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# 1.1 INTRODUCTION

It is a familiar, fundamental truth of modern Physics that light is composed of *photons*, entities known as elementary particles despite possessing both particle- and wave-like attributes. Readers of these volumes on Photonics will accede with the view that numerous properties and interactions of light are only fully comprehensible when represented and formulated in terms of photons. Indeed, in order to understand certain processes, there simply appears to be no viable alternative. This is not only the case with observations such as the photoelectric effect that have played a pivotal role in familiar scientific history; in optics there is a wealth of recent and developing applications that also hinge on other, specifically photon-borne properties. Many properties of light appear to manifest directly corresponding attributes of the individual photons. Other qualities, of course, reflect ensemble characteristics that can emerge only in a beam comprising numerous photons—but there too we find phenomena dependent on statistical properties which can only make sense by reference to photon distributions. There is a rich and deeply embedded relationship between photons and the modern science of light.

The word *photon* has not yet reached its centenary; it was in fact coined in 1926 by a thermodynamics researcher named Lewis [1], who surprisingly introduced it to describe "not light but [that which] plays an essential part in every process of radiation." However, the emergence of a reasonably fully fledged photon concept can be traced back much earlier, to 1905, where it first surfaced in Einstein's explanation of a frequency threshold for photoelectric emission. Using the term *Lichtquant*, "light

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quantum," Einstein wrote: "When a light ray spreads out from a point source, the energy is not distributed continuously over an increasing volume [wave theory of light], but consists of a finite number of energy quanta that are localized at points in space, move without dividing, and can only be absorbed or generated as complete units" [2].

With subsequent success in explaining the line spectra of atoms, this quantum concept rapidly gained an apparently incontrovertible status—and given the passage of time, one might suppose that there would now be little left to discover, little scope for debate over what the photon truly is. However, the nature of that reality has never been simple to explain. It is telling that Lamb, an early pioneer of modern optical spectroscopy, would jest that it should be necessary to be granted a license before being allowed to use the word *photon* [3]. Following the arrival of the laser [4] and the subsequent emergence of the derivative term "photonics" in the 1960s, the distinctive and sometimes paradoxical nature of the photon has become more than ever evident. The state of knowledge even a quarter of a century ago is a pale shadow of the current understanding; many of the issues discussed in the following were unheard of even at that time [5]. What we now understand about photons has certainly become very much richer, certainly less simple, than Einstein's original conception.

It powerfully illustrates the wide diversity of interpretation that, as more and more exotic phenomena have been identified, specially coined descriptors have been introduced to identify and qualify particular kinds of behavior with which photons can be associated. So we now find in the literature terms that would seem to signify various distinctive kinds of photon, if such a thing were possible. Examples abound: we read of photons whose character is *dressed* [6], *ballistic* or *snake-like* [7,8], *electric* or magnetic [9, 10], entangled and heralded [11, 12], dipole or quadrupole [13, 14], real or virtual [15, 16] ... there is literature on biphotons [17]-and so the list goes on. Different kinds of system or physical effect certainly manifest different attributes of the photon, but it is evident that various scientific communities and practitioners who share the use of the term would not find themselves in full agreement on every aspect of what a photon is. Far from becoming a fixture in modern physics, the notion of a photon has, if anything, become more of a quandary as time goes by. Many of those who research such matters would be drawn to agree with Loudon, author of one of the classic books on the quantum theory of light, that "it is no longer so straightforward to explain what is meant by a photon" [18].

It is also remarkable that, in the centenary year of the photon concept, 2005, a major international conference with the title *What is a Photon?* could attract wide-ranging contributions and stimulate debate amongst leading scientists from across the globe [19]. Indeed, that initial meeting spawned an ongoing series of conferences and discussions in which the truth and character of the photon continues to be the subject of highly active deliberation. So it seems fitting, in this first chapter of the series on Photonics, to attempt an objective assessment of which, if any, of those photon attributes are incontrovertible, representing a common ground for interpretation—and also to provide a certain perspective on some of the more intricate and less well-settled issues. As we shall see, even the momentum or information content of a photon, or the existence of its wavefunction, are not entirely uncontroversial.

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# **1.2 FOUNDATIONS**

#### **1.2.1** Modes of Optical Propagation

To lay the foundations for a discussion of the most unequivocal photon properties, it will first be helpful to recall some established ground from classical optics which will also serve to introduce some key definitions. It has been known, since the pioneering work of Maxwell, that light entails the propagation of mutually associated, oscillatory electric and magnetic induction fields, which we shall call e and b, respectively (lowercase symbols being used, as is common, to signify fields in a microscopic regime). In free space these fields propagate at a speed c, and they oscillate at a common frequency v, in phase with each other. The simplest case, polarized monochromatic light, can be regarded as a fundamental mode of excitation for the radiation field—an optical mode that is conveniently characterized by two quantities: a wavevector k and a polarization  $\eta$ . The former is a vector in conventional three-dimensional space, pointing in the direction of propagation such that the triad of vectors e, b, and k together form a right-handed, mutually orthogonal set; its magnitude is  $k = 2\pi v/c = 2\pi/\lambda$ , where  $\lambda$  is the optical wavelength.

The second mode attribute, the polarization  $\eta$ , which designates the disposition of the electromagnetic oscillations, is most often a label rather than a directly quantifiable variable. Plane (or linear) and circular polarizations are the most familiar forms: circular polarizations in particular occupy a privileged position with regard to some of the ancillary properties to be examined below. The terms "linear" and "circular" are usually taken as referring to a projection of the electric field vector locus, in a plane normal to the direction of propagation. Any more general, conventional form of polarization can be conceived as a variant of an elliptical form, and defined as such by the values of three classical *Stokes parameters*  $S_1$ ,  $S_2$ , and  $S_3$ . The corresponding polarization state can then be regarded as mapping the coordinates of a particular position on the surface of a Poincaré sphere [20] whose axes are defined by those three parameters. The fourth Stokes parameter  $S_0$ , representing the extensive property of intensity, does not carry over to the case of individual photons, since the intensity of light will necessarily be related to the population of photons in a given volume. Equally, any degree of polarization will signify the relative photon populations of different polarization states. More intricate forms of polarization state exist in vector beams, such as those with electric vectors that are radially or azimuthally directed with respect to the direction of beam propagation. Structured light of this and other kinds is to be discussed in Section 1.6.

