CHAPTER 1

Introduction

1.1 OBJECTIVES OF LEGGED LOCOMOTION AND CHALLENGES IN CONTROLLING DYNAMIC WALKING AND RUNNING

The most effective type of locomotion in rough terrains is legged locomotion. During the past three decades enormous advances have occurred in robot control and motion planning of dynamic walking and running locomotion. In particular, hundreds of walking mechanisms have been built in research laboratories and companies throughout the world. The desire to study legged locomotion has been motivated by the need to assist people with disabilities to walk and replace humans in hazardous environments. Underactuation, impulsive nature of the impact between the lower limbs and the environment, the existence of foot structure and the large number of degrees of freedom (DOF) are basic problems in controlling legged robots. Underactuation is naturally associated with dexterity. For example, headstands are considered dexterous [1]. In this case, the contact point between the body and the ground is acting as a pivot without actuation. The nature of the impact between the lower limbs of legged robots and the environment causes the dynamics of the system to be hybrid and impulsive. The impact between the foot and the ground is one of the main difficulties in designing control laws for walking and running robots. Unlike robotic manipulators, legged robots are always free to detach from the walking surface, thereby leading to various types of motions. Finally, the existence of many degrees of freedom in the mechanism of legged robots causes the coordination of the links to be difficult. As a result of these latter issues, the design of practical controllers for legged robots remains to be a challenging problem. Also, these features complicate the application of traditional stability margins. Consequently, the major issues in the control of dynamic walking and running are as follows:

1. *Limb coordination*. Legged robots are high degree of freedom mechanisms, and consequently, coordination of their links to achieve dynamic walking and running locomotion is complex.

Hybrid Control and Motion Planning of Dynamical Legged Locomotion, Nasser Sadati, Guy A. Dumont, Kaveh Akbari Hamed, and William A. Gruver.

2. *Hybrid nature of locomotion due to presence of impact and liftoff.* The presence of impact, foot touchdown, and liftoff leads to *models with impulse effects* and *hybrid systems* consisting of multiple *continuous* and *discrete phases*. In particular, mathematical models describing the evolution of legged robots during walking and running include both discrete and continuous phenomena. Instantaneous discrete phases arise when feet impact the ground or feet liftoff the ground, whereas ordinary differential equations based on classical Lagrangian mechanics describe the evolution of legged robots during continuous phases of locomotion.
3. *Underactuation*. During certain phases of walking and running such as *single support* (one leg on the ground) in walking and *flight* (no leg on the ground) in running, legged robots have fewer actuators than degrees of freedom.
4. *Overactuation*. During *double support phase* of bipedal walking (both legs on the ground), biped robots have fewer degrees of freedom than actuators. Due to overactuation, the control input corresponding to a specific trajectory in the state space is not unique.
5. *Inability to apply the Zero Moment Point criterion*. Most past work in the literature of legged robots emphasizes the *quasi-static stability criteria* and *flat-footed walking* based on the *Zero Moment Point (ZMP)* [2–16] and the *Foot Rotation Indicator (FRI) point* [17]. The ZMP is defined as the point on the ground where the net moment generated from ground reaction forces has zero moment about two axes that lie in the plane of ground [3]. The ZMP is contained in the robot's *support polygon*, where the support polygon is defined as the convex hull formed by all contact points with the ground. The ZMP criterion states that when the ZMP is contained within the *interior* of the support polygon, the robot is stable so that it will not topple. Thus, in this kind of stability, as long as the ZMP lies strictly inside the support polygon of the foot the trajectories are feasible. If the ZMP lies on the edge of the support polygon, then the trajectories may not be feasible. The center of pressure (COP) is a standard notion in mechanics which was renamed as the ZMP by Vukobratovic [3]. The FRI point is a concept defined when the foot is in rotation with respect to the ground [17]. The FRI is the point on the ground where the net ground reaction force would have to act to keep the foot stationary. Thus, if FRI is within the convex hull of the stance foot, the robot can walk and it does not roll over its extremities, such as the heel or the toe. This type of walking is called as *fully actuated walking*. If the FRI is not in the projection of the foot on the ground, the stance foot rotates about the extremities. Such an event is also known as *underactuated walking*. As long as the foot does not rotate about its extremities, the ZMP, COP, and FRI points are equivalent [15] (see Fig. 1.1). In the literature of legged locomotion, a *statically stable gait* is a periodic locomotion in which the robot's center of mass (COM) does not leave the support polygon. A *quasi-statically stable gait* is a periodic locomotion in which the COP of the robot is within the *interior* of the support polygon. Moreover, a *dynamically stable gait* is a periodic locomotion where the robot's COP is on the boundary of

