1

Introduction

1.1 Historical Overview of Lightning Electromagnetic-Field and Surge Computations

Lightning return-stroke electromagnetic fields have been calculated using analytical expressions, derived for a vertical lightning channel (e.g., Uman et al. 1975). Effects of finite ground conductivity on lightning electromagnetic fields have also been studied using analytical expressions (e.g., Rachidi et al. 1996). These analytical expressions are still being used. Lightning-induced voltages on an overhead power distribution line or telecommunication line have been calculated using an engineering model of the lightning return stroke (e.g., Uman et al. 1975) and a field-to-wire coupling model (e.g., Rachidi 1993). Horizontal electric fields above a finitely conducting ground, which are needed for calculating lightning-induced voltages, have been evaluated using approximate expressions such as the Cooray–Rubinstein formula (Rubinstein 1996). Note that the Cooray–Rubinstein formula is given in the frequency domain, although its time-domain counterparts also exist. Lightning surges due to a direct lightning strike to an overhead power transmission or distribution line have been analyzed using distributed-circuit simulation methods such as the electromagnetic transients program (EMTP) (Dommel 1969). EMTP and other similar programs are still widely used in lightning surge simulations.

Around 1990, electromagnetic computation methods were first applied to lightning electromagnetic and surge simulations. One of the advantages of electromagnetic computation methods, in comparison with circuit simulation methods, is that they allow a self-consistent full-wave solution for both the transient current distribution in a 3D conductor system and resultant electromagnetic fields, although they are computationally expensive. Podgorski and Landt (1987) applied the method of moments (MoM) in the time domain (Van Baricum and Miller 1972; Miller et al. 1973) to analyze the lightning current along a tall object struck by lightning. Grcev and Dawalibi (1990) applied the MoM in the frequency domain (Harrington 1968) to analyze the

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surge characteristics of a grounding electrode. Since then, the MoM in the frequency domain has been frequently used in lightning surge simulations (e.g., Baba and Rakov 2008 and references therein).

Tanabe (2001) applied the finite-difference time domain (FDTD) method (Yee 1966), which is one of the electromagnetic computation methods, to studying the surge characteristics of a grounding electrode. Baba and Rakov (2003) used the FDTD method to compute lightning electromagnetic fields. More than 60 journal papers and a large number of conference papers, which use the FDTD method in lightning electromagnetic-field and surge simulations, have been published during the last 15 years (e.g., Baba and Rakov 2014 and references therein). Interest in using the FDTD method continues to grow. The FDTD method is presently the most widely used electromagnetic computation method in lightning electromagnetic-field and surge simulations.

Other electromagnetic computation methods such as the finite-element method (FEM) (e.g., Sadiku 1989), the partial-element equivalent-circuit (PEEC) method (Ruehli 1974), the hybrid electromagnetic model (HEM) (Visacro and Soares 2005), the transmission line matrix or modeling (TLM) method (Johns and Beurle 1971), and the constrained interpolation profile (CIP) method (Takewaki et al. 1985) have been recently applied to analyzing lightning electromagnetic fields and surge simulations (e.g., Yutthagowith et al. 2009 (PEEC); Silveira et al. 2010 (HEM); Smajic et al. 2011 (FEM); Tanaka et al. 2014a (TLM); Tanaka et al. 2014b (CIP)).

In the following section, we briefly introduce each of these electromagnetic computation methods.

1.2 Overview of Existing Electromagnetic Computation Methods

1.2.1 Method of Moments

The MoM in the time domain (Van Baricum and Miller 1972; Miller et al. 1973) has been used to analyze responses of thin-wire conducting structures to external transient electromagnetic fields. The entire conducting structure is modeled by a combination of cylindrical wire segments whose radii are much smaller than the wavelengths of interest. The so-called electric-field integral equation for a perfectly conducting thin wire in air (shown in Figure 1.1) is given below, assuming that current I and charge q are confined to the wire axis (thin-wire approximation) and that the boundary condition on the tangential electric field on the surface of the wire (this field must be equal to zero) is fulfilled:

$$\hat{\boldsymbol{s}}\cdot\boldsymbol{E}_{inc}(\boldsymbol{r},t)=\frac{\mu_0}{4\pi}\int_C\left[\frac{\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}}'}{R}\frac{\partial I(s',t')}{\partial t'}+c\frac{\hat{\boldsymbol{s}}\cdot\boldsymbol{R}}{R^2}\frac{\partial I(s',t')}{\partial s'}-c^2\frac{\hat{\boldsymbol{s}}\cdot\boldsymbol{R}}{R^3}q(s',t')\right]ds'$$
$$\text{with}\qquad(1.1)$$
$$q(s',t')=-\int_{-\infty}^{t'}\frac{\partial I(s',\tau)}{\partial s'}d\tau,$$

where C is an integration path along the wire axis; $\boldsymbol{E}_{inc}$ denotes the incident electric field that induces current I; $\boldsymbol{r}$ and t denote the observation location (a point on the wire surface) and time, respectively; $\boldsymbol{r}'$ and t' denote the source location (a point on the wire axis) and time, respectively; $\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}'$; s and s' denote the distance along the wire surface at $\boldsymbol{r}$ and that along the wire axis at $\boldsymbol{r}'$, respectively; $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}}'$ denote unit vectors tangential to path $\boldsymbol{C}$ in Eq. (1.1) at $\boldsymbol{r}$ and $\boldsymbol{r}'$,

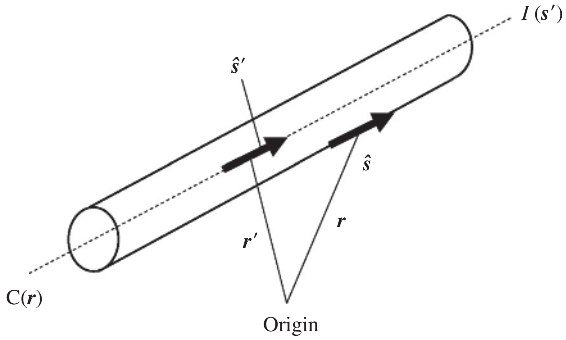

Figure 1.1 Thin-wire cylindrical segment for method of moment (MoM)-based computation. Current is confined to the wire axis, and the tangential electric field on the surface of the wire is set to zero.

respectively; μ_0 is the permeability of vacuum; and c is the speed of light. By numerically solving Eq. (1.1), which is based on Maxwell's equations, the time-dependent current distribution along the wire structure excited by external field is obtained.

The thin-wire time domain (TWTD) code (Van Baricum and Miller 1972), developed at the Lawrence Livermore National Laboratory, is based on the MoM in the time domain. One of the advantages of the time-domain MoM is that it can incorporate nonlinear effects such as the lightning attachment process (Podgorski and Landt 1987) or the back-flashover at an overhead-power transmission line tower struck by lightning (Mozumi et al. 2003), although it does not allow lossy ground and wires buried in lossy ground to be incorporated. Other representative applications of the time domain MoM to lightning electromagnetic-field or surge simulations are found in Moini et al. (1998, 2000), Kato et al. (1999), Kordi et al. (2002, 2003), Pokharel and Ishii (2007), and Bonyadi-Ram et al. (2008).

The MoM in the frequency domain (Harrington 1968) has been widely used in analyzing responses of thin-wire conducting structures to incident electromagnetic fields. In order to obtain the time-varying responses, Fourier and inverse Fourier transforms are employed. The electric-field integral equation derived for a perfectly conducting thin wire in air, as shown in Figure 1.1, in the frequency domain is given by

$$-\hat{\boldsymbol{s}}\cdot\boldsymbol{E}_{inc}(\boldsymbol{r}) = \frac{j\eta}{4\pi k}\int_C I(s')\left(k^2\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}}' - \frac{\partial^2}{\partial s\partial s'}\right)g(\boldsymbol{r},\boldsymbol{r}')\ ds'$$

with

$$g(\boldsymbol{r},\boldsymbol{r}') = \exp\left(\frac{-jk|\boldsymbol{r}-\boldsymbol{r}'|}{|\boldsymbol{r}-\boldsymbol{r}'|}\right), \quad k=\omega\sqrt{\mu_0\varepsilon_0}, \quad \eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}, \tag{1.2}$$

where ω is the angular frequency, μ_0 is the permeability of vacuum, and ε_0 is the permittivity of vacuum. Other quantities in Eq. (1.2) are the same as those in Eq. (1.1). Current distribution along the thin-wire conducting structure can be obtained by numerically solving Eq. (1.2). Note that triangular and/or rectangular patches based on a surface-current formulation could also be used in the MoM in the frequency domain.

This method allows lossy ground and wires in lossy ground to be incorporated into the model. The numerical electromagnetic codes such as NEC-2 (Burke and Poggio 1980) and

NEC-4 (Burke 1992), developed at the Lawrence Livermore National Laboratory, are based on the MoM in the frequency domain. Representative applications of the MoM in the frequency domain to lightning electromagnetic-field or surge simulations are found in Grcev and Dawalibi (1990), Baba and Ishii (2000, 2003), Pokharel et al. (2003, 2004), Shoory et al. (2005, 2010), Geranmayeh et al. (2006), Pokharel and Ishii (2007), Miyazaki and Ishii (2008a, 2008b), Sheshyekani et al. (2008), Aniserowicz and Maksimowicz (2011), Khosravi-Farsani et al. (2013), and Miyamoto et al. (2015). The MoM is the second most frequently used electromagnetic computation method in lightning electromagnetic-field and surge simulations.

1.2.2 Partial-Element Equivalent-Circuit Method

The PEEC method (Ruehli 1974) provides a full-wave solution to Maxwell's equations. The method is applicable to both time (Wang et al. 2010) and frequency domains. A significant difference from the MoM is that the conductor system subject to analysis is transformed into its equivalent circuit. Although the PEEC method is based on exact field theory, it was originally developed not for electromagnetic-field computations but for the analysis of interconnect and packaging structures. In the 1990s, field retardation, external field excitation, and the treatment of dielectric materials were incorporated (Ruehli and Heeb 1992). This method has been recently employed in lightning-surge simulations (e.g., Yutthagowith et al. 2009).

1.2.3 Finite-Element Method

The FEM (e.g., Sadiku 1989) is a technique for solving partial differential equations. This method has the ability to deal with complex geometries using unstructured grids, commonly with triangles in a 2D simulation and tetrahedrons in a 3D simulation. Tetrahedral shapes allow one to represent curved media or objects, which are difficult to represent using cubic or rectangular parallelepiped cells in the FDTD method. Although both time domain and frequency domain formulations have been derived, most implementations of FEM have been performed in the frequency domain. Some specific applications of FEM in the frequency domain are found in Smajic et al. (2011) and Shoory et al. (2012).

1.2.4 Transmission Line Modeling Method

The TLM method (Johns and Beurle 1971) has been applied to lightning electromagnetic-field and surge simulations (Mattos 2005; Yuda et al. 2013). The TLM method is based on Huygen's principle and the analogy between electromagnetic-wave propagation in a 3D space and voltage-wave propagation through a 3D grid composed of short transmission lines. Figure 1.2 illustrates a 3D symmetrical condensed node (SCN) (Johns 1987; Christopoulos 1995), which is the fundamental element of the 3D-TLM method. It is composed of mutually connected short transmission lines. Each face of a 3D-SCN has two orthogonal ports. The working volume is divided into 3D-SCNs, each of which is connected to six adjacent nodes. At all nodes of the 3D network, reflections and refractions of incident voltage waves are computed in the time domain for simulating the 3D propagation of electromagnetic waves.

A reflected voltage wave ${}_kV_n{}^r$ (k is a time step, and n is an integer indicating its port number from 1 to 12), reflected at each port of an SCN, is computed from its incident voltage ${}_kV_n{}^i$ and the scattering matrix, using the following equation:

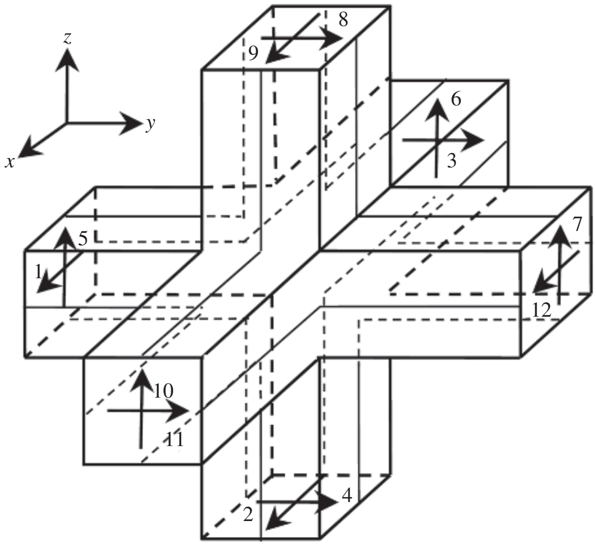

Figure 1.2 Three-dimensional symmetrical condensed node.

$$_k\boldsymbol{V}^r = \boldsymbol{S}_k\boldsymbol{V}^i \tag{1.3}$$

where $_k\boldsymbol{V}^r = [_kV_1{}^r,\ _kV_2{}^r,\ \ldots,\ _kV_{12}{}^r]^T$, $_k\boldsymbol{V}^i = [_kV_1{}^i,\ _kV_2{}^i,\ \ldots,\ _kV_{12}{}^i]^T$, and $\boldsymbol{S}$ is the scattering matrix. Incident voltage waves $_{k+1}V_n{}^i$ at the next time step are the voltage waves reflected from its adjacent SCNs. By repeating similar computations at each node, one can simulate the 3D propagation of electromagnetic waves in the working volume. The presence of dielectric is considered by adding three open-circuit ports (in the x-, y-, and z-directions) to an SCN (Tong and Fujino 1991). The presence of magnetic material is considered by adding three short-circuit nodes to the SCN (Tong and Fujino 1991), and the presence of lossy medium is considered by adding three lossy ports to the SCN (German et al. 1990; Naylor and Desai 1990).

The TLM method has advantages similar to those of the FDTD method: it is capable of dealing with nonlinear effects/components and complex structures with arbitrary geometries. Furthermore, the TLM method is generally less sensitive to numerical dispersion and more stable than the FDTD method, because the TLM method is not based on the central difference scheme. The only significant drawback of the 3D-TLM method is that it requires more computation time and memory than the 3D-FDTD method, although the computational cost for the TLM method in the 2D cylindrical coordinate system is comparable to that for the FDTD method in the same coordinate system (Tanaka et al. 2014a).

1.2.5 Constrained Interpolation Profile Method

The CIP method (Takewaki et al. 1985) is a sort of finite-difference method for numerically solving the advection equation. It has been applied to analyzing lightning electromagnetic