Secured as solutions to Maxwell's equations, the propagating electric and magnetic fields of each optical mode oscillate with a phase factor  $\exp[i(\mathbf{k}.\mathbf{r} - \omega t)]$ , where  $\omega = 2\pi v$ . The form of this factor is such that a complete set of optical modes  $(\mathbf{k}, \eta)$ —a set of infinite extent for light in free space—can be regarded as the individual components of a Fourier expansion for the radiation field. Summed over all modes, the total electromagnetic energy is accordingly expressible as a Hamiltonian cast as a volume integral of the energy density: see ① in Table 1.1, where  $\varepsilon_0$  is the vacuum electric permittivity. The transverse designation of the electric field vector

 TABLE 1.1
 Key equations and significance

Reference	Equation	Significance
1	$H_{\rm rad} = \frac{\varepsilon_0}{2} \int \{ \boldsymbol{e}^{\perp 2}(\boldsymbol{r}) + c^2 \boldsymbol{b}^2(\boldsymbol{r}) \} d^3 \boldsymbol{r}$	Hamiltonian is a volume integral of an energy density that is quadratic in the transverse electric and magnetic fields
2	$E = nh\nu \ (\equiv n\hbar ck)$	Electromagnetic energy is quantized in units whose magnitude is determined by the optical frequency
3	$I = (n/V) hc^2 / \lambda$	For a given wavelength, irradiance is linearly proportional to the number of photons per unit volume
4	$H_{\text{rad}}  n(\boldsymbol{k}, \eta)\rangle = \left(n_{\boldsymbol{k}, \eta} + \frac{1}{2}\right) \hbar c k  n(\boldsymbol{k}, \eta)\rangle$	Schrödinger equation for light; each mode delivers an integer number of photon energies plus a ground state energy
5	$\boldsymbol{S}_{\text{rad}} = \boldsymbol{\varepsilon}_0 \int \left\{ \boldsymbol{e}^{\perp}(\boldsymbol{r}) \times \boldsymbol{a}(\boldsymbol{r}) \right\} d^3 \boldsymbol{r}$	Spin angular momentum is the volume integral of an operator cross product of the electric field and vector potential
6	$E^2 = p^2 c^2 + m_0^2 c^4$	General equation from special relativity theory, linking energy with momentum and rest mass
T	$v_g = c/[n(v) + v  dn(v)/dv]$	Group velocity differs from the phase velocity by an additional term in the denominator
8	$H = H_{\rm mat} + H_{\rm int} + H_{\rm rad}$	Quantum electrodynamical Hamiltonian comprising terms for matter, matter-radiation coupling, and the radiation itself
9	$L = \varepsilon_0 \int \boldsymbol{e}_i^{\perp}(\boldsymbol{r}) \left(\boldsymbol{r} \times \nabla\right) a_i(\boldsymbol{r}) \ d^3 \boldsymbol{r}$	Orbital angular momentum operator which, together with spin, accounts for the total angular momentum
0	$\chi = \frac{\varepsilon_0}{2} \left\{ \boldsymbol{e}\left(\boldsymbol{r}\right) \cdot \nabla \times \boldsymbol{e}\left(\boldsymbol{r}\right) + c^2 \boldsymbol{b}\left(\boldsymbol{r}\right) \cdot \nabla \times \boldsymbol{b}\left(\boldsymbol{r}\right) \right\}$	Optical chirality density derives from projections of the curls of both the electric and magnetic fields

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is a reminder that the defining expression applies in charge-free regions so that there are no static (longitudinal) electric fields: the magnetic induction field is necessarily transverse, too. Consequently, in each free-space mode, both fields oscillate in directions perpendicular to the propagation wavevector.

It is interesting to observe the imperative for the energy density to take the mathematical structure exhibited by <sup>①</sup>. One clear interpretive consequence of the photoelectric effect is that photons have a directly additive energy, requiring a set of evenly spaced energy levels for each optical mode. In fact, the quadratic dependence of  $\mathbb D$ on conjugate pair variables  $e^{\perp}$  and **b** is necessary in order for quantization to provide energy levels forming exactly such an evenly spaced, infinite set. Without this, the notion of a photon could not arise: there would not be a linear relationship between the quantum number and the total electromagnetic energy. So, it has transpired that the very existence of photons proves the energy density to be quadratic in both the electric and magnetic fields [21].

#### 1.2.2 **Quantum Foundations**

In principle the theory of quantized light begins with the simple electromagnetic energy equation @, E = nhv, signifying a number n of discrete quanta, with each individual photon energy given by  $hv \equiv \hbar ck$  ( $\hbar = h/2\pi$  is the Dirac constant, and h is Planck's constant). This formula represents the first step toward the modern, comprehensive theory of quantum optics—a theory that in fully fledged form is nonetheless as different in character from its origins as is modern quantum mechanics from Bohr's atomic theory. Equation 2 also provides the basis for another, equally useful but lesser known formula  $\Im$ , that directly relates beam irradiance I (optical power per unit area) to the mean number of photons n within a volume V. For example, how likely is it, at any instant, to find a single photon within a cube the size of the wavelength? This equation shows that for a laser emitting 532 nm wavelength radiation, the necessary conditions would require an intensity of around  $7 \times 10^8$  W m<sup>-2</sup>; that is about half a million times more intense than the average intensity of sunlight on the Earth's surface. The yet much smaller instantaneous probability of finding a photon in the volume of a molecule, under ambient conditions, goes some way to explaining why our comprehension of the photon has been so much advanced by the invention of the laser.

The optical wavevector and polarization, which together define the optical mode  $(k, \eta)$ , characterize photons in terms of five degrees of freedom. Typically, three (for example, three Cartesian components) can serve to define the wavevector and the other two the polarization (such as a latitude and longitude on the Poincaré sphere). We now recognize that specifying these mode parameters will serve to determine the unit values of all *extensive* quantities—those that directly scale with the number of quanta within a given volume. This is true not only for energy, but also *linear momentum* and *spin angular momentum*. The photon momentum is given by  $\hbar k$ , whilst the spin is determined by polarization state. In Section 1.3, we shall note issues that arise for photons propagating through material media. For the moment, we simply observe that beam attributes, such as measures of linewidth

or coherence, are not extensive quantities as they are meaningful only for photons collectively.