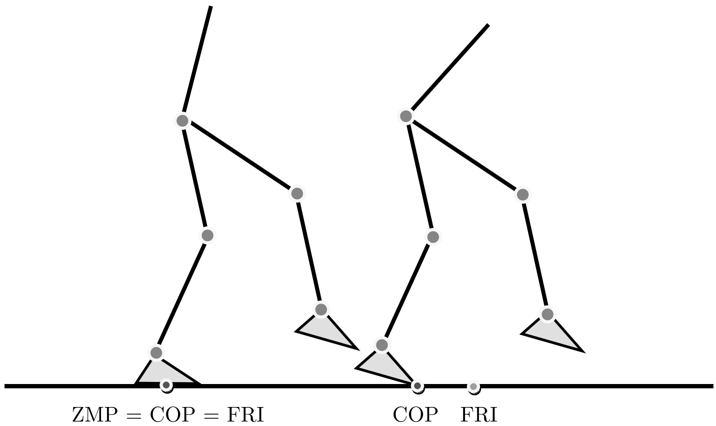

Figure 1.1 Two planar bipedal models to compare the COP, ZMP, and FRI points. The FRI point is a point on the ground contact surface, within or outside the convex hull of the foot support area, at which the resultant moment of the force/torque impressed on the foot is normal to the surface [17]. In the left figure, the foot does not rotate about its extremities, thus, the ZMP, COP, and FRI points are equivalent. At the right figure, the foot is starting to rotate since the FRI point is outside the convex hull of the stance foot. We note that the COP is at the tip of the stance foot about which the foot rotates.

the support polygon for at least part of the walking cycle [18]. Thus, during dynamic walking and running cycles, the location of the robot's COP is on the boundary of the support polygon and, as a result, this will prohibit the use of the ZMP criterion. To make this notion more precise, the ZMP criterion is a sufficient and necessary condition for the stance foot not to rotate. However, this does not imply that the walking motion is asymptotically stable in the sense of Lyapunov [18, Chapter 11].

6. *Lack of algorithms to achieve feasible period-one orbits and limit cycles*. The main problem in control of legged locomotion is how to design a feedback law that guarantees the existence of a stable *limit cycle* for the closed-loop system. Underactuation and *unilateral constraints* must be included in order to design a *feasible* periodic orbit for legged locomotion. Unilateral constraints are constraints on the state and control inputs of the mechanical system that represent feasible contact conditions between the leg ends and the ground. In particular, leg ends, whether they are terminated with feet or points, are not attached to the ground. Hence, the ground reaction forces must lie in the *friction cone* to prevent slippage and foot liftoff. Thus, normal forces at the leg ends can only act in one direction, and are unilateral. In addition, if the foot is to remain flat on the ground and not rotate about its extremities, then the FRI must be between the heel and toe, a condition that can be expressed as a pair of unilateral constraints. These facts combined with underactuation during the single support and flight phases complicate the design of motion planning algorithms to generate feasible periodic locomotion.

7. *Conservation of angular momentum about the robot's COM during flight phases.* During flight phases of running, conservation of angular momentum about the robot's COM is a *nonintegrable Pfaffian constraint* which complicates the path planning and control of the robot's configuration during landing (flight to stance phase).

1.2 LITERATURE OVERVIEW

1.2.1 Tracking of Time Trajectories

Most existing control algorithms in the literature of legged robots are *time-dependent* approaches based on tracking of *predetermined* time trajectories generated by the ZMP criterion, *inverted pendulum model* and *nonlinear oscillators* as central pattern generators of the spinal cord. The ZMP criterion [2–16] has been used for trajectory tracking in ASIMO [2] and WABIAN [5, 6]. The *Linear Inverted Pendulum Model (LIPM)* [19, 20] and ZMP criterion-based approaches for stable walking reference generation have been reported in the literature. In these techniques, generally, the ZMP reference during a stepping motion is kept fixed in the middle of the supporting foot. Erbatur and Kurt [9] proposed a reference generation algorithm based on the LIPM and moving support foot ZMP. In addition, they made use of a simple inverse kinematics-based joint space controller to test the reference trajectory for simulation of a 3D, 12-DOF biped robot model. By allowing a variation of ZMP over the convex hull of foot polygon, Bum-Joo et al. [10] proposed an algorithm to modify the walking period and the step length in both the sagittal and lateral planes of the humanoid robot HanSaRam-VII. Motoi et al. [11] presented a real-time gait planning algorithm based on ZMP for pushing motion of humanoid robots to deal with an object with unknown mass. Kajita et al. [12] presented a ZMP-based running pattern generation algorithm for running of the humanoid robot HRP-2LR. ZMP-based online jumping pattern generation for running of a monopedal robot has also been reported in Ref. [13]. Sato et al. [14] proposed walking trajectory planning on stairs for biped robots using the method of virtual slope and the ZMP criterion. Sardain and Bessonnet [15] proved the coincidence of COP and ZMP and they examined related control aspects. In this latter reference, the virtual COP–ZMP was also defined to extend the concept for walking on uneven terrain. Ref. [21] approximated the biped model as an *inverted pendulum* and made use of trajectory tracking to control dynamic walking locomotion in Biper-3. Katoh and Mori used PID controllers to track reference trajectories generated by a van der Pol oscillator in the control of BIPMAN [22]. To control walking in Kenkyaku, Furusho and Masubuchi applied PID controllers for tracking joint reference trajectories [23]. Furusho and Sano also applied a decoupled control approach for control of motions in the frontal and sagittal planes during walking of the three-dimensional bipedal prototype BLR-G2 [24, 25]. PID controllers were employed to track the time trajectories generated by a length-varying inverted pendulum during walking of Meltran II by Kajita et al. [26, 27]. In these latter references, to maintain the biped's COM at a constant height, the pendulum's length is assumed to vary in a proper manner. In Ref. [28], PID control was also used to track predetermined trajectories to improve the stability behavior. A computed torque method was used to control a planar, 5-DOF bipedal robot in Ref. [29]. The performance of three control techniques including PID, computed torque, and sliding mode control in the tracking of joint trajectories during walking by a planar, 5-DOF biped was compared by Raibert et al. [30]. Tracking of time trajectories, based on computed torque with gravity compensation and a length-varying inverted pendulum model, has also been applied during walking by a three-dimensional bipedal robot in Refs. [8, 31, 32]. Trajectories generated by an inverted pendulum was also used by Kajita et al., to control walking of HRP-2 [33, 34].

The ZMP criterion has become a very powerful tool for trajectory generation in walking of legged robots. However, it needs a stiff joint control of the prerecorded trajectories that leads to poor robustness in unknown rough terrains whereas humans and animals show exceptional robustness when walking on irregular terrains. It is well known in biology that there are *Central Pattern Generators (CPG)* in spinal cord coupling with the musculoskeletal system [35–37]. The CPG and feedback networks can coordinate the body links of the vertebrates during locomotion. There are several mathematical models that have been proposed for a CPG. Among them, Matsuoka [38–41] has studied an approach in which a CPG is modeled by a *Neural Oscillator* consisting of two mutually inhibiting neurons. Each neuron is represented by a nonlinear differential equation. Matsuoka's approach has been used by Taga [36, 37] and Miyakoshi et al. [42] for biped robots. Kimura has also used this approach at the hip joints of quadruped robots [43, 44]. Ref. [45] presented a hybrid CPG–ZMP controller for the real-time balance of a simulated flexible spine humanoid robot. The CPG component of the controller allows the mechanical spine and feet to exhibit rhythmic motions using two control parameters. By monitoring the measured ZMP location, the control scheme modulates the neural activity of the CPG to allow the robot to maintain balance on the sagittal and frontal planes in real time.

1.2.2 Poincaré Return Map and Hybrid Zero Dynamics

As stated in Section 1.1, the main problem in controlling legged locomotion is how to design a controller that guarantees the existence of an *asymptotically stable limit cycle* for the closed-loop mechanical system. A classical technique for analyzing stability of periodic orbits for *time-invariant* dynamical systems described by ordinary differential equations is the *method of Poincaré sections*. This method establishes an equivalence between the stability analysis of the *periodic orbit* for an nth-order *continuous-time system* and that of the corresponding *equilibrium point* for an $(n-1)$th-order *discrete-time system*. Grizzle et al. [46] extended the method of Poincaré sections to *systems with impulse effects*. A system with impulse effects consists of a continuous phase described by an ordinary differential equation and a discrete phase described by an instantaneous reinitialization rule for the differential equation. To simplify the application of the Poincaré sections method in the design of *time-invariant controllers* for walking by an underactuated three-link biped robot and *instantaneous* double support phase, Grizzle et al. [46] created *zero dynamics* *manifolds* that are *forward invariant* under the flow of the continuous phase of walking (i.e., single support phase). However, since the zero dynamics manifolds are not invariant under the flow of the discrete phase (i.e., impact model), the approach of Ref. [46] resulted in a *restricted Poincaré return map* (i.e., the Poincaré return map restricted to the zero dynamics manifolds) that cannot be expressed in a closed-form. The outputs in Ref. [46], corresponding to the zero dynamics manifolds, were selected as *holonomic* functions referred to as *virtual constraints*. Virtual constraints are a set of holonomic output functions defined on the configuration space of the mechanical system. They are forced to be zero by within-stride feedback laws to reduce the dimension of the Poincaré return map and to coordinate the links of biped robots during walking [47]. The method of virtual constraints for designing time-invariant controllers in walking of planar biped robots with one degree of underactuation, point feet and instantaneous double support has been studied in Refs. [18, Chapter 6, 46–52]. Westervelt et al. [52] created virtual constraints to ensure that the corresponding zero dynamics manifold is *hybrid invariant* under the closed-loop hybrid model of walking and introduced the notion of *hybrid zero dynamics* (HZD). The zero dynamics manifolds are said to be hybrid invariant if they are both forward invariant (i.e., invariant under the flow of the continuous phase) and *impact invariant* (i.e., invariant under the flow of the discrete phase). During walking of a planar biped robot with an underactuated *cyclic* variable [1, 53], HZD results in a two-dimensional zero dynamics manifold, and consequently, a one-dimensional restricted Poincaré return map that can be expressed in closed form. This approach was also extended by Choi and Grizzle [54] for creating a two-dimensional zero dynamics manifold during walking of a planar fully actuated biped robot in fully actuated and underactuated phases. To reduce the dimension of the full-order hybrid model of running, which in turn simplifies the stabilization problem of the desired orbit, Ref. [55] proposed that the configuration of the mechanical system should be transferred from a specified initial pose (immediately after the takeoff) to a specified final pose (immediately before the landing) during flight phases. This problem is referred to as *landing in a fixed configuration* or *configuration determinism at landing* [18, p. 252]. By using the virtual constraints approach and the configuration determinism at landing, Ref. [55] obtained a closed-form expression for the one-dimensional restricted Poincaré return map of running by the five-link, four-actuator planar bipedal robot, RABBIT [47]. Moreover, to ensure that the stance phase zero dynamics manifold is hybrid invariant under the closed-loop hybrid model of running, an additional constraint was imposed on the vector of generalized velocities at the end of flight phases. To satisfy the configuration determinism at landing and hybrid invariance, Ref. [55] utilized the approach of *parameterized* HZD. In particular, using the Implicit Function Theorem and a *numerical* nonlinear optimization problem with an equality constraint, the parameters of the virtual constraints of the *flight phase* were updated in a *step-by-step fashion* during the discrete transition from stance to flight (i.e., takeoff). However, the stance phase controller was assumed to be fixed. For running of RABBIT, an alternative parameterized control law was proposed by Morris et al. [56]. However, their approach did not create HZD. The use of *event-based* control laws to update the parameters of time-invariant controllers for stabilization of periodic orbits in systems with impulse effects was presented in Refs. [57–59]. When the amount of underactuation during locomotion of biped robots is increased, it becomes difficult to create hybrid invariant manifolds. Morris and Grizzle [60] proposed a method to generate an open-loop *augmented* system with impulse effects, a new holonomic output function for the resultant system and an event-based update law for the parameters of the output such that the zero dynamics manifold associated with this output is hybrid invariant under the closed-loop augmented system. This latter approach has been used in design of time-invariant controllers for walking of a 3D biped robot in Refs. [61, 62] and also for walking and running of planar bipedal robots with springs, MABEL [63–65] and ATRIAS [66]. Hürmüzlü also applied the method of Poincaré sections to a planar, five-link bipedal robot and imposed a mix of holonomic and nonholonomic constraints on the mechanical system to obtain a closed-form expression for the robot's trajectory [67].

