CHAPTER 1

The Bond Instrument

Bonds are debt-capital market instruments that represent a cash flow payable during a specified time period heading into the future. This cash flow represents the interest payable on the loan and the loan redemption. So, essentially, a bond is a loan, albeit one that is tradable in a secondary market. This differentiates bond-market securities from commercial bank loans.

In the analysis that follows, bonds are assumed to be default-free, which means that there is no possibility that the interest payments and principal repayment will not be made. Such an assumption is reasonable when one is referring to government bonds such as U.S. Treasuries, UK gilts, Japanese JGBs, and so on. However, it is unreasonable when applied to bonds issued by corporates or lower-rated sovereign borrowers. Nevertheless, it is still relevant to understand the valuation and analysis of bonds that are default-free, as the pricing of bonds that carry default risk is based on the price of risk-free securities. Essentially, the price investors charge borrowers that are not of risk-free credit standing is the price of government securities plus some credit risk premium.

BOND-MARKET BASICS

All bonds are described in terms of their issuer, maturity date, and coupon. For a default-free conventional, or plain-vanilla, bond, this will be the essential information required. Nonvanilla bonds are defined by further characteristics such as their interest basis, flexibilities in their maturity date, credit risk, and so on.

Figure 1.1 shows screen DES from the Bloomberg system. This page describes the key characteristics of a bond. From Figure 1.1, we see a description of a bond issued by the Singapore government, the 4.625% of 2010. This tells us the following bond characteristics:

Issue date	July 2000
Coupon	4.625%
Maturity date	1 July 2010
Issue currency	Singapore dollars
Issue size	SGD 3.4 million
Credit rating	AAA/Aaa

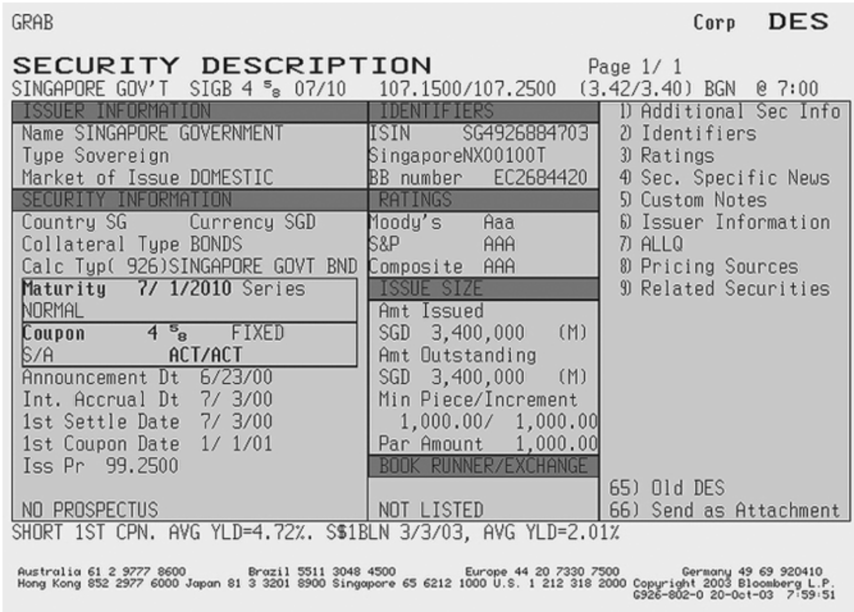

FIGURE 1.1 Bloomberg Screen DES Showing Details of $4\frac{5}{8}$ % 2010 Issued by Republic of Singapore as of 20 October 2003

Calling up screen DES for any bond, provided it is supported by Bloomberg, will provide us with its key details. Later on, we will see how nonvanilla bonds include special features that investors take into consideration in their analysis.

We will consider the essential characteristics of bonds later in this chapter. First, we review the capital market, and an essential principle of finance, the time value of money.

CAPITAL MARKET PARTICIPANTS

The debt capital markets exist because of the financing requirements of governments and corporates. The source of capital is varied, but the total supply of funds in a market is made up of personal or household savings, business savings, and increases in the overall money supply. Growth in the money supply is a function of the overall state of the economy, and interested readers may wish to consult the references at the end of this chapter, which include several standard economic texts. Individuals save out of their current income for future consumption, while business savings represent retained earnings. The entire savings stock represents the capital available in a market. The requirements of savers and borrowers differ significantly, in that savers have a short-term investment horizon while borrowers prefer to take a longer-term view. The constitutional weakness of what would otherwise be *unintermediated* financial markets led, from an early stage, to the development of financial intermediaries.

Financial Intermediaries

In its simplest form a financial intermediary is a *broker* or *agent*. Today we would classify the broker as someone who acts on behalf of the borrower or lender, buying or selling a bond as instructed. However, intermediaries originally acted between borrowers and lenders in placing funds as required. A broker would not simply on-lend funds that have been placed with it, but would accept deposits and make loans as required by its customers. This resulted in the first banks. A *retail bank* deals mainly with the personal financial sector and small businesses, and in addition to loans and deposits also provides cash transmission services. A retail bank is required to maintain a minimum cash reserve, to meet potential withdrawals, but the remainder of its deposit base can be used to make loans. This does not mean that the total size of its loan book is restricted to what it has taken in deposits: loans can also be funded in the wholesale market. An *investment bank* will deal with governments, corporates, and institutional investors. Investment banks perform an agency role for their customers, and are the primary vehicle through which a corporate will borrow funds in the bond markets. This is part of the bank's corporate finance function; it will also act as wholesaler in the bond markets, a function known as *market making*. The bond-issuing function of an investment bank, by which the bank will issue bonds on behalf of a customer and pass the funds raised to this customer, is known as *origination*. Investment banks will also carry out a range of other functions for institutional customers, including export finance, corporate advisory, and fund management.

Other financial intermediaries will trade not on behalf of clients but for their own *book*. These include *arbitrageurs* and speculators. Usually such market participants form part of investment banks.

Investors

There is a large variety of players in the bond markets, each trading some or all of the different instruments available to suit their own purposes. We can group the main types of investors according to the time horizon of their investment activity.

Short-term institutional investors. These include banks and building societies, money-market fund managers, central banks, and the treasury desks of some types of corporates. Such bodies are driven by short-term investment views, often subject to close guidelines, and will be driven by the total return available on their investments. Banks will have an additional requirement to maintain *liquidity,* often in fulfilment of regulatory authority rules, by holding a proportion of their assets in the form of easily tradable short-term instruments.

Long-term institutional investors. Typically these types of investors include pension funds and life assurance companies. Their investment horizon is long term, reflecting the nature of their liabilities; often they will seek to match these liabilities by holding long-dated bonds.

Mixed horizon institutional investors. This is possibly the largest category of investors and will include general insurance companies, most corporate bodies, and sovereign wealth funds. Like banks and financial-sector companies, they are also very active in the primary market, issuing bonds to finance their operations.

Market professionals. This category includes the banks and specialist financial intermediaries mentioned earlier, firms that one would not automatically classify as "investors" although they will also have an investment objective. Their time horizon will range from one day to the very long term. Proprietary traders will actively position themselves in the market in order to gain trading profit, for example in response to their view on where they think interest rate levels are headed. These participants will trade directly with other market professionals and investors, or via brokers. Market makers or *traders* (called *dealers* in the United States) are wholesalers in the bond markets; they make two-way prices in selected bonds. Firms will not necessarily be active market makers in all types of bonds; smaller firms often specialise in certain sectors. In a two-way quote the *bid price* is the price at which the market maker will buy stock, so it is the price the investor will receive when selling stock. The *offer price* or *ask price* is the price at which investors can buy stock from the market maker. As one might expect, the bid price is always higher than the offer price, and it is this *spread* that represents the theoretical profit to the market maker. The bid-offer spread set by the market maker is determined by several factors, including supply and demand, and liquidity considerations for that particular stock, the trader's view on market direction and *volatility* as well as that of the stock itself and the presence of any market intelligence. A large bid-offer spread reflects low liquidity in the stock, as well as low demand.

As mentioned earlier, *brokers* are firms that act as intermediaries between buyers and sellers and between market makers and buyers/sellers. Floor-based stock exchanges such as the New York Stock Exchange (NYSE) also feature *specialists,* members of the exchange who are responsible for maintaining an orderly market in one or more securities. These are known as *locals* on the London International Financial Futures and Options Exchange (LIFFE). Locals trade securities for their own account to counteract a temporary imbalance in supply and demand in a particular security; they are an important source of *liquidity* in the market. Locals earn income from brokerage fees and also from pure trading, when they sell securities at a higher price than the original purchase price.

Markets

Markets are that part of the financial system where capital market transactions, including the buying and selling of securities, takes place. A market can describe a traditional stock exchange; that is, a physical trading floor where securities trading occurs. Many financial instruments are traded over the telephone or electronically; these markets are known as *over-the-counter* (OTC) markets. A distinction is made between financial instruments of up to one year's maturity and instruments of over one year's maturity. Short-term instruments make up the *money market* while all other instruments are deemed to be part of the *capital market*. There is also a distinction made between the *primary market* and the *secondary market*. A new issue of bonds made by an investment bank on behalf of its client is made in the primary market. Such an issue can be a *public* offer, in which anyone can apply to buy the bonds, or a *private* offer where the customers of the investment bank are offered the stock. The secondary market is the market in which existing bonds and shares are subsequently traded.

WORLD BOND MARKETS

The origin of the spectacular increase in the size of global financial markets was the rise in oil prices in the early 1970s. Higher oil prices stimulated the development of a sophisticated international banking system, as they resulted in large capital inflows to developed country banks from the oil-producing countries. A significant proportion of these capital flows were placed in *eurodollar* deposits in major banks. The growing trade deficit and level of public borrowing in the United States also contributed. The past 20 years has seen tremendous growth in capital markets volumes and trading. As capital controls were eased and exchange rates moved from fixed to floating, domestic capital markets became internationalised. Growth was assisted by the rapid advance in information technology and the widespread use of financial engineering techniques. Today we would think nothing of dealing in virtually any liquid currency bond in financial centres around the world, often at the touch of a button. Global bond issues, underwritten by the subsidiaries of the same banks, are commonplace. The ease with which transactions can be undertaken has also contributed to a very competitive market in liquid currency assets.

The world bond market has increased in size more than 15 times in the past 30 years. As at the end of 2013, outstanding volume stood at over $25 trillion.

The market in U.S. Treasury securities is the largest bond market in the world. Like the government bond markets in the United Kingdom, Germany, France, and other developed economies, it also very liquid and transparent. Of the major government bond markets in the world, the U.S. market makes up nearly half of the total. The Japanese market is second in size, followed by the German market. A large part of the government bond market is concentrated therefore in just a few countries. Government bonds are traded on major exchanges as well as *over-the-counter* (OTC). Generally OTC refers to trades that are not carried out on an exchange but directly between the counterparties. Bonds are also listed on exchanges.

Companies finance their operations in a number of ways, from equity to short-term debt such as bank overdrafts. It is often advantageous for companies to fix longer-term finance, which is why bonds are so popular. Bonds are also attractive as a means of raising finance because the interest payable on them to investors is tax deductible for the company. Dividends on equity are not tax deductible. A corporate needs to get a reasonable mix of debt versus equity in its funding however, as a high level of interest payments will be difficult to service in times of recession or general market downturn. For this reason the market views unfavourably companies that have a high level of debt. Corporate bonds are also traded on exchanges and OTC. One of the most liquid corporate bond types is the *Eurobond,* which is an international bond issued and traded across national boundaries. Sovereign governments have also issued Eurobonds.

OVERVIEW OF THE MAIN BOND MARKETS

So far we have established that bonds are debt capital market instruments, which means that they represent loans taken out by governments and corporations. The duration of any particular loan will vary from 2 years to 30 years or longer. In this chapter we introduce just a small proportion of the different bond instruments that trade in the market, together with a few words on different country markets. This will set the scene for later chapters, where we look at instruments and markets in greater detail.

Domestic and International Bonds

In any market there is a primary distinction between *domestic* bonds and other bonds. Domestic bonds are issued by borrowers domiciled in the country of issue, and in the currency of the country of issue. Generally they trade only in their original market. A *Eurobond* is issued across national boundaries and can be in any currency, which is why they are also sometimes called *international* bonds. It is now more common for Eurobonds to be referred to as international bonds, to avoid confusion with "euro bonds", which are bonds denominated in *euros,* the currency of 17 countries of the European Union (EU). As an issue of Eurobonds is not restricted in terms of currency or country, the borrower is not restricted as to its nationality either. There are also *foreign* bonds, which are domestic bonds issued by foreign borrowers. An example of a foreign bond is a *Bulldog,* which is a sterling bond issued for trading in the United Kingdom (UK) market by a foreign borrower. The equivalent foreign bonds in other countries include *Yankee* bonds (United States), *Samurai* bonds (Japan), *Alpine* bonds (Switzerland) and *Matador* bonds (Spain).

There are detail differences between these bonds, for example in the frequency of interest payments that each one makes and the way the interest payment is calculated. Some bonds such as domestic bonds pay their interest *net,* which means net of a withholding tax such as income tax. Other bonds including Eurobonds make *gross* interest payments.

Government Bonds As their name suggests, government bonds are issued by a government or *sovereign.* Government bonds in any country form the foundation for the entire domestic debt market. This is because the government market will be the largest in relation to the market as a whole. Government bonds also represent the best *credit risk* in any market as people generally do not expect the government to go bankrupt. As we see in a later chapter, professional institutions that analyse borrowers in terms of their credit risk always rate the government in any market as the highest credit available. While this may sometimes not be the case, it is usually a good rule of thumb.[1] The government bond market is usually also the most *liquid* in the domestic market due to its size and will form the benchmark against which other borrowers are rated. Generally, but not always, the yield offered on government debt will be the lowest in that market.

United States Government bonds in the United States are known as *Treasuries.* Bonds issued with an original maturity of between 2 and 10 years are known as *notes* (as in "Treasury note") while those issued with an original maturity of over

[1] Occasionally one may come across a corporate entity that one may view as better rated in terms of credit risk compared to the government of the country in which the company is domiciled. However, the main credit rating agencies will not rate a corporate entity at a level higher than its country of domicile.

10 years are known as *bonds*. There is no difference between notes and bonds, and they trade the same way in the market. Treasuries pay semiannual coupons. The U.S. Treasury market is the largest single bond market anywhere and trades on a 24-hour basis all around the world. A large proportion of Treasuries are held by foreign governments and corporations. It is a very liquid and *transparent* market.

United Kingdom The UK government issues bonds known as *gilt-edged securities* or *gilts*.[2] The gilt market is another very liquid and transparent market, with prices being very competitive. Many of the more esoteric features of gilts such as "tick" pricing (where prices are quoted in 32nds and not decimals) and *special ex-dividend* trading have been removed in order to harmonise the market with other European Union sovereign bonds. Gilts still pay coupon on a semiannual basis though, unlike eurozone paper. The UK government also issues bonds known as *index-linked* gilts, whose interest and redemption payments are linked to the rate of inflation. There are also older gilts with peculiar features such as no redemption date and quarterly paid coupons.

Germany Government bonds in Germany are known as *bunds,* BOBLs, or *Schatze*. These terms refer to the original maturity of the paper and have little effect on trading patterns. Bunds pay coupon on an annual basis and are of course now denominated in euros.

Table 1.1 summarises the main characteristics of a selected sample of sovereign bond markets. A set of sovereign yield curve data as of December 2013 is given in Table 1.2.

Nonconventional Bonds

The definition of bonds given earlier in this chapter referred to conventional or *plain-vanilla* bonds. There are many variations on vanilla bonds and we can introduce a few of them here.

Floating rate notes. The bond market is often referred to as the *fixed income* market, or the *fixed interest* market in the United Kingdom. Floating rate notes (FRNs) do not have a fixed coupon but instead link their interest payments to an external reference, such as the three-month bank lending rate. Bank interest rates will fluctuate constantly during the life of the bond and so an FRN's cash flows are not known with certainty. Usually FRNs pay a fixed margin or *spread* over the specified reference rate; occasionally the spread is not fixed and such a bond is known as a *variable rate note*.

Because FRNs pay coupons based on the three-month or six-month bank rate, they are essentially money-market instruments and are treated by bank dealing desks as such.

Index-linked bonds. An index-linked bond has its coupon and redemption payment, or possibly just either one of these, linked to a specified index. When governments issue index-linked bonds the cash flows are linked to a price index such as consumer or commodity prices. Corporates have issued index-linked bonds that are connected to inflation or a stock market index.

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[2] This is because early gilt issues are said to have been represented by certificates that were edged with gold leaf, hence the term gilt-edged. In fact the story is almost certainly apocryphal, and it is unlikely that gilt certificates were ever edged with gold.

TABLE 1.1 Selected Government Bond Market Characteristics

	Credit Rating	Maturity Range	Dealing Mechanism	Benchmark Bonds	Issuance	Coupon and Day-Count Basis
Australia	AAA	2–15 years	OTC Dealer network	5, 10 years	Auction	Semiannual, act/act
Canada	AAA	2–30 years	OTC Dealer network	3, 5, 10 years	Auction, subscription	Semiannual, act/act
France	AAA	BTAN: 1–7 years OAT: 10–30 years	OTC Dealer network Bonds listed on Paris Stock Exchange	BTAN: 2, 5 years OAT: 10, 30 years	Dutch auction	BTAN: Semiannual, act/act OAT: Annual, act/act
Germany	AAA	OBL: 2, 5 years BUND: 10, 30 years	OTC Dealer network Listed on Stock Exchange	The most recent issue	Combination of Dutch auction and proportion of each issue allocated on fixed basis to institutions	Annual, act/act
South Africa	A	2–30 years	OTC Dealer network Listed on Johannesburg SE	2, 7, 10, 20 years	Auction	Semiannual, act/365
Singapore	AAA	2–15 years	OTC Dealer network	1, 5, 10, 15 years	Auction	Semiannual, act/act
Taiwan	AA–	2–30 years	OTC Dealer network	2, 5, 10, 20, 30 years	Auction	Annual, act/act
United Kingdom	AAA	2–50 years	OTC Dealer network	5, 10, 30 years	Auction, subsequent issue by "tap" subscription	Semiannual, act/act
United States	AAA	2–20 years	OTC Dealer network	2, 5, 10 years	Auction	Semiannual, act/act

TABLE 1.2 Selected Government Bond Markets, Yield Curves as at 2 December 2013

Term (years)	Australia	Germany	Japan	United Kingdom	United States
1				0.43	0.110
2	2.74	0.12	0.08	0.48	0.250
3					
4					
5	3.49	0.66	0.18	1.58	1.251
7					
10	4.32	1.7	0.61	2.83	2.753
15	4.63			3.17	
20				3.36	
30		2.62	1.65	3.64	3.756

Zero-coupon bonds. Certain bonds do not make any coupon payments at all, and these are known as *zero-coupon bonds*. A zero-coupon bond or *strip* has only one cash flow, the redemption payment at maturity. If we assume that the maturity payment is 100% or *par*, the issue price will be at a discount to par. Such bonds are also known therefore as *discounted* bonds. The difference between the price paid on issue and the redemption payment is the interest realised by the bondholder. As we will discover when we look at strips, this has certain advantages for investors, the main one being that there are no coupon payments to be invested during the bond's life. Both governments and corporates issue zero-coupon bonds. Conventional coupon-bearing bonds can be *stripped* into a series of individual cash flows, which would then trade as separate zero-coupon bonds. This is a common practice in government bond markets where the borrowing authority does not actually issue strips, and they have to be created via the stripping process.

Amortised bonds. A conventional bond will repay on maturity the entire nominal sum initially borrowed on issue. This is known as a *bullet* repayment (which is why vanilla bonds are sometimes known as bullet bonds). A bond that repays portions of the borrowing in stages during its life is known as an *amortised* bond.

Bonds with embedded options. Some bonds include a provision in their offer particulars that gives either the bondholder and/or the issuer an option to enforce early redemption of the bond. The most common type of option embedded in a bond is a *call feature*. A call provision grants the issuer the right to redeem all or part of the debt before the specified maturity date. An issuing company may wish to include such a feature as it allows it to replace an old bond issue with a lower coupon rate issue if interest rates in the market have declined. As a call feature allows the issuer to change the maturity date of a bond, it is considered harmful to the bondholder's interests; therefore the market price of the bond at any time will reflect this. A bond issue may also include a provision that allows the investor to change the maturity of the bond. This is known as a *put feature* and gives the bondholder the right to sell the bond back to the issuer at par on specified dates. The advantage to the bondholder is that if interest rates rise after the issue date, thus depressing the bond's value, the investor can realise par value by *putting* the bond back to the issuer. A *convertible* bond is an issue giving the bondholder the right to exchange the bond for a specified amount of shares (equity) in the issuing company. This feature allows the investor to take advantage of favorable movements in the price of the issuer's shares. The presence of embedded options in a bond makes valuation more complex compared to plain-vanilla bonds, and will be considered separately.

Bond warrants. A bond may be issued with a *warrant* attached to it, which entitles the bondholder to buy more of the bond (or a different bond issued by the same borrower) under specified terms and conditions at a later date. An issuer may include a warrant in order to make the bond more attractive to investors. Warrants are often detached from their host bond and traded separately.

Finally there is a large class of bonds known as *asset-backed securities*. These are bonds formed from pooling together a set of loans such as mortgages or car loans and issuing bonds against them. The interest payments on the original loans serve to back the interest payable on the asset-backed bond. We will look at these instruments in some detail in a later chapter.

TIME VALUE OF MONEY

The principles of pricing in the bond market are exactly the same as those in other financial markets, which state that the price of any financial instrument is equal to the present (today's) value of all the future cash flows from the instrument. Bond prices are expressed as per 100 nominal of the bond, or "percent". So for example, if the price of a U.S. dollar–denominated bond is quoted as "98.00", this means that for every \$100 nominal of the bond a buyer would pay \$98.[3] The interest rate or discount rate used as part of the present value (price) calculation is key, as it reflects where the bond is trading in the market and how it is perceived by the market. All the determining factors that identify the bond, including the nature of the issuer, the maturity, the coupon and the currency, influence the interest rate at which a bond's cash flows are discounted, which will be similar to the rate used for comparable bonds. First, we consider the traditional approach to bond pricing for a plain-vanilla instrument, making certain assumptions to keep the analysis simple, and then we present the more formal analysis commonly encountered in academic texts.

Present Value 101

Bonds or *fixed-income*[4] instruments are debt-capital market securities and therefore have maturities longer than one year. This differentiates them from money-market

[3] The convention in certain markets is to quote a price per 1,000 nominal, but this is less common.

[4] The term "fixed income" originated at a time when bonds were essentially plain-vanilla instruments paying a fixed coupon per year. In the United Kingdom, the term "fixed interest" was used. These days, many bonds do not necessarily pay a fixed coupon each year, for instance, asset-backed bond issues were invariably issued in a number of tranches, with each tranche paying a different fixed or floating coupon. The market is still commonly referred to as the fixed-income market, however.

securities. Bonds have more intricate cash flow patterns than money-market securities, which usually have just one cash flow at maturity. This makes bonds more involved to price than money-market instruments, and their prices more responsive to changes in the general level of interest rates. There is a large variety of bonds. The most common type is the plain-vanilla (or *straight, conventional,* or *bullet*) bond. This is a bond paying a regular (annual or semiannual) fixed interest payment or coupon over a fixed period to maturity or redemption, with the return of principal (the par or nominal value of the bond) on the maturity date. All other bonds are variations on this.

The key identifying feature of a bond is its issuer, the entity that is borrowing funds by issuing the bond into the market. Issuers are generally categorised as one of four types: governments (and their agencies), local governments (or municipal authorities), supranational bodies such as the World Bank, and corporates. Within the municipal and corporate markets there is a wide range of issuers, each assessed as having differing abilities to maintain the interest payments on their debt and repay the full loan on maturity. This ability is identified by a *credit rating* for each issuer. The *term to maturity* of a bond is the number of years over which the issuer has promised to meet the conditions of the debt obligation. The maturity of a bond refers to the date that the debt will cease to exist, at which time the issuer will redeem the bond by paying the principal. The practice in the bond market is to refer to the "term to maturity" of a bond as simply its "maturity" or "term". Some bonds contain provisions that allow either the issuer or the bondholder to alter a bond's term. The term to maturity of a bond is its other key feature. First it indicates the time period over which the bondholder can expect to receive coupon payments and the number of years before the principal is paid back. Secondly, it influences the yield of a bond. Finally, the price of a bond will fluctuate over its life as yields in the market change. The volatility of a bond's price is dependent on its maturity. All else being equal, the longer the maturity of a bond, the greater its price volatility resulting from a change in market interest rates.

The principal of a bond is the amount that the issuer agrees to repay the bondholder on maturity. This amount is also referred to as the *redemption value, maturity value, par value,* or *face value.* The coupon rate, or nominal rate, is the interest rate that the issuer agrees to pay each year during the life of the bond. The annual amount of interest payment made to bondholders is the coupon. The cash amount of the coupon is the coupon rate multiplied by the principal of the bond. For example, a bond with a coupon rate of 8% and a principal of $1,000 will pay annual interest of $80. In the United States, the usual practice is for the issuer to pay the coupon in two semiannual installments. All bonds make periodic coupon payments, except for zero-coupon bonds. Such bonds are issued at a discount and redeemed at par. The holder of a zero-coupon bond realises interest by buying the bond at this discounted value, below its principal value. Interest is therefore paid on maturity, with the exact amount being the difference between the principal value and the discounted value paid on purchase.

Present Value and Discounting

As fixed-income instruments are essentially a collection of cash flows, we begin by reviewing the key concept in cash-flow analysis, that of discounting and *present* *value*. It is essential to have a firm understanding of the main principles of this before moving on to other areas. When reviewing the concept of the time value of money, assume that the interest rates used are the market-determined rates of interest.

Financial arithmetic has long been used to illustrate that £1 received today is not the same as £1 received at a point in the future. Faced with a choice between receiving £1 today or £1 in one year's time, we would be indifferent given a rate of interest of, say, 10% that was equal to our required nominal rate of interest. Our choice would be between £1 today or £1 plus 10p—the interest on £1 for one year at 10% per annum. The notion that money has a time value is a basic concept in the analysis of financial instruments. Money has time value because of the opportunity to invest it at a rate of interest. A loan that has one interest payment on maturity is accruing simple interest. On short-term instruments, there is usually only the one interest payment on maturity, hence simple interest is received when the instrument expires. The terminal value of an investment with simple interest is given by (1.1).

$$F = P(1 + r) \tag{1.1}$$

where F is the terminal value or future value
P is the initial investment or present value
r is the interest rate.

The market convention is to quote interest rates as annualised interest rates, which is the interest that is earned if the investment term is one year. Consider a three-month deposit of \$100 in a bank, placed at a rate of interest of 6%. In such an example, the bank deposit will earn 6% interest for a period of 90 days. As the annual interest gain would be \$6, the investor will expect to receive a proportion of this, which is calculated as follows:

$$\$6.00 \times 90/360$$

Therefore, the investor will receive \$1.50 interest at the end of the term. The total proceeds after the three months is therefore \$100 plus \$1.50. Note, we use 90/360 as that is the convention in the U.S. markets. For a small number of currencies, including Hong Kong dollars and Sterling, a 365-day denominator is used. If we wish to calculate the terminal value of a short-term investment that is accruing simple interest, we use the following expression:

$$F = P(1 + r \times \text{days/year}) \tag{1.2}$$

The fraction days/year refers to the numerator, which is the number of days the investment runs, divided by the denominator which is the number of days in the year. The convention in most markets (including the dollar and euro markets) is to have a 360-day year. In the sterling markets, the number of days in the year is taken to be 365. For this reason, we simply quote the expression as "days" divided by "year" to allow for either convention.

Let us now consider an investment of $100 made for three years, again at a rate of 6%, but this time fixed for three years. At the end of the first year, the investor will be credited with interest of $6. Therefore for the second year the interest rate of 6% will be accruing on a principal sum of $106, which means that at the end of year 2 the interest credited will be $6.36. This illustrates how compounding works, which is the principle of earning interest on interest. The outcome of the process of compounding is the future value of the initial amount. The expression is given in (1.3).

$$FV = PV(1+r)^n \tag{1.3}$$

where FV is the future value
PV is initial outlay or *present value*
r is the periodic rate of interest (expressed as a decimal)
n is the number of periods for which the sum is invested.

When we compound interest, we have to assume that the reinvestment of interest payments during the investment term is at the same rate as the first year's interest. That is why we stated that the 6% rate in our example was fixed for three years. We can see, however, that compounding increases our returns compared to investments that accrue only on a simple-interest basis.

Now let us consider a deposit of $100 for one year, at a rate of 6% but with quarterly interest payments. Such a deposit would accrue interest of $6 in the normal way, but $1.50 would be credited to the account every quarter, and this would then benefit from compounding. Again assuming that we can reinvest at the same rate of 6%, the total return at the end of the year will be:

$$100 \times [(1+0.015) \times (1+0.015) \times (1+0.015) \times (1+0.015)] = 100 \times [(1+0.015)^4]$$

which gives us 100 × 1.06136, a terminal value of $106.136. This is some 13 cents more than the terminal value using annual compounded interest.

In general, if compounding takes place m times per year, then at the end of n years mn interest payments will have been made and the future value of the principal is given by (1.4).

$$FV = PV(1+r/m)^{mn} \tag{1.4}$$

As we showed in our example, the effect of more frequent compounding is to increase the value of the total return when compared to annual compounding. The effect of more frequent compounding is shown in the following table, where we consider the annualised interest-rate factors, for an annualised rate of 6%.

$$\text{Interest-rate factor} = (1 + r/m)^m$$

Compounding Frequency		*Interest-Rate Factor*
Annual	$(1 + r)$	= 1.060000
Semiannual	$(1 + r/2)^2$	= 1.060900
Quarterly	$(1 + r/4)^4$	= 1.061364
Monthly	$(1 + r/12)^{12}$	= 1.061678
Daily	$(1 + r/365)^{365}$	= 1.061831

This shows us that the more frequent the compounding, the higher the interest-rate factor. The last case also illustrates how a limit occurs when interest is compounded continuously. Equation 1.4 can be rewritten as follows:

$$
\begin{aligned}
FV &= PV\left[\left(1+\frac{r}{m}\right)^{m/r}\right]^{rn} \\
&= PV\left[\left(1+\frac{1}{m/r}\right)^{m/r}\right]^{rn} \\
&= PV\left[\left(1+\frac{1}{k}\right)^{k}\right]^{rn}
\end{aligned}
\tag{1.5}
$$

where $k = m/r$. As compounding becomes continuous and m and hence n approach infinity, the expression in the square brackets in (1.5) approaches a value known as e, which is shown in the following equation.

$$e = \lim_{K\to\infty}\left(1+\frac{1}{k}\right)^{k} = 2.718281\ldots$$

If we substitute this into (1.5) this gives us:

$$FV = PVe^{rn} \tag{1.6}$$

where we have continuous compounding. In (1.6), e^{rn} is known as the exponential function of rn, and it tells us the continuously compounded interest rate factor. If $r = 6\%$ and $n = 1$ year then:

$$e^{r} = (2.718281)^{0.06} = 1.061837$$

This is the limit reached with continuous compounding.

The convention in both wholesale and personal (retail) markets is to quote an annual interest rate. A lender who wishes to earn the interest at the rate quoted has to place her funds on deposit for one year. Annual rates are quoted irrespective of the maturity of a deposit, from overnight to 50 years. For example if one opens a bank account that pays interest at a rate of 3.5% but then closes it after six months, the actual interest earned will be equal to 1.75% of the sum deposited.

The actual return on a three-year building society bond (fixed deposit) that pays 6.75% fixed for three years is 21.65% after three years. The quoted rate is the annual one-year equivalent. An overnight deposit in the wholesale or interbank market is still quoted as an annual rate, even though interest is earned for only one day.

The convention of quoting annualised rates is to allow deposits and loans of different maturities and different instruments to be compared on the basis of the interest rate applicable. We must be careful when comparing interest rates for products that have different payment frequencies. As we have seen from the foregoing paragraphs, the actual interest earned will be greater for a deposit earning 6% on a semiannual basis than one earning 6% on an annual basis. The convention in the money markets is to quote the equivalent interest rate applicable when taking into account an instrument's payment frequency.

We saw how a future value could be calculated given a known present value and rate of interest. For example, \$100 invested today for one year at an interest rate of 6% will generate $100 \times (1 + 0.06) = \$106$ at the end of the year. The future value of \$100 in this case is \$106. We can also say that \$100 is the present value of \$106 in this case.

In (1.3) we established the following future-value relationship:

$$FV = PV\ (1 + r)^n$$

By reversing this expression we arrive at the present-value calculation given in (1.7).

$$PV = \frac{FV}{(1+r)^n} \tag{1.7}$$

where the symbols represent the same terms as before. Equation 1.7 applies in the case of annual interest payments and enables us to calculate the present value of a known future sum.

To calculate the present value for a short-term investment of less than one year, we will need to adjust what would have been the interest earned for a whole year by the proportion of days of the investment period. Rearranging the basic equation, we can say that the present value of a known future value is:

$$PV = \frac{FV}{\left(1 + r \times \frac{\text{days}}{\text{year}}\right)} \tag{1.8}$$

Given a present value and a future value at the end of an investment period, what then is the interest rate earned? We can rearrange the basic equation again to solve for the *yield*.

When interest is compounded more than once a year, the formula for calculating present value is modified, in shown (1.9).

$$PV = \frac{FV}{\left(1 + \frac{r}{m}\right)^{mn}} \tag{1.9}$$

where, as before, FV is the cash flow at the end of year n, m is the number of times a year interest is compounded, and r is the rate of interest or discount rate. Illustrating this, therefore, the present value of $100 that is received at the end of five years at a rate of interest rate of 5%, with quarterly compounding is:

$$\begin{aligned} PV &= \frac{100}{\left(1+\frac{0.05}{4}\right)^{(4)(5)}} \\ &= \$78.00 \end{aligned}$$

Interest rates in the money markets are always quoted for standard maturities—for example, overnight, tom next (the overnight interest rate starting tomorrow, or "tomorrow to the next"), spot next (the overnight rate starting two days forward), one week, one month, two months, and so on, up to one year. If a bank or corporate customer wishes to deal for nonstandard periods, an interbank desk will calculate the rate chargeable for such an "odd date" by interpolating between two standard-period interest rates. If we assume that the rate for all dates in between two periods increases at the same steady state, we can calculate the required rate using the formula for straight-line interpolation, shown in (1.10).

$$r = r_1 + (r_2 - r_1) \times \frac{n - n_1}{n_2 - n_1} \tag{1.10}$$

where r is the required odd-date rate for n days
r_1 is the quoted rate for n_1 days
r_2 is the quoted rate for n_2 days

Let us imagine that the one-month (30-day) offered interest rate is 5.25% and that the two-month (60-date) offered rate is 5.75%. If a customer wishes to borrow money for a 40-day period, what rate should the bank charge? We can calculate the required 40-day rate using the straight-line interpolation process. The increase in interest rates from 30 to 40 days is assumed to be 10/30 of the total increase in rates from 30 to 60 days. The 40-day offered rate would therefore be:

$$5.25\% + (5.75\% - 5.25\%) \times 10/30 = 5.4167\%$$

What about the case of an interest rate for a period that lies just before or just after two known rates and not roughly in between them? When this happens we extrapolate between the two known rates, again assuming a straight-line relationship between the two rates and for a period after (or before) the two rates. So if the one-month offered rate is 5.25% while the two-month rate is 5.75%, the 64-day rate is:

$$5.25 + (5.75 - 5.25) \times 34/30 = 5.8167\%$$

Discount Factors

An n-period discount factor is the present value of one unit of currency (£1 or \$1) that is payable at the end of period n. Essentially, it is the present-value relationship expressed in terms of \$1. If $d(n)$ is the n-year discount factor, then the five-year discount factor at a discount rate of 6% is given by

$$d(5) = \frac{1}{(1+0.06)^5} = 0.747258$$

The set of discount factors for every time period from one day to 30 years or longer is termed the *discount function.* Discount factors may be used to price any financial instrument that is made up of a future cash flow. For example, what would be the value of \$103.50 receivable at the end of six months if the six-month discount factor is 0.98756? The answer is given by:

$$0.98756 \times 103.50 = 102.212$$

In addition, discount factors may be used to calculate the future value of any present investment. From the earlier example, \$0.98756 would be worth \$1 in six months' time, so by the same principle a present sum of \$1 would be worth

$$1/d(0.5) = 1/0.98756 = 1.0126$$

at the end of six months.

It is possible to obtain discount factors from current bond prices. Assume a hypothetical set of bonds and bond prices at the date of 7 Dec 2000 as given in Table 1.3, and assume further that the first bond in the table matures in precisely six months' time (these are semiannual coupon bonds).

Taking the first bond, this matures in precisely six months' time, and its final cash flow will be 103.0, comprising the \$3.50 final coupon payment and the \$100 redemption payment. The price or present value of this bond is 101.65, which allows us to calculate the six-month discount factor as:

$$d(0.5) \times 103.50 = 101.65$$

which gives $d(0.5)$ equal to 0.98213.

TABLE 1.3 Hypothetical Set of Bonds and Bond Prices at the Date of 7 Dec 2000

Coupon	Maturity Date	Price
7%	7-Jun-01	101.65
8%	7-Dec-01	101.89
6%	7-Jun-02	100.75
6.50%	7-Dec-02	100.37

TABLE 1.4 Discount Factors Calculated Using Bootstrapping Technique

Coupon	Maturity Date	Term (years)	Price	*d(n)*
7%	7-Jun-01	0.5	101.65	0.98213
8%	7-Dec-01	1.0	101.89	0.94194
6%	7-Jun-02	1.5	100.75	0.92211
6.50%	7-Dec-02	2.0	100.37	0.88252

From this first step we can calculate the discount factors for the following six-month periods. The second bond in Table 1.3, the 8% 2001, has the following cash flows:

- \$4 in six months' time
- \$104 in one year's time

The price of this bond is 101.89, which again is the bond's present value, and this comprises the sum of the present values of the bond's total cash flows. So we are able to set the following:

$$101.89 = 4 \times d(0.5) + 104 \times d(1)$$

However, we already know $d(0.5)$ to be 0.98213, which leaves only one unknown in the preceding expression. Therefore we may solve for $d(1)$, and this is shown to be 0.94194.

If we carry on with this procedure for the remaining two bonds, using successive discount factors, we obtain the complete set of discount factors as shown in Table 1.4. The continuous function for the two-year period from today is known as the discount function, shown in Figure 1.2.

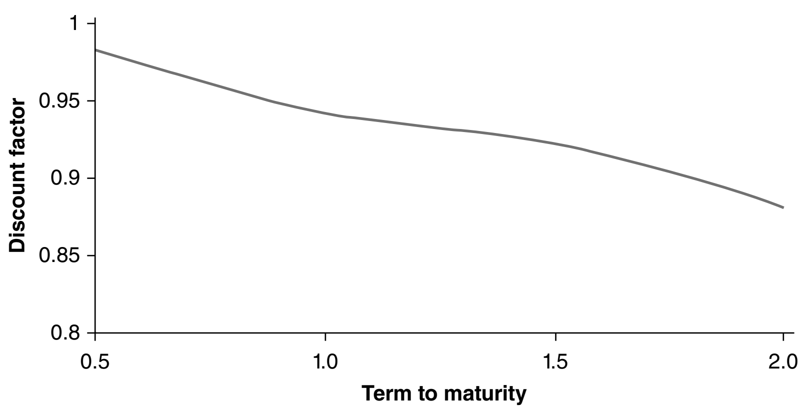

FIGURE 1.2 Hypothetical Discount Function

This technique, which is known as *bootstrapping,* is conceptually neat but presents problems when we do not have a set of bonds that mature at precise six-month intervals. In addition, liquidity issues connected with specific individual bonds can also cause complications. However, it is still worth being familiar with this approach.

Note from Figure 1.2 how discount factors decrease with increasing maturity: this is intuitively obvious, since the present value of something to be received in the future diminishes the further into the future we go.

BOND PRICING AND YIELD: THE TRADITIONAL APPROACH

Bond Pricing

The interest rate that is used to discount a bond's cash flows (and therefore called the discount rate) is the rate required by the bondholder. This is therefore known as the bond's yield. The yield on the bond will be determined by the market and is the price demanded by investors for buying it, which is why it is sometimes called the bond's return. The required yield for any bond will depend on a number of political and economic factors, including what yield is being earned by other bonds of the same class. Yield is always quoted as an annualised interest rate, so that for a bond paying semiannually exactly half of the annual rate is used to discount the cash flows.

The fair price of a bond is the present value of all its cash flows. Therefore, when pricing a bond, we need to calculate the present value of all the coupon interest payments and the present value of the redemption payment, and sum these. The price of a conventional bond that pays annual coupons can therefore be given by (1.11).

$$\begin{aligned} P &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ldots \frac{C}{(1+r)^N} + \frac{M}{(1+r)^N} \\ &= \sum_{n=1}^{N} \frac{C}{(1+r)^n} + \frac{M}{(1+r)^N} \end{aligned} \tag{1.11}$$

where P is the price
C is the annual coupon payment
r is the discount rate (therefore, the required yield)
N is the number of years to maturity (therefore, the number of interest periods in an annually paying bond)
M is the maturity payment or par value (usually 100% of currency)

Note that (1.11) applies only for fixed coupon bonds where the "recovery rate" (RR) on default of the issuer is zero. In other words, we can only assume it for default-risk free bonds. The RR term will be explained and considered in later chapters.

For long-hand calculation purposes, the first half of (1.11) is usually simplified and is sometimes encountered in one of the two ways shown in (1.12).

$$\sum_{n=1}^{N} \frac{C}{(1+r)^n} = C\left[\frac{1-\left[\frac{1}{(1+r)^N}\right]}{r}\right]$$

or

$$\sum_{n=1}^{N} \frac{C}{(1+r)^n} = \frac{C}{r}\left[1-\frac{1}{(1+r)^N}\right] \tag{1.12}$$

The price of a bond that pays semiannual coupons is given by the expression in (1.13), which is our earlier expression modified to allow for the twice-yearly discounting:

$$\begin{aligned} P &= \frac{C/2}{\left(1+\tfrac{1}{2}r\right)} + \frac{C/2}{\left(1+\tfrac{1}{2}r\right)^2} + \frac{C/2}{\left(1+\tfrac{1}{2}r\right)^3} + \ldots\ldots \frac{C/2}{\left(1+\tfrac{1}{2}r\right)^{2N}} + \frac{M}{\left(1+\tfrac{1}{2}r\right)^{2N}} \\ &= \sum_{n=1}^{2N} \frac{C/2}{\left(1+\tfrac{1}{2}r\right)^n} + \frac{M}{\left(1+\tfrac{1}{2}r\right)^{2N}} \\ &= \frac{C}{r}\left[1-\frac{1}{\left(1+\tfrac{1}{2}r\right)^{2N}}\right] + \frac{M}{\left(1+\tfrac{1}{2}r\right)^{2N}} \end{aligned} \tag{1.13}$$

Note how we set $2N$ as the power to which to raise the discount factor, as there are two interest payments every year for a bond that pays semiannually. Therefore, a more convenient function to use might be the number of interest periods in the life of the bond, as opposed to the number of years to maturity, which we could set as n, allowing us to alter the equation for a semiannually paying bond as:

$$P = \frac{C}{r}\left[1-\frac{1}{(1+\tfrac{1}{2}r)^{2n}}\right] + \frac{M}{(1+\tfrac{1}{2}r)^{2n}} \tag{1.14}$$

The formula in (1.14) calculates the fair price on a coupon-payment date, so that there is no accrued interest incorporated into the price. It also assumes that there is an even number of coupon-payment dates remaining before maturity. The concept of accrued interest is an accounting convention, and treats coupon interest as accruing every day that the bond is held; this amount is added to the discounted present value of the bond (the *clean* price) to obtain the market value of the bond, known as the *dirty* price.

The date used as the point for calculation is the settlement date for the bond, the date on which a bond will change hands after it is traded. For a new issue of bonds, the settlement date is the day when the stock is delivered to investors and payment is received by the bond issuer. The settlement date for a bond traded in the secondary market is the day when the buyer transfers payment to the seller of the bond and when the seller transfers the bond to the buyer. Different markets will have different settlement conventions. For example, Australian government bonds normally settle two business days after the trade date (the notation used in bond markets is "T + 2"), whereas Eurobonds settle on T + 3. The term *value date* is sometimes used in place of settlement date. However, the two terms are not strictly synonymous. A settlement date can only fall on a business date, so that an Australian government bond traded on a Friday will settle on a Tuesday. However, a value date can sometimes fall on a non-business day; for example, when accrued interest is being calculated.

The standard formula also assumes that the bond is traded for a settlement on a day that is precisely one interest period before the next coupon payment. The price formula is adjusted if dealing takes place in between coupon dates. If we take the value date for any transaction, we then need to calculate the number of calendar days from this day to the next coupon date. We then use the following ratio i when adjusting the exponent for the discount factor:

$$i = \frac{\text{Days from value date to next coupon date}}{\text{Days in the interest period}}$$

The number of days in the interest period is the number of calendar days between the last coupon date and the next one, and it will depend on the day-count basis used for that specific bond. The price formula is then modified as shown in (1.15).

$$P = \frac{C}{(1+r)^{i}} + \frac{C}{(1+r)^{1+i}} + \frac{C}{(1+r)^{2+i}} + \ldots\ldots \frac{C}{(1+r)^{n-1+i}} + \frac{M}{(1+r)^{n-1+i}} \tag{1.15}$$

where the variables C, M, n and r are as before. Note that (1.15) assumes r for an annually paying bond and is adjusted to $r/2$ for a semiannually paying bond.

EXAMPLE 1.1

In these examples we illustrate the long-hand price calculation, using both expressions for the calculation of the present value of the annuity stream of a bond's cash flows.

1.1 (A)

Calculate the fair pricing of a U.S. Treasury, the 4% of February 2014, which pays semiannual coupons, with the following terms:

C = \$4.00 per \$100 nominal

M = \$100

N = 10 years (that is, the calculation is for value the 17th February 2004)

r = 4.048%

(continued)

EXAMPLE 1.1 (*Continued*)

$$
\begin{aligned}
P &= \frac{\$4.00}{0.04048}\left\{1-\frac{1}{[1+\frac{1}{2}(0.04048)]^{20}}\right\}+\frac{\$100}{[1+\frac{1}{2}(0.04048)]^{20}}\\
&= \$32.628+\$66.981\\
&= \$99.609 \text{ or } 99-19+
\end{aligned}
$$

The fair price of the Treasury is \$99 – 19+, which is composed of the present value of the stream of coupon payments (\$32.628) and the present value of the return of the principal (\$66.981).

This yield calculation is shown at Figure 1.3, the Bloomberg YA page for this security. We show the price shown as 99 – 19+ for settlement on 17 Feb 2004, the date it was issued.

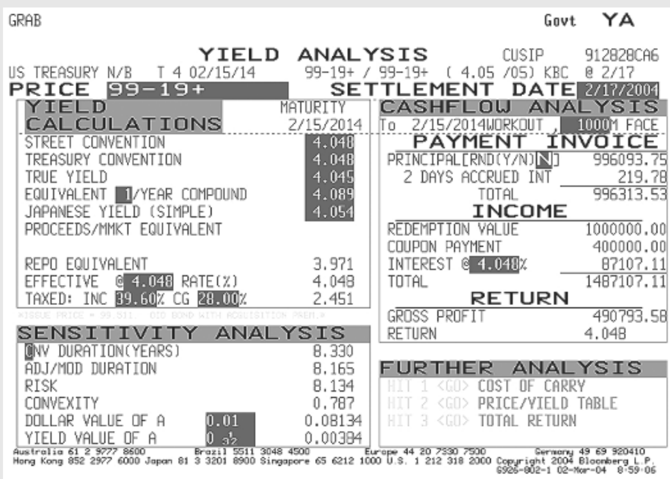

FIGURE 1.3 Bloomberg YA Page for Yield Analysis

1.1(B)

What is the price of a 5% coupon sterling bond with precisely five years to maturity, with semiannual coupon payments, if the yield required is 5.40%?

As the cash flows for this bond are 10 semiannual coupons of £2.50 and a redemption payment of £100 in 10 six-month periods from now, the price of the bond can be obtained by solving the following expression, where we substitute $C = 2.5$, $n = 10$, and $r = 0.027$ into the price equation (the values for C and r reflect the adjustments necessary for a semiannual paying bond).

EXAMPLE 1.1 (*Continued*)

$$P = 2.5\left[\frac{1-\left[\frac{1}{(1.027)^{10}}\right]}{0.027}\right]+\frac{100}{(1.027)^{10}}$$

$$= 21.65574 + 76.61178$$

$$= \$98.26752$$

The price of the bond is \$98.2675 per \$100 nominal.

1.1(C)

What is the price of a 5% coupon euro bond with five years to maturity paying annual coupons, again with a required yield of 5.4%?

In this case there are five periods of interest, so we may set $C = 5$, $n = 5$, with $r = 0.05$.

$$P = 5\left[\frac{1-\left[\frac{1}{(1.054)^{5}}\right]}{0.054}\right]+\frac{100}{(1.054)^{5}}$$

$$= 21.410121 + 76.877092$$

$$= £98.287213$$

Note how the annual-paying bond has a slightly higher price for the same required annualised yield. This is because the semiannual paying sterling bond has a higher effective yield than the euro bond, resulting in a lower price.

1.1(D)

Consider our 5% sterling bond again, but this time the required yield has risen and is now 6%. This makes $C = 2.5$, $n = 10$, and $r = 0.03$.

$$P = 2.5\left[\frac{1-\left[\frac{1}{(1.03)^{10}}\right]}{0.03}\right]+\frac{100}{(1.03)^{10}}$$

$$= 21.325507 + 74.409391$$

$$= £95.7349$$

(*continued*)

EXAMPLE 1.1 (*Continued*)

As the required yield has risen, the discount rate used in the price calculation is now higher, and the result of the higher discount is a lower present value (price).

1.1(E)

Calculate the price of our sterling bond, still with five years to maturity but offering a yield of 5.1%.

$$P = 2.5\left[\frac{1-\left[\frac{1}{(1.0255)^5}\right]}{0.0255}\right] + \frac{100}{(1.0255)^5}$$
$$= 21.823737 + 77.739788$$
$$= £99.563523$$

To satisfy the lower required yield of 5.1%, the price of the bond has fallen to £99.56 per £100.

1.1(F)

Calculate the price of the 5% sterling bond one year later, with precisely four years left to maturity and with the required yield still at the original 5.40%. This sets the terms in 1.1(b) unchanged, except now $n = 8$.

$$P = 2.5\left[\frac{1-\left[\frac{1}{(1.027)^8}\right]}{0.027}\right] + \frac{100}{(1.027)^8}$$
$$= 17.773458 + 80.804668$$
$$= £98.578126$$

The price of the bond is £98.58. Compared to 1.1(B) this illustrates how, other things being equal, the price of a bond will approach par (£100 percent) as it approaches maturity.

There also exist *perpetual* or *irredeemable* bonds which have no redemption date, so that interest on them is paid indefinitely. They are also known as undated bonds. An example of an undated bond is the 3½% War Loan, a UK gilt originally issued in 1916 to help pay for the 1914–1918 war effort. Most undated bonds date from a long time in the past, and it is unusual to see them issued today. In structure, the cash flow from an undated bond can be viewed as a continuous annuity. The fair price of such a bond is given from (1.11) by setting $N = \infty$, such that:

$$P = \frac{C}{r} \tag{1.16}$$

In most markets, bond prices are quoted in decimals, in minimum increments of 1/100ths. This is the case with Eurobonds, euro-denominated bonds, and gilts, for example. Certain markets—including the U.S. Treasury market and South African and Indian government bonds, for example—quote prices in ticks, where the minimum increment is 1/32nd. One tick is therefore equal to 0.03125. A U.S. Treasury might be priced at "98-05" which means "98 and five ticks". This is equal to 98 and 5/32nds which is 98.15625.

Bonds that do not pay a coupon during their life are known as zero-coupon bonds or strips, and the price for these bonds is determined by modifying (1.11) to allow for the fact that $C = 0$. We know that the only cash flow is the maturity payment, so we may set the price as:

$$P = \frac{M}{(1+r)^N} \tag{1.17}$$

where M and r are as before and N is the number of years to maturity. The important factor is to allow for the same number of interest periods as coupon bonds of the same currency. That is, even though there are no actual coupons, we calculate prices and yields on the basis of a quasi-coupon period. For a U.S. dollar or a

EXAMPLE 1.2

What is the total consideration for £5 million nominal of a gilt, where the price is 114.50?

The price of the gilt is £114.50 per £100, so the consideration is:

$$1.145 \times 5{,}000{,}000 = £5{,}725{,}000$$

What consideration is payable for \$5 million nominal of a U.S. Treasury, quoted at an all-in price of 99-16?

The U.S. Treasury price is 99-16, which is equal to 99 and 16/32, or 99.50 per \$100. The consideration is therefore:

$$0.9950 \times 5{,}000{,}000 = \$4{,}975{,}000$$

If the price of a bond is below par, the total consideration is below the nominal amount; whereas if it is priced above par, the consideration will be above the nominal amount.

EXAMPLE 1.3

1.3(A)

Calculate the price of a gilt strip with a maturity of precisely five years, where the required yield is 5.40%.

These terms allow us to set $N = 5$ so that $n = 10$, $r = 0.054$ (so that $r/2 = 0.027$), with $M = 100$ as usual.

$$
\begin{aligned}
P &= \frac{100}{(1.027)^{10}} \\
&= £76.611782
\end{aligned}
$$

1.3(B)

Calculate the price of a French government zero-coupon bond with precisely five years to maturity, with the same required yield of 5.40%.

$$
\begin{aligned}
P &= \frac{100}{(1.054)^{5}} \\
&= £76.877092
\end{aligned}
$$

sterling zero-coupon bond, a five-year zero coupon bond would be assumed to cover 10 quasi-coupon periods, which would set the price equation as:

$$P = \frac{M}{(1 + \tfrac{1}{2}r)^n} \tag{1.18}$$

We have to note carefully the quasi-coupon periods in order to maintain consistency with conventional bond pricing.

An examination of the bond price formula tells us that the yield and price for a bond are related. A key aspect of this relationship is that the price changes in the opposite direction to the yield. This is because the price of the bond is the net present value of its cash flows; if the discount rate used in the present value calculation increases, the present values of the cash flows will decrease. This occurs whenever the yield level required by bondholders increases. In the same way, if the required yield decreases, the price of the bond will rise. This property was observed in Example 1.2. As the required yield decreased, the price of the bond increased, and we observed the same relationship when the required yield was raised.

The relationship between any bond's price and yield at any required yield level is illustrated in a stylised manner in Figure 1.4, which is obtained if we plot the yield against the corresponding price; this shows a convex curve. In practice the curve is not quite as perfectly convex as illustrated in Figure 1.4, but the diagram is representative.

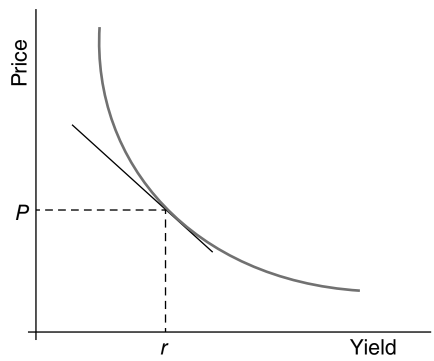

FIGURE 1.4 The Price/Yield Relationship

SUMMARY OF THE PRICE/YIELD RELATIONSHIP

At issue, if a bond is priced at par, its coupon will equal the yield that the market requires from the bond.

If the required yield rises above the coupon rate, the bond price will decrease.

If the required yield goes below the coupon rate, the bond price will increase.

BOND YIELD

We have observed how to calculate the price of a bond using an appropriate discount rate known as the bond's yield. We can reverse this procedure to find the yield of a bond where the price is known, which would be equivalent to calculating the bond's internal rate of return (IRR). The IRR calculation is taken to be a bond's yield to maturity or redemption yield and is one of various yield measures used in the markets to estimate the return generated from holding a bond. In most markets, bonds are generally traded on the basis of their prices, but because of the complicated patterns of cash flows that different bonds can have they are generally compared in terms of their yields. This means that a market maker will usually quote a two-way price at which she will buy or sell a particular bond, but it is the yield at which the bond is trading that is important to the market maker's customer. This is because a bond's price does not actually tell us anything useful about what we are getting. Remember, that in any market there will be a number of bonds with different issuers, coupons, and terms to maturity. Even in a homogenous market such as the Treasury market, different bonds and notes will trade according to their own specific characteristics. To compare bonds in the market, therefore, we need the yield on any bond, and it is yields that we compare, not prices.

The yield on any investment is the interest rate that will make the present value of the cash flows from the investment equal to the initial cost (price) of the investment. Mathematically, the yield on any investment, represented by r, is the interest rate that satisfies (1.19), which is simply the bond price equation we've already reviewed.

$$P = \sum_{n=1}^{N} \frac{C_n}{(1+r)^n} \tag{1.19}$$

But as we have noted there are other types of yield measure used in the market for different purposes. The simplest measure of the yield on a bond is the current yield, also known as the flat yield, interest yield or running yield. The running yield is given by (1.20).

$$rc = \frac{C}{P} \times 100 \tag{1.20}$$

where rc is the current yield.

In (1.20) C is not expressed as a decimal. Current yield ignores any capital gain or loss that might arise from holding and trading a bond and does not consider the time value of money. It essentially calculates the bond coupon income as a proportion of the price paid for the bond, and to be accurate would have to assume that the bond was more like an annuity rather than a fixed-term instrument.

The current yield is useful as a rough-and-ready interest-rate calculation; it is often used to estimate the cost of or profit from a short-term holding of a bond. For example, if other short-term interest rates such as the one-week or three-month rates are higher than the current yield, holding the bond is said to involve a running cost. This is also known as *negative carry* or *negative funding*. The term is used by bond traders and market makers and leveraged investors. The carry on a bond is a useful measure for all market practitioners as it illustrates the cost of holding or funding a bond. The funding rate is the bondholder's short-term cost of funds. A private investor could also apply this to a short-term holding of bonds.

The yield to maturity or gross redemption yield is the most frequently used measure of return from holding a bond.[5] Yield to maturity (YTM) takes into account the pattern of coupon payments, the bond's term to maturity, and the capital gain (or loss) arising over the remaining life of the bond. We saw from our bond price formula in the previous section that these elements were all related and were important components determining a bond's price. If we set the IRR for a set of cash flows to be the rate that applies from a start-date to an end-date we can assume the IRR to be the YTM for those cash flows. The YTM therefore is equivalent to the internal rate of return on the bond, the rate that equates the value of the discounted cash flows on the bond to its current price. The calculation

[5] In this book the terms *yield to maturity* and *gross redemption yield* are used synonymously. The latter term is encountered in sterling markets.

assumes that the bond is held until maturity, and therefore it is the cash flows to maturity that are discounted in the calculation. It also employs the concept of the time value of money.

As we would expect, the formula for YTM is essentially that for calculating the price of a bond. For a bond paying annual coupons, the YTM is calculated by solving (1.11). Note that the expression in (1.11) has two variable parameters, the price P and yield r. It cannot be rearranged to solve for yield r explicitly, and, in fact, the only way to solve for the yield is to use the process of numerical iteration. The process involves estimating a value for r and calculating the price associated with the estimated yield. If the calculated price is higher than the price of the bond at the time, the yield estimate is lower than the actual yield, and so it must be adjusted until it converges to the level that corresponds with the bond price.[6] For the YTM of a semiannual coupon bond, we have to adjust the formula to allow for the semiannual payments, shown in (1.13).

EXAMPLE 1.4 YIELD TO MATURITY FOR SEMIANNUAL COUPON BOND

A semiannual paying bond has a dirty price of \$98.50, an annual coupon of 6%, and there is exactly one year before maturity. The bond therefore has three remaining cash flows, comprising two coupon payments of \$3 each and a redemption payment of \$100. Equation 1.12 can be used with the following inputs:

$$98.50 = \frac{3.00}{(1 + \frac{1}{2}rm)} + \frac{103.00}{(1 + \frac{1}{2}rm)^2}$$

Note that we use half of the YTM value *rm* because this is a semiannual paying bond. The preceding expression is a quadratic equation, which is solved using the standard solution for quadratic equations, which is noted in the following equations.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our expression, if we let $x = (1 + rm/2)$, we can rearrange the expression as follows:

$$98.50x^2 - 3.0x - 103.00 = 0$$

(*continued*)

[6] Bloomberg also uses the term *yield-to-workout*, where *workout* refers to the maturity date for the bond.

EXAMPLE 1.4 (*Continued*)

We then solve for a standard quadratic equation, and there will be two solutions, only one of which gives a positive redemption yield. The positive solution is $rm/2 = 0.037929$ so that $rm = 7.5859\%$.

As an example of the iterative solution method, suppose that we start with a trial value for rm of $r_1 = 7\%$ and plug this into the right-hand side of (1.12). This gives a value for the right-hand side of:

$$\text{RHS}_1 = 99.050$$

which is higher than the left-hand side (LHS = 98.50); the trial value for rm was therefore too low. Suppose then that we try next $r_2 = 8\%$ and use this as the right-hand side of the equation. This gives:

$$\text{RHS}_2 = 98.114$$

$$rm = r_1 + (r_2 - r_1)\frac{RHS_1 - LHS}{RHS_1 - RHS_2}$$

our linear approximation for the redemption yield is $rm = 7.587\%$, which is near the exact solution.

To differentiate redemption yield from other yield and interest-rate measures described in this book, we henceforth refer to it as *rm*.

Note that the redemption yield, as discussed earlier in this section, is the gross redemption yield, the yield that results from payment of coupons without deduction of any withholding tax. The net redemption yield is obtained by multiplying the coupon rate C by (1 – marginal tax rate). The net yield is what will be received if the bond is traded in a market where bonds pay coupon net, which means net of a withholding tax. The net redemption yield is always lower than the gross redemption yield.

We have already alluded to the key assumption behind the YTM calculation, namely that the rate *rm* remains stable for the entire period of the life of the bond. By assuming the same yield, we can say that all coupons are reinvested at the same yield *rm*. For the bond in Example 1.4, this means that if all the cash flows are discounted at 7.59% they will have a total net present value of 98.50. This is patently unrealistic since we can predict with virtual certainty that interest rates for instruments of similar maturity to the bond at each coupon date will not remain at this rate for the life of the bond. In practice, however, investors require a rate of return that is equivalent to the price that they are paying for a bond, and the redemption yield is, to put it simply, as good a measurement as any. A more accurate measurement might be to calculate present values of future cash flows using the discount rate that is equal to the forward interest rates at that point, known as the forward interest rate. However, forward rates are simply interest rates today for execution at a future date, and so a YTM measurement calculated using forward rates can be as speculative as one calculated using the conventional formula. So a YTM calculation made using forward rates would not be realised in practice either. We shall see later how the zero-coupon interest rate is the true interest rate for any term to maturity. However, despite the limitations presented by its assumptions, the YTM is the main measure of return used in the markets.

We have noted the difference between calculating redemption yield on the basis of both annual and semiannual coupon bonds. Analysis of bonds that pay semiannual coupons incorporates semiannual discounting of semiannual coupon payments. This is appropriate for most UK and U.S. bonds. However, government bonds in most of continental Europe and most Eurobonds pay annual coupon payments, and the appropriate method of calculating the redemption yield is to use annual discounting. The two yields measures are not therefore directly comparable. We could make a Eurobond directly comparable with a UK gilt by using semi-annual discounting of the Eurobond's annual coupon payments. Alternatively we could make the gilt comparable with the Eurobond by using annual discounting of its semiannual coupon payments. The price/yield formulae for different discounting possibilities we encounter in the markets are listed in the following equations (as usual we assume that the calculation takes place on a coupon payment date so that accrued interest is zero).

Semiannual discounting of annual payments:

$$\mathrm{P}_d = \frac{C}{(1+\frac{1}{2}rm)^2} + \frac{C}{(1+\frac{1}{2}rm)^4} + \frac{C}{(1+\frac{1}{2}rm)^6} + \ldots \frac{C}{(1+\frac{1}{2}rm)^{2N}} + \frac{M}{(1+rm)^{2N}} \tag{1.21}$$

Annual discounting of semiannual payments:

$$\mathrm{P}_d = \frac{C/2}{(1+rm)^{\frac{1}{2}}} + \frac{C/2}{(1+rm)} + \frac{C/2}{(1+rm)^{\frac{3}{2}}} + \ldots \frac{C/2}{(1+rm)^N} + \frac{M}{(1+rm)^N} \tag{1.22}$$

Consider a bond with a dirty price of 97.89, a coupon of 6%, and five years to maturity. This bond would have the following gross redemption yields under the different yield-calculation conventions:

Discounting	Payments	Yield to Maturity (%)
Semiannual	Semiannual	6.500
Annual	Annual	6.508
Semiannual	Annual	6.428
Annual	Semiannual	6.605

This proves what we have already observed: namely, that the coupon and discounting frequency will affect the redemption yield calculation for a bond. We can see that increasing the frequency of discounting will lower the yield, while increasing the frequency of payments will raise the yield. When comparing yields for bonds that trade in markets with different conventions, it is important to convert all the yields to the same calculation basis. Intuitively we might think that doubling a semiannual yield figure will give us the annualised equivalent; in fact, this will result in an inaccurate figure due to the multiplicative effects of discounting and one that is an underestimate of the true annualised yield. The correct procedure for producing an annualised yields from semiannual and quarterly yields is given by the following expressions.

The general conversion expression is given by (1.23):

$$rm_a = (1 + \text{interest rate})^m - 1 \tag{1.23}$$

where m is the number of coupon payments per year.

Specifically we can convert between yields using the expressions given in (1.24) and (1.25).

$$\begin{aligned} rm_a &= [(1 + \tfrac{1}{2} rm_s)^2 - 1] \\ rm_s &= [(1 + rm_a)^{\frac{1}{2}} - 1] \times 2 \end{aligned} \tag{1.24}$$

$$\begin{aligned} rm_a &= [(1 + \tfrac{1}{4} rm_q)^4 - 1] \\ rm_q &= [(1 + rm_a)^{\frac{1}{4}} - 1] \times 4 \end{aligned} \tag{1.25}$$

where rm_q, rm_s, and rm_a are, respectively, the quarterly, semiannually, and annually compounded yields to maturity.

The market convention is sometimes simply to double the semiannual yield to obtain the annualised yields, despite the fact that this produces an inaccurate result. It is only acceptable to do this for rough calculations. An annualised yield obtained by multiplying the semiannual yield by two is known as a bond equivalent yield.

While YTM is the most commonly used measure of yield, it has one major disadvantage. The disadvantage is that implicit in the calculation of the YTM is the assumption that each coupon payment as it becomes due is reinvested at the rate rm.

EXAMPLE 1.5

A UK gilt paying semiannual coupons and a maturity of 10 years has a quoted yield of 4.89%. A European government bond of similar maturity is quoted at a yield of 4.96%. Which bond has the higher effective yield?

The effective annual yield of the gilt is:

$$rm = (1 + {}^1/_2 \times 0.0489)^2 - 1 = 4.9498\%$$

Therefore, the gilt does indeed have the lower yield.

This is clearly unlikely, due to the fluctuations in interest rates over time and as the bond approaches maturity. In practice, the measure itself will not equal the actual return from holding the bond, even if it is held to maturity. That said, the market standard is to quote bond returns as yields to maturity, bearing the key assumptions behind the calculation in mind.

Another disadvantage of this measure of return arises where investors do not hold bonds to maturity. The redemption yield measure will not be of great value where the bond is not being held to redemption. Investors might then be interested in other measures of return, which we can look at later.

To reiterate then, the redemption yield measure assumes that:

1. The bond is held to maturity;
2. All coupons during the bond's life are reinvested at the same (redemption yield) rate.

Therefore the YTM can be viewed as a prospective yield if the bond is purchased on issue and held to maturity. Even then the actual realised yield on maturity would be different from expected or anticipated yield and is closest to reality only where an investor buys a bond on first issue and holds the YTM figure because of the inapplicability of the second condition in the preceding list.

In addition, as coupons are discounted at the yield specific for each bond, it actually becomes inaccurate to compare bonds using this yield measure. For instance, the coupon cash flows that occur in two years time from both a two-year and five-year bond will be discounted at different rates (assuming we do not have a flat yield curve). This would occur because the YTM for a five-year bond is invariably different from the YTM for a two-year bond. However, it would clearly not be correct to discount a two-year cash flow at different rates, because we can see that the present value calculated today of a cash flow in two years' time should be the same whether it is sourced from a short- or long-dated bond. Even if the first condition noted earlier for the YTM calculation is satisfied, it is clearly unlikely for any but the shortest maturity bond that all coupons will be reinvested at the same rate. Market interest rates are in a state of constant flux and would thus affect money reinvestment rates. Therefore, although yield to maturity is the main market measure of bond levels, it is not a true interest rate. This is an important result, and we shall explore the concept of a true interest rate in Chapter 2.

Market Yield Measures

In Figure 1.3 we saw the Bloomberg page YA, used for yield analysis of fixed income instruments. Readers will notice from this figure that there are a number of different measures of yield shown on the page, all related to the standard yield-to-maturity calculation. This is because different markets and instruments use slightly different conventions when calculating yield-to-maturity.

All market yield measures define a bond's maturity date as the date when the final principal amount becomes due. This is almost invariably the stated maturity date of the bond, but in certain cases this date may fall on a non–business day, so the next working date is used instead.

Otherwise there are subtle differences in the way the different measures are calculated, although they may still give the same result. Here we explain these differences:

- **Street convention:** the standard yield-to-maturity calculation;
- **True yield:** this is the standard yield-to-maturity calculated with coupon dates moved whenever they fall on a non–business day, to the next valid business day. Moving this date is pertinent to the yield measure because it affects the number of days in an interest period;
- **Treasury convention:** the yield calculated using simple interest for the first coupon period, and compounded interest for subsequent interest periods. Assume an actual/actual accrued interest basis (this term is explained in the next section);
- **Consortium yield:** this is used for bonds that use an actual/365 day–count basis; it assumes 182.5 days in each (semiannual) interest period;
- **DMO yield:** a yield associated with United Kingdom gilts, the street convention equivalent as defined by the UK Debt Management Office;
- **Equivalent/year compound:** this is the street convention method adjusted for actual cash flow and compounding frequency; it uses the actual date of cash flows as they would be received;
- **Japanese yield:** this is a simple yield calculation using the annualised cash flow, expressed as a percentage of the original clean price used at purchase;
- **Money-market equivalent:** this is the yield of the bond but adjusted to make it equate to money-market yield convention. It would only be used for a bond that had less than 365 days left to maturity. This calculation compounds the remaining coupon payments to the maturity date, and this total amount is used to calculate a yield, based on simple interest, quoted using the present value of the total cash flows. Assumes an actual/360-day count basis;
- **Repo equivalent:** this is the yield calculated with interest accumulated on an overnight basis, with bond prices fixed, and assuming actual/360-day basis;
- **Effective rate:** the bond yield realised by investing coupon income at a specified reinvestment rate from now until maturity.

Investors will use the yield measure appropriate to the market and instrument they are analysing.

Accrued Interest, Clean and Dirty Bond Prices

Our discussion of bond pricing up to now has ignored coupon interest. All bonds accrue interest on a daily basis, and this is then paid out on the coupon date. The calculation of bond prices using present-value analysis does not account for coupon interest or *accrued interest*. In all major bond markets, the convention is to quote price as a *clean price*. This is the price of the bond as given by the net present value of its cash flows, but excluding coupon interest that has accrued on the bond since the last dividend payment. As all bonds accrue interest on a daily basis, even if a bond is held for only one day, interest will have been earned by the bondholder. However, we have referred already to a bond's *all-in* price, which is the price that is actually paid for the bond in the market. This is also known as the *dirty price* (or *gross price),* which is the clean price of a bond plus accrued interest.

In other words, the accrued interest must be added to the quoted price to get the total consideration for the bond.

Accruing interest compensates the seller of the bond for giving up all of the next coupon payment even though she will have held the bond for part of the period since the last coupon payment. The clean price for a bond will move with changes in market interest rates; assuming that this is constant in a coupon period, the clean price will be constant for this period. However, the dirty price for the same bond will increase steadily from one interest payment date until the next. On the coupon date, the clean and dirty prices are the same and the accrued interest is zero. Between the coupon payment date and the next ex-dividend date the bond is traded cum dividend, so that the buyer gets the next coupon payment. The seller is compensated for not receiving the next coupon payment by receiving accrued interest instead. This is positive and increases up to the next ex-dividend date, at which point the dirty price falls by the present value of the amount of the coupon payment. The dirty price at this point is below the clean price, reflecting the fact that accrued interest is now negative. This is because after the ex-dividend date the bond is traded "ex-dividend"; the seller not the buyer receives the next coupon, and the buyer has to be compensated for not receiving the next coupon by means of a lower price for holding the bond.

The net interest accrued since the last ex-dividend date is determined as follows:

$$AI = C \times \left[\frac{N_{xt} - N_{xc}}{Day\,Base} \right] \tag{1.26}$$

where AI is the next accrued interest
C is the bond coupon
N_{xc} is the number of days between the ex-dividend date and the coupon payment date (seven business days for UK gilts)
N_{xt} is the number of days between the ex-dividend date and the date for the calculation
$Day\ Base$ is the day-count base (365 or 360)

Certain bonds do not have an ex-dividend period (for example, Eurobonds) and accrue interest right up to the coupon date.

Interest accrues on a bond from and including the last coupon date up to and excluding what is called the value date. The value date is almost always the settlement date for the bond, or the date when a bond is passed to the buyer and the seller receives payment. Interest does not accrue on bonds whose issuer has subsequently gone into default. Bonds that trade without accrued interest are said to be trading flat or clean. By definition therefore,

$$\text{Clean price of a bond} = \text{Dirty price} - \text{Accrued interest}$$

For bonds that are trading ex-dividend, the accrued coupon is negative and would be subtracted from the clean price. The calculation is given by (1.27).

$$AI = -C \times \left[\frac{\text{Days to next coupon}}{Day\,Base} \right] \tag{1.27}$$

As we noted, certain classes of bonds—for example, U.S. Treasuries and Eurobonds—do not have an ex-dividend period and therefore trade cum dividend right up to the coupon date.

The accrued-interest calculation for a bond is dependent on the day-count basis specified for the bond in question. When bonds are traded in the market, the actual consideration that changes hands is made up of the clean price of the bond together with the accrued that has accumulated on the bond since the last coupon payment; these two components make up the dirty price of the bond. When calculating the accrued interest, the market will use the appropriate day-count convention for that bond. A particular market will apply one of five different methods to calculate accrued interest:

Actual/365	Accrued = Coupon × Days/365
Actual/360	Accrued = Coupon × Days/360
Actual/actual	Accrued = Coupon × Days/actual number of days in the interest period
30/360	See following text
30E/360	See following text

When determining the number of days in between two dates, include the first date but not the second; thus, under the actual/365 convention, there are 37 days between 4th August and 10th September. The last two conventions assume 30 days in each month; so, for example, there are "30 days" between 10th February and 10th March. Under the 30/360 convention, if the first date falls on the 31st, it is changed to the 30th of the month, and if the second date falls on the 31st *and* the first date is on the 30th or 31st, the second date is changed to the 30th. The difference under the 30E/360 method is that if the second date falls on the 31st of the month, it is automatically changed to the 30th.

The accrued interest day-count basis for selected country bond markets is given in Table 1.5.

Van Deventer (1997) presents an effective critique of the accrued interest concept, believing essentially that it is an arbitrary construct that has little basis in economic reality. He states:

> The amount of accrued interest bears no relationship to the current level of interest rates.
>
> *Van Deventer, 1997, p. 11*

This is quite true; the accrued interest on a bond that is traded in the secondary market at any time is not related to the current level of interest rates, and is the same irrespective of where current rates are. As Example 1.6 makes clear, the accrued interest on a bond is a function of its coupon, which reflects the level of interest rates at the time the bond was issued. Accrued interest is therefore an accounting concept only, but at least it serves to recompense the holder for interest earned during the period the bond was held. It is conceivable that the calculation could be adjusted for present value, but, at the moment, accrued interest is the convention that is followed in the market.

TABLE 1.5 Selected Country Market Accrued Interest Day-Count Basis

Market	Coupon Frequency	Day-Count Basis	Ex-Dividend Period
Australia	Semiannual	Actual/actual	Yes
Austria	Annual	Actual/actual	No
Belgium	Annual	Actual/actual	No
Canada	Semiannual	Actual/actual	No
Denmark	Annual	30E/360	Yes
Eurobonds	Annual	30/360	No
France	Annual	Actual/actual	No
Germany	Annual	Actual/actual	No
Eire	Annual	Actual/actual	No
Italy	Annual	Actual/actual	No
New Zealand	Semiannual	Actual/actual	Yes
Norway	Annual	Actual/365	Yes
Spain	Annual	Actual/actual	No
Sweden	Annual	30E/360	Yes
Switzerland	Annual	30E/360	No
United Kingdom	Semiannual	Actual/actual	Yes
United States	Semiannual	Actual/actual	No

EXAMPLE 1.6

1.6(A): ACCRUAL CALCULATION FOR 7% TREASURY 2002

This gilt has coupon dates of 7th June and 7th December each year. £100 nominal of the bond is traded for value 27th August 1998. What is accrued interest on the value date?

On the value date, 81 days have passed since the last coupon date. Under the old system for gilts, act/365, the calculation was:

$$7 \times 81/365 = 1.55342$$

Under the current system of act/act, which came into effect for gilts in November 1998, the accrued calculation uses the actual number of days between the two coupon dates, giving us:

$$7 \times 81/183 \times 0.5 = 1.54918$$

(*continued*)

EXAMPLE 1.6 (*Continued*)

1.6(B)

Mansur buys £25,000 nominal of the 7% 2002 gilt for value on 27th August 1998, at a price of 102.4375. How much does he actually pay for the bond?

The clean price of the bond is 102.4375. The dirty price of the bond is $102.4375 + 1.55342 = 103.99092$.

The total consideration is therefore

$$1.0399092 \times 25{,}000 = £25{,}997.73$$

EXAMPLE 1.6(C)

A Norwegian government bond with a coupon of 8% is purchased for settlement on 30th July 1999 at a price of 99.50. Assume that this is seven days before the coupon date and therefore the bond trades ex-dividend. What is the all-in price?

$$\text{The accrued interest } = -8 \times 7 / 365 = -0.153424$$

$$\text{The all-in price is therefore } 99.50 - 0.1534 = 99.3466$$

EXAMPLE 1.6(D)

A bond has coupon payments on 1st June and 1st December each year. What is the day-base count if the bond is traded for value date on 30th October, 31st October and 1st November 1999, respectively? There are 183 days in the interest period.

	30th October	31st October	1st November
Act/365	151	152	153
Act/360	151	152	153
Act/Act	151	152	153
30/360	149	150	151
30E/360	149	150	150

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The following readings are recommended; Sundaresan (1997) presents a high-quality overview of the debt markets as a whole, and there is much practical application as well as theoretical treatment. Higson (1995) is a very accessible treatment that places fixed-income instruments in the context of the capital markets, and is also very good for an introduction to financial arithmetic.

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