1

Introduction and the Concept of Effective Stress

1.1 Preliminary Remarks

The engineer designing such soil structures as embankments, dams, or building foundations should be able to predict the safety of these against collapse or excessive deformation under various loading conditions which are deemed possible. On occasion, he may have to apply his predictive knowledge to events in natural soil or rock outcrops, subject perhaps to new, man-made conditions. Typical of this is the disastrous collapse of the mountain (Mount Toc) bounding the Vajont reservoir which occurred on 9 October 1963 in Italy (Müller 1965). Figure 1.1 shows both a sketch indicating the extent of the failure and a diagram indicating the cross section of the encountered ground movement.

In the above collapse, the evident cause and the "straw that broke the camel's back" was the filling and the subsequent drawdown of the reservoir. The phenomenon proceeded essentially in a static (or quasi-static) manner until the last moment when the moving mass of soil acquired the speed of "an express train" at which point, it tumbled into the reservoir, displacing the water dynamically and causing an unprecedented death toll of some 4000 people from the neighboring town of Longarone.

Such *static* failures which occur, fortunately at a much smaller scale, in many embankments and cuttings are subjects of typical concern to practicing engineers. However, dynamic effects such as those frequently caused by earthquakes are more spectacular and much more difficult to predict.

We illustrate the dynamic problem by the near-collapse of the Lower San Fernando dam near Los Angeles during the 1971 earthquake (Figure 1.2) (Seed, 1979; Seed et al. 1975). This failure, fortunately, did not involve any loss of life as the level to which the dam "slumped" still contained the reservoir. Had this been but a few feet lower, the overtopping of the dam would indeed have caused a major catastrophe with the flood hitting a densely populated area of Los Angeles.

It is evident that the two examples quoted so far involved the interaction of pore water pressure and the soil skeleton. Perhaps the particular feature of this interaction, however, escapes immediate attention. This is due to the "weakening" of the soil–fluid composite during the periodic motion such as that which is involved in an earthquake. However, it is this

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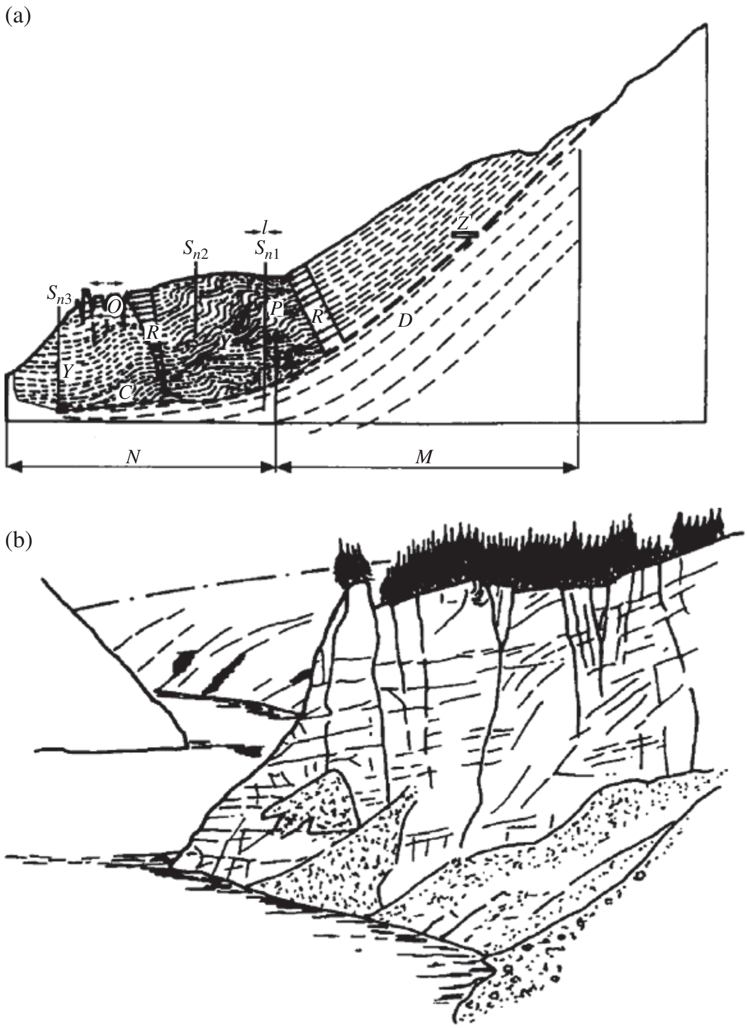


Figure 1.1 The Vajont reservoir, failure of Mant Toc in 1963 (9 October): (a) hypothetical slip plane; (b) downhill end of the slide (Müller, 1965). Plate 1 shows a photo of the slides (front page).

rather than the overall acceleration forces which caused the collapse of the Lower San Fernando dam. What appears to have happened here is that during the motion, the interstitial pore pressure increased, thus reducing the interparticle forces in the solid phase of the soil and its strength.[1]

This phenomenon is well documented and, in some instances, the strength can drop to near-zero values with the soil then behaving almost like a fluid. This behavior is known as *soil liquefaction* and Plate 2 shows a photograph of some buildings in Niigata, Japan taken after the 1964 earthquake. It is clear here that the buildings behaved as if they were floating during the active part of the motion.