# 1.2.3 Developing Quantum Optics

To put the theory on a firmer footing, since the photon is unequivocally a quantum entity, it is now appropriate to formulate a proper quantum mechanical depiction, cast in terms of state vectors and wavefunctions. The majority of light beams in the real world do not have a well-defined value for the number of photons (we shall return to this in Section 1.7), but it will be helpful to focus at the outset on states that do-noting that a variety of other radiation states are nonetheless possible [22]. For a specific optical mode  $(k, \eta)$ , a number state with n photons is designated  $|n(k, \eta)\rangle$  in the standard Dirac notation. Here, it is important to recognize that the wavefunction for the two-photon state,  $|2(\mathbf{k}, \eta)\rangle$ , will *not* be expressible as a product of one-photon  $|1(k, \eta)\rangle$  state functions. This is not necessarily surprising; in much the same way, the wavefunction for a 2s electron in the hydrogen atom cannot be represented as a product of 1s wavefunctions. The latter refutation only seems more obvious because atomic energy levels have non-uniform energy spacing. In this respect, the fact that the energy of a state with n photons scales directly with n can be potentially misleading, if wrongly taken to indicate that each of those *n* photons has a separate identity [23].

The quantum optical equations for light propagating in a vacuum, in which the electric and magnetic fields are promoted to operator status, are based on developing the radiation Hamiltonian into operator form, from the usual classical expression for the spatially integrated energy density, 1. The Schrödinger equation 4 for a single electromagnetic mode can then be constructed. The quantum radiation states  $|n(\mathbf{k}, \eta)\rangle$  are eigenstates of the radiation Hamiltonian; for each mode they form a complete basis set, in terms of which other states of the same optical mode can be expressed through use of the quantum expansion postulate. More generally, the eigenstate basis for any polychromatic radiation can be cast as a product,  $|n_1(k_1,\eta_1)\rangle \otimes |n_2(k_2,\eta_2)\rangle \otimes |n_3(k_3,\eta_3)\rangle \cdots$ . To develop the terminology, it follows that the number of photons  $n_{k,n}$  can be interpreted as the occupancy, or occupation number, of the corresponding optical mode. It also transpires that each term in the freespace equation ①, of electric and magnetic origin, delivers an equal half-contribution to the total mode energy  $(n_{k,n} + \frac{1}{2})\hbar ck$ . It is therefore reasonable to consider the photon "conveying" quanta that comprise conjoined electric and magnetic elements of the electromagnetic field.

Just as the electromagnetic energy density is promoted to operator form in quantum optics, so too are other key observables. It can be shown that all physically meaningful observables—properties whose determination can leave the radiation field unchanged—are associated with operators that are bilinear in the electromagnetic fields [24, 25]. An important, further example is the operator for spin angular momentum (5), the volume integral of a vector product between the transverse electric field  $e^{\perp}(r)$  and the vector potential a(r): the latter is related to the electric and magnetic fields by  $b(r) = \nabla \times a(r)$ ;  $e(r) = -\dot{a}(r)$ . The spin operator and the quantum

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Hamiltonian together support a set of common optical eigenstates, the latter comprising the two circular polarization states for each wavevector k. In the usual optics convention, the angular momentum is taken to have a positive sense of rotation for left-circular polarization (the electric and magnetic vectors describing an anticlockwise helix, as the light propagates towards the observer) and a negative sign for right circularity. It is only photons of specifically left- and/or right-handed circularity, for which both energy and spin can be precisely quantified. The spin angular momentum operator then delivers an eigenvalue that is an integer multiple of  $\hbar$ , the multiplier corresponding to the difference in numbers of left- and right-hand photons in the beam. The result is consistent with spin angular momentum values of  $\pm\hbar$ , per photon, in the direction of k.

#### 1.2.4 Boson Statistics

The hallmark integer spin associated with the circular polarization states for the photon, its value one unit of  $\hbar$ , has a powerful foundational significance. It marks out the photon as a particular kind of elementary particle: a boson, subject to Bose–Einstein distribution laws. In the parlance of elementary particle physics, the photon is the gauge boson for electromagnetism. This designation is distinct from fermions (half-integer spin particles such as electrons) which satisfy Fermi–Dirac statistics. The contrast is stark: boson states have wavefunctions that are symmetric on exchange of any two components. In consequence, Bose–Einstein statistics impose no limitation on the number of particles that can be accommodated in a single quantum state. In the case of photons, this is one of the principal reasons that strongly coherent laser light is achievable. In the nature of boson character we also find a deeper reason for the previously noted fact that the quantum state vector for a beam conveying two or more identical photons is not separable as a product of wavefunctions for photons individually—this despite the fact that photon energies appear directly additive, and there is no electrodynamic coupling between the photons themselves.

As an aside, this categorization of photons as a type of elementary *particle* raises the question of whether they have any other "material" attributes. Certainly, the photon is a stable elementary particle: there is no limit on how long a photon can live, and we can detect light from a distant star that has taken billions of years to reach the earth. In the realm of optics, two other fundamental properties would be charge and mass: both are zero. The former is unsurprising; the latter is reconciled with non-zero momentum on recognition of the special relativistic equation 6 connecting energy E, rest mass  $m_0$ , and momentum p. For the single photon E = hv; with  $m_0 = 0$  we simply retrieve p = hv/c, consistent with the magnitude  $h/\lambda$  of the momentum vector  $\hbar k$ . One does have to exercise caution in this regard, however; Einstein famously showed that mass will indeed be conveyed (in the form of energy) from an emitter to an absorber [26]. While relativistic issues are in scope, it is also interesting to note a curious implication of the *proper time* formula from special relativity. This shows that for a hypothetical observer moving along with the light, the photon lifetime is actually zero: at the speed of light, all times are equal. So the times of photon emission and detection, however remote, are in this sense simultaneous.

# **1.3 MEDIUM ISSUES**

Before proceeding further, it is instructive to consider how some of the above material, essentially formulated for propagation in a vacuum, needs to be extended—indeed to some extent reinterpreted—for light passing through a material medium. The issues that arise at this stage are not specifically associated with the quantum nature of light; such concerns will be addressed in later sections. However, the modification by matter of electromagnetic propagation does impinge on the way in which photons are understood. It also transpires that one must take special care with the interpretation of issues that might seem to impinge on causality—the tenet of special relativity that no information can be transmitted faster than the vacuum speed of light, that is, the signal velocity  $\neq c$ .

