CHAPTER 1

MEASURING THE STRENGTH OF METALS

The strength of metals is a fascinating topic that carries engineering and scientific implications. You cannot build a bridge, design a turbine blade, or construct a transmission line tower without understanding the strength of the materials of construction in balance with the requirements of the system.

1.1 HOW IS STRENGTH MEASURED?

Let us take a piece of metal and machine a *test specimen* of either the geometry shown in Figure 1.1 or Figure 1.2. Note that two geometries are shown. At the center point of these specimens, one has a circular cross section, whereas the other has a rectangular cross section. Either geometry can be used according to the material stock available.

After machining, this sample is attached to the "grips" of a test machine. An example of a testing machine is shown in Figure 1.3. This is an "electromechanical" machine in that the moving crosshead is attached to very large screws (in the columns) that move the crosshead up or down with great force. The test specimen is mounted between the two grips.

Figure 1.4 shows a test specimen mounted between two grips. It also shows a "deflection measurement device" attached to the specimen. As force is applied to the top grip and the specimen elongates, the deflection

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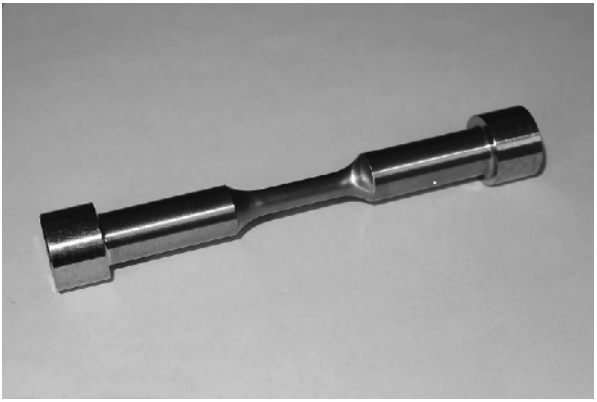


FIGURE 1.1 Test specimen machined from a plate thick enough to give a circular cross section. Courtesy of Westmoreland Mechanical Testing and Research, Inc. (WMT&R, Inc.).

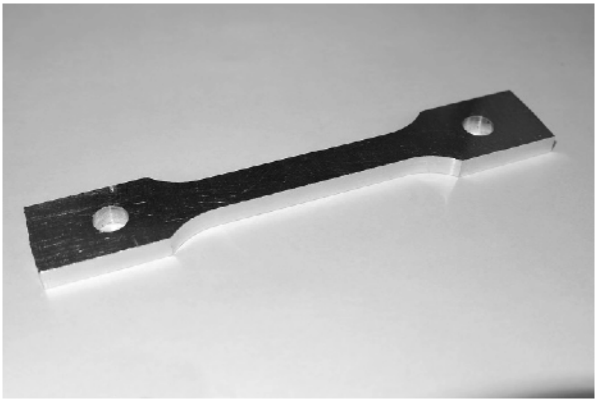


FIGURE 1.2 Test specimen machined from a plate that is too thin to give a large-enough circular cross section. Courtesy of Westmoreland Mechanical Testing and Research, Inc.

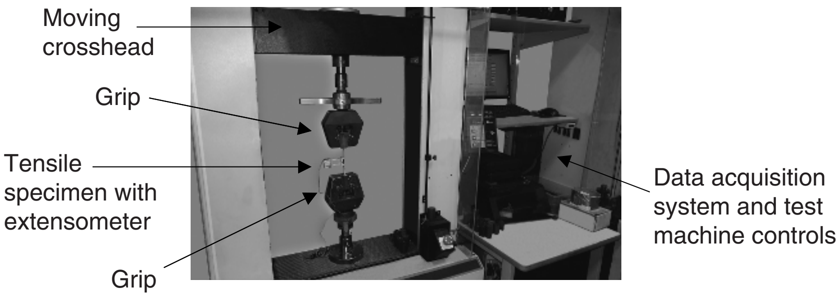


FIGURE 1.3 Mechanical test frame with top and bottom grips. Courtesy of Westmoreland Mechanical Testing and Research, Inc.

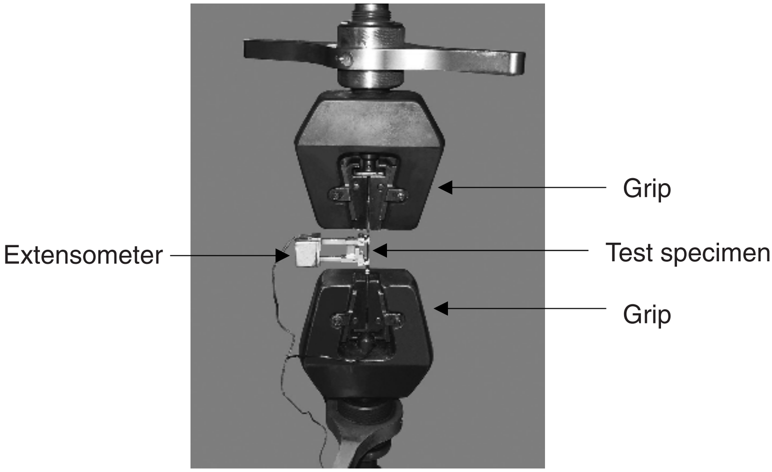


FIGURE 1.4 Tensile specimen mounted in and ready for testing. Also shown is a deflection measurement device attached to two points on the specimen. Courtesy of Westmoreland Mechanical Testing and Research, Inc.

measurement device gives a precise measurement of how much the specimen has elongated.

Along with specimen elongation, another measurement is required in this test, which is the force applied to the top grip (and thus the top end of the specimen) required to elongate it. This measurement is supplied by a "load cell" usually mounted between the upper, moving crosshead and the top grip. Because the elongations in the specimen are so tiny, the measurement of elongation must be made on the actual test specimen. It is easier to measure the motion of the top crosshead, but this might be confounded by other deflections in the "load train," which is the ensemble of hardware between the moving crosshead and the stationary base; thus, the measurement movement of the top crosshead would likely be somewhat larger than the deflection in the test specimen. The measurement of force applied to the specimen, however, can be measured anywhere along the load train. In this sense, it is an easier measurement to make in materials testing.

1.2 THE TENSILE TEST

After machining the test specimen, mounting it in a test machine, and setting up the load and displacement measurements, the test is ready to perform. This section describes what happens when a test specimen is mounted and a force is applied. Assume a cylindrical test specimen machined from mild steel with a machined cross-sectional diameter of 0.250 in. (6.35 mm) and a length of 1.0 in. (25.4 mm). Figure 1.5 illustrates the test configuration, including

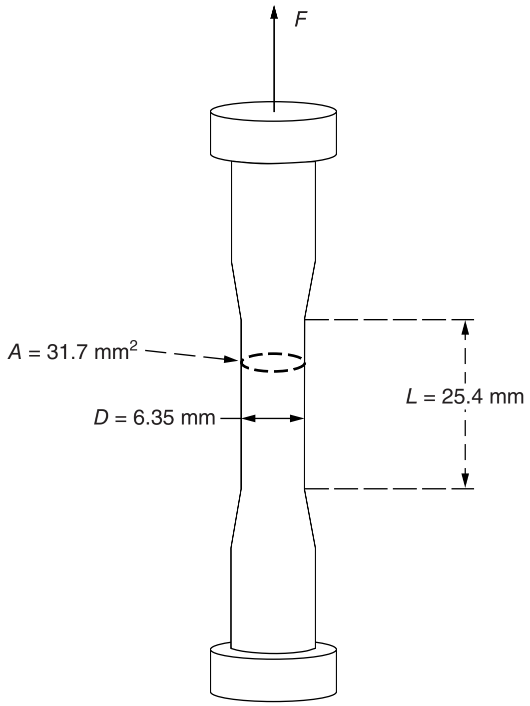


FIGURE 1.5 Schematic tensile test with dimensions.

TABLE 1.1 Load and Deflection Measured for the Specimen Shown in Figure 1.5

Load		Deflection	
Pounds (lb)	Newtons (N)	Inches (in)	Meters (m)
100	445	0.000066	0.00000169
200	890	0.000133	0.00000337
400	1179	0.000265	0.00000674
1000	4448	0.000664	0.0000169
1500	6672	0.000995	0.0000253
2000	8896	0.00133	0.0000337

dimensions, for a mechanical test using a specimen such as shown in Figure 1.1. For the test specimen indicated, the load F is applied to the end of the specimen, thereby stretching it. This is a "tensile" test in that the test specimen is placed into "tension." If load were instead applied that led to shortening of the test specimen—and if the specimen could be kept from buckling—then the test is referred to as a "compression" test.

Table 1.1 shows the deflection in the specimen illustrated in Figure 1.5 that would be measured as a function of the applied load. It is immediately evident

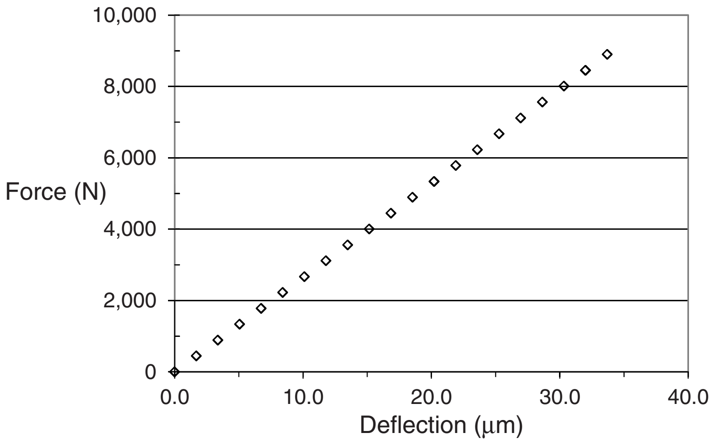


FIGURE 1.6 Force (as MN or 10^6 N) plotted versus deflection (as μm or 10^{-6} m) for the specimen shown in Figure 1.5.

that even with what appears to be very large forces on this test specimen, the deflections are quite small. With 1 ton of force pulling on the specimen, the specimen only elongates 1.33 mils (33.7 μm).

Figure 1.6 plots the load versus the deflection—as if a deflection measurement were taken for every 100-lb increase in load. (Even though deflection has been defined as the dependent variable, which would normally be plotted on the y-axes, Figure 1.6 plots deflection on the x-axis; this will be made clear later.)

It is evident in Figure 1.6 that the force-versus-deflection measurements plot on a straight line. Recall from general physics that the behavior illustrated in Figure 1.6 is analogous to the behavior of a spring, defined by the familiar equation

$$F = K_S \Delta X, \tag{1.1}$$

where F is the applied force and ΔX is the extension of the spring. In fact, the test specimen does behave like a spring. However, this test specimen is indeed a strong spring. Simple springs used in general physics laboratory experiments typically have spring constants on the order of ~10 N/m. As a comparison, the test specimen is a spring with a spring constant of

$$K_S = \frac{F}{\Delta X} = \frac{8896\ \text{N}}{3.37 \times 10^{-5}\ \text{m}} = 2.64 \times 10^8\ \frac{\text{N}}{\text{m}}.$$

Thus, as a spring, the test specimen has a spring constant ~20 million times larger than the springs used in a common physics laboratory. When materials behave like this during a tensile test, they are said to be behaving as an "elastic" solid.

Just as with any spring, if the force on our tensile specimen is reduced, the elongation decreases until it reaches zero extension when the force goes to zero.

1.3 STRESS IN A TEST SPECIMEN

Imagine that the test specimen had a diameter of 0.500 in. (12.7 mm) instead of 0.250 in. (6.35 mm). One would expect that the spring constant of our test specimen would get even larger, and the same force would cause less elongation. To eliminate the effect of specimen size, the force, F, is divided by the cross-sectional area, A, to yield a "stress," σ_e, where

$$\sigma_e = \frac{F}{A}. \tag{1.2}$$

Of course, stress in general is force divided by area. When deformation increases as shown in Figure 1.6—which implies the specimen geometry changes—but stress is computed by dividing the force by the original area, then the stress is referred to as the *engineering stress*, which is why a subscript "e" is used in Equation (1.2). Later (Section 1.10), the stress will be based on the current area. If F has the unit of newton and A has the unit of square meter, then the stress σ_e has the unit newton per square meter, which is known as a pascal:

$$\mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2}.$$

A pascal is a very small stress. Recall that the units of pressure are also force per unit area. Standard atmospheric air pressure is $\sim 10^5$ Pa. Thus, stresses in test specimens are more commonly stated in terms of 10^6 Pa or MPa. The engineering units for stress are pounds per square inch, or psi. Standard atmospheric air pressure is 14.7 psi. Accordingly, when using engineering units, stresses in test specimens are commonly stated in terms of 10^3 psi or ksi. The conversion between MPa and ksi is

$$\mathrm{MPa} = \frac{\mathrm{ksi}}{0.145}. \tag{1.3}$$

1.4 STRAIN IN A TEST SPECIMEN

Suppose the test specimen was machined with a length of 2.0 in. (50.8 mm) instead of 1.0 in. (25.4 mm). The same force would produce an extension in the 2.0-in.-long test specimen that is twice that in the 1.0-in.-long test specimen. To eliminate the effect of specimen length, L, on the test result, the extension, ΔL, is measured relative to the specimen's length:

$$\varepsilon_e = \frac{\Delta L}{L}. \tag{1.4}$$

This relative extension is referred to as the "strain" ε_e. Note that since ΔL and L have the same units, the ratio ε_e is dimensionless. As in the calculation of stress, when strain is based on the initial specimen length, the value is referred to as the *engineering strain* denoted by the subscript "e."

1.5 THE ELASTIC STRESS VERSUS STRAIN CURVE

Since the cross-sectional area of the test specimen (with a diameter of 0.250 in. or 6.35 mm) and the length of the test specimen (1.0 in. or 25.4 mm) are known, the force versus extension curve can easily be converted to stress versus strain for the tensile test shown earlier. The resulting stress versus strain curve is shown in Figure 1.7.

Note that there are no units shown for strain, which is a dimensionless quantity, and that the units for stress are MPa (or 10^6 Pa or 10^6 N/m^2). Other than this, the curve appears identical to that shown in Figure 1.6. The difference is that if a specimen with a diameter of 0.500 in. instead of 0.250 in. or a specimen with a length of 2.0 in. instead of 1.0 in. had been used, the resulting stress versus strain curves would have been identical; that is, converting from force to stress and extension to strain has eliminated specimen geometry effects.

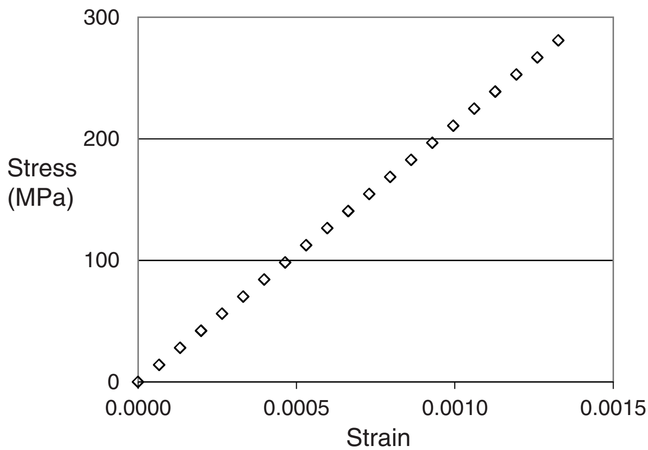


FIGURE 1.7 Stress versus strain for the specimen shown in Figure 1.5.

1.6 THE ELASTIC MODULUS

Given the curve shown in Figure 1.7 is geometry invariant, the slope of this line rather than the slope of the line in Figure 1.6 takes on special meaning. This slope is defined as the "elastic modulus." It is also referred to as Young's modulus, named after a nineteenth century British scientist. In fact, the equation for the line in Figure 1.7 is

$$\sigma_e = E\varepsilon_e \tag{1.5}$$

$$\sigma_e = \frac{F}{A} = E\varepsilon_e = E\frac{\Delta L}{L}, \tag{1.6}$$

where E is the elastic modulus. Because ε_e in this equation is dimensionless, E has the unit of σ_e, which, as discussed earlier, is newton per square meter or pascal. From the plot in Figure 1.6, E for this steel is 30.7×10^6 psi (212×10^9 Pa).

It turns out that E is a fundamental physical property of metals and ceramics. It is defined by interatomic forces and varies from material to material. Table 1.2 lists some values (at room temperature) of E for various materials.

The variation in the elastic modulus from material to material is evident. Note the stipulation that these measurements are at room temperature (RT). Indeed the elastic modulus is a function of temperature—generally E decreases with increasing T. This variation will become meaningful when the temperature dependence of strength is discussed.

Figure 1.8 shows results from the tensile test for a steel test specimen compared with what these results would have been for a beryllium, aluminum alloy, tungsten, and tantalum test specimen. Again, the test is run by incrementing the force in steps of 100 lb for the same specimen (0.25-in. diameter). The slope of each of these lines is the elastic modulus for the material specified. Note that the slope of the plot for the tungsten specimen is much higher than the slope of the line for the aluminum alloy, which correlates with the values of the elastic modulus in the above-mentioned table.

TABLE 1.2 Elastic Modulus of Some Common Materials at Room Temperature

Material	E (10^6 psi)	E (10^9 Pa)
Aluminum	10.2	70.3
Copper	18.8	130
Beryllium	45.1	311
Molybdenum	47.1	325
Tantalum	26.9	186
Gold	11.5	79
Tungsten	59.6	411
Al_2O_3	50	345
Glass (silica)	10	69

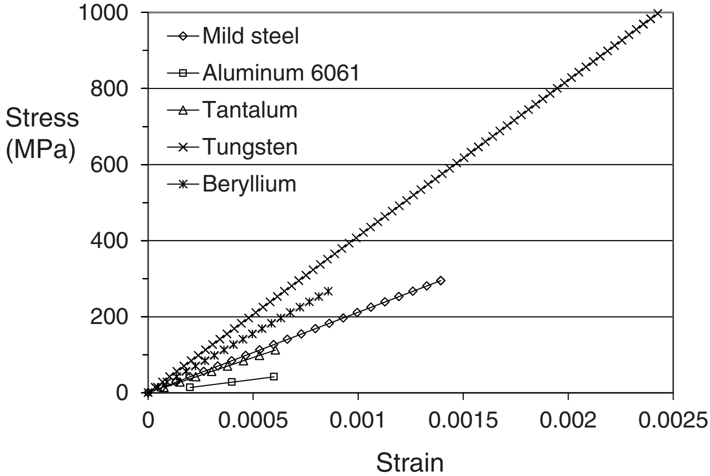


FIGURE 1.8 Stress versus strain for five metals with differing values of the elastic modulus.

1.7 LATERAL STRAINS AND POISSON'S RATIO

For the tensile tests described earlier, a deflection measuring device—also known as an *extensometer*—was attached to measure axial deflection, which was converted to strain. How about the strain in the radial direction (for the case of a cylindrical test specimen)? If volume is conserved during the test, then this can be estimated. Assume the initial and final diameters of the test specimen are D and D_f and that the initial and final lengths are L and L_f. Accordingly, when volume is conserved,

$$L\frac{\pi}{4}D^2 = L_f\frac{\pi}{4}D_f^2$$

$$\frac{L_f}{L} = \left(\frac{D}{D_f}\right)^2$$

$$\frac{L_f - L}{L} = \frac{\Delta L}{L} = \varepsilon_e = \left(\frac{D}{D_f}\right)^2 - 1$$

$$\frac{D}{D_f} = (1+\varepsilon_e)^{1/2}$$

$$\frac{D_f}{D} = \frac{1}{(1+\varepsilon_e)^{1/2}}$$

$$\frac{D_f - D}{D} = \varepsilon_{eD} = \frac{1}{(1+\varepsilon_e)^{1/2}} - 1,$$

where ε_{eD} is defined as the strain in the direction of the diameter of the test specimen. The first term in the equation can be estimated using a binomial series approximation:

$$\varepsilon_{eD} = \frac{1}{(1+\varepsilon_e)^{1/2}} - 1 \cong \left(1 - \frac{\varepsilon_e}{2}\right) - 1 = -\frac{\varepsilon_e}{2}. \tag{1.7}$$

When volume is conserved, the strain in the direction of the diameter—the *diametrical* strain—is one-half the axial strain—and it is of opposite sign. The ratio of the diametrical strain to the axial strain is

$$\frac{\varepsilon_{eD}}{\varepsilon_e} = -\frac{1}{2}. \tag{1.8}$$

The negative of this ratio is called *Poisson's ratio*, ν, named after the French mathematician Siméon Poisson:

$$\nu = -\frac{\varepsilon_{eD}}{\varepsilon_e}. \tag{1.9}$$

This derivation, however, assumed that the volume remains constant throughout the test. Materials that behave like this are referred to as "incompressible." However, most materials are not incompressible but have a very small change in volume when elastically deformed. In fact, Table 1.3 summarizes typical values of Poisson's ratio for metals, ceramics, and polymers.

How much does the volume and density change? Taking the example tensile test on a steel test specimen (which has a Poisson's ratio of 0.33) shown in Figure 1.7, the volume would be

$$\begin{aligned}\frac{V_F}{V_0} &= \frac{\pi D_F^2 L_F}{\pi D_0^2 L_0} = \left(\frac{D_F}{D_0}\right)^2 \left(\frac{L_F}{L_0}\right) \\ &= (1 - \nu\varepsilon_{e_{max}})^2 (1 + \varepsilon_{e_{max}}),\end{aligned}$$

where V_0 and V_F are the initial and final volumes, respectively, and $\varepsilon_{e_{max}}$ is the maximum strain. Taking $\varepsilon_{e_{max}} = 0.0013$ from Figure 1.7,

TABLE 1.3 Poisson's Ratios for Metals, Ceramics, and Polymers Differ and Are in the Ranges Shown

Material	Poisson's Ratio ν
Metals	~0.33
Ceramics	~0.25
Polymers	~0.40

$$\frac{V_F}{V_0} = (1 - 0.33 \times 0.0013)^2 (1 + 0.0013) = 1.00044,$$

which shows the volume increases (thus, the density decreases) by 0.044%. Thus, the metal is compressible during elastic loading, but the change in volume is very small.

1.8 DEFINING STRENGTH

One should wonder now whether a tensile specimen can be stretched elastically even further than shown in Figure 1.6, Figure 1.7, and Figure 1.8. Intuitively, the answer of course is no; eventually, the test specimen will break. Figure 1.9 shows a test sample before and after testing. Notice that the failed tensile specimen is longer than at start. This is one hint that something other than elastic strain occurred, since, as described earlier, elastic strain is reversible, meaning the specimen length would have returned to its initial length. Thus, the strain experienced by the test specimen was permanent—not recoverable. From inspection of Figure 1.9, the sample elongates ~50% upon failure. Thus, the (permanent) strain experienced by this specimen is

$$\varepsilon_e = \frac{\Delta L}{L} = \frac{\approx L/2}{L} = 0.5,$$

which is much larger than the elastic strains plotted in Figure 1.7 and Figure 1.8. Also evident in the photo is that the test specimen formed a *neck* (a location where the cross-sectional area decreased locally) and eventually broke. Thus, when a force is initially applied, the test specimen elongates elastically,

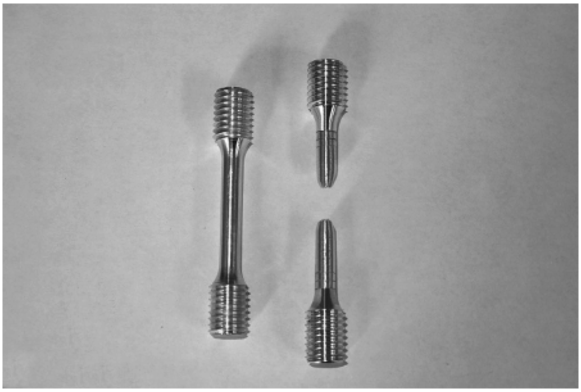


FIGURE 1.9 Tensile test specimen before and after testing showing in the tested specimen the necked region and location of final fracture. Courtesy of Westmoreland Mechanical Testing and Research, Inc.

but eventually, the specimen experiences permanent strain. The transition between these two processes is one definition of strength.

There was actually a hint regarding this definition of strength in Figure 1.8, when the lines were only plotted to a specific value of strain (or stress). The aluminum line was plotted only to a strain of only ~0.00065, whereas the tungsten line was plotted all the way to a strain of ~0.0025. The reason is that this maximum strain is the point on each curve where this transition occurs.

1.9 STRESS–STRAIN CURVE

To better understand the full stress–strain curve, it is useful to view an actual data set. First, since the example used is from a compression test rather than a tension test, it is necessary to describe how these tests differ. Figure 1.10 shows a schematic of a compression test. The test specimen is a solid cylinder with an aspect ratio (L/D) of ~1 to 1.5. It is sandwiched between two "platens" and usually a lubricant is coated on the mating surfaces. The crosshead in the testing machine is moved downward to compress the test specimen. An extension measurement system is used to measure ΔL.

In this case, ΔL is a negative quantity. Thus,

$$\varepsilon_{\mathrm{e}} = \frac{\Delta L}{L}$$

is also a negative quantity. Because the force is in an opposite direction than that in a tension test, the stress is also a negative quantity. In fact, the "stress state," which can be tension, compression, shear, or mixed mode, is an important consideration in strength analysis (see Box 1.1).

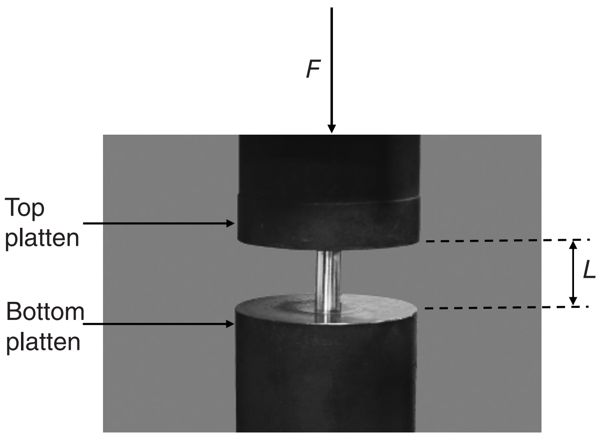
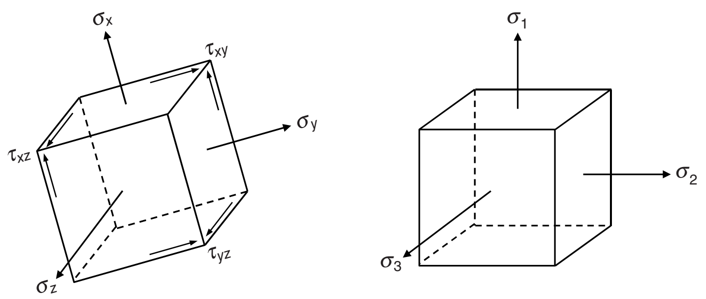


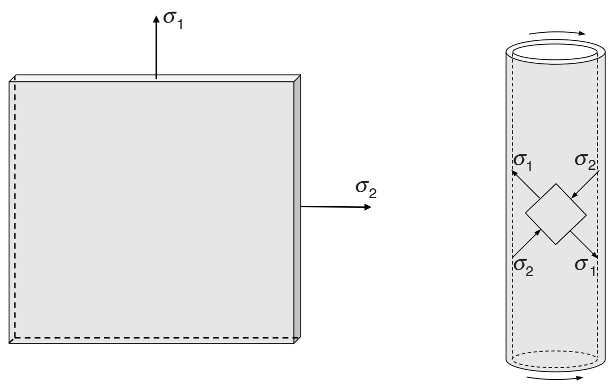
FIGURE 1.10 Schematic of a compression test showing a solid cylinder between two platens. Note the direction of force is opposite that in the tensile shown in Figure 1.5. Courtesy of Westmoreland Mechanical Testing and Research, Inc.

Box 1.1 Stress State

As illustrated in the figure on the left, stress is a tensor quantity comprised in a general x, y, z coordinate system of three normal stresses—σ_x, σ_y, and σ_z—and three shear stresses—τ_{xy}, τ_{yz}, and τ_{xz}.



The principal coordinate system (on the right) is one where the *principal stresses* σ_1, σ_2, and σ_3 are normal to the three planes and the shear stress components on these planes are zero. Uniaxial tension and compression tests represent the case where the stress axis is a principal stress axis and σ_2 and σ_3 equal zero. Of course, in a tensile test, σ_1 is a positive quantity, whereas in a compression test, it is a negative quantity. There are a myriad of stress states that are used in mechanical testing. The following figure shows the case on the left of a plane stress test on a thin sheet, where $\sigma_3 = 0$ and σ_1 and σ_2 can take on any (positive) values, and on the right, a torsion test on a tube, where $\sigma_1 = -\sigma_2 = \tau$ and $\sigma_3 = 0$.



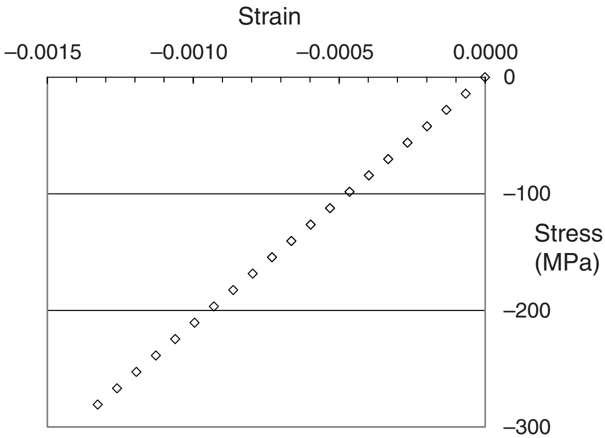


FIGURE 1.11 Stress versus strain in the elastic regime for a compression test in 1018 steel.

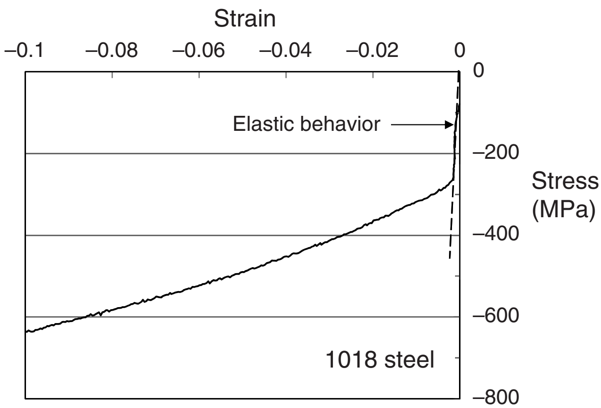


FIGURE 1.12 Compression test in 1018 steel showing stress versus strain up to a strain of −10%.

The elastic loading behavior for our steel compression specimen is as shown in Figure 1.11. Per convention, the stress and strain start at zero, but as the sample compresses, the stress and strain increase in a negative direction.

Figure 1.12 shows the measured stress–strain curve in a 1018 steel compression test specimen [1]. The solid line is the measurement of force and extension converted to stress and strain as discussed earlier. The dashed line is the elastic loading line plotted in Figure 1.11. Because the full-scale value of strain in Figure 1.12 is −0.10 whereas that in Figure 1.11 was −0.0015, the elastic line in Figure 1.12 seems to have a much higher slope. It actually is the same line.

How about the "bumps and wiggles" on the measured curve? More than likely in this case, these represent "noise," for example, electronic noise, in the measurement system, although there are examples where features such as these may actually reflect a metallurgical process.

The key point in Figure 1.12 is that the measured behavior follows the elastic line to a stress of ~−265 MPa, at which point a marked transition in behavior occurs and the strain begins to build rapidly while the stress increases slowly—opposite the trend during elastic loading. This point of transition is referred to as the "yield stress." It is the point at which the strain becomes nonrecoverable. As mentioned earlier, the yield stress is one definition of material strength.

The strain axis in Figure 1.12 was limited to a value of −0.10, but in fact, this test was carried out to a much higher strain, as shown in Figure 1.13. In this case, the elastic loading line is barely discernible from the ordinate. However, the blip at a strain of ~−0.20 is very interesting. For this test, the load was momentarily released. Figure 1.14 shows the same data but with the

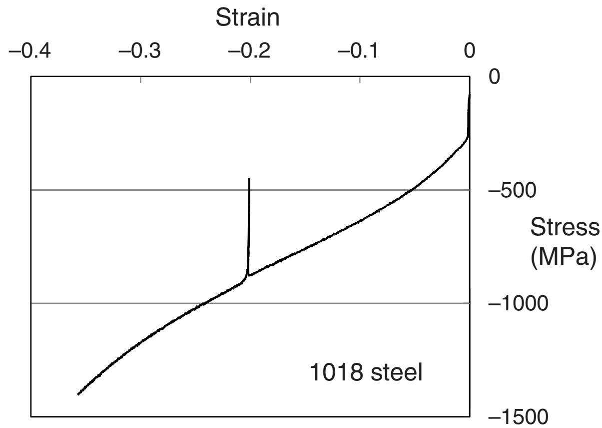


FIGURE 1.13 Compression test in 1018 steel showing stress versus strain for the entire test. Included is a partial unload and reload step.

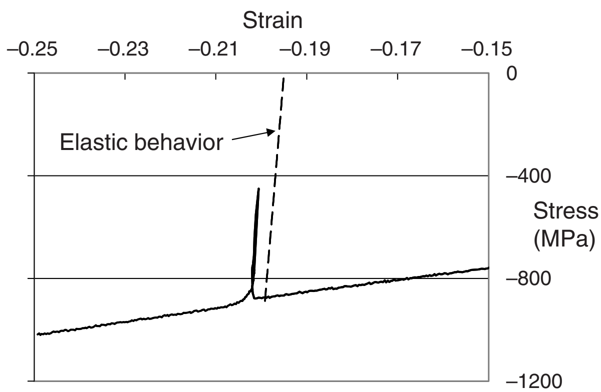


FIGURE 1.14 The unload and reload step in the test shown in Figure 1.13 plotted with a strain range that highlights the elastic behavior.

x-axis adjusted to focus on the unloading/loading behavior. In Figure 1.14, a line with the slope of the elastic loading line has been drawn to illustrate that the unloading/loading line follows the slope of the elastic line; that is, when the load was decreased, the strain decreased along the elastic line rather than along the solid line. And when the load was once again increased, the strain increased along the elastic line until the previous maximum stress was reached. At this point, the stress–strain curve followed the trend of the previous curve.

If the load had been decreased all the way to zero, then the strain would have relaxed elastically by

$$\varepsilon_e = \frac{\sim 875\ \text{MPa}}{212{,}000\ \text{MPa}} = 0.004,$$

which is the maximum (absolute value of the) stress reached before unloading divided by the elastic modulus. In the case of the compression test, the strain is negative; thus, when the specimen is unloaded, it becomes less negative. The permanent strain in the test specimen would have been

$$\varepsilon_{ep} = -0.202 + 0.004 = -0.198,$$

which is the maximum (negative) strain reached before unloading plus the elastic strain that is recovered. The compression test specimen would have been 19.8% shorter.

The strain past the yield stress has been referred to earlier as permanent and nonrecoverable strain. The more traditional term is "plastic strain" as opposed to "elastic strain" during the initial elastic portion of the loading cycle.

1.10 THE TRUE STRESS–TRUE STRAIN CONVERSION

Recall that all of the calculations of stress and strain have been based on the initial cross-sectional area of the specimen (for the stress calculation) and the initial length of the specimen (for the strain calculation). The specimen, however, is deforming and the cross-sectional area changes. In a tension test, the cross-sectional area decreases, whereas in a compression test, the area increases. The stress–strain curves based on original dimension—as plotted in Figure 1.13—are referred to as "engineering" stress–strain curves. When the stress during the test is based on the current (or actual) area and the strain is based on the current length, the resulting curve is referred to as a "true" stress–strain curve; that is, the incremental change in true strain, ε, is defined as

$$d\varepsilon = \frac{dl}{l}, \tag{1.10}$$

and the *true strain*, ε, in a test where the specimen's length begins as L_o and ends as L_f is

$$\varepsilon = \int_{L_o}^{L_f} \frac{dl}{l} = \ln L_f - \ln L_o = \ln\left(\frac{L_f}{L_o}\right) = \ln(1+\varepsilon_e). \tag{1.11}$$

The true strain is the logarithm of one plus the engineering strain. The *true stress*, σ, is defined as

$$\sigma = \frac{F}{A},$$

where A is now taken as the current area:

$$\sigma = \frac{F}{A}\frac{A_o}{A_o} = \frac{F}{A_o}\frac{A_o}{A} = \sigma_e \frac{A_o}{A}.$$

One distinction between elastic deformation and plastic deformation is that whereas (as discussed in Section 1.7) volume is not conserved in the former, it is conserved in the latter. Thus, plastic deformation is *incompressible*, and

$$\sigma = \sigma_e \frac{A_o}{A} = \sigma_e \frac{L}{L_o} = \sigma_e(1+\varepsilon_e). \tag{1.12}$$

The true stress is simply the engineering stress multiplied by one plus the engineering strain. Here the distinction between a tension test where ε_e is positive and a compression test where ε_e is negative becomes critically important.

For the compression test shown in Figure 1.13, the conversion from an engineering stress–strain curve to a true stress–strain curve yields the curve shown in Figure 1.15. Compare this curve with that in Figure 1.13. The absolute

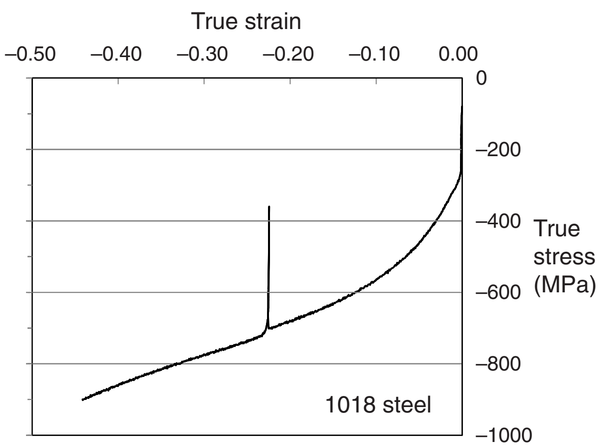


FIGURE 1.15 The stress–strain curve in Figure 1.13 (plotted as engineering stress versus engineering strain) converted to true stress versus true strain.

value of the maximum stress is lower in the true stress–strain curve because the diameter of the compression specimen increases during testing and the force divided by the current area gives a smaller stress. The peak strain has increased from ~−0.35 on engineering coordinates to ~−0.43 on true stress–strain coordinates, which follows from $\varepsilon = \ln(1 - 0.35)$.

In this monograph, only true stress–strain properties will be considered. However, engineers need to check to verify whether it is engineering or true stress–strain curves that are being reported and to be explicit themselves when reporting test results.

1.11 EXAMPLE TENSION TESTS

Figure 1.15 showed a real compression test (in 1018 steel) on true stress–true strain axes. Figure 1.16 shows such a tensile test in a 304L stainless steel published by Krempl and Kallianpur [2]. This test specimen was only stretched to a strain of ~0.0038, which is barely past the elastic loading region, then unloaded. Elastic loading and unloading curves (with $E = 195.2$ GPa) are included in the figure. Note that the measured curves do not follow exactly the elastic lines, although they do approximate them. This could be due to errors associated with measuring very small extensions or it may be a real material effect, for example, viscoelasticity (nonlinear elasticity), commonly observed in polymeric materials.

If deviation from elastic loading is interpreted as a measure of strength, the curve in Figure 1.16 illustrates the difficulty in precisely defining this point. One definition is called the "proportional limit," which is the point where the stress departs the elastic loading line. Although, as mentioned earlier, there is

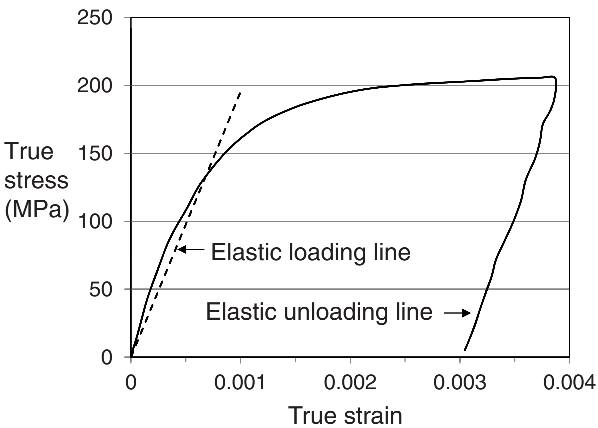


FIGURE 1.16 Stress versus strain curve in AISI 304L stainless steel measured by Krempl and Kallianpur [2] using very precise deflection measurements.

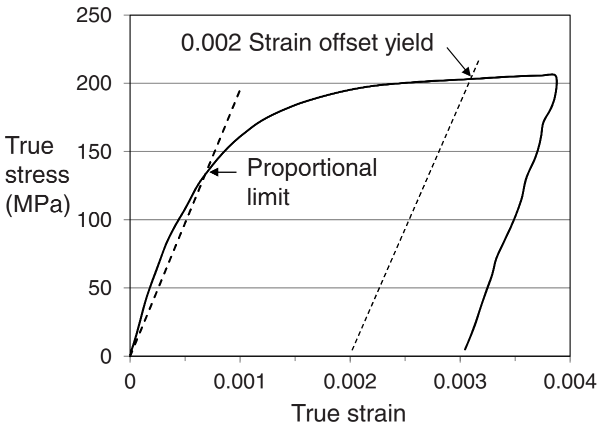


FIGURE 1.17 The curve of Figure 1.16 shown with the 0.002 strain offset and a possible proportional limit to illustrate practicing definitions of the yield stress.

ideally no elastic behavior in the loading curve of Figure 1.16, Figure 1.17 shows where one might identify the proportional limit.

Engineers have adopted a convention for the yield stress as the stress at a strain of 0.002, referred to as the "0.002 offset." This also is shown in Figure 1.17. While in this figure the 0.002 offset strain looks to be far from the elastic line, in fact, this test specimen was tested to a very low strain. The 0.002 offset strain would be almost coincident with the elastic loading line in Figure 1.13. Notice that the yield stress defined by the proportional limit would be ~140 MPa, whereas that defined by the 0.002 offset strain would be ~205 MPa, which is a big difference. When quoting yield stress, engineers need to verify how this is defined and measured.

Box 1.1 emphasized that stress is a tensor quantity. While the uniaxial tension and compression tests are often used to characterize strength, a scalar representation of stress that is independent of stress state has long been sought. Box 1.2 introduces some commonly used scalar stress and strain measures.

Box 1.2 Scalar Stress and Strain Representations

Figure 1.15 showed a compression stress–strain curve in 1018 steel with a yield stress of ~−265 MPa and Figure 1.17 showed a tensile stress–strain curve in AISI 304LSS with a (0.002 offset) yield stress of ~205 MPa. The question arises what would be the yield stresses in these materials for a different stress state; that is, what would be the yield stress of the 1018 steel in tension or torsion? There are two ways to consider this. The second will be addressed in Box 3.4 in Chapter 3.

The common model for the stress-state dependence of the yield stress is based on the von Mises criterion. In the principal axis coordinate system, J_2—the second invariant of the stress deviator—is

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right],$$

where J_2 is a scalar combination of the principal stresses. The von Mises yield criterion states that yield occurs when J_2 reaches the critical value of k^2:

$$J_2 = k^2.$$

If σ_o is the yield stress in a tension test, where $\sigma_1 = \sigma_o$ and $\sigma_2 = \sigma_3 = 0$, then

$$k = \frac{\sigma_o}{\sqrt{3}},$$

which defines the value of k. Note that due to the squared terms, the yield stress in compression has the same absolute value as that in tension. In a torsion test where $\sigma_1 = -\sigma_2 = \tau$ and $\sigma_3 = 0$, then

$$J_2 = k^2 = \frac{1}{6}\left[(\sigma_1 + \sigma_1)^2 + \sigma_1^2 + \sigma_1^2\right] = \sigma_1^2$$

$$\sigma_1 = \tau = k = \frac{\sigma_o}{\sqrt{3}}; \quad \tau = \frac{\sigma_o}{\sqrt{3}}.$$

Note that the yield stress in a torsion test is predicted to be 0.577, the yield stress in uniaxial tension. The yield stress in tension σ_o is also referred to as the von Mises stress, σ_v. Rearranging the above-mentioned equation defining J_2 gives (for principal stress coordinate system)

$$\sigma_v = \left[\frac{1}{2}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}\right]^{1/2}.$$

The von Mises strain (in principal axes) is defined as

$$\varepsilon_v = \frac{2}{3}\left[\frac{1}{2}\left\{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right\}\right]^{1/2}.$$

Recall in a tension test (with the tensile axis along the principal direction "1") that

$$\varepsilon_2 = \varepsilon_3 = -\frac{\varepsilon_1}{2}.$$

In this case, it can be shown that $\varepsilon_v = \varepsilon_1$; the von Mises strain is equal to the tensile strain.

For pure shear, $\varepsilon_1 = -\varepsilon_3 = \gamma/2$ and $\varepsilon_2 = 0$, where γ is the shear strain. It can be shown that

$$\text{pure shear: } \varepsilon_v = \frac{2}{\sqrt{3}}\varepsilon_1 = \frac{\gamma}{\sqrt{3}}.$$

The von Mises stress is one scalar stress definition. Another is the *octahedral shear stress*, τ_{oct}, defined as follows. Note the relation between this scalar and the von Mises stress:

$$\tau_{oct} = \sqrt{\frac{2}{3}J_2} = \sqrt{\frac{2}{3}\frac{1}{3}\sigma_O^2} = \frac{\sqrt{2}}{3}\sigma_O = \frac{\sqrt{2}}{3}\sigma_v.$$

The octahedral shear strain, γ_{oct}, is defined as

$$\gamma_{oct} = \frac{2}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}.$$

In a uniaxial tensile test, γ_{oct} relates to the tensile strain ε_1 through

$$\gamma_{oct} = \frac{2}{3}\left[\left(\varepsilon_1 + \frac{\varepsilon_1}{2}\right)^2 + \left(-\frac{\varepsilon_1}{2} - \varepsilon_1\right)^2\right]^{1/2} = \frac{2}{3}\left[\left(\frac{3\varepsilon_1}{2}\right)^2 + \left(-\frac{3\varepsilon_1}{2}\right)^2\right]^{1/2} = \frac{2}{3}\left[\frac{9}{2}\varepsilon_1^2\right]^{1/2} = \sqrt{2}\varepsilon_1.$$

Still another common scalar measure is the effective stress, $\bar{\sigma}$:

$$\bar{\sigma} = \frac{\sqrt{2}}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2},$$

and the effective strain, $\bar{\varepsilon}$:

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}.$$

Both the effective stress and the effective strain reduce to the uniaxial stress and strain, which is why the von Mises stress is often referred to as the von Mises effective stress and, similarly, the von Mises strain is often referred to as the von Mises effective strain.

While the von Mises yield criterion explicitly refers to yield, stress–strain curves often plot the entire curve on von Mises stress and strain axes or octahedral shear stress and strain axes.

1.12 ACCOUNTING FOR STRAIN MEASUREMENT ERRORS

The measurement of strain in a tension or compression specimen can be challenging. A deflection measurement device such as that shown in Figure 1.4 is costly and can be difficult to install—particularly when experiments at cryogenic temperatures or elevated temperatures are desired. One common practice is to use the linear variable displacement transducer (LVDT) attached to the machine axis to record displacement in the test specimen, but as mentioned earlier, this measurement could represent displacements beyond those in the test specimen. Recent development of noncontact strain measurement devices based on laser interferometry offers an attractive alternative [3]. Even these new techniques, though, can introduce measurement artifacts. Thus, when viewing stress–strain measurements in the published literature, one must be cognizant of potential effects of errors in the strain measurement system.

An example of a published measurement that raises questions is shown in Figure 1.18. The curves in this figure are compression stress–strain curves measured by Qu et al. on samples machined from a block of 304L SS that had been equal-channel angular pressing (ECAP) processed [4]. In this case, the material is extruded at 700°C through 90° dies such that the material is severely strained but without change in shape. (Chapter 12 includes a more detailed discussion of this processing method.) Whereas the (absolute value of the) yield strength in annealed 304L SS is ~200 MPa, note that the yield strength in the equal-channel angular extrusion (ECAE)-processed material is on the order of 500 MPa. An elastic loading line for a sample subjected to a single pressing is drawn as a dotted line in Figure 1.18. The slope of this line, the

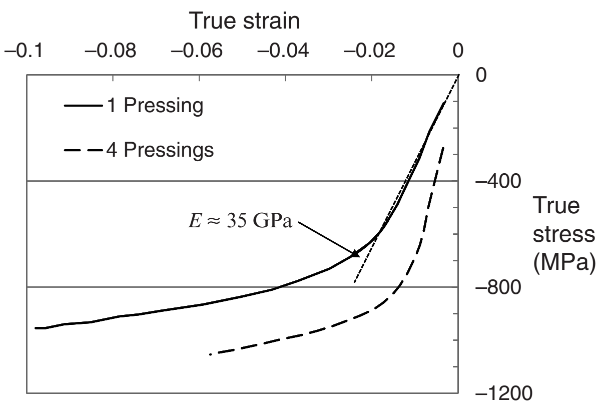


FIGURE 1.18 Compression stress versus strain curves measured by Qu et al. [4] on 304L SS that had been equal-channel angular extrusion processed 1 and 4 passes. The slopes of the elastic loading lines are unusually high.

apparent elastic modulus, is ~35 MPa. The typical elastic modulus of stainless steels, however, is ~192 MPa. Since strain alone—even high levels of strain produced by ECAP processing—does not alter the interatomic forces, the apparent modulus must either be an artifact of the strain measurement or reflect unusual loading behavior.

Although in this case Qu et al. are confident that the strain measurements reported by them are accurate and that the low initial slope is real* (perhaps indicating microplasticity in a few grains), this measurement serves as an example of the effect of a measurement system that indicates displacements beyond actual elastic displacements in the deforming specimen.

To illustrate the issue, Figure 1.19 is a schematic two-spring system, which is a model for the elastic displacements of a mechanical test specimen attached to a load frame. In this model, the test specimen has a spring constant (equivalent to an elastic modulus) of E_1, whereas the load frame has a spring constant of E_2, which is assumed to be a constant. A force F is applied to the load axis and the total displacement, ΔY, is measured with an LVDT. In this case, the total displacement is

$$\Delta Y = \frac{F}{E_1} + \frac{F}{E_2} \tag{1.13}$$

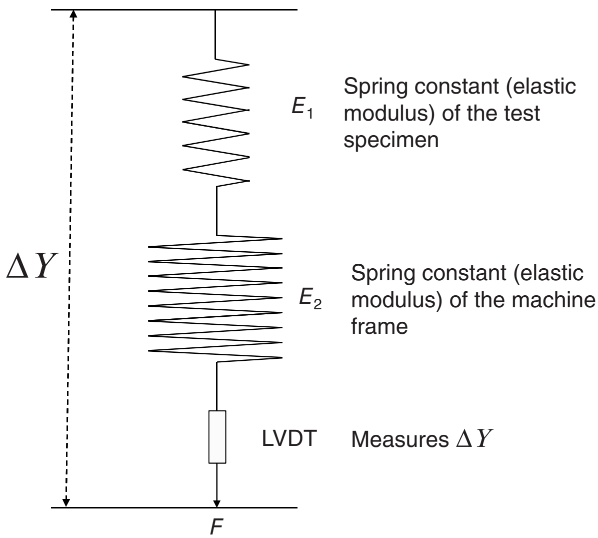


FIGURE 1.19 Schematic spring analogy illustrating how the stiffness (elastic modulus) of the machine can add to displacements measuring using an LVDT.

* This was communicated in a private conversation with a member of the Qu et al. research group.

and the displacement within the test specimen is

$$\Delta Y_1 = \Delta Y - \frac{F}{E_2}. \tag{1.14}$$

If the displacements within the machine frame are not subtracted from the total displacements, the (absolute value of the) presumed displacements within the test specimen will be too large. Once the test specimen begins to plastically deform, the elastic displacements within the machine frame will continue to add to the measured displacements. Inaccurate measurement of displacements within the test specimen offers one source of error in the stress–strain curve.

This error compounds in the true stress calculation using Equation (1.12) since the conversion from engineering to true stress involves multiplication of the engineering stress by $(1 + e)$, which relies on an accurate measurement of the engineering strain using Equation (1.4).

Fortunately, a simple correction is possible under certain idealized conditions. When, for instance, the machine modulus is constant and the errors to the measurement of strain solely arise from displacements in the machine frame, the fictitious elastic displacements (engineering strains) can be subtracted from the measured displacements.*

In this case, the actual elastic strains can be computed using the known specimen modulus:

$$\varepsilon_{\mathrm{e_{act}}} = \varepsilon_{\mathrm{e_{app}}} - \frac{\sigma_{\mathrm{e}}}{E_{\mathrm{app}}} + \frac{\sigma_{\mathrm{e}}}{E_{\mathrm{act}}}, \tag{1.15}$$

where the subscript "app" refers to "apparent" and "act" refers to "actual." Note that the correction is performed on the engineering values of stress and strain and that the signage is very important.

Figure 1.20 shows the stress–strain curves from Figure 1.18 corrected using Equation (1.15). In this case, the actual measured apparent moduli for the 1-pass and 4-pass curves are used. As indicated in Figure 1.18, this was 35 GPa for the 1-pass curve and 74 GPa for the 4-pass curve. The actual modulus was assumed to equal 192 GPa for both cases. A comparison between the raw data in Figure 1.18 and the corrected curves in Figure 1.20 shows that it is mostly the strain values that have changed. The stresses show less of a change since the strains in these tests were fairly low (compared, for instance, with those in Figure 1.13 and Figure 1.15) It is worth reemphasizing that Qu et al. believe that the strains are accurate, in which case the stress–strain curves in Figure 1.18 are accurate and there is no reason to apply the above-mentioned correction. It is done only

* The fact that the slopes of the elastic portions of the 1-pressing and 4-pressing curves are not identical suggests that the high slope does not simply arise from a machine compliance effect. Assuming a constant machine modulus, thus, is an approximation.

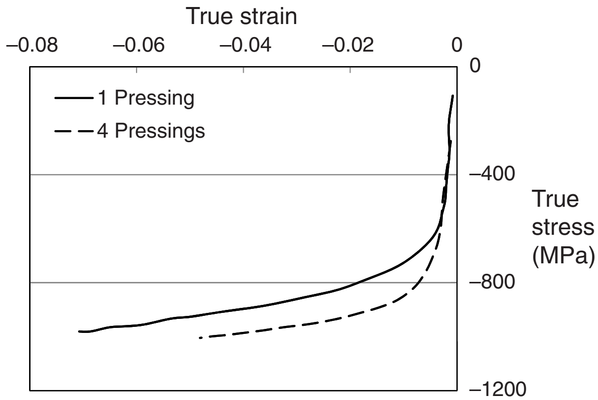


FIGURE 1.20 True stress versus true strain curves from Figure 1.18 with apparent elastic strains not associated with those in the test specimen removed using Equation (1.15).

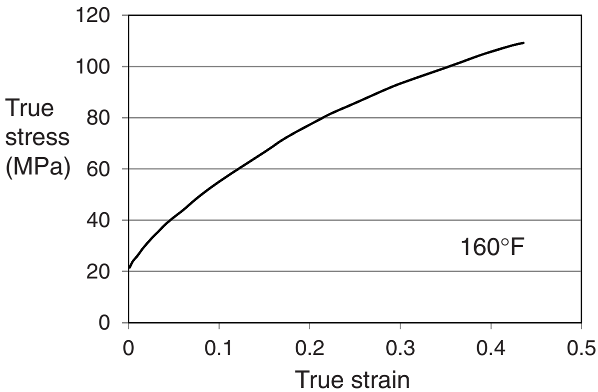


FIGURE 1.21 Stress versus strain (tension) measured in 304 SS by Antoun [5] at 160°F (71°C).

as an illustration. For tests carried to large strains, the errors introduced by inaccurate strain measurements can be significant.

1.13 FORMATION OF A NECK IN A TENSILE SPECIMEN

Figure 1.21 shows another tensile test measurement. This one was reported by Antoun [5] and is also in a 304 stainless steel—but at 160°F instead of room temperature. This stress–strain curve was taken all the way to a true strain of

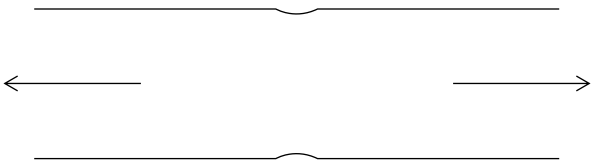


FIGURE 1.22 Cross section of a tensile test specimen with an incipient neck region.

~0.44. The unloading portion of the curve (as load is reduced to zero) is not shown. It is likely that this test specimen formed a neck and broke (as in Figure 1.9).

To examine formation of a neck during a tensile test, Figure 1.22 shows schematically a cross section of a specimen with an "incipient" neck. This could have initiated from a machining defect or could simply be a region that for some reason strained slightly nonuniformly. The area within the necked region is less than the area in the bulk of the test specimen. Thus, the stress will increase locally, which will lead to an even more localized strain and will accentuate the growth of the neck. This will be countered by the fact that the local region that strains nonuniformly will be a little stronger due to the increase in the stress–strain curve.

When the increased stress along the stress—strain curve is no longer sufficient to counter the increased stress due to the area reduction in the neck, the neck will grow. This is a classic instability process. Recall the force F, defined as

$$F = \sigma A.$$

Differentiating the force with respect to the strain gives

$$\frac{dF}{d\varepsilon} = \sigma \frac{dA}{d\varepsilon} + A \frac{d\sigma}{d\varepsilon}.$$

At the point of instability,

$$\frac{dF}{d\varepsilon} = 0.$$

From the constant volume condition,

$$\begin{aligned}
&V = AL \\
&\frac{dV}{d\varepsilon} = 0 = A\frac{dL}{d\varepsilon} + L\frac{dA}{d\varepsilon} \\
&\frac{dA}{d\varepsilon} = -\frac{A}{L}\frac{dL}{d\varepsilon} = -A\frac{\frac{dL}{L}}{d\varepsilon} = -A\frac{d\varepsilon}{d\varepsilon} = -A
\end{aligned}$$

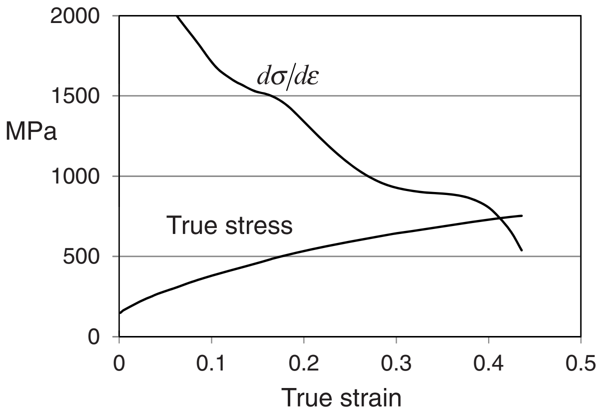


FIGURE 1.23 Strain hardening rate (slope of the stress–strain curve) versus true stress for the tensile test shown in Figure 1.21. The point of intersection of these curves is the point of tensile instability.

$$\frac{dF}{d\varepsilon} = 0 = \sigma\frac{dA}{d\varepsilon} + A\frac{d\sigma}{d\varepsilon} = -\sigma A + A\frac{d\sigma}{d\varepsilon}.$$

Therefore, at the point of instability,

$$\sigma = \frac{d\sigma}{d\varepsilon}. \tag{1.16}$$

When the true stress equals the slope of the true stress versus true strain curve, the specimen is prone to undergoing unstable deformation via formation of a neck. The instability condition represented by Equation (1.16) is referred to as the Considère criterion in recognition of the 1885 experimental observations in iron and steel by this French researcher. Figure 1.23 shows the stress–strain data of Figure 1.21 plotted along with $d\sigma/d\varepsilon$. Recall that since ε is dimensionless, $d\sigma/d\varepsilon$ has the units of stress.

The two curves in Figure 1.23 coincide at a strain of ~0.41. Thus, per the instability condition, a neck likely begins to form at this point and the tensile specimen will soon break. In addition, data are not reported past this point since the stress and strain are no longer uniform in the test specimen. This prediction is consistent with the measurement shown in Figure 1.21.

1.14 STRAIN RATE

In the test results highlighted in Figure 1.13, