	$n = -1$	$15 - n = 15$	$0 \div 10 = n$	$6 \div n = 3$

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Expressions and Equations: 6.EE.6

"Reason about and solve one-variable equations and inequalities."

> 6. "Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set."

BACKGROUND

A variable represents an unknown quantity. It is usually expressed as a letter. An expression is a group of mathematical symbols, for example, numbers, variables, and operations that represent a number, or can represent a number if values are assigned to the variables. Variables and expressions are essential for solving mathematical problems.

ACTIVITY: A SLICE OF LIFE WITH VARIABLES AND EXPRESSIONS

Students are to write a story in which a problem can be modeled mathematically. They will model the problem using an expression that contains at least one variable.

MATERIALS

Optional: computers and printers.

PROCEDURE

1. Explain to your students that a "slice of life" is a short piece of writing that depicts a relatively minor but nonetheless important aspect of life. For this activity, students are to write a slice-of-life piece in which math plays a part. They will use variables and expressions to model the math in their writing.
2. Explain that an algebraic expression contains a variable. Expressions can be used to model and solve problems. For example, Maria is 5 years older than her brother Paulo, who is 8 years old. Maria's age, represented by the variable M, can be expressed as $M = 8 + 5$. Here is another example. Alicia is 10 years old. She is four years older than her brother. Alicia's age may be expressed as 10 or $b + 4$, where b represents the age of Alicia's brother.

3. Instruct your students to write a slice-of-life story. Their story may be entirely fictional or it may be based on a real-life event. Their story should include a problem or situation that can be modeled and solved by using a mathematical expression. Brainstorm with your students to generate some possible ideas that can help them get started in developing ideas for their stories.
    - A trip to the mall to buy new clothes and eat at the food court
    - A visit to an amusement park
    - A part-time job
    - The school spring dance
    - A sporting event
    - A favorite hobby or activity
4. Offer this excerpt of a possible story to your students. "I babysit each week for my neighbor's little girl, Rachel. Last week I earned $50 for working five hours. I read to Rachel, we colored, and we watched a little bit of TV." Ask your students how they might model the writer's earnings. One possibility is $n =$ the hourly wage, therefore $5n$ is an expression that represents the amount of money earned. $5n = \$50$ is an equation that models this situation. The solution is $n = \$10$.
5. Encourage your students to use correct grammar and mechanics in their stories, as well as be mathematically accurate.
6. After students have completed their writing, they should select an event of the story they will model mathematically using expressions and variables. They should write the expression, and the meaning of each variable, at the end of their story.

CLOSURE

Ask volunteers to read their stories to the class and explain how math describes a situation in the story. Discuss how math can be used to model events in life. Display the stories.

Expressions and Equations: 6.EE.7

"Reason about and solve one-variable equations and inequalities."

> 7. "Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers."

BACKGROUND

Mathematical equations are commonly used to solve real-world problems. When students use equations, they must understand that variables stand for letters, and they must understand the steps necessary for solving one-step equations.

To solve equations in the form of $x + p = q$, students should subtract the value of p from both sides of the equation.

To solve equations in the form of $px = q$, students should divide both sides of the equation by the value of p, provided that the value of p is not equal to zero.

ACTIVITY: EQUATIONS, EQUATIONS, EQUATIONS

Students will write problems and use equations to solve problems. You will present individual student's problems to the class to solve.

MATERIALS

One transparency per student; markers; tissues or erasers.

PROCEDURE

1. Explain to your students that you will provide them with various real-life situations that they will use as the foundation for writing problems that can be solved using the equations $x + p = q$ and $px = q$. x, p, and q are nonnegative, rational numbers. If necessary, review the steps for solving these types of equations.
2. List the following examples of situations and encourage your students to offer more:
   - The Little League bake sale
   - A party
   - Going to a movie

- Shoveling snow for neighbors for a fee
- Saving money to buy a new laptop
- Taking public transportation
- Going shopping

3. Explain that each of these broad situations presents possibilities for writing word problems that can be solved using the equations stated above. For example, a store offers a $5 refund on your next purchase if you buy three bottles of shampoo. Sheila has purchased two bottles of shampoo. How many more bottles does she need to qualify for the refund? This problem can be solved using the equation $x + p = q$ where x equals the number of bottles left to purchase, p equals the bottles of shampoo Sheila purchased, and q equals the amount of bottles needed to qualify for the refund. $x + 2 = 3$. $x = 1$. Here is another example. A supermarket offers a discount for each shopping bag customers bring to the store to bag their purchases. Mike brought four bags to the store and saved 20 cents. What was the discount? This problem can be solved using the equation of the form $px = q$ where p stands for the number of bags, x stands for the discount per bag, and q stands for the discount. This can be written as $4x = \$0.20$. $x = \$0.05$. You might want to point out to your students that at first glance, these equations might be considered easy; however, they serve as a means for helping students to recognize real-life situations that may be expressed mathematically.

4. Explain that students are to choose one or two real-life situations, either those you listed or their own, and write two problems, one that can be solved using the equation $x + p = q$ and the other using the equation $px = q$. Students should write their problems near the top of the transparency. At the bottom of the transparency, they should write the equations that can be used to solve the problems, the solutions to the problems, and their names. Remind them to write neatly.

5. After students have completed their problems, collect their transparencies. Present some of your students' problems to the class, keeping the equations and students' names on the bottom of the transparencies covered. The class should solve the problem. While keeping the name on the transparency covered, compare the answer on the transparency to the answer the class believes is correct.

CLOSURE

Discuss how equations can be used to solve problems. Ask students to describe the steps they used for solving problems with the two equations they used in this activity.

Expressions and Equations: 6.EE.8

"Reason about and solve one-variable equations and inequalities."

> 8. "Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams."

BACKGROUND

An inequality is a mathematical expression that shows two quantities are not equal. The following symbols are used in inequalities:

- Greater than: $>$
- Less than: $<$
- Greater than or equal to: $\geq$
- Less than or equal to: $\leq$
- Not equal to: $\neq$

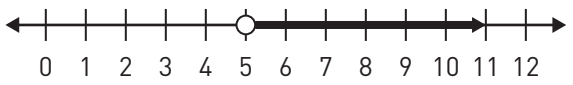
(Note: This Standard only addresses "greater than" or "less than" inequalities; however, students should also understand the symbols for "greater than or equal to," "less than or equal to," and "not equal to.")

An inequality can have infinitely many solutions. The solution to an inequality is the value or values that make the inequality true. For example, the solutions to the inequality $x > 5$ are all real numbers that are larger than 5.

The solutions to inequalities can be represented on a number line. The following guidelines summarize the steps for graphing inequalities that contain the symbols $>$ and $<$.

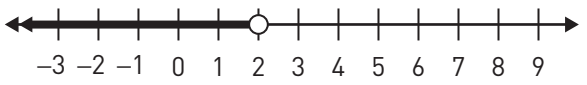
- Rewrite the inequality so that the variable is isolated on the left in the number sentence (if necessary).
- Draw a number line and label it with the appropriate numbers.
- For inequalities that are greater than or less than a number, the number is not included in the solution set. Draw an open circle on the point that corresponds to the number on the number line.
- Draw an arrow along the number line, pointing to the numbers that are included in the solution set.

Example: $x > 5$



5 is not included in the solution set and is marked as an open circle. The arrow points to the right because numbers to the right of 5 satisfy the inequality.

Example: $x < 2$



2 is not included in the solution set and is marked as an open circle. The arrow points to the left because numbers to the left of 2 satisfy the inequality.

(Note: The procedure for graphing inequalities that contain $\geq$ and $\leq$ is the same as the examples above, except that the circle is closed instead of open.)

ACTIVITY: FIND YOUR MATCH

Students will be given a card that describes a situation that can be modeled by an inequality or a card that contains the graph of an inequality. Students who have the description card must write an inequality using symbols and find the graph that matches the card they have. At the same time, students who have the graph card must write the inequality that is graphed and find the description of the situation modeled by the graph.

MATERIALS

Index cards; 2-page reproducible, ''Inequality and Graph Cards.''

PREPARATION

Make one copy of the reproducible and cut out the description cards and the graph cards. Use the original as an answer key. Each graph is to the right of the situation it models.

PROCEDURE

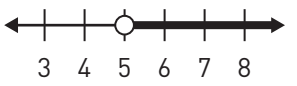
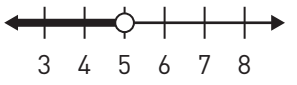
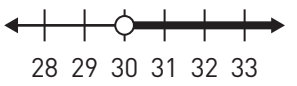
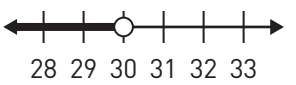
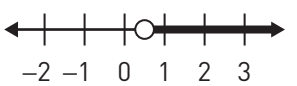
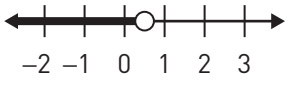
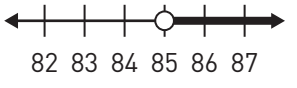
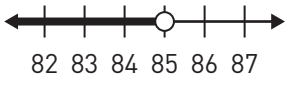
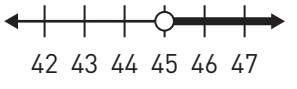
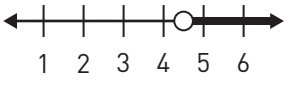
1. Review the concepts of inequalities with your students, including how to graph inequalities on a number line.
2. Explain that half of the students in class will receive a card that contains a description of an inequality. (If you have an odd number of students, one student may be given two cards.) The other half of the class will receive graph cards that correspond to the inequalities.

3. Randomly distribute the description cards and the graph cards, one per student. Instruct them to write an inequality to describe the situation or graph, depending on the type of card they received. Instruct them to use *x* to represent the variable.
4. Instruct students to walk around the room and sit with the person who has a matching card.
5. Once all students have found their match, select a few students to share their descriptions, the inequalities they wrote, and their graphs.
6. You may repeat this activity by shuffling the cards and passing them out to students again.

CLOSURE

Provide each of your students with an index card. On the front of the card, they are to write a situation that can be expressed as an inequality. On the back, they are to graph the inequality they wrote. Collect the cards as your students leave. You may wish to use these as a review or follow-up activity.

INEQUALITY AND GRAPH CARDS

Descriptions Cards	Graph Cards
To buy a ticket for the county fair, you must have more than $5.	
To enter the kids' pool, you must be less than 5 feet tall.	
You must order more than 30 cupcakes for your birthday party.	
Luckily, your math test has fewer than 30 questions.	
Your walk to school is more than 0.5 mile.	
Gym class is less than a half hour.	
To receive an A in Social Studies, you need a grade greater than 85% on your test.	
The weather person said the temperature would be lower than 85°F today.	
Your older sister gets paid more than $45 for tutoring.	
You must be more than 4.5 feet tall to ride a roller coaster.	

(continued)

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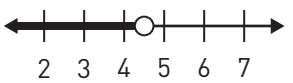
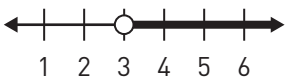
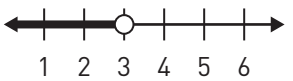
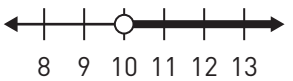
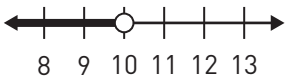
INEQUALITY AND GRAPH CARDS (continued)

Descriptions Cards	Graph Cards
The admission price of the matinee is less than $4.50.	
The blizzard brought more than 3 feet of snow!	
There are fewer than 3 days of school left!	
I went to bed after 10:00 last night.	
My birthday is fewer than 10 days away!	

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Expressions and Equations: 6.EE.9

"Represent and analyze quantitative relationships between dependent and independent variables."

> 9. "Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation."

BACKGROUND

A variable is defined as a quantity that can change. It can be an event, object, measurement, or any other item that can be measured. There are two types of variables: independent and dependent. An independent variable is the variable that determines the value of the other variable. It is the variable that stands alone and is not affected by changes in the other variable. For example, in the formula $d = 45t$, t represents time, which is an independent variable. A dependent variable is the variable that depends upon the value of the independent variable. In the formula $d = 45t$, d represents distance and is the dependent variable because distance depends on the length of time traveled. In an experiment, the dependent variable is usually the variable that is being tested or measured. The relationship between these variables can be expressed in tables, graphs, and equations.

ACTIVITY 1: EXAMINING RELATIONSHIPS

Working in groups of three or four, students will be given two variables that they must analyze. They will show the relationship in a table, graph, and equation.

MATERIALS

Graph paper; rulers.

PROCEDURE

1. Assign each group one of the following situations:
   - Joey can run 1 mile in 8 minutes.
   - Sam works at a pizza shop and earns $9 an hour.

- Kylie's family drives to Florida at an average speed of 50 miles per hour.
- Rebecca swims 3 laps in 2 minutes.
- Tim's cell phone bill costs $0.49 per minute for long distance.

2. Instruct each group to use the information presented to them to identify the independent and dependent variables. They will then create a table to show the relationship between the variables by selecting ten values for the independent variable. Using the information in their table, they will construct a graph of their data, making sure to label the independent and dependent variables. Lastly, students will write an equation showing the relationship between the variables.

CLOSURE

Have students present their work to the class. Students should explain how they determined which variable was independent and which was dependent. They should also discuss how they created the table, graph, and equation and how each represents the relationship between the two variables. Discuss each situation, describing how one variable changes in relation to another.

ACTIVITY 2: COMPARING HEART RATES

Students will compare their resting heart rate to their heart rate after 1 minute of exercise. They will organize their results in a table and graph, and write an equation.

MATERIALS

Graph paper; stopwatch.

PROCEDURE

1. Explain to your students that a person's heart rate is the number of times his heart beats per minute. Explain that in this activity, students will find their resting heart rate and compare it to their heart rate after a minute of exercise. Explain that a resting heart rate is the number of times a heart beats in one minute when a person is at rest.
2. Instruct your students to calculate their resting heart rate. They can find their pulse in one of two places: the wrist or neck. Instruct them to place their second and third fingers either on the palm side of the other wrist right below the thumb or on their lower neck just below the windpipe. Tell your students to apply a small amount of pressure until they feel their pulse. Once everyone has found their pulse, tell students to begin counting while you keep track of the time for 10 seconds. After 10 seconds, have

your students write the number they counted on their paper. Because heart rate is defined as the number of heartbeats per minute, have your students multiply that number by 6 to calculate their resting heart rate for 1 minute.

3. Explain that your students are going to conduct an experiment to determine how their resting heart rate compares to their heart rate after 1 minute of exercise. Watch the clock and instruct your students to do jumping jacks. They may jump as fast or as slowly as they want. The purpose is to simply elevate their heart rate to identify a difference between their resting heart rate and their heart rate after exercise. (Caution: Check with your school's nurse before doing any type of exercise with the class. Some students may have a medical condition that prohibits them from exercising. If so, these students can use a classmate's data. You might also allow students to modify jumping jacks to marching in place, if necessary.) After 1 minute, tell your students to stop and immediately find their pulse. Tell them to begin counting while you keep time for another 10 seconds. At the end of 10 seconds, have your students write the number they counted on their paper. Have them multiply that number by 6 to calculate their heart rate per minute after exercise.

4. Using their two heart rates as their data, students should create a three-column table that displays the time (ranging from 1 to 10 minutes), the number of their resting heartbeats, and the number of their heartbeats after exercising. To do this, they must first identify the variables in this situation, which are heartbeats and time.

5. After students have completed their tables, instruct them to graph their data. Depending on the abilities of your students, you may find it necessary to help them select an appropriate scale. Note that each point on the table corresponds to an ordered pair on the graph.

6. Finally, instruct your students to write an equation based on the table and graph for their resting heart rate and heart rate after 1 minute of exercise.

CLOSURE

Select a few students to present their work to the class. Discuss how students determined the independent and dependent variables and how they used their heart rates to create tables, graphs, and equations. Discuss how the relationship between the two variables is represented in the table, graph, and equation. Also discuss what factors may influence heart rates and how this experiment could be applied to real-world situations. (A good example is the individual who exercises and wants to monitor his heart rate.)

Geometry: 6.G.1

"Solve real-world and mathematical problems involving area, surface area, and volume."

1. "Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems."

BACKGROUND

The areas of many two-dimensional figures can be derived from other area formulas. The area of a right triangle, for example, can be found by dividing a rectangle into two congruent parts. Similarly, the area of a trapezoid can be found by dividing a parallelogram into two congruent parts. Realizing how formulas can be derived often makes it easier for students to understand and remember the formulas.

Irregular figures are so varied that the best way to find their areas is to divide them into other geometric figures whose areas can easily be found.

ACTIVITY 1: IT'S HALF

Students will discover the formula for finding the area of a right triangle by using the formula for finding the area of a rectangle.

MATERIALS

Rulers; scissors; rectangular sheets of unlined paper.

PROCEDURE

1. Explain that some area formulas can be found if other area formulas are known. For example, finding the area of a right triangle can be found by using the formula for finding the area of a rectangle.
2. Instruct your students to label their rectangular paper with an "L" for length and a "W" for width.
3. Ask for a volunteer to state the formula for finding the area of a rectangle. ($A = l \times w$)
4. Instruct your students to draw a diagonal line from one vertex of their paper to the opposite vertex. They should cut the paper along the line they drew.

5. Ask what figures are formed. (Two congruent right triangles)
6. Ask how the area of each triangle compares with the area of the rectangle. (Each area is half of the area of the rectangle.)

CLOSURE

Ask your students to write the formula for finding the area of a right triangle. ($A = \frac{1}{2} \times l \times w$)
Review how they derived this formula.

ACTIVITY 2: IT'S AWE-SUM

Students will use virtual interactive geoboards to create irregular polygons. They will find the areas by decomposing the figures into triangles and other shapes.

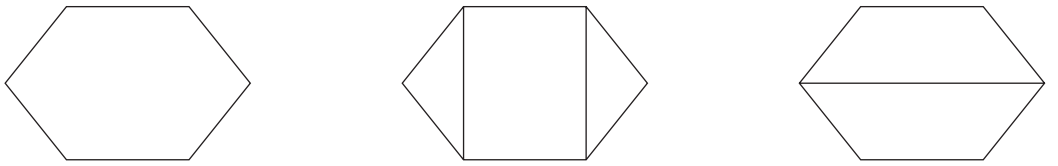
MATERIALS

Digital projector; computers with Internet access; graph paper.

PROCEDURE

1. Instruct your students to go to the Web site http://nlvm.usu.edu/en/nav/vlibrary.html. They should go to Geometry and click on grades 6–8. Next, they should scroll down and click on Geoboard.
2. Explain to your students that they will use a virtual geoboard to draw irregular polygons. They will then find the area of the figure by dividing it into smaller shapes for which they know area formulas. Note that the sum of the areas of these smaller figures is equal to the area of the irregular polygon.
3. Demonstrate the use of the virtual geoboards.
   - Start by clicking on ''Clear'' to clear the board.
   - Click on ''Bands'' and drag a band to the virtual geoboard.
   - Click on the band and drag it to a peg, repeating this process to create an irregular polygon.
4. Instruct your students to make an irregular figure whose sides are line segments on their virtual geoboards. After students have made their figure, ask them to draw it on graph paper. Explain that each line on the graph paper is 1 unit long and the area

of each box is 1 square unit. They can divide the figure they made into triangles, or other figures for which they know area formulas. By adding the areas of these figures, they will be able to find the area of the irregular figure they created. An example is shown below.



The hexagon can be divided into two triangles and a rectangle or two trapezoids.

5. After they have found the area of their figure on their graph paper, students should go back to their virtual geoboards, click on their figure and then click on "Measure" to find the area of the figure. They can now check that the area they found for their irregular figure is correct. If their answer is incorrect, students should check their sketch and check their work.
6. Allow students time to create additional figures and check their answers.

CLOSURE

Ask your students how this procedure of decomposing irregular figures may apply to real-world problems. (Answers may vary. Finding the area of an L-shaped room is one example.)

Geometry: 6.G.2

"Solve real-world and mathematical problems involving area, surface area, and volume."

2. "Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems."

BACKGROUND

Volume is the number of cubic units needed to fill a container. The formula for finding the volume of a right rectangular prism (a typical box) is $V = lwh$, where l stands for the length of the rectangular base, w stands for the width of the base, and h stands for the height of the prism. Another formula that gives the same result is $V = Bh$, where B stands for the area of the rectangular base and h stands for the height of the rectangular prism.

ACTIVITY: IT'S VOLUMINOUS

Working in pairs or groups of three, students will "fill" a rectangular prism with cubes by making a sketch, then find the volume of the prism by using formulas. They will also find the volume of other rectangular prisms.

MATERIALS

Rulers; unlined paper; reproducible, "Volumes of Rectangular Prisms."

PROCEDURE

1. To start this activity, explain to your students that they will find the volume of a rectangular prism by sketching cubes that are needed to fill the prism.
2. Distribute a copy of the reproducible to each student. Explain that the figure at the top of the sheet is a rectangular prism, for example, a cardboard box. Students are to imagine filling the box with cubes that measure $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$. To find the volume of the prism they should follow the guidelines under Step 1 on the reproducible.

3. After students have found the volume of the prism in the diagram, they are to complete the rest of the steps on the reproducible. If necessary, review the meaning of the variables in the volume formulas.

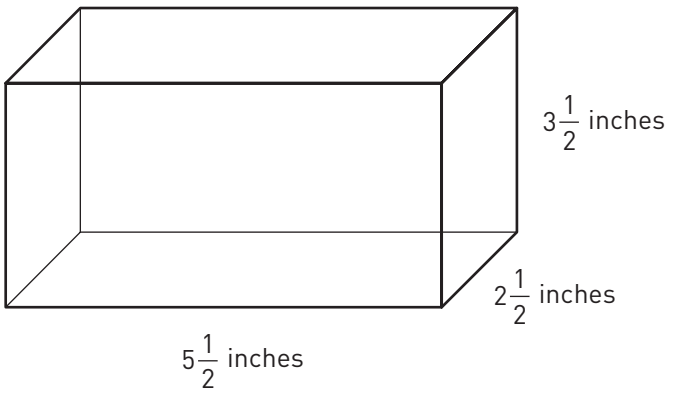
CLOSURE

Correct and discuss the answers students found.