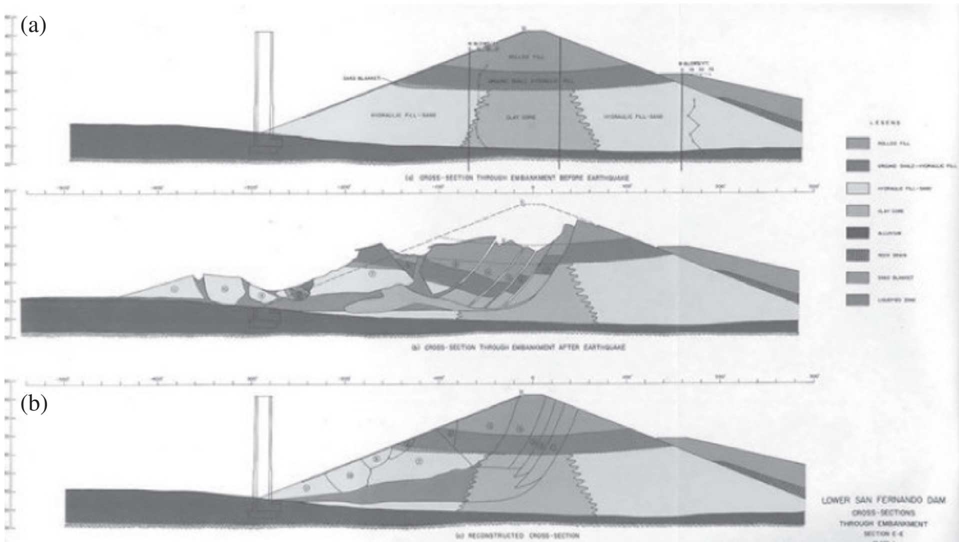


Figure 1.2 Failure and reconstruction of original conditions of Lower San Fernando dam after 1971 earthquake, according to Seed (1979): (a) cross section through embankment after the earthquake; (b) reconstructed cross section. *Source:* Based on Seed (1979).

In this book, we shall discuss the nature and detailed behavior of the various static, quasi-static and dynamic phenomena which occur in soils and will indicate how a computer-based, finite element, analysis can be effective in predicting all these aspects quantitatively.

1.2 The Nature of Soils and Other Porous Media: Why a Full Deformation Analysis Is the Only Viable Approach for Prediction

For single-phase media such as those encountered in structural mechanics, it is possible to predict the ultimate (failure) load of a structure by relatively simple calculations, at least for static problems. Similarly, for soil mechanics problems, such simple, limit-load calculations are frequently used under static conditions, but even here, full justification of such procedures is not generally valid. However, for problems of soil dynamics, the use of such simplified procedures is almost never admissible.

The reason for this lies in the fact that the behavior of soil or such a rock-like material as concrete, in which the pores of the solid phase are filled with one fluid, cannot be described by behavior of a single-phase material. Indeed, to some, it may be an open question whether such porous materials as shown in Figure 1.3 can be treated at all by the methods of continuum mechanics. Here we illustrate two apparently very different materials. The first has a granular structure of loose, generally uncemented, particles in contact with each other. The second is a solid matrix with pores that are interconnected by narrow passages.

From this figure, the answer to the query concerning the possibility of continuum treatment is self-evident. Provided that the dimension of interest and the so-called

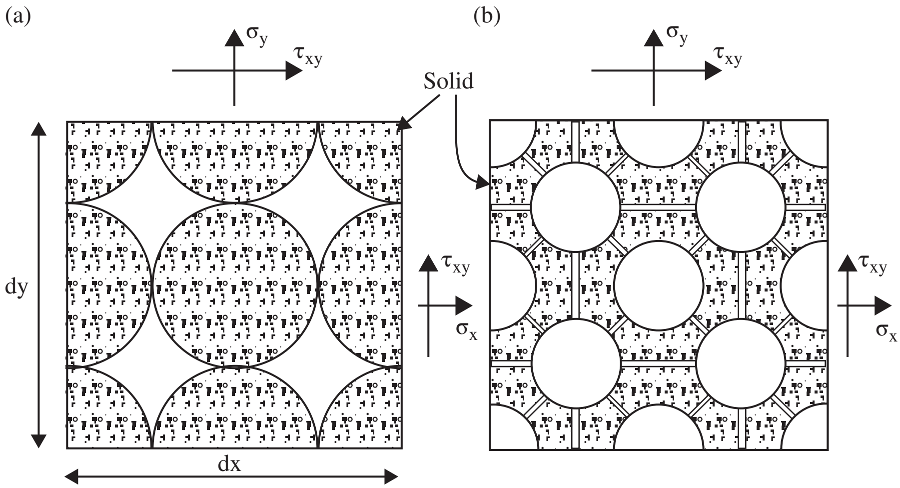


Figure 1.3 Various idealized structures of fluid-saturated porous solids: (a) a granular material; (b) a perforated solid with interconnecting voids.

"infinitesimals" dx, dy, etc., are large enough when compared to the size of the grains and the pores, it is evident that the approximation of a continuum behavior holds. However, it is equally clear that the intergranular forces will be much affected by the pressures of the fluid–p in single phase (or p_1, p_2, etc., if two or more fluids are present). The strength of the solid, porous material on which both deformations and failure depend can thus only be determined once such pressures are known.

Using the concept of *effective stress*, which we shall discuss in detail in the next section, it is possible to reduce the soil mechanics problem to that of the behavior of a single phase once all the pore pressures are known. Then we can again use the simple, single-phase analysis approaches. Indeed, on occasion, the limit load procedures are again possible. One such case is that occurring under long-term load conditions in the material of appreciable permeability when a steady-state drainage pattern has been established and the pore pressures are independent of the material deformation and can be determined by uncoupled calculations.

Such *drained behavior*, however, seldom occurs even in problems that we may be tempted to consider as static due to the slow movement of the pore fluid and, theoretically, the infinite time required to reach this asymptotic behavior. In very finely grained materials such as silts or clays, this situation may never be established even as an approximation.

Thus, in a general situation, the complete solution of the problem of solid material deformation coupled to a transient fluid flow needs to be solved generally. Here no shortcuts are possible and full *coupled analyses* of equations which we shall introduce in Chapter 2 become necessary.

We have not mentioned so far the notion of the so-called *undrained behavior*, which is frequently assumed for rapidly loaded soil. Indeed, if all fluid motion is prevented, by zero permeability being implied or by extreme speed of the loading phenomena, the pressures developed in the fluid will be linked in a unique manner to deformation of the solid material and a single-phase behavior can again be specified. While the artifice of simple undrained behavior is occasionally useful in static studies, it is not applicable to dynamic phenomena such as those which occur in earthquakes as the pressures developed will, in general, be linked again to the straining (or loading) history and this must always be taken into account. Although in early attempts to deal with earthquake analyses and to predict the damage and response, such undrained analyses were invariably used, adding generally a linearization of the total behavior and a heuristic assumption linking the pressure development with cycles of loading and the behavior predictions were poor. Indeed, comparisons with centrifuge experiments confirmed the inability of such methods to predict either the pressure development or deformations (VELACS – Arulanandan and Scott 1993). For this reason, we believe that the only realistic type of analysis is of the type indicated in this book. This was demonstrated in the same VELACS tests to which we shall frequently refer in Chapter 7.

At this point, perhaps it is useful to interject an observation about the possible experimental approaches. The question which could be addressed is whether a scale model study can be made relatively inexpensively in place of elaborate computation. A typical civil engineer may well consider here the analogy with hydraulic models used to solve such problems as spillway flow patterns where the cost of a small-scale model is frequently small compared to equivalent calculations.

Unfortunately, many factors conspire to deny in geomechanics a readily accessible model study. Scale models placed on shaking tables cannot adequately model the main force acting on the soil structure, i.e. that of gravity, though, of course, the dynamic forces are reproducible and scalable.

To remedy this defect, centrifuge models have been introduced and, here, though, at considerable cost, gravity effects can be well modeled. With suitable fluids substituting water, it is indeed also possible to reproduce the seepage timescale and the centrifuge undoubtedly provides a powerful tool for modeling earthquake and consolidation problems in fully saturated materials. Unfortunately, even here a barrier is reached which appears to be insurmountable. As we shall see later under conditions when two fluids, such as air and water, for instance, fill the pores, capillary effects occur and these are extremely important. So far, no significant success has been achieved in modeling these and, hence, studies of structures with free (phreatic) water surface are excluded. This, of course, eliminates the possible practical applications of the centrifuge for dams and embankments in what otherwise is a useful experimental procedure.

