PART I

Bond Yields, Bond Prices, and Bond Investment

CHAPTER 1

Interest on Interest

The recent high level of bond yields and the uncertainty whether yields will be high in the years ahead emphasizes the importance of interest-on-interest, that is to say, the rate at which receipts from coupons can be reinvested in the future. An original investment compounds automatically at the purchase yield only until the funds are paid back in the form of coupons and finally of principal. However, some investors mistakenly expect that a bond purchased at a given yield will always produce that rate as a realized compound yield over the whole life of the bond. If future reinvestment rates during the life of the bond are less than the purchase yield, then the realized compound yield for the whole life of the bond will be less than the purchase yield; if future rates are higher than the purchase yield, then the realized compound yield will be more than the purchase yield.

Long-Term Par Bonds

For most long-term bonds, the interest-on-interest is a surprisingly important part of the total compounded return to the bondholder: typically over half.

Table 1 shows that for an 8 percent 20-year bond bought at 100 to yield 8 percent the total return over the twenty-year period may vary from 1,600 per 1,000 invested (4.84 percent) to 4,832 (9.01 percent) depending upon whether the interest-on-interest is 0 percent (interest spent–in a financially non-productive way–as coupons are paid) or 10 percent. When the reinvestment rate is also 8 percent, the coupons over the twenty years will total 1,600 per 1,000 invested and the interest on this interest will total 2,201 or 58 percent of the total return of 3,801. If the rate of interest-on-interest is 6 percent, the interest-on-interest will decline to 1,416, and the total return to 3,016 per 1,000 invested, bringing the total realized compound yield

TABLE 1 An 8 percent non-callable 20-year bond bought at 100 to yield 8 percent

Interest-on-Interest						
Reinvestment Rate	% of Total Return	Amount	Coupon Income	Discount	Total Return	Total Realized Compound Yield
0%	0%	0	1,600	0	1,600	4.84%
5	41	1,096	1,600	0	2,696	6.64
6	47	1,416	1,600	0	3,016	7.07
7	53	1,782	1,600	0	3,382	7.53
8*	58*	2,201*	1,600*	0	3,801*	8.00*
9	63	2,681	1,600	0	4,281	8.50
10	67	3,232	1,600	0	4,832	9.01

*Yield from Yield Book.

to the purchaser down from the original 8 percent to 7.07 percent. On the other hand, if the rate of interest-on-interest rises to 10 percent, interest-on-interest will rise to 3,232 and total return to 4,832 per 1,000 invested bringing the total realized compound yield to the purchaser up to 9.01 percent.

It follows that a present purchaser of a long-term 8 percent non-callable bond at 100 is by no means assured of a realized yield of 8 percent for the life of his investment if by yield is meant interest compounded on the entire original investment for the entire life of the bond: it might tum out to be 6.64 percent or 9.01 percent or more or less, depending on the future trend of bond yields. The uncertainty is entirely confined to the compounding factor. In terms of simple interest, the investor is sure to get 8 percent from this 8 percent bond, i.e., 80 a year per 1,000 if the bond is not called or defaulted.

The Yield Book

The Yield Book serves the essential function of providing a uniform basis for comparing the market values of bonds having different coupons, maturities, dollar prices and, consequently, different cash flows over their life. To achieve this uniformity, the Yield Book in essence refers every dollar of every bond's cash flow to the standard of an initial investment allowed to accumulate compound interest semiannually at the Yield Book rate until it is paid off in the form of coupon or principal. For example, suppose one has two 20-year

bonds, one with a coupon of 8 percent and the other with a 4 percent coupon, both priced "to yield 8 percent." This 8 percent figure can be taken to mean that both bonds are equivalent to the standard of an 8 percent semiannually compounded investment, which would realize a return of 3,801 per 1,000 invested over the twenty-year period.

It is not so well known that to obtain this objective it is necessary that the bonds' coupon income be reinvested so as to gather "interest-on-interest" at a rate exactly equal to the yield-to-maturity itself. The two 8 percent yield-to-maturity bonds in the above example would realize the standardized 8 percent return of 3,801 per 1,000 invested only if each and every coupon were itself reinvested at an exact 8 percent rate. If the coupons cannot be reinvested at the Yield Book rate, then the realized compound yield over the bond's life of the dollars originally invested may vary widely from the Yield Book figure. For this reason, when facing future periods involving possible major swings in yield levels, it becomes vitally important to distinguish between the yield-to-maturity (as stated in the Yield Book) and the realized compound yield that will actually be obtained if the bond is held to maturity.