1.3 THE OBJECTIVE OF THE BOOK

In this book we provide a comprehensive overview of hybrid models describing the evolution of planar and 3D legged robots during dynamical legged locomotion and also propose hybrid control schemes to asymptotically stabilize desired periodic orbits for the closed-loop systems. The topics include (i) hybrid systems, (ii) systems with impulse effects, (iii) offline and online motion planning algorithms to generate desired feasible periodic walking and running motions, (iv) two-level control schemes, including within-stride feedback laws to reduce the dimension of the hybrid systems, (v) continuous-time update laws to minimize a general cost function online, and (vi) event-based update laws to asymptotically stabilize the desired periodic orbits. This book also provides a comprehensive presentation of issues and challenges faced by researchers and practicing engineers in motion planning and hybrid control of dynamical legged locomotion. Furthermore, we describe the current state of the art and future directions across all domains of dynamical legged locomotion so that readers can extend the proposed motion planning algorithms and hybrid control methodologies to other planar and 3D legged robots. The main objectives of this book are as follows.

1.3.1 Hybrid Zero Dynamics in Walking with Double Support Phase

There has been little attention given to control of biped robots during the double support phase with unilateral constraints. Such constraints present challenges for the design of controllers. The objective of Chapter 3 is to develop an analytical approach for designing a continuous feedback law that realizes a desired period-one trajectory as an asymptotically stable orbit for a planar biped robot. The robot is assumed to be a five-link, four-actuator planar mechanism in the sagittal plane with point feet. The fundamental assumption is that the double support phase is *not* instantaneous. Hence, bipedal walking can be represented by a hybrid model with two continuous phases, including a single support phase and a double support phase, and discrete transitions between the continuous phases. In the single support phase, the mechanical system has one degree of underactuation, whereas it is overactuated in the double support phase. Chapter 3 shows how to design a continuous time-invariant feedback law that asymptotically stabilizes a feasible periodic trajectory using an extension of HZD for a hybrid model of walking [68, 69]. The main contribution is to develop a continuous time-invariant control law for walking of a planar biped robot during the double support phase. Since the mechanical system in the double support phase has three degrees of freedom and four actuators, a *constrained dynamics approach* [70, p. 157] is used to describe the reduced-order dynamics of the system. Then, two virtual constraints are proposed as holonomic outputs for the constrained system and an output zeroing problem with two control inputs is solved. This results in a *nontrivial* two-dimensional zero dynamics manifold corresponding to the virtual constraints in the state manifold of the constrained system. Moreover, the corresponding zero dynamics has two control inputs that are not employed for output zeroing. Instead, they are used to satisfy the unilateral constraints. Furthermore, these inputs are obtained such that the control has *minimum norm* on the desired periodic trajectory. It can be shown that the constrained dynamics of the double support phase is completely feedback linearizable on an open subset of the state manifold. However, since our objective is to design a continuous time-invariant controller based on nontrivial HZD, in contrast to Ref. [71] we do not use input-state linearization nor a discontinuous time optimal control for tracking trajectories. An analogous approach is used in Refs. [54, 72] for creating a two-dimensional zero dynamics manifold in the state space of a fully actuated phase of walking where the fully actuated dynamics is completely feedback linearizable. The control strategy is presented at the following two levels. At the first level, we employ *within-stride controllers* including single and double support phase controllers. These are continuous time-invariant and parameterized feedback laws that create a *family of two-dimensional finite-time attractive and invariant submanifolds* on which the dynamics of the mechanical system is restricted. At the second level, the parameters of the within-stride controllers are updated at the end of the single support phase (in a stride-to-stride manner) by an event-based update law to achieve *hybrid invariance* and *stabilization*. As a consequence, the stability properties of the desired periodic orbit can be analyzed using a one-dimensional restricted Poincaré return map.