1.3 Concepts of Effective Stress in Saturated or Partially Saturated Media

1.3.1 A Single Fluid Present in the Pores – Historical Note

The essential concepts defining the stresses which control the strength and constitutive behavior of a porous material with internal pore pressure of fluid appear to have been defined, at least qualitatively two centuries ago. The work of Lyell (1871), Boussinesq (1876), and Reynolds (1886) was here of considerable note for problems of soils. Later, similar concepts were used to define the behavior of concrete in dams (Levy 1895 and

Fillunger 1913a, 1913b, 1915) and indeed for other soil or rock structures. In all of these approaches, the concept of division of the total stress between the part carried by the solid skeleton and the fluid pressure is introduced and the assumption made that the strength and deformation of the skeleton is its intrinsic property and not dependent on the fluid pressure.

If we thus define the *total stress* $\boldsymbol{\sigma}$ by its components σ_{ij} using indicial notation, these are determined by summing the appropriate forces in the i-direction on the projection, or cuts, $\mathrm{d}x_j$ (or $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$ in conventional notation). The surfaces of cuts are shown for two kinds of porous material structure in Figure 1.3 and include the total area of the porous skeleton.

In the context of the finite element computation, we shall frequently use a vectorial notation for stresses, writing

$$\boldsymbol{\sigma} \equiv [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]^{\mathrm{T}} \tag{1.1a}$$

or

$$\boldsymbol{\sigma} \equiv \left[\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}\right]^{\mathrm{T}} \tag{1.1b}$$

This notation reduces the components to six rather than nine and has some computational merit.

Now if the stress in the solid skeleton is defined as the *effective stress* σ' again over the whole cross sectional area, then the hydrostatic stress due to the pore pressure, p acting, only on the pore area should be

$$-\delta_{ij} np \tag{1.2}$$

where n is the porosity and δ_{ij} is the Kronecker delta. The negative sign is introduced as it is a general convention to take tensile components of stress as positive.

The above, plausible, argument leads to the following relation between total and effective stress with total stress

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} np \tag{1.3}$$

or if the vectorial notation is used, we have

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - \mathbf{m} np \tag{1.4}$$

where $\mathbf{m}$ is a vector written as

$$\mathbf{m} = [1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0]^{\mathrm{T}} \tag{1.5}$$

The above arguments do not stand the test of experiment as it would appear that, with values of porosity n with a magnitude of 0.1–0.2, it would be possible to damage a specimen of a porous material (such as concrete, for instance) by subjecting it to external and internal pressures simultaneously. Further, it would appear from Equation (1.3) that the strength of the material would be always influenced by the pressure p.

Fillunger introduced the concepts implicit in (1.3) in 1913 but despite conducting experiments in 1915 on the tensile strength of concrete subject to water pressure in the pores, which gave the correct answers, he was not willing to depart from the simple statements made above.

It was the work of Terzaghi and Rendulic (1934) and by Terzaghi (1936) which finally modified the definition of effective stress to

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - \mathbf{m} n_w p \tag{1.6}$$

where n_w is now called the *effective area coefficient* and is such that

$$n_w \approx 1 \tag{1.7}$$

Much further experimentation on such porous solids as the concrete had to be performed before the above statement was generally accepted. Here the work of Leliavsky (1947), McHenry (1948), and Serafim (1954, 1964) made important contributions by experiments and arguments showing that it is more rational to take sections for determining the pore water effect through arbitrary surfaces with minimum contact points.

Bishop (1959) and Skempton (1960) analyzed the historical perspective and, more recently, de Boer (1996) and de Boer et al. (1996) addressed the same problem showing how an acrimonious debate between Fillunger and Terzaghi terminated in the tragic suicide of the former in 1937.

Zienkiewicz (1947, 1963) found that interpretation of the various experiments was not always convincing. However, the work of Biot (1941, 1955, 1956a, 1956b, 1962) and Biot and Willis (1957) clarified many concepts in the interpretation of effective stress and indeed of the coupled fluid and solid interaction. In the following section, we shall present a somewhat different argument leading to Equations (1.6) and (1.7).

If the quantity $\boldsymbol{\sigma}'$ of (1.3) and (1.4) is interpreted as the volume-averaged solid stress $(1 - n)\,\mathbf{t}_s$ used in the mixture theory (partial stress), see Gray et al. (2009), then we recover the stress split introduced in Biot (1955). There the fluid pressure, as opposed to the effective stress concept, is weighted by the porosity. Biot (1955) declares that "the remaining components of the stress tensor are the forces applied to that portion of the cube faces occupied by the solid." In this book, we use the much more common concept of effective stress.

1.3.2 An Alternative Approach to Effective Stress

Let us now consider the effect of the simultaneous application of a total external hydrostatic stress and a pore pressure change, both equal to Δp, to any porous material. The above requirement can be written in tensorial notation as requiring that the total stress increment is defined as

$$\Delta\sigma_{ij} = -\delta_{ij}\Delta p \tag{1.8a}$$

or, using the vector notation

$$\Delta\boldsymbol{\sigma} = -\mathbf{m}\Delta p \tag{1.8b}$$

In the above, the negative sign is introduced since "pressures" are generally defined as being positive in compression, while it is convenient to define stress components as positive in tension.