# 1.3.1 Speed of Propagation

To introduce this topic we can begin by focusing on the simplest and most familiar aspect, the reduction of light's speed in inverse proportion to the refractive index n(v). As each photon propagates into a material medium, it continues onward with an unchanged energy and frequency, but its wavelength is generally shorter so that the product of frequency and wavelength is diminished. Within the medium, the light thus has a *phase velocity* given by c/n(v), and generally n(v) > 1. In some condensed matter, phase velocities in the X-ray region can in fact exceed c, through a refractive index that is a little below unity. However, measurements based on detection at different locations will not escape from the strict conditions of relativistic causality: information cannot be transmitted faster than the speed of light. It is usually argued that the positional uncertainty associated with a precise wavelength (and hence precise photon momentum) precludes causality violation.

The frequency or wavelength dependence of the refractive index means that the velocity varies in a characteristic way across the absorption spectrum of each material. The intricate form of this velocity dependence is captured by the familiar *dispersion curves*, which plot optical frequency against wavenumber k. In frequency regions well away from material absorption bands, these curves display an approximately linear response with a slope approaching the vacuum speed of light c—but this line separates off into paired asymptotes of zero slope in the vicinity of each absorption frequency, namely,  $\neq$ . The quantum interpretation is instructive, for in the regions of diminished slope which appear above and below each resonant frequency, photon behavior seamlessly changes to that of a *polariton* [27].

Polaritons, also sometimes termed *dressed* or *medium-dressed* photons, are associated with strong interactions between the propagating radiation and electronic excitations of the material, usually through electric dipole coupling. In photonic terms, we can understand the onset of this change in behavior: as the optical frequency approaches resonance with an optical absorption band, the local electronic polarizability (or linear susceptibility) correspondingly exhibits a rapid escalation in magnitude. In consequence, the optical fields drive increasingly strong, oscillatory motions in the local electronic distribution, slowing down forward propagation. When

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exact resonance is reached and the radiation can be halted by actual absorption, the photon energy is taken up in effecting a local electronic transition to a corresponding excited state. This excitation may itself propagate through the material by successive processes of resonant energy transfer, the fulfillment of local energy conservation at each step permitting a small delay in the forward progression. What is remarkable is that there is a continuum of such behavior, the photon acquiring a progressively modified character as its own electromagnetic fields are "dressed" by those of the material it encounters.

In passing it is worth noting that, as its name suggests, the group velocity T is a concept designed for application to beams of light with a frequency spread. It is here, for media exhibiting exotic dispersion features, that the much-vaunted cases of *slow light* arise. It is indeed remarkable that, since the speed of light is generally so far beyond the reach of any material motion, beams of light can be slowed to essentially zero velocity [28]. At the other extreme is a phenomenon that has earned the oxymoron superluminal light, in which it is observed that the peak intensity components of short radiation pulses entering and emerging from suitably tailored media are separated by a shorter time interval than the vacuum speed of light would allow. Nonetheless, one cannot meaningfully interpret this as the composite effect of individually "superluminal photons"; it is an ensemble effect associated with the interference between photons in modes spanning the broad range of wavelengths present in short pulses of light. So the term "superluminal" might be applied in a limited sense to the beam, but not its constituent photons. The need for this caution appears salutary; as Chiao and Milonni have pointed out "[In many] classic texts on optics and electromagnetism ... it is incorrectly implied that a group velocity greater than c would contradict the special theory of relativity" [29]. Nothing could be further from reality; no information can be conveyed at this speed. Issues that arise in the less "real" world of virtual photons will be considered in Section 1.5.

#### 1.3.2 Momentum

It is somewhat alarming to find that a persuasive resolution has only very recently been found, to a long-standing controversy over the electromagnetic momentum of light propagating through a material medium. The controversy dates back to the early years of the twentieth century, when the photon concept was yet in its infancy. The full development of that concept made the central question much simpler to frame, but not to solve. In modern parlance the essence of the issue can be posed thus: is the linear momentum of a photon diminished or increased from its free space value? Specifically, one might argue that since the speed of light is reduced by a factor of n(v) in a medium, the linear momentum of each photon should suffer a corresponding loss, and the momentum per photon would be hv/n(v)c. On the other hand, since that reduced speed is associated with a proportionate decrease in the wavelength, it might equally be supposed that the inverse relationship between the wavelength and momentum should signify an increase in the latter, giving a photon momentum n(v)hv/c. The argument over which is the correct formula has become known as the Abraham-Minkowski controversy, respectively named after the two individuals

whose formulations would support each of these apparently irreconcilable conclusions. The recently proffered solution to this enigma [30] centers on an assertion that the Abraham momentum signifies a *kinetic* quantity directly relating time derivative of a positional measurement, whilst the Minkowski version is the correct mathematical formula for *canonical* momentum, in the sense of Lagrangian mechanics. It is not yet clear how widely accepted this resolution will prove to be.

### 1.3.3 Directedness of Propagation

In the sphere of time-gated imaging, other descriptors signifying a distinctive manner of propagation can be found associated with photons. The terms *ballistic* and *snake-like* [7,8] were originally introduced to signify a distinction between components of light that, in the former case, emerged from transmission through a complex (often biological) sample without significant scattering, compared to components emerging after multiple scattering events within the sample. The longer, more meandering pathway of the optical energy in the latter case, which suggests the sinuous description, also correlates with a delayed emergence from the sample. This enables the "ballistic" signal—which of course has a far greater capacity to produce images that resolve interior structures—to be cleanly captured by time gating. The classic illustration is imaging bone through tissue [31].

The terminology was never meant to suggest that there are indeed two different kinds of photon in the source radiation; the difference in behavior simply represents two opposite extremes within a statistical distribution of photon passage events. Nevertheless, there is a clear inference that still demands attention: interpreting every scattering event as a photon path deviation implies photons that continue to exist, but with changed propagation vector. Yet, since the initial and emergent states for each such interaction relate to quanta of excitation in different radiation modes, it is essentially meaningless to regard the input and output as the *same* photon. More correctly, one has to regard each scattering event as the annihilation of one photon and the creation of another.