1.3.2 Hybrid Zero Dynamics in Running with an Online Motion Planning Algorithm

Chapter 4 presents an analytical approach for designing a *two-level control law* to asymptotically stabilize a desired period-one orbit during running by a planar monopedal robot. The monopedal robot is a three-link, two-actuator planar mechanism in the sagittal plane with point foot. The desired periodic orbit is generated by the method developed in Ref. [73]. It is assumed that the model of monopedal running can be expressed by a hybrid system with two continuous phases, including stance phase and flight phase, and discrete transitions between the continuous phases, including takeoff and landing (impact). The configuration of the mechanical system is specified by the *absolute orientation* with respect to an inertial world frame and by the joint angles determining the shape of the robot. During the flight phase, the angular momentum of the mechanical system about its COM is conserved. To reduce the dimension of the full-order hybrid model of running, which in turn simplifies the stabilization problem of the desired orbit, as proposed in Ref. [55], the configuration determinism at landing should be solved. However, the flight time and angular momentum about the COM may differ during consecutive steps. Consequently, the *reconfiguration problem* must be solved online. A number of control problems for reconfiguration of a planar multilink robot with zero angular momentum have been considered in the literature, for example, Refs. [74–78]. For the case that the angular momentum is not necessarily zero, a method based on the Averaging Theorem [79, Theorem 2.1] was presented in Ref. [80] such that for any value of the angular momentum, joint motions can reorient the multilink arbitrarily over an arbitrary time interval. However, when the angular momentum is not zero, this method cannot be employed online for solving the reconfiguration problem for monopedal running. For this reason, Chapter 4 presents an *online reconfiguration algorithm* that solves this problem for given flight times and angular momenta [81, 82]. The algorithm proposed in Chapter 4 is expressed using the methodology of *reachability* and *optimal control* for time-varying linear systems with input and state constraints. The main contribution of this chapter is to present an *analytical* approach for online generation of twice continuously differentiable (C^2) *modified reference trajectories* during flight phases of running to satisfy the configuration determinism at landing [81]. Moreover, by relaxing the constraint of Ref. [55] on the vector of generalized velocities at the end of the flight phases, Chapter 4 presents a two-level control scheme based on the reconfiguration algorithm to asymptotically stabilize a desired periodic orbit. In this scheme, within-stride controllers, including stance and flight phase controllers, are employed at the first level. The stance phase controller is chosen as a time-invariant and *parameterized* feedback law to generate a *family* of finite-time attractive zero dynamics manifolds. An alternative approach based on continuous feedback law is employed here to track the modified reference trajectories generated by the reconfiguration algorithm during the flight phases. To generate a family of hybrid invariant manifolds, an event-based controller updates the parameters of the *stance phase controller* during the transition from flight to stance (i.e., impact) [81]. Consequently, the stability properties of the desired periodic orbit can be analyzed and modified by a one-dimensional discrete-time system defined on the basis of a restricted Poincaré return map.