It is evident that for the loading mentioned, only a very uniform and small volumetric strain will occur in the skeleton and the material will not suffer any damage provided that the grains of the solid are all made of identical material. This is simply because all parts of the porous medium solid component will be subjected to identical compressive stress.

However, if the microstructure of the porous medium is composed of different materials, it appears possible that nonuniform, localized stresses, can occur and that local grain damage may be suffered. Experiments performed on many soils and rocks and rock-like materials show, however, that such effects are insignificant. Thus, in general, the grains and, hence, the total material will be in a state of pure volumetric strain

$$\Delta\varepsilon_v \approx \Delta\varepsilon_{ii} = \Delta\varepsilon_{11} + \Delta\varepsilon_{22} + \Delta\varepsilon_{33} = -\frac{1}{K_s}\Delta p \tag{1.9}$$

where K_s is the average material bulk modulus of the solid components of the skeleton. Alternatively, adopting a vectorial notation for strain in a manner involved in (1.1)

$$\Delta\varepsilon_v = \mathbf{m}^{\mathrm{T}}\Delta\boldsymbol{\varepsilon} = -\frac{1}{K_s}\Delta p \tag{1.10a}$$

where $\boldsymbol{\varepsilon}$ is the vector defining the strains in the manner corresponding to that of stress increment definition. Again, assuming that the material is isotropic, we shall have

$$\Delta\boldsymbol{\varepsilon} = -\mathbf{m}\frac{1}{3K_s}\Delta p \tag{1.10b}$$

Those not familiar with soil mechanics may find the following hypothetical experiment illustrative. A block of porous, sponge-like rubber is immersed in a fluid to which an increase in pressure of Δp is applied as shown in Figure 1.4. If the pores are connected to the fluid, the volumetric strain will be negligible as the solid components of the sponge rubber are virtually incompressible.

If, on the other hand, the block is first encased in a membrane and the interior is allowed to drain freely, then again a purely volumetric strain will be realized but now of a much larger magnitude.

The facts mentioned above were established by the very early experiments of Fillunger (1915) and it is surprising that so much discussion of "area coefficients" has since been necessary.

From the preceding discussion, it is clear that if the material is subject to a simultaneous change of total stress $\Delta\sigma$ and of the total pore pressure Δp, the resulting strain can always be written incrementally in tensorial notation as

$$\Delta\varepsilon_{kl} = C_{klij}\left(\Delta\sigma_{ij} + \delta_{ij}\Delta p\right) - \delta_{kl}\frac{1}{3K_s}\Delta p + \Delta\varepsilon_{kl}^0 \tag{1.11a}$$

or in vectoral notation

$$\Delta\boldsymbol{\varepsilon} = \mathbf{D}^{-1}(\Delta\boldsymbol{\sigma} + \mathbf{m}\Delta p) - \mathbf{m}\frac{1}{3K_s}\Delta p + \Delta\boldsymbol{\varepsilon}^0 \tag{1.11b}$$