ANSWERS

1. 55 cubes in an 11 by 5 arrangement of cubes; 7 layers; 385 cubes; $\frac{1}{8}$ cubic inch; $385 \times \frac{1}{8} = 48\frac{1}{8}$ cubic inches
2. $5\frac{1}{2} \times 2\frac{1}{2} \times 3\frac{1}{2} = 48\frac{1}{8}$ cubic inches
3. $13\frac{3}{4} \times 3\frac{1}{2} = 48\frac{1}{8}$ cubic inches
4. All of the answers are the same; explanations may vary.
5. $15\frac{5}{8}$ cubic inches; $166\frac{7}{8}$ cubic inches; $131\frac{1}{4}$ cubic inches

VOLUMES OF RECTANGULAR PRISMS



1. Find the volume of the $5\frac{1}{2}$-inch by $2\frac{1}{2}$-inch by $3\frac{1}{2}$-inch rectangular prism by filling it with $\frac{1}{2}$-inch cubes. Follow the guidelines below.

- Sketch the number of cubes that can be placed in the base of the rectangular prism. This is the number of cubes in one layer. The base measures $5\frac{1}{2}$ inches by $2\frac{1}{2}$ inches. How many cubes, in one layer, would fit in the base?
- How many layers of cubes are needed to fill a rectangular prism that is $3\frac{1}{2}$ inches high?
- How many $\frac{1}{2}$-inch cubes are needed to fill the box?
- What is the volume of each $\frac{1}{2}$-inch cube?
- Find the volume of the prism by multiplying the total number of cubes by the volume of each cube. What is the volume of the prism?

2. Find the volume of the rectangular prism using the formula $V = lwh$.

3. Find the volume of the rectangular prism using the formula $V = Bh$, where B is the area of the base.

4. How do your answers to questions 1, 2, and 3 compare? Explain your reasoning.

5. Use a formula to find the volume of the following:

- A cube whose side measures $2\frac{1}{2}$ inches.
- A cereal box with the following dimensions: $7\frac{1}{2}$ inches by 2 inches by $11\frac{1}{8}$ inches.
- A container with the following dimensions: $3\frac{3}{4}$ inches by 5 inches by 7 inches.

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Geometry: 6.G.3

"Solve real-world and mathematical problems involving area, surface area, and volume."

> 3. "Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems."

BACKGROUND

A polygon is a closed figure whose sides are line segments. A vertex of a polygon is the point where two line segments meet. To draw a polygon in the coordinate plane, students must graph the coordinates of the vertices and draw line segments to connect the vertices.

ACTIVITY: INITIALS

Students will draw their initials represented by polygons in the coordinate plane. They will label the vertices on their graphs and find the lengths of the sides of the polygons used in their initials. After listing their vertices on a separate sheet of paper, students will hand in their lists, which will be randomly redistributed among other students. Students then graph each other's initials in the coordinate plane and confirm the lengths of the sides of the polygons by subtracting the coordinates of the vertices.

MATERIALS

Graph paper; reproducible, "Guidelines for 'Initial' Graphing in the Coordinate Plane."

PROCEDURE

1. Explain that your students will use graph paper to draw their initials in the form of polygons in the coordinate plane.
2. Distribute copies of the reproducible and review the information for graphing points, as well as the steps for finding the lengths of line segments by subtracting the coordinates of the vertices. Note the sample initials, emphasizing how polygons were used to create the initials.

3. Explain that students must use all four quadrants of the coordinate plane when graphing their initials. Note that they may only use polygons to graph their initials. They are to label the vertices on their graphs, and then find the length of each line segment of the polygons that make up their initials. Emphasize that they should only use polygons composed of rectangles and squares with sides that are parallel to an axis in graphing their initials, because this will allow them to find the lengths of the line segments easily by subtracting the appropriate coordinates. They should avoid using diagonal lines.
4. After finding the lengths of the line segments, they are to list the vertices in order on another sheet of paper. Students should not write their names on this sheet.
5. Review the example provided on the reproducible. Make sure that students understand how to list the coordinates of the vertices.
6. After students have completed these steps, they should hand their papers that list vertices to you. After collecting all of the papers, mix them randomly and redistribute them to your students. Explain that students are to now graph the vertices listed on the paper they received, and connect the points in the order the points are listed to form polygons that will reveal the initials of another student. After completing the graphs, they are to verify the lengths of the line segments of the polygons by subtracting the coordinates of the vertices. These second graphs should be identical to the original graphs.

CLOSURE

Ask students to identify the person whose initials they graphed.

GUIDELINES FOR "INITIAL" GRAPHING IN THE COORDINATE PLANE

To graph a point in the coordinate plane, do the following:

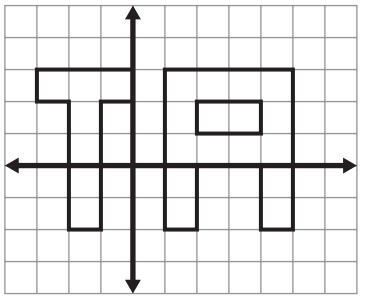
1. Start at the origin.
2. Move along the *x*-axis. If the *x*-coordinate is positive, move right. If it is negative, move left. If the *x*-coordinate is zero, do not move.
3. Stop at the point that represents the *x*-coordinate.
4. From this point, move parallel to the *y*-axis. If the *y*-coordinate is positive, move up. If it is negative, move down. If the *y*-coordinate is zero, do not move.
5. Mark the point.

To graph a polygon, connect the vertices in order.

Graph: (0, 3), (−3, 3), (−3, 2), (−2, 2), (−2, −2), (−1, −2), (−1, 2), (0, 2), (0, 3) Stop

Graph: (1, 3), (5, 3), (5, −2), (4, −2), (4, 0), (2, 0), (2, −2), (1, −2), (1, 3) Stop

Graph: (2, 1), (4, 1), (4, 2), (2, 2), (2, 1) Stop



To find the length of a vertical line segment, subtract the *y*-coordinates. (If the value is negative, find the absolute value.)