1.3.3 Online Motion Planning Algorithms for Flight Phases of Running

Following the results of Chapter 4, to asymptotically stabilize the desired periodic orbit for the hybrid model of running using a one-dimensional restricted Poincaré return map and HZD approach, the configuration of the mechanical system should be transferred from a predetermined initial pose (immediately after takeoff) to a predetermined final pose (immediately before landing) during the flight phases of running. The objective of Chapter 5 is to present *modified* online motion planning algorithms for generation of continuous (C^0) and continuously differentiable (C^1) open-loop trajectories in the body configuration space of the mechanical system such that the reconfiguration problem is solved [82, 83]. The algorithms presented in Chapter 5 are extensions of that presented in Chapter 4. In particular, the generated trajectories in Chapter 4 were C^2 while the reachable sets associated with the algorithms of Chapter 5 are larger than that of Chapter 4. We address the motion planning problem for general planar open kinematic chains composed of $N \geq 3$ rigid links interconnected with frictionless and rotational joints. The main contribution of Chapter 5 is to present online motion planning algorithms based on *virtual time* for generation of joint motions to satisfy configuration determinism at transitions. In particular, it is assumed that the time trajectory of a desired joint motion, precomputed offline, solves the reconfiguration problem. By replacing the time argument of the desired motion by a strictly increasing function of time called the *virtual time*, Chapter 5 shows how to determine continuous and continuously differentiable joint motions in an online manner during consecutive steps of running so that they solve the reconfiguration problem.

1.3.4 Hybrid Zero Dynamics in 3D Running

Chapter 6 presents a motion planning algorithm to generate periodic time trajectories for running by a 3D monopedal robot. In order to obtain a *symmetric* gait along a straight line, the overall open-loop model of running can be expressed as a hybrid system with four continuous phases consisting of two stance phases and two flight phases and discrete transitions among them (takeoff and impact). The robot is assumed to be a 3D, three-link, three-actuator, monopedal mechanism with a point foot. During the stance phases, the robot has three degrees of underactuation, whereas it has six degrees of underactuation in the flight phases. The motion planning algorithm is developed on the basis of a *finite-dimensional nonlinear optimization problem* with equality and inequality constraints and extends the results of Refs. [73, 84] for planar bipedal robots. The main objective of Chapter 6 is to develop time-invariant feedback scheme to exponentially stabilize a desired periodic orbit generated by the motion planning algorithm for the hybrid model of running.

Chapter 6 shows how to create hybrid invariant manifolds during 3D running [85]. By assuming that the control inputs of the mechanical system have discontinuities during discrete transitions between continuous phases, the takeoff switching hypersurface can be expressed as a zero level set of a scalar holonomic function. In other words, takeoff occurs when a scalar quantity, a strictly increasing function of time on the desired gait, passes through a threshold value. The virtual constraints during stance phases are defined as the summation of two terms including a *nominal* holonomic output function vanishing on the periodic orbit and an *additive* parameterized *Bézier polynomial*, both in terms of the latter strictly increasing scalar. By properties of Bézier polynomials, an update law for the parameters of the stance phase virtual constraints is developed, which in turn results in a *common* intersection of the parameterized stance phase zero dynamics manifolds and the takeoff switching hypersurface. By this approach, creation of hybrid invariance can be easily achieved by updating the other parameters of the Bézier polynomial. Consequently, a parameterized restricted