# 1.4 PHOTON LOCALIZATION AND WAVEFUNCTION

There is a concise measurement-based interpretation, unattributed but popularly ascribed to one of the pioneers of optical coherence theory, asserting that: "A photon is what a photodetector does." It is a viewpoint that is not far removed from Bohr's initial stance on the status of quantized radiation [32]. In the spirit of this succinct depiction, various calculational methods have been demonstrated that can deliver a spatially localized visualization of photon propagation—there are numerous examples [33]. Some such representations have didactic value. Nonetheless, a more accurate description of what any conventional (or quantum) photodetector does is that it responds to the electromagnetic field of impinging radiation, thereby registering its quantum energy. It is indeed entirely consistent with the modern path integral formulation of quantum mechanics in which theoretical derivations do not

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presume to describe what intervenes between setup and measurement, but instead make allowance for all possibilities. The post-interpretation of such calculations in terms of specific photon attributes is notoriously misleading; it is impossible to measure the path a photon has taken.

### 1.4.1 Localization

The results of double-slit experiments conducted at low intensities, which reveal interference effects even when a single photon is present, unequivocally show that photons have an essential spatial delocalization. Any more direct measurement that is localized by the physical extent of a detector cannot be interpreted as signifying localization of the photon itself. For example, since the process of light absorption involves photon capture, it clearly effects a collapse of the state function for the radiation field. It is salutary to recall the view of Mandel and Wolf, who pointed out that attempts for whatever purposes to build a picture of "localized photons" are fraught with danger, for "a photon has no precise position no matter what the state may be" [34]. It is essential to keep in mind the notion of photons having a significant degree of delocalization; one has to resist any temptation to think of the photon as in any sense a point particle.

The most significant exceptions to the apparently inexorable process of state collapse in most photon measurements are experiments designed to provide "quantum non-demolition" measurements on the optical field [35, 36], specifically aiming to circumvent the usual limitations associated with quantum measurement. One means of achieving this is to deploy a form of interaction whose effective Hamiltonian commutes with the Hamiltonian driving the system dynamics. In one such scheme based on the optical Kerr effect [37], a beam of throughput radiation emerges unchanged despite bringing about a modification to a second beam, coincident with it in the nonlinear medium. Such a process can both serve to signify the presence of photons in the former beam and also in principle to quantify, without changing, their number. It has been suggested that non-demolition principles can be extended to the detection of single photons, [38, 39] but the viability of some techniques at this level has been contested [40]. Certainly, no such measurements can determine with arbitrary precision in space or time exactly where any individual photon can be found.

### 1.4.2 Wavefunction

Authors who deploy the term "photon wavefunction" usually seem to have in mind a quantum amplitude relating to a spatial distribution of the associated electric field, though we have to bear in mind that there is more to the photon than its electric field! To conceive a more specific "electric field wavefunction" basically indicates a confusion between wavefunctions and operators.

Certainly, the quantum state of radiation comprising a single photon must have a wavefunction—as indeed will a radiation state with any number of photons—yet it is not in general correct or meaningful to conceive a *photon wavefunction*. The important difference between the two propositions, seemingly minor in their verbal

expression, was first touched on in Section 1.2.3. The notion is almost defensible for a one-photon state, where the distinction from state vector does not produce major problems, and in this sense it can be found utilized in areas of application concerned with quantum effects that are significant only at very low levels of intensity. But as soon as we have two or more photons, the notion of a wavefunction for either component is inadmissible. It would seem inadvisable to use, as if it were generally applicable, a concept that is defensible only for one-photon states.

Direct insights into the specifically quantum nature of light are afforded by observations of processes such as spontaneous parametric down-conversion—essentially the time-inverse of sum-frequency or second harmonic generation. When single photons are suitably converted into pairs that propagate off in different directions, the detection of non-classical correlations between the polarization states of the two components signifies quantum entanglement [41,42]. Radiation produced in this manner is often referred to as comprising *entangled photons* [11, 43] or *biphotons* [17]; the possibility of deducing information concerning one photon component, from a measurement made on the other, also leads to the latter being described as a *heralded photon* [12].

Numerous other experiments in quantum optics underscore the more general inseparability of two-photon states, perhaps most categorically in the Hong-Ou-Mandel effect [44]. When a single photon is intercepted by a beam splitter, the result is a state of the radiation field with one well-defined quantum of energy: this state can be cast as a linear superposition of "reflected" and "transmitted" states, for which each has a distinct photon character since two different modes are involved. Experiments on the output generally provide results consistent with a linear combination of outcomes, again signifying quantum entanglement [45]. What is observed is certainly the collapse of a quantum state, but arguably not "a photon" in the sense we have established. Recognizing that linear combinations of states are not the same as linear combinations of particles gets to the heart of the distinction between the wavefunction for a state comprising one photon, and a "photon wavefunction."

All such results undermine the best intentioned attempts to portray the quantum mechanical nature of the photon by reference to "its" wavefunction: even the most ardent proponents of this concept will usually admit to the implication of a significant degree of controversy [46]. It has in fact been shown that the contrivance of any "photon wavefunction" approach can indeed only produce results already described by standard quantum electrodynamics [47].

# 1.5 THE QUANTUM VACUUM AND VIRTUAL PHOTONS

# 1.5.1 Vacuum Fluctuations

The  $\frac{1}{2}\hbar ck = \frac{1}{2}\hbar v$  energy term on the right in ④ invites attention, for it represents the residual energy of an optical mode with zero occupancy, that is, a state in which no photons are present. The totality of such ground-state contributions signifies the energy associated with *vacuum fluctuations*, the physical consequences of which extend from spontaneous emission to Casimir forces [48]. For any given mode

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 $(\mathbf{k}, \eta)$ , the vacuum energy of the state  $|n(\mathbf{k}, \eta)\rangle$ , though linearly dependent on *n*, is *not proportional* to *n*. For example, that energy is exactly the same,  $\frac{1}{2}\hbar v$ , for both the  $|1(\mathbf{k}, \eta)\rangle$  and  $|2(\mathbf{k}, \eta)\rangle$  states. This non-additivity of ground-state energies is yet another feature serving to underscore the fact that there is not a separate wavefunction for each of the photons, in a multiphoton state.

The implication of the non-zero vacuum term is that every radiation mode has an irreducible ground state energy, consistent with quantum fluctuations in the electric and magnetic fields, and representing as such the vacuum expectation value of the radiation Hamiltonian ① when no observable light is present. The fact that this vacuum energy is infinite, when summed over all radiation modes and integrated over space, raises some interesting philosophical issues, though it is not a practical problem. A directly significant implication is that, consistent with quantum uncertainty, it is possible to create and then destroy photons for brief intervals of time. These photons, whose fleeting lives begin and end as energy is borrowed from and returned to the vacuum fluctuations, are not experimentally observable: they are accordingly termed *virtual photons*.