Simple Interest vs. Compound Interest

Many investors, like university endowment funds and foundations and private investors, simply collect and spend their coupons. They tend to ignore the variability of compound interest. Others, like pension funds, accumulate interest receipts, merge them with principal and reinvest them; these are vitally affected by the future rate of interest and ordinarily cannot, when they invest, obtain any assurance as to just what their total return will be.*

Maturity

As maturity is reduced, the importance of interest-on-interest declines sharply. This is illustrated by Table 2 which shows that for a 1-year 8 percent bond at 100, interest-on-interest will account for only 2 percent of total return, while for a 40-year 8 percent bond it will account for 86 percent of total return. The uncertainty can be said to be basic only for longer maturities.

*It would be possible to design a bond issue that would guarantee a rate of compound interest by paying coupons in debt rather than cash, but it has rarely if ever been done. Savings bonds do provide guaranteed compound interest.

TABLE 2 Effect of maturity on the importance of interest-on-interest (assuming reinvestment at yield rate)

	% of Total Return Represented by Interest-on-Interest	
Maturity	8% Bonds Bought at 100 to Yield 8%	4% Bonds Bought at 100 to Yield 4%
1 Year	2%	1%
5 Years	17	9
10 "	33	18
20 "	58	34
30 "	75	47
40 "	86	59

It is obvious, however, that shorter maturities, while reducing or almost eliminating the uncertainty of the rate of interest-on-interest do not solve the problems of maintaining future income. Indeed, the uncertainty is larger with shorter term bonds because in the reinvestment of shorts at maturity the coupon, in addition to the compounding factor, is uncertain. Thus, the old rule will usually hold: if future rates are to rise, shorts bought now will be better than longs; if rates decline, longs will be better than shorts.

Long-Term Discount Bonds

Table 3 shows the same calculation for a deep discount bond, a 20-year 4 percent bond selling at about $60^{3}/_{8}$ to yield 8 percent. The top panel shows total return in dollars from one bond, and the bottom panel translates the same figures on the basis of each 1,000 invested, so that the returns can be compared with the 20-year 8 percent bond in Table 1. Here we find that the variation of total return based on changes in interest-on-interest is also large, but not as large as in the case of the par bonds. This is because the discount (eventual capital gain) is a fixed component of total return that does not vary with future interest rates. When coupon income is a smaller proportion of total return, interest-on-interest must also be a smaller portion of total return. The difference, however, between the par bond and the discount bond is not so large as to provide an absolute guide for selection. If interest-on-interest varies, between 6 percent and 10 percent, the total realized compound yield of the 8 percent bond will vary between 7.07 percent and 9.01 percent, while that of the 4 percent bond will

TABLE 3 A 4 percent 20-year bond bought at 60.414 to yield 8 percent

Interest-on-Interest						
Reinvestment Rate	% of Total Return	Amount	Coupon Income	Discount	Total Return	Total Realized Compound Yield
		A: Per Bond				
0%	0%	0	800	396	1,196	5.53%
5	31	548	800	396	1,744	6.90
6	37	708	800	396	1,904	7.25
7	43	891	800	396	2,087	7.61
8*	48*	1,100*	800*	396*	2,296*	8.00*
9	53	1,341	800	396	2,536	8.41
10	57	1,616	800	396	2,812	8.85
		B: Per 1,000 Invested				
0	0	0	1,325	655	1,980	5.53
5	31	907	1,325	655	2,877	6.90
6	37	1,172	1,325	655	3,152	7.25
7	43	1,474	1,325	655	3,454	7.61
8*	48*	1,820*	1,325*	655*	3,800*	8.00*
9	53	2,218	1,325	655	4,198	8.41
10	57	2,674	1,325	655	4,654	8.85

*Yield from Yield Book.

vary between 7.25 percent and 8.85 percent. At lower future rates, the 4 percent bond will yield more than the 8 percent boned because the fixed discount substitutes for some variable interest; at higher future rates the 8 percent bond will yield more than the 4 percent bond because interest-on-interest is a larger component of total return and there is no discount.