Poincaré return map can be defined on the common intersection for studying the stabilization problem. Thus, the overall feedback scheme can be considered at two levels. At the first level, within-stride controllers including stance and flight phase controllers, which are continuous time-invariant and parameterized feedback laws, are employed to create a family of attractive zero dynamics manifolds in each of the continuous phases. At the second level, the parameters of the within-stride controllers are updated by event-based update laws during discrete transitions between continuous phases to achieve hybrid invariance and stabilization. By this means, the stability analysis of the periodic orbit for the full-order hybrid system can be treated in terms of a reduced-order hybrid system with a five-dimensional Poincaré return map.

1.3.5 Hybrid Zero Dynamics in Walking with Passive Knees

In Chapter 7, a motion planning algorithm to generate time trajectories of a periodic walking motion by a five-link, two-actuator planar bipedal robot is presented. In order to reduce the number of actuated joints for walking on a flat ground and restore the walking motion in the disabled, it is assumed that the robot has passive point feet and unactuated knee joints. In other words, only the hip joints of the robot are assumed to be actuated. The motion planning algorithm is developed on the basis of a finite-dimensional nonlinear optimization problem with equality and inequality constraints. The equality constraints are necessary and sufficient conditions by which the impulsive model of walking has a period-one orbit. Whereas the inequality constraints are introduced to guarantee (i) the feasibility of the periodic motion and (ii) capability of applying the proposed two-level control scheme for stabilization of the orbit. The main objective of Chapter 7 is to present a time-invariant two-level feedback law based on the notion of virtual constraints and HZD to exponentially stabilize a desired periodic motion generated by the motion planning algorithm [86]. The studied mechanical system has three degrees of underactuation during single support. Chapter 7 presents a control methodology for creation of hybrid invariant manifolds and stabilization of a desired periodic orbit for the impulsive model of walking. In particular, for a given integer number $M \geq 2$, we introduce $M - 1$ *within-stride switching hypersurfaces* and thereby split the single support phase into M *within-stride phases*. The within-stride switching hypersurfaces are defined as level sets of a scaler holonomic quantity that is strictly increasing function of time on the desired walking motion. To stabilize the desired orbit, the overall controller is chosen as a two-level feedback law. At the first level, during a within-stride phase, a parameterized holonomic output function is defined for the dynamical system and imposed to be zero by using a continuous-time feedback law. The output function is expressed as the difference between the actual values of the angle of hip joints and their desired evolutions, in terms of the latter increasing holonomic quantity. At the second level, the parameters of continuous-time feedback laws are updated during within-stride transitions by event-based update laws. The purpose of updating the parameters is (i) achieving hybrid invariance, (ii) continuity of continuous-time feedback laws during within-stride transitions, and (iii) stabilization of the desired orbit. From the construction procedure of the parameterized output functions and event-based update laws, it is shown that the intersections of the corresponding zero dynamics manifolds and within-stride switching hypersurfaces are independent of the parameters. Consequently, by choosing one of these common intersections as the Poincaré section, stabilization can be addressed on the basis of a five-dimensional restricted Poincaré return map.

1.3.6 Hybrid Zero Dynamics with Continuous-Time Update Laws

To improve the convergence rate, the idea of updating the parameters of time-invariant stabilizing controllers by event-based update laws has been described in Refs. [57–59]. The contribution of Chapter 8 is to develop a method for designing a class of continuous-time update laws to update the parameters of stabilizing controllers *during continuous phases* of locomotion such that (i) a general cost function (such as the energy of the control input over single support) can be minimized in an online manner, and (ii) the exponential stability behavior of the limit cycle for the closed-loop system is not affected [87]. In addition, Chapter 8 introduces a class of continuous-time update laws with *radial basis step length* to minimize a desired cost function in terms of the controller parameters and initial states.