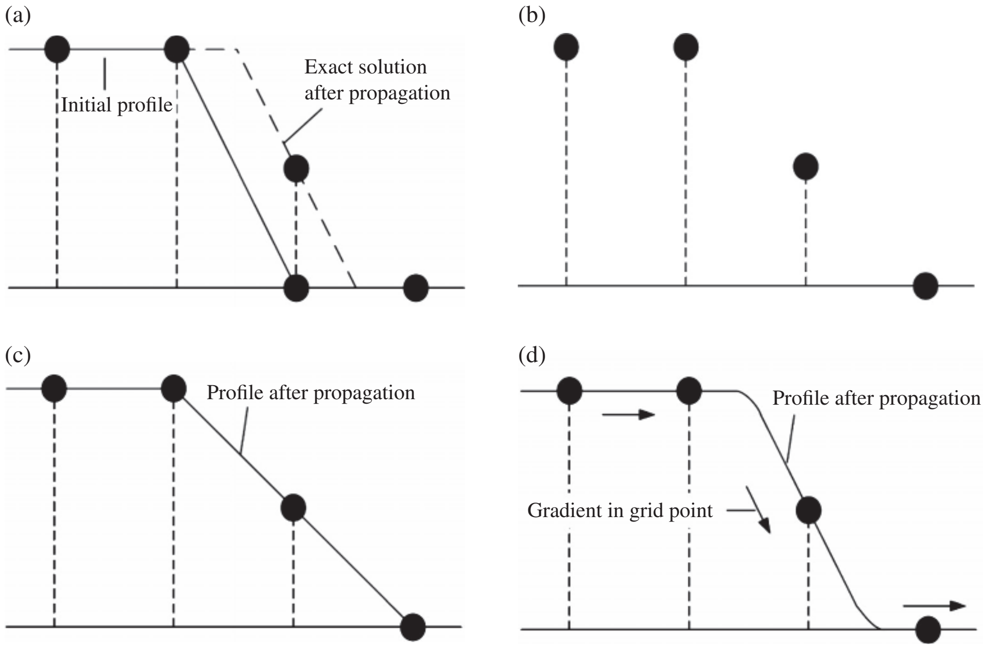

Figure 1.3 Conceptual picture of the CIP interpolation. In (a), the solid line represents the initial profile, the dashed line represents the expected profile after propagation, and solid circles represent values computed at discretized points. In (b), the solid circles represent values computed at discretized points. In (c), the solid circles are linearly interpolated by a piece-wise line, and a numerical dispersion due to coarse gridding appears. In (d), the numerical dispersion due to coarse gridding is reduced because, in the CIP method, the spatial-derivative values are also considered.

fields (Okubo and Takeuchi 2007; Kajita et al. 2014). In contrast with the FDTD method (Yee 1966), the CIP method considers not only electric- and magnetic-field values at each grid point, but also their spatial derivative values. Therefore, in principle, it can suppress numerical dispersion, as schematically shown in Figure 1.3, and instability even when a relatively coarse grid is employed.

1.2.6 Finite-Difference Time Domain Method

The FDTD method (Yee 1966) is one of the most widely used electromagnetic computation methods for a variety of electromagnetic problems. The FDTD method uses the central difference approximation to Maxwell's curl equations, which are Faraday's law and Ampere's law, in the time domain. It solves the resultant update equations for electric and magnetic fields at each time step and at each discretized spatial point in the working volume using the leapfrog method. For the analysis of the electromagnetic response of a structure in an unbounded space, an absorbing boundary condition, which suppresses unwanted reflections, needs to be applied.

Advantages of the FDTD method in comparison with other electromagnetic computation methods can be summarized as follows:

1. It is based on a simple procedure in electric and magnetic-field computations, and therefore its programming is relatively easy.
2. It is capable of treating complex geometries and inhomogeneities.
3. It is capable of incorporating nonlinear effects and components.
4. It can handle wideband quantities from one run with a time-to-frequency transforming tool.

Its disadvantages are:

1. It is computationally expensive compared to other methods such as the MoM.
2. It cannot deal with oblique boundaries that are not aligned with the Cartesian grid when the standard orthogonal grid is employed, and needs a staircase approximation for oblique boundaries.
3. It would require a complex procedure for incorporating dispersive materials.

Additional details on the FDTD method are given in works of Kunz and Luebbers (1993), Taflove (1995), Uno (1998), Sullivan (2000), Hao and Mittra (2009), Yu et al. (2009), and Inan and Marshall (2011).

The first peer-reviewed paper in which the FDTD method was used in a surge simulation was published in 2001 (Tanabe 2001), and the first peer-reviewed paper in which it was applied to a lightning electromagnetic-field analysis was published in 2003 (Baba and Rakov 2003). As mentioned earlier in this chapter, more than 60 journal papers and a large number of conference papers, which use the FDTD method in lightning electromagnetic-field and surge simulations, have been published during the last 15 years (e.g., Baba and Rakov 2014 and references therein). Interest in using the FDTD method for lightning electromagnetic-field and surge simulations continues to grow.

1.3 Summary

In this chapter, we have provided a historical overview of lightning electromagnetic-field and surge computations. The MoM and the FDTD method were first used in lightning electromagnetic-field and surge simulations around 1990 and 2000, respectively. The latter method is the most widely used electromagnetic computation tool. We have briefly explained these two methods and other existing electromagnetic computation methods—the PEEC method, the FEM, the TLM method, and the CIP method—which have been applied to lightning electromagnetic-field or surge simulations.

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