Another way of viewing this effect is to compare the different Yield Book values giving rise to the same realized compound yield under the same reinvestment assumption. For example, the 8 percent par bond (Table 1) with coupons reinvested at 6 percent results in a realized compound yield of 7.07 percent. To obtain this same realized compound yield of 7.07 percent under the same reinvestment assumption (6 percent), the 4 percent coupon

bond (Table 3) would have to be priced to yield 7.70 percent by the Yield Book. In other words, one could "give up" as much as 30 basis points in yield at cost, and the 4 percent discount bond would still prove to be as good a buy from the standpoint of realized compound yield. This comparison would, of course, not be valid in case of stable or rising interest rates.

Long-Term Bonds at Lower Yields

These considerations show that long-term bonds bought a few years ago at yields of 4 percent to 5 percent are actually permitting the purchasers to receive a much higher compound yield than the expected rate if the purchasers have been reinvesting their coupons. This is because those lower yields at cost assumed future reinvestment rates of 4 percent to 5 percent, while their coupons have recently been reinvested at rates as high as 8 percent to 9 percent. Table 4 below is comparable to Table 1 except that the original investment is in a 4 percent bond at 100 to yield 4 percent. Discount rates are tabulated all the way from 0 to 10 percent. It will be seen that the proportion

TABLE 4 A 4 percent non-callable 20-year bond bought at 100 to yield 4 percent

Interest-on-Interest						
Reinvestment Rate	% of Total Return	Amount	Coupon Income	Discount	Total Return	Total Realized Compound Yield
0%	0%	0	800	0	800	2.96%
1	9	83	800	0	883	3.19
2	18	178	800	0	978	3.44
3	26	285	800	0	1,085	3.71
4*	34*	408*	800*	0	1,208*	4.00*
5	41	548	800	0	1,348	4.31
6	47	708	800	0	1,508	4.65
7	53	891	800	0	1,691	5.01
8	58	1,101	800	0	1,901	5.40
9	63	1,341	800	0	2,141	5.80
10	67	1,616	800	0	2,416	6.24

*Yield from Yield Book.

of total return depending on interest-on-interest is exactly the same as that of the 8 percent bond in Table 1 if the reinvestment rates are the same while, of course, the total returns and yields are very much less.

Timing of Rate Changes

In all of the above tables, the future rate of interest-on-interest is stated as one figure which might seem like an average for the twenty-year period, but it is not: It is an artificially fixed rate of reinvestment for all coupons from first to last at the indicated rate assumed by the tables. In real life, rates vary widely from year to year, and a simple average would be fallacious because of the time factor. A low reinvestment rate a year or two after investment followed by higher rates would bring much bigger total interest-on-interest than an early period of high rates followed by low rates. This is because the high reinvestment rate later would earn much more interest from the larger accumulation of funds being reinvested. Thus, the maximum income benefit to today's purchaser would accrue from a rapid rise in interest rates soon after his purchase and thereafter sustained high rates. This, of course, is just the opposite to his profits or losses from principal fluctuations.

Investment Implications

1. The purchaser of long-term bonds who plans to compound his return has not achieved a certainty as to just what his total compounded realized yield will be even if the bonds are non-callable. The area of doubt is perhaps a yield range of 2 percent.
2. Conversely, the purchaser of non-callable long-term bonds who plans to spend his income can count on a predetermined rate of return provided only that the bonds remain in good standing.
3. For the compounding investor who expects interest rates to average lower over a long period of years than at time of purchase, there is a structural advantage in discount bonds over par bonds because that part of his return represented by the discount is fixed and cannot decline with interest rates. This advantage is supplementary to other advantages such as superior call protection and (for taxpayers) a lower tax rate. However, these advantages are often offset when discount bonds yield substantially less than high coupon bonds.
4. Conversely, for those who expect high or higher interest rates in the years ahead but who are constrained to stay with long-term bonds, there is a

structural advantage in par or premium bonds because their total return will rise more rapidly with rising rates. Also, they usually yield more at time of purchase.

5. Short-term bonds provide much greater certainty of the compounded rate of return than do long-term bonds, but only for limited periods corresponding to the short-term maturities. Thereafter, because their entire principal amount must be reinvested at the then prevailing rates, the area of uncertainty is very much larger than in the case of long-term bonds.