with

$$C_{ijkl}D_{mnop} = \delta_{im}\delta_{jn}\delta_{ko}\delta_{lp} \tag{11.11c}$$

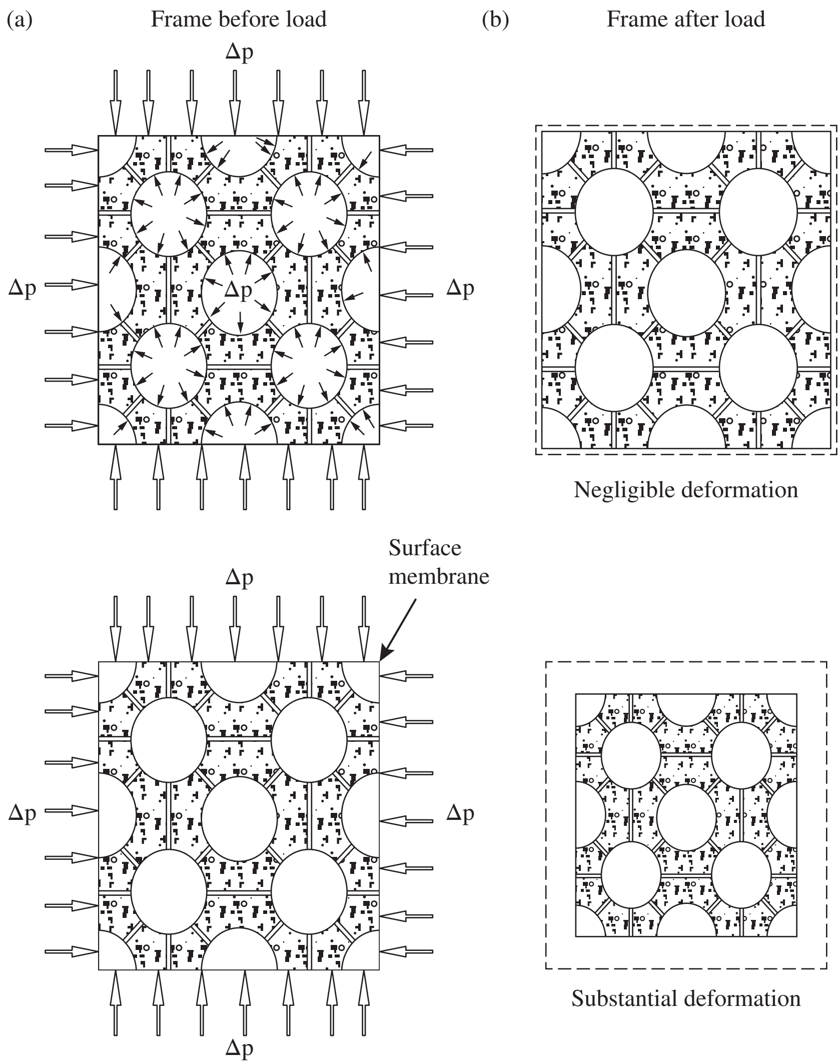


Figure 1.4 A porous material subject to external hydrostatic pressure increases Δ_p, and (a) internal pressure increment Δ_p; (b) internal pressure increment of zero.

The last term in (1.11a) and (1.11b), $\Delta\varepsilon^0$, is simply the increment of an initial strain such as may be caused by temperature changes, etc., while the penultimate term is the strain due to the grain compression already mentioned, viz. Equation (1.10). **D** is a tangent matrix of the solid skeleton implied by the constitutive relation with corresponding compliance coefficient matrix $\mathbf{D}^{-1} = \mathbf{C}$. These, of course, could be matrices of constants, if linear elastic behavior is assumed, but generally will be defined by an appropriate nonlinear relationship of the type which we shall discuss in Chapter 4 and this behavior can be established by fully drained ($p = 0$) tests.

Although the effects of skeleton deformation due to the effective stress defined by (1.6) with $n_w = 1$ have been simply added to the uniform volumetric compression, the principle of superposition requiring linear behavior is not invoked and in this book, we shall almost exclusively be concerned with nonlinear, irreversible, elastoplastic and elastoviscoplastic responses of the skeleton which, however, we assume incremental properties.

For assessment of the strength of the saturated material, the effective stress previously defined with $n_w = 1$ is sufficient. However, we note that the *deformation* relation of (1.11) can always be rewritten incorporating the small compressive deformation of the particles as (1.12).

It is more logical at this step to replace the finite increment by an infinitesimal one and to invert the relations in (1.11) writing these as

$$\mathrm{d}\sigma''_{ij} = \mathrm{d}\sigma_{ij} + \alpha\delta_{ij}\mathrm{d}p = D_{ijkl}\left(\mathrm{d}\varepsilon_{kl} - \mathrm{d}\varepsilon^0_{kl}\right) \tag{1.12a}$$

or

$$\mathrm{d}\sigma'' = \mathrm{d}\sigma + \alpha\mathbf{m}\mathrm{d}p = \mathbf{D}(\mathrm{d}\boldsymbol{\varepsilon} - \mathrm{d}\boldsymbol{\varepsilon}_0) \tag{1.12b}$$

where a new "effective" stress, σ'', is defined. In the above

$$\alpha\delta_{ij} = \delta_{ij} - D_{ijkl}\delta_{kl}\frac{1}{3K_s} \tag{1.13a}$$

or

$$\alpha\mathbf{m} = \mathbf{m} - \mathbf{D}\mathbf{m}\frac{1}{3K_s} \tag{1.13b}$$

and the new form eliminates the need for separate determination of the volumetric strain component. Noting that, in three dimensions,

$$\delta_{ij}\delta_{ij} = 3$$

or

$$\mathbf{m}^{\mathrm{T}}\mathbf{m} = 3$$

we can write

$$\alpha\mathbf{m}^{\mathrm{T}}\mathbf{m} = \mathbf{m}^{\mathrm{T}}\mathbf{m} - \mathbf{m}^{\mathrm{T}}\mathbf{D}\mathbf{m}\frac{1}{3K_s} \tag{1.14a}$$

or simply

$$\alpha = 1 - \frac{\mathbf{m}^T\mathbf{D}\mathbf{m}}{K_s}$$

Alternatively, in tensorial form, the same result is obtained as

$$\alpha\delta_{ij}\delta_{ij} = \delta_{ij}\delta_{ij} - \delta_{ij}D_{ijkl}\delta_{kl}\frac{1}{3K_s} \tag{1.14b}$$