#### **1.5.2** Virtual Photons in Action

There are essentially two distinct types of application based on virtual photon calculations. One is for energies, the other for interactions: both arise from the following. In general, the quantum electrodynamical Hamiltonian for a region of space containing matter of any kind, (a), comprises three terms. Two terms designate the quantum energy operators for matter and for the radiation field (the latter only arises in quantum operator form because we are dealing with photons). The presence of the other term, which represents coupling between the matter and the radiation, means that neither the eigenstates of  $H_{mat}$ , nor those of  $H_{rad}$ , are stationary states of the full Hamiltonian *H*. Accordingly,  $H_{int}$  will act as a perturbation on those eigenstates to produce energy shifts and transitions. Each action of  $H_{int}$  will create or annihilate a photon—and since no photon created from the vacuum can be sustained beyond the spatial and temporal limits decreed by the Uncertainty Principle, it follows that measurable energy shifts and rates of transition between states of the same total energy must be associated with the emergence and disappearance of virtual photons.

In the realm of elementary particles, all electromagnetic interactions are considered to be mediated by the interparticle propagation of such virtual photons. The absence of any interparticle term in the Hamiltonian <sup>®</sup>—which is exact when cast in multipolar form [49]—means that there is no other means of coupling between separate particles. In the sphere of single-particle interactions, calculations based on the virtual photon premise also provide values for fundamental properties in precise agreement with experiment, for widely varying physical parameters. The most notable example is the fine-structure constant—one of the fundamental constants in the Standard Model of particle physics—whose value based on virtual photon calculations agrees with experiment at an unparalleled level of precision, better than 1 ppb (part per billion) [50].

One key feature of the virtual photon representation is its intrinsic accommodation of causal constraints from special relativity, which precludes instantaneous or any

other superluminal forms of coupling. In this context it is worth clarifying a misleading remark by Feynman, suggesting that by virtue of quantum uncertainty a virtual photon might propagate faster than light [51]. In fact, by evaluating measurable quantities associated with the duly retarded electromagnetic fields, an analysis by Power and Thirunamachandran [24] proved strict adherence to relativistic causality. Another notable feature is a quantum mechanical aspect, enshrined in the mathematics of mode summation for each virtual photon. Enforcing summation over all wavevectors and polarizations is consistent with the principle of summing over every quantum variable for unobserved intermediate states. In some "self-energy" applications, the term *virtual photon cloud* [52] is sometimes used as a reminder that for such calculations there is no constraint over the position or momentum of each virtual photon interaction—indeed it is as if such photons constantly emerge from, and retreat into, the space surrounding each material particle. Matter is universally suffused with virtual light. Moreover, the diffuse spatial character of the photon means that it is conceivable for a photon to be created and annihilated by the same material particle.

#### 1.5.3 Virtual Photon Propagation

To better appreciate the propagative characteristics associated with virtual photon coupling, it proves instructive to consider the result of calculations on the simplest two-site process. Known as *resonance energy transfer*, this affords the mechanism for electronic excitation to pass between electrically neutral, independent particles [53]. The initial and final states for the system thus differ through vector displacement of the electronic energy  $\Delta E$  from the donor to the acceptor site. The principal contribution to the quantum amplitude entails the creation of a virtual photon at one center and its subsequent annihilation at the other. It highlights the seemingly paradoxical nature of virtual photons that the calculation includes important contributions from virtual photons of all energies and all directions of propagation, not only those travelling in the same direction as the displacement of the acceptor from the energy donor. Most counterintuitively, account must also be taken of virtual photons propagating from the acceptor to the donor [15, 54].

The result reveals that the character of the energy transfer slowly changes between asymptotic forms, as the distance *R* between the donor and acceptor increases. In the *near-field* region, where  $R \ll \hbar c/\Delta E$ , the net quantum amplitude exhibits an inverse cubic dependence on distance, the rate thus falling off as  $R^{-6}$ . This is the result familiarly known as *Förster* (or *fluorescence*) *energy transfer* [55], often termed "radiationless" because there is no direct observation of light emission by the donor. But for distances  $R \gg \hbar c/\Delta E$ , the amplitude moves toward an  $R^{-1}$  asymptote, leading to a rate that exhibits an inverse square dependence on distance. The latter result exactly accords with the acceptor capturing a "real" photon—one that has been released in spontaneous emission by the donor. In fact, in not only these features, but every respect, it turns out that as distance increases there is a smooth transition from virtual to real photon behavior [56, 57]. This can be understood as another manifestation of the Uncertainty Principle, corresponding to the increasingly tight constraints on energy (and momentum) conservation as the coupling photons travel

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for longer (and over larger distances). The result lends a fresh interpretation to the supposed distinction between real and virtual photons. By considering the life of a photon as it propagates from its source of creation toward the site of its annihilation at a detector, it becomes evident that every such photon in principle exhibits virtual traits. As has been remarked in the context of elementary particle physics, "In a sense every photon is virtual, being emitted and then sooner or later being absorbed" [58].

Such a perspective also lends new insights into the physical origin and form of the electromagnetic fields in the immediate vicinity of a photon emitter. For example, it emerges that the electric field produced by an electric dipole emitter has some properties that disappear on propagation beyond the near-field region. An example of such evanescent features is the fact that the electric field has longitudinal, as well as the conventional transverse, character with respect to the displacement of a detector from the source [15]. Related, and yet more striking, is the fact that there is a small but significant effective variation of wavelength close to the source [59]. Caution must be exercised in interpreting such near-field behavior as evidence of exotic properties of the virtual photon itself: in vacuum, the virtual photons always propagate with transverse electric fields and a constant wavelength—but measurable near-field properties will always be a result of summing over a range of virtual photons with different directions and wavelengths [60].

#### **1.5.4 Casimir Forces**

A fascinating link between the seemingly disconnected topics of vacuum fluctuations and virtual photons is afforded by the phenomena known as Casimir forces which are observed to occur between electrically neutral, non-polar species separated by microor nano-scale distances. One particular form of this coupling, which arises between molecules, is commonly designated the Casimir–Polder interaction [61]. This is manifested as a modification to the more familiar London or "dispersion" potential, changing it from an inverse sixth power dependence on separation R to an inverse seventh power—an exotic feature that has been convincingly proven by experiment [62, 63]. Another manifestation of similar origins, manifested as a sharply distancedependent attractive force between parallel conducting surfaces [64], represents a feature that has to be corrected for in the design of nanoelectromechanical systems [65–67]. Here, the result is a negative pressure that varies with the inverse fourth power of R. Surprisingly, it is possible to derive equivalent expressions for such forces either by considering the effect of matter on the vacuum radiation field, or by formulating the interaction potential in terms of virtual photon exchange [49, 68].

# 1.6 STRUCTURED LIGHT

#### 1.6.1 Complex Modes and Vector Beams

In recent years, the field of optics has undergone what can only be regarded as little short of a revolution, through the development of new structural forms of optical beam. Discovering a capacity to engineer beams with non-uniform, intricately crafted

wave-fronts, polarization behavior, and phase structures has stimulated the emergence of new theory and visions of wide-ranging new applications [69]. Although it might have been supposed that some attributes could only be supported in beams comprising a high density of photons, it transpires that most novel features are interpretable as reflecting the properties of individual photons. Indeed, there are experiments demonstrating that individual photons convey full information on the structure of complex beams [70, 71]. Recalling the intrinsic delocalization of photons makes it a little easier to comprehend how one photon might have the capacity to convey such properties. Some of the most important examples of wave-front structures are the complex optical modes associated with Laguerre–Gaussian, Airy, and Bessel beams [72, 73]; many similar photonic issues arise in connection with so-called polarized "vector" beams, as for example those with radial or azimuthal polarization [74].

### 1.6.2 Chirality and Angular Momentum

The most widely studied forms of structured beam are Laguerre–Gaussian modes, often known as *optical vortices*, which propagate with a helically twisted wave-front. The helical structure is characterized by an azimuthal quantum number, l, the integer number of twists in the wave-front per unit wavelength. Detailed analysis shows that any such mode with an occupation number n carries an axial angular momentum of  $n\hbar l$ , equivalent to  $\hbar l$  per photon. Thus, l acquires the status of an orbital angular momentum quantum number, for which a positive sign once again denotes left helicity and a negative sign, right. Generally, in order to convey orbital angular momentum, light must have a topological charge [75], and experiments demonstrate in a direct way that single photons carry the helical-mode information [76]. Surprisingly, it also transpires that not just integer but even fractional values of the orbital angular momentum per photon are attainable [77, 78].

The observation of optical beams conveying such quantized orbital angular momentum, independent of any circularity of polarization, is now understood as exhibiting a partitioning of a total angular momentum J for the optical field. The quantum operator for J not only comprises the spin term given by  $\mathfrak{S}$ , as shown by Mandel and Wolf [34], but also includes an orbital term  $L \mathfrak{D}$ . It is encouraging to observe that issues of gauge dependence in the standard formulation, deployed to effect this separation of spin and orbital angular momentum components, can in fact be fully circumvented in a photon-based representation [79].

The association between helical wave-front structure and orbital angular momentum correctly suggests that both are involved with intrinsic measures of electromagnetic chirality. First addressed in work by Lipkin, a variety of such optical chirality measures have been discovered and all of them shown to be connected with the conservation of polarization in the electromagnetic field [80]. Foremost amongst these measures is an "optical chirality density" <sup>(1)</sup>. In a quantum optical analysis, it has recently been proven that an essentially infinite hierarchy of such helicity measures [81] reduces to a common physically meaningful quantity: all are linearly dependent on the difference between the number of photons with opposite polarization circularity [25].

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#### 1.6.3 Multipole Emission

There exists much confusion in the literature over the issue of whether a photon conveys any kind of imprint of the symmetry character for the electronic decay transition in which it was originally released. For example, there is a prevalent notion that a photon emitted in an electric dipole transition might have an undefined yet measurably different character from the one emitted in an electric quadrupole transition, even given identical wavelengths and polarizations. This has given rise to the cavalier use of terms such as *dipole* or *quadrupole* photon [13, 14]. Similar conjectures also give rise to the terms *electric* photon or *magnetic* photon [9, 10]. The latter are more obviously misnomers; Maxwell's electromagnetism equations ensure that it is not possible for light to have one kind of field without the other. The origin of all such descriptors is mostly attributable to the theory of atomic transitions, in which the spherical symmetry of atoms generally leads to mutually exclusive selection rules for electric dipole (E1) and electric quadrupole (E2) transitions. Although similar kinds of mutual exclusion occur in any centrosymmetric systems, most molecules exhibit transitions that can be both E1 and E2 allowed, with no distinction between the resulting final states. However, there are much deeper and more general factors that undermine the notion of photons with multipolar character.

The decay of an atom or molecule by a particular multipolar transition can be detected, through absorption of the released photon, in an excitation process of a potentially different multipolar character [82–84]. This is a direct physical interpretation of a finite quantum amplitude, despite a non-correspondence in multipoles. In fact, retracting a detector from the immediate vicinity of the source produces a decreasing angular uncertainty in photon propagation direction, consistent with an angular quantum uncertainty principle, [85] and thus reflected in an increasing range of integer values for the measured angular momentum. Indeed, the emission of a photon in an electric quadrupole decay will generally produce a larger signal for dipole detection than for quadrupole detection. The corollary that follows, on considering time-reversal arguments, is that it is no more meaningful to designate such a photon as a "quadrupole" photon than a "dipole" photon: both are equally redundant descriptors. Of course there are differences according to the multipolar order of each process, emission, and detection, in terms of spatial distribution and not least the intensity (signified by the detection rate); the theory reveals the detail. But the inference of a particular multipolarity in any particular detected photon has no basis.

# 1.6.4 Information in a Photon

In the field of spin optics, standard methods provide for two independent degrees of freedom to be communicated by photon polarization. Circular states of opposing handedness are the most obvious candidates, though any two states corresponding to diametrically opposite positions on the Poincaré sphere would provide a suitable basis. However, complex light with structured wave-fronts or vector polarization offer additional degrees of freedom, enabling individual photons to convey a far greater

information content than was previously considered possible [86]. Nonetheless, one must entertain reservations about the extent to which any individual photon can convey structural information. The truth that there is an infinite range of mathematically orthogonal Laguerre–Gaussian modes, for example, has been taken by some to suggest that in principle each photon might convey the information content of an entire image. Such notions have to be tempered by an appreciation of the practical requirement for a detector that is already preconfigured with information on the sought image.

There are many more realistic propositions for methods that could deploy individual photons to encode information on beam structure-particularly topological charge. It has been shown that the necessary modal information can be encoded in single-photon states and resolved by beam tomography [87]. Such studies have stimulated the conception of numerous schemes for quantum communication and data handling applications, although there are daunting practical issues to be taken into account; a useful survey has been provided by Franke-Arnold and Jeffers [88]. One attractive possibility is to selectively divert a mixed-mode beam into components with different orbital angular momentum content; a variety of innovative sorting and detection schemes have been reported [34–36]. One frequently overlooked issue is the outworking of the previously mentioned uncertainty relation that links angle and orbital angular momentum, and which is associated with a non-zero commutator for the two corresponding operators [85, 89]. All schemes for quantum information processing based on photon orbital angular momentum [90] are ultimately limited by this feature, which has been convincingly demonstrated by photon correlations in downconversion [91]. It is surprising that this principle, particularly relevant to low-number states, has seemingly received very little consideration in the literature to date.

# 1.7 PHOTON NUMBER FLUCTUATIONS AND PHASE

As noted earlier, it should not be inferred that all of the properties of light beams are, like intensity and other extensive quantities, equivalent to the summative effect of individual photons; an obvious exception is the conventional linewidth. Nonetheless, there are numerous other beam attributes with specific photonic measures and interpretations, such as phase, degrees of coherence, and autocorrelation functions.

### 1.7.1 Coherence and Fluctuations

For a variety of reasons, every beam of light exhibits fluctuations of intensity. It is clear enough that a constant beam would require an unattainable degree of control over the temporal regularity of individual photon emission events in its source. However, the nature of the intensity fluctuations reflects important statistical features of the source. For example, a perfect thermal source releases photons with a chaotic phase, whose number distribution (directly corresponding to an intensity distribution) accordingly follows Bose–Einstein statistics. Conversely, a perfectly coherent source delivers photons with a Poisson number distribution [92].

#### PHOTON NUMBER FLUCTUATIONS AND PHASE 19

It also has to be recognized that no source has the capacity to deliver photons that all have precisely the same wavelength—not least because this would fall foul of quantum uncertainty in the energy released through each photon emission event. In consequence there is a coherence length, essentially determined by the inverse of the beam linewidth expressed in wavenumber terms [93]; this represents a maximum distance over which the electromagnetic field oscillations of the copropagating photons are meaningfully held in phase. At any instant in time, different portions of the beam will therefore experience different degrees of constructive and destructive interference, correlating with the observed intensity fluctuations. This is indeed the principle that is exploited to achieve laser mode locking, a technique that is widely used to produce ultrashort pulses of light [94].

Under the high levels of intensity produced by ultrashort pulsed lasers, where multiphoton absorption and other nonlinear optical process can occur, interactions with matter generally depend not only on the mean irradiance but also on other higher moments of the intensity distribution. For example, two-photon absorption processes depend on the degree of second-order coherence,  $g^{(2)}$  [18]. To understand the quantum basis for such features, we have to go beyond a simple number state representation.

### 1.7.2 Phase

Whilst it is possible to ascribe fluctuations in beam intensity to essentially stochastic interference between modes of different wavelength and frequency, there are other, more fundamental aspects of phase that arise at the quantum level. Each photon carries a phase  $\exp(i\mathbf{k}\cdot\mathbf{r} - \omega t)$ , referring the space coordinate  $\mathbf{r}$  and time t to the position and time for photon creation (or annihilation, if the displacements in space and time are taken with a negative sign). This "absolute" phase, which is conveyed by the electromagnetic field operators in the Heisenberg representation [92], nonetheless has little practical significance unless optical interference is engaged in some observable. When two or more photons are present in a system, one can consider issues of relative phase between them—yet only if the individual number of photons in each relevant mode is not completely determined: see p. 495 in Reference 34. This constraint arises for an important fundamental reason: the quantum operators for photon number and phase do not commute. The mathematical formulation of a quantum phase operator is itself fraught with difficulties [95]. Partly, but not wholly, this is due to the physical equivalence of waveforms shifted by an integer number of wavelengths; this is a subject of active research [96].

Loudon [18] gives a nice picture of the implications: if the exact number of photons in a given optical mode is precisely known, then one can extract precisely no phase information. Conversely, if one entertains quantum modes with an exact phase, then their photon number occupancy becomes infinitely uncertain. There are, however, quantum optical states that lie between the extremes represented by number and phase states. An important example is the "minimum uncertainty" state known as a coherent state [92]. Generally considered closest in form to a classical wave, and taken as a good representation of a stable laser beam, it leads to a Poisson distribution

of photon numbers. Although it is perfectly straightforward to formulate theory for processes such as photon absorption, using a coherent state representation for the light, there are curious implications for the photon concept. For example, the mean number of photons in the beam after photon absorption appears to be unchanged. On the other hand, observed quantum interferences between different processes, such as between single- and two-photon absorption in molecules [97], or three- and five-photon ionization of atoms [98], cannot be represented by number states precisely because the final states for each alternative possibility differ.

# **1.8 THE REALITY OF PHOTONICS**

In the specific technical sense encountered earlier, it becomes apparent that no photon can ever be regarded as entirely "real." But, in the broader everyday sense of reality, one can also pose the more philosophical question of whether the photon is real. Perhaps surprisingly, not all exponents of quantum optics or quantum electrodynamics are willing to be drawn on what such photon reality means in the case of optical effects. Power (author of a classic textbook [99] on QED), for example, held that the electromagnetic fields should ultimately be considered closest to irreducible reality. Yet whether it is fields or photons in terms of which we elect to describe phenomena, neither can represent what is actually measured. The surest ground is where theory can explain or predict actual observations, based on given conditions. We need not venture into the quagmire territory of quantum measurement theory [100]; it suffices that all of the observations filling the pages of these volumes, and the applications on which they are based, are fully explicable in photonic terms. The modern concept of the photon, with multiple interpretations and widely differing subsidiary connotations, supports an astonishingly rich and diverse range of phenomena. A strong case can be made that in the modern world, Photonics has become more real than the photon.

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