To find the length of a horizontal line segment, subtract the *x*-coordinates. (If the value is negative, find the absolute value.)

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Geometry: 6.G.4

"Solve real-world and mathematical problems involving area, surface area, and volume."

4. "Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems."

BACKGROUND

A *net* is a two-dimensional drawing of connected polygons and/or circles in a plane that can be folded along adjacent sides to form a polyhedron.

A *polyhedron* is a three-dimensional shape with surfaces called faces that are two-dimensional. Each face consists of a polygon and the interior of the polygon.

A *prism* is a polyhedron with two parallel faces, called bases, that are the same size and shape. Prisms are classified according to the shape of their bases. If the bases are triangular, for example, the prism is called a triangular prism. If the bases are rectangular, the prism is called a rectangular prism.

A *pyramid* is a polyhedron with one face, called a base, that is a polygon. The other faces are triangles with a common vertex. A pyramid is classified according to the shape of its bases.

The *Platonic solids* are three-dimensional figures whose faces are regular polygons. A regular polygon is a polygon that has congruent sides and congruent angles. The five Platonic solids are listed below:

- *Tetrahedron:* 4 faces are equilateral triangles.
- *Cube:* 6 faces are squares.
- *Octahedron:* 8 faces are equilateral triangles.
- *Icosahedron:* 20 faces are equilateral triangles.
- *Dodecahedron:* 12 faces are regular pentagons.

To find the surface area of a three-dimensional figure, students should find the sum of the areas of the faces and bases.

ACTIVITY: FINDING THE SURFACE AREA OF NETS

This is a two-day activity. Students may work individually or in small groups. On the first day, students will watch a simulation of folding a net into a three-dimensional figure. They will also select a polyhedron that they would like to make. On the second day they will construct their polyhedron and find its surface area.

MATERIALS

Card stock paper; scissors; rulers; glue sticks; computers with Internet access; digital projector.

PREPARATION

Print the nets that students will use to form their models.

PROCEDURE

1. Explain that nets are two-dimensional figures that will form a three-dimensional figure when they are folded.
2. Project the simulations of nets being used to create figures from the Web site http://www.mathsnet.net/geometry/solid/nets.html. Students will be able to view nets being folded into various three-dimensional figures.
3. Instruct your students to go to the Web site http://www.korthalsaltes.com to view models of three-dimensional figures. Clicking on a model will allow them to view the model and scroll down to view its net.
4. Instruct students to work either individually or in small groups at their computers. They are to select a model they would like to construct from a net.
5. Allow time for your students to select a figure they would like to make, limiting them to the Platonic solids, pyramids, and prisms. Ask them to write their selections on a sheet of paper with their name on it at the end of the class. If necessary, offer explanations about the types of figures listed in the Background for this activity.
6. After class, go to the Web site http://www.korthalsaltes.com and print your students' selections of nets on card stock paper.
7. On the second day, distribute materials—scissors, rulers, and glue sticks—and students' nets. Explain that the tabs on the nets are not a face of the figure, but are used to be glued and will hold the model together.
8. Explain that students are to find the area of each polygon that is a face or a base of the model. If necessary, review the formulas for finding the areas of squares ($A = s^2$); rectangles ($A = l \times w$); triangles $\left(A = \frac{1}{2}bh\right)$; and pentagons (divide the pentagon into five triangles, find the area of each, and find the sum of the areas). Emphasize that the sum of the areas of the faces and base(s) is equal to the surface area of the polyhedron.
9. Instruct your students to write their name on their model, find the area of each face and base, and record the areas on the model. Then they are to find the sum of the areas, and write the surface area on the paper on which they originally wrote their selection of the net.

CLOSURE

Ask your students to brainstorm situations where finding the surface area is necessary in everyday life. (Answers might include the following: wrapping a package, painting the walls and ceiling of a room, or the amount of paint to use on products.)

Statistics and Probability: 6.SP.1

"Develop understanding of statistical variability."

1. "Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers."

BACKGROUND

Statistics is a branch of mathematics that focuses on the collection, organization, and analysis of data. In collecting statistical data, care must be taken to obtain thorough and accurate information. Recognizing the difference between a statistical question—a question to which the answers making up the data are expected to vary—and a nonstatistical question is fundamental to efficient collection of data.

ACTIVITY: STATISTICAL QUESTIONS VERSUS NONSTATISTICAL QUESTIONS

Working in groups of four or five, students will generate examples of statistical and nonstatistical questions. They will present their questions to the class, providing explanations as to why their questions are statistical or not.

MATERIALS

One transparency per group; markers; erasers.

PROCEDURE

1. Explain to your students that a statistical question is a question designed to collect data that expects the data related to it to vary and takes account of the variability in its answer. This is unlike questions that have specific answers.
2. Offer these examples of statistical questions:
   - How tall are the members of the high school basketball team?
   - How old are the students in the school band?
   - What are the ethnic backgrounds of the students in my school?

   Note that in each case, the question anticipates various answers.

3. Offer these examples of nonstatistical questions:
   - Whose portrait is on a one-dollar bill?
   - Who is the president of the United States?
   - What is today's date?

   Note that in each case, a specific answer is anticipated.
4. Instruct your students to work in their groups to brainstorm and generate examples of statistical questions and nonstatistical questions. They are to choose one statistical question and write it neatly on their transparency. They should then choose one of the nonstatistical questions they generated and write it beneath their statistical question. They should be prepared to explain why these are, in fact, examples of a statistical and nonstatistical question.
5. Have each group present their questions to the class via the overhead projector. They should support their selection of these questions.