and

$$\alpha = 1 - \frac{\delta_{ij} D_{ijkl} \delta_{kl}}{9K_s}$$

For isotropic materials, we note that,

$$\frac{\mathbf{m}^{\mathrm{T}}\mathbf{D}\mathbf{m}}{9} = \frac{\delta_{ij} D_{ijkl} \delta_{kl}}{9} = \frac{\delta_{ij}\left(\lambda \delta_{ij} \delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\right)\delta_{kl}}{9} = \frac{9\lambda + 6\mu}{9} = K_T \tag{1.15a}$$

which is the tangential bulk modulus of an isotropic elastic material with λ and μ being the Lamé's constants. Thus we can write

$$\alpha = 1 - \frac{K_T}{K_s} \tag{1.15b}$$

The reader should note that in (1.12), we have written the definition of the effective stress increment which can, of course, be used in a non-incremental state as

$$\sigma''_{ij} = \sigma_{ij} + \alpha \delta_{ij} p \tag{1.16a}$$

or

$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma} + \alpha \mathbf{m}\, p \tag{1.16b}$$

assuming that all the stresses and pore pressure started from a zero initial state (for example, material exposed to air is taken as under zero pressure). The above definition corresponds to that of the effective stress used by Biot (1941) but is somewhat more simply derived. In the above, α is a factor that becomes close to unity when the bulk modulus K_s of the grains is much larger than that of the whole material. In such a case, we can write, of course

$$\sigma''_{ij} = \sigma'_{ij} \equiv \sigma_{ij} + \delta_{ij} p \tag{1.17a}$$

or

$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma}' \equiv \boldsymbol{\sigma} + \mathbf{m} p \tag{1.17b}$$

recovering the common definition used by many in soil mechanics and introduced by Terzaghi (1936). Now, however, the meaning of α is no longer associated with an effective area.

It should have been noted that in some materials such as rocks or concrete, it is possible for the ratio K_T/K_s to be as large as 1/3 with $\alpha = 2/3$ being a fairly common value for determination of deformation.

We note that in the preceding discussion, the only assumption made, which can be questioned, is that of neglecting the local damage due to differing materials in the soil matrix. We have also implicitly assumed that the fluid flow is such that it does not separate the contacts of the soil grains. This assumption is not totally correct in soil liquefaction or flow in the soil-shearing layer during localization; therefore, it is not clear if Terzaghi's definition of effective stress still applies when the soil is liquefied.

1.3.3 Effective Stress in the Presence of Two (or More) Pore Fluids – Partially Saturated Media

The interstitial space, or the pores, may, in a practical situation, be filled with two or more fluids. We shall, in this section, consider only two fluids with the degree of saturation by each fluid being defined by the proportion of the total pore volume n (porosity) occupied by each fluid. In the context of soil behavior discussed in this book, the fluids will invariably be *water* and *air*, respectively. Thus, we shall refer to only two saturation degrees, S_w that for water and S_a that for air, but the discussion will be valid for any two fluids.

It is clear that if both fluids fill the pores completely, we shall always have

$$S_w + S_a = 1 \tag{1.18}$$

Clearly, this relation will be valid for any other pair of fluids, e.g. oil and water and indeed the treatment described here is valid for any fluid conditions.

The two fluids may well present different areas of contact with the solid grains of the material in the manner illustrated in Figure 1.5a and b. The average *pressure* reducing the interstitial contact and relevant to the definition of effective stress found in the previous section (Equations (1.16) and (1.17)) can thus be taken as

$$p = \chi_w p_w + \chi_a p_a \tag{1.19}$$

where the coefficients χ_w and χ_a refer to water and air, respectively, and are such that

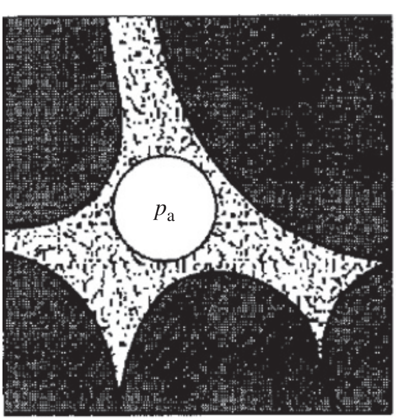
$$\chi_w + \chi_a = 1 \tag{1.20}$$

The individual pressures p_w and p_a are again referring to water and air and their difference, i.e.

$$p_c = p_a - p_w \tag{1.21}$$

is dependent on the magnitude of surface tension or capillarity and on the degree of saturation (p_c is often referred to, therefore, as capillary pressure).

(a)



(b)

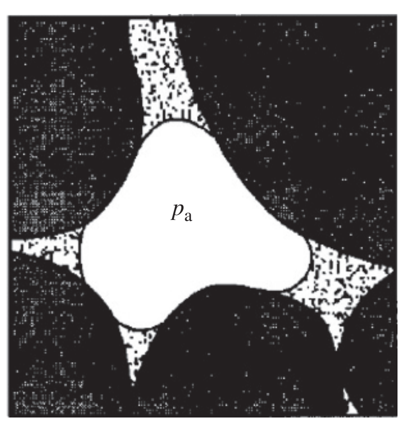


Figure 1.5 Two fluids in pores of a granular solid (water and air). (a) Air bubble not wetting solid surface (effective pressure $p = p_w$); (b) Both fluids in contact with solid surfaces (effective pressure $p = \chi_w p_w + \chi_a p_a$).

Depending on the nature of the material surface, the contact surface may take on the shapes shown in Figure 1.5 with

$$\chi_w = \chi_w(S_w) \tag{1.22a}$$

and

$$\chi_a = \chi_a(S_a) \tag{1.22b}$$

Occasionally, the contact of one of the phases and the solid may disappear entirely as shown in Figure 1.5a giving isolated air bubbles and making in this limit

$$\chi_a = 0 \;\; \chi_w = 1 \tag{1.23}$$

In many situations, in soil mechanics, it is sufficient to take χ equal to the respective degrees of saturation (Lewis and Schrefler 1982; Nuth and Laloui 2008).

Whatever the nature of the contact, we shall find, neglecting the hysteresis during the wetting and drying cycles, that a unique relationship between p_c and the saturation S_w can be written, i.e.

$$p_c = p_c(S_w) \tag{1.24}$$

Indeed, the degree of saturation will similarly affect flow parameters such as the permeability k to which we shall make reference in the next chapter, giving

$$\begin{aligned} k_w &= k_w(S_w) \\ k_a &= k_a(S_a) \end{aligned} \tag{1.25}$$

Many studies of such relationship are reported in the literature (Liakopoulos 1965; Neuman 2017; Van Genuchten et al. 1977; Narasimhan and Witherspoon 1978; Safai and Pinder 1979; Lloret and Alonso 1980; Bear et al. 1984; Alonso et al. 1987). Figure 1.6 shows a typical relationship.

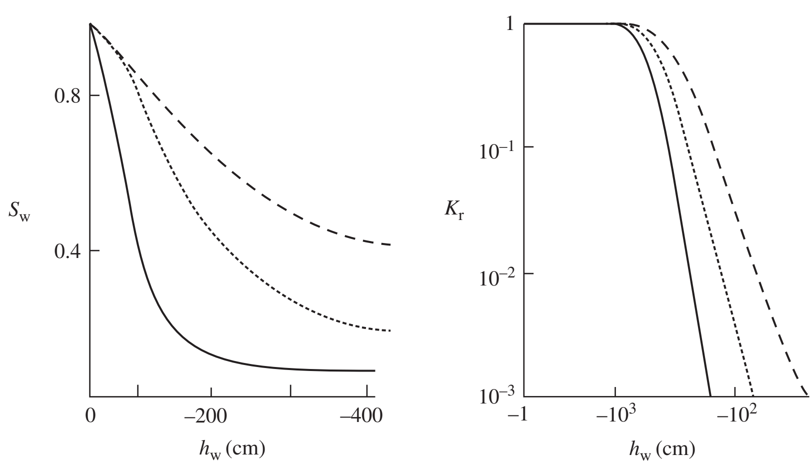


Figure 1.6 Typical relations between pore pressure head, $h_w = p_w/\chi_w$, saturation, S_w, and relative permeability, $k_r = k_w(S_w)/k_w(1)$ (Safai and Pinder 1979). Note that relative permeability decreases very rapidly as saturation decreases. *Source:* From Safai and Pinder (1979).

The concepts of dealing with the effects of multiple pore pressure by introducing an *average pressure* and using the standard definition of *effective stress* (1.19, 1.16, and 1.17) were first introduced by Bishop (1959). Certainly, the arguments for thus extending the original concepts are less clear than is the case when only a single fluid is present. However, the results obtained by this extension are quite accurate. We shall, therefore, use such a definition in the study of partially saturated media.

In many cases occurring in practice, the air pressure is close to zero (atmospheric datum) as the pores are interconnected. Alternatively, negative air pressure occurs as cavitation starts and here the datum is the vapor pressure of water. In either case, the effect of p_a can be easily neglected as the water pressure simply becomes negative from Equation (1.24). Such negative pressures are responsible for the development of certain cohesion by the soil and are essential in the study of free surface conditions occurring in embankments, as we shall see later.

Note

1 Such strength reduction phenomena are mainly evident in essentially non-cohesive materials such as sand and silt. Clays in which negative capillary pressure provide an apparent cohesion are less liable to such strength reduction.

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