CLOSURE

Instruct your students to write a brief summary of the difference between a statistical question and a nonstatistical question.

Statistics and Probability: 6.SP.2

"Develop understanding of statistical variability."

2. "Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape."

BACKGROUND

A statistical question is one that is expected to have variability in the data related to it and accounts for this variability in its answers. For example, "How far do you live from your school?" is a statistical question because students will no doubt live various distances from their school. Another example of a statistical question is "What is your favorite ice cream?" Respondents to the question will likely have various types of favorite ice cream. The distribution of the answers to a statistical question can be described by its center (the mean, median, and mode), its spread (how far data is from the center), and overall shape (the shape of the graph of the data).

ACTIVITY: AND THE ANSWER IS...

This activity requires two days. Working in pairs or groups of three, students will write a statistical question that they will pose to ten other students in the class. They will use their data to describe the center, spread, and overall shape of the distribution.

MATERIALS

Rulers.

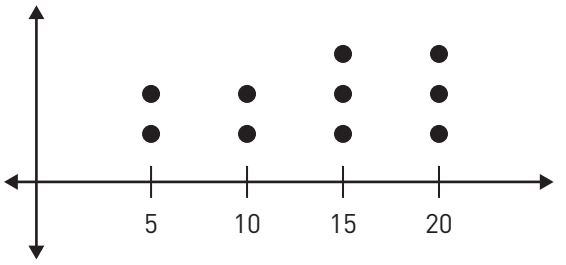
PREPARATION

A day before you present this activity, ask each pair or group of students to write a statistical question that they wish to ask other students. Collect the questions to check that they are, in fact, statistical questions and that the questions are appropriate.

PROCEDURE

1. On the first day, instruct each pair or group of students to write a statistical question that they will later ask ten other students. If necessary, provide examples and counter-examples of statistical questions. Collect the questions and make sure that they are written correctly. (Any incorrect questions should be corrected before the next part of the activity.)

2. On the second day, hand back the statistical questions your students wrote. Instruct the pairs or groups to ask their question of ten other students in the class.
3. Instruct students to describe the data they collected in the following manner:
    - *By its center*. Students may use the mean, median, or mode. Note that the mean is the average number of the data. The median is the middle value of the data, arranged in ascending or descending order. In this case, because there is an even number of data, the median is the average of the two middle values. The mode is the number that occurs most often in the data, and is particularly useful when students must identify favorites or the most popular.
    - *By its spread*. Students may use the range, which is the highest value minus the lowest value. The spread may also be described as the values being close to the center or far apart.
    - *By its shape*. Students may sketch a dot plot to describe the shape of the data.
4. If necessary, review the steps for sketching a dot plot. Note that students should do the following:
    - Record each item of data on a number line.
    - Place each dot above the number line to represent the frequency of the data points.



This dot plot represents 5, 5, 10, 10, 15, 15, 15, 20, 20, 20.

CLOSURE

Have your students share their results with the class. Ask and discuss the following question: Did any group pose a statistical question that could not be analyzed by its center, range, or shape? (The answer is no, because all data gathered from statistical questions can be analyzed in this manner.)

Statistics and Probability: 6.SP.3

"Develop understanding of statistical variability."

3. "Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number."

BACKGROUND

A measure of center (mean, median, and mode) summarizes the values of a set of data with a single number. A measure of variation describes how spread out or scattered the set of data is. It includes the range and mean absolute deviation.

ACTIVITY: MEASURE OF CENTER VERSUS MEASURE OF VARIATION

Working in pairs or groups of three, students will be given three sets of data. They will also be given cards identifying various measures of center and various measures of variation. They are to select which measures match their data.

MATERIALS

Scissors; glue sticks; reproducible, "Data Sets."

PROCEDURE

1. Distribute one copy of the reproducible to each pair or group of students. Explain that the reproducible contains three Data Sets. Eighteen cards are located at the bottom of the sheet, each representing either a measure of center or a measure of variation.
2. Explain that students are to find the mean, median, mode, range, and absolute mean deviation for each Data Set. If necessary, review the process for finding these measures, particularly how to find the mean absolute deviation, which is found by finding the mean and then finding the distance between each value and the mean. The sum of the distances divided by the number of data values is the mean absolute deviation. It is a measure of how far, on average, each value is from the mean.
3. Explain that students are to cut out the cards found at the bottom of the sheet, then match and glue the cards beneath the appropriate Data Sets, starting with measures of center and ending with measures of variation. Note that some cards will not be used.

CLOSURE

Provide the answers to your students.

ANSWERS

The values are listed in this order: mean, median, mode, range, mean absolute deviation.
Data Set 1: 70, 67.5, 65, 40, 9; Data Set 2: 85, 90, 90, 35, 10; Data Set 3: 75, 70, 70, 60, $14\frac{2}{7}$

DATA SETS

Data Set 1 Measures of Center	80, 70, 60, 50, 90, 70, 65, 65, 65, 85	Measures of Variation
	(Place matching cards here)	
Data Set 2 Measures of Center	80, 70, 95, 90, 90, 65, 90, 100	Measures of Variation
	(Place matching cards here)	
Data Set 3 Measures of Center	40, 90, 85, 100, 70, 70, 70	Measures of Variation
	(Place matching cards here)	

The mean is 60	The median is 90	The median is 67.5	The range is 40	The mean is 70	The mean absolute deviation is 9
The median is 70	The mode is 65	The range is 60	The mean is 87	The mode is 70	The mean absolute deviation is $14\frac{2}{7}$
The mean is 85	The range is 35	The median is 76	The mode is 90	The mean is 75	The mean absolute deviation is 10

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Statistics and Probability: 6.SP.4

"Summarize and describe distributions."

4. "Display numerical data in plots on a number line, including dot plots, histograms, and box plots."

BACKGROUND

Data can be organized and displayed in a variety of ways. A dot plot, histogram, and box plot are three common graphs used for summarizing and displaying data.

ACTIVITY: CREATING DATA DISPLAYS

Working in groups of three or four, students will gather and display data in the form of a dot plot, histogram, and box plot.

MATERIALS

Rulers; unlined paper; reproducible, "Displaying Data."

PREPARATION

A few days before the activity, instruct your students to ask at least five adults (family members or friends) about how much time they need to travel to work each day. The answers students obtain will be the data they use for this activity.

PROCEDURE

1. Explain that each group of students is to combine the data they obtained and use the data to create a dot plot, histogram, and box plot. (For example, a group of four should have at least 20 items of data.)
2. Provide the following guidelines for organizing their data:
   - Express all times in minutes.
   - Round each time to the nearest multiple of 5.
   - Arrange data from the least to the greatest.

3. Distribute and review the information on the reproducible with your students. Note that the reproducible contains a description of each graph and instructions for constructing the graphs.
4. Remind your students that their graphs should be neat, accurate, and labeled correctly.

CLOSURE

Ask your students the following questions: Which graph did they find easiest to construct? Why? Which was hardest? Why?

(Note: Instruct your students to retain their data and graphs, which will be used in the next activity.)

DISPLAYING DATA

Dot plots, histograms, and box plots are three ways data can be displayed. Use the data the members of your group obtained to create these graphs.

A DOT PLOT

Also called a line plot, a dot plot represents a set of data by using dots placed over a number line. To make a dot plot, do the following:

1. Draw a number line, labeling the numbers. (For your dot plot in this activity, label the numbers in multiples of 5.)
2. Place a dot above the number line each time a number representing data occurs.

A HISTOGRAM

A special type of bar graph, a histogram displays the frequency of data that has been organized in equal distributions. To make a histogram, do the following:

1. Determine the number of bars you will include on your graph.
2. Subtract the smallest value of the data from the largest value of the data, and divide that answer by the number of bars to obtain the bar width.
3. Draw horizontal and vertical axes. Label the horizontal axis with the data and the vertical axis with the frequency.
4. Draw each bar (with no spaces between them), using the frequency as the height.

A BOX PLOT

Also called a box-and-whisker plot, a box plot displays the median of a set of data, a median of each half of the data, and the least and greatest values of the data. To make a box plot, to the following:

1. Find the median of the data. (This divides the data into two parts.)
2. Find the median of the first half of the data.
3. Find the median of the second half of the data.
4. Draw a box using the median of the first half of the data as the left end, and the median of the second half of the data as the right end. Mark the median of the data with a verical line.
5. Draw a line from the box to the smallest number.
6. Draw a line from the box to the largest number.

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Statistics and Probability: 6.SP.5

"Summarize and describe distributions."

5. "Summarize numerical data sets in relation to their context, such as by:
   - **a.** "Reporting the number of observations.
   - **b.** "Describing the nature of the attribute under investigation, including how it was measured and its unit of measurement.
   - **c.** "Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   - **d.** "Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered."

BACKGROUND

Analyzing data is essential for data display. An explanation of how the data was collected, the number of data, how accurate the data is, how the measures of central tendencies and/or variability are used, the identification of outliers, and the identification of patterns can enhance any numerical display. (Note that for this activity, students will work with the same data they obtained for the previous activity, "Creating Data Displays," for 6.SP.4.)

ACTIVITY: SUMMARIZING DATA

Working in groups of three or four (preferably the same groups students worked in for the prior activity), students will write a description of the data they previously obtained and displayed.

MATERIALS

Data regarding travel time to work; reproducible, "Guidelines for Summarizing Data."

PROCEDURE

1. Instruct your students that they are to use the data they obtained previously. They may also use their data displays (from the previous activity) for reference.
2. Explain that each group is to write a summary of their data.
3. Distribute copies of the reproducible. Explain that it is a reference sheet that includes the topics students should include in their summaries. It also includes definitions and how to determine the values they should include in their summaries. Discuss the definitions and processes, offering any necessary examples.

CLOSURE

Exhibit students' displays and discuss their summaries.

GUIDELINES FOR SUMMARIZING DATA

Your group is to write a summary of the data you collected about the traveling time of adults going to work. You may use any dot plot, histogram, and box plot you created previously to support your summary.

Your summary should include answers to the following questions:

1. How much data did your group collect?
2. How did you collect your data?
3. Was the data exact?
4. Are there any values that are unusually large or small compared with the rest of the values? (These are called outliers.)
5. How close is your data to its center? This can be found by using measures of central tendencies: the mean, median, and mode.
   - The mean is an average and can be found by adding all of the data and dividing the sum by the number of data. It is not a good measure to use if there are outliers, because outliers can have a major effect on the mean.
   - The median is the middle number of the data when the numbers are arranged in order from the least to the greatest (or the greatest to the least). The median is less affected by outliers than the mean and mode.
   - The mode is the number of the data that occurs most often.
6. How spread out is your data? This can be found by using measures of variability, the range, the interquartile range, and/or mean absolute deviation.
   - The range of the data is the highest value minus the lowest value.
   - The interquartile range is the middle half of the data and tells how spread out the middle values are.
   - The mean absolute deviation (also known as the mean deviation) uses each piece of data in its computation to find out how far on average the data deviates from the mean. To calculate the mean absolute deviation, first find the mean. Then subtract the mean from each piece of data. Find the absolute value of each difference. Add the total of the absolute values and divide the sum by the number of data.
7. Do you notice any patterns? If yes, what are they? What might they be a result of?
8. How do the measures of central tendencies and variation relate to the shape of your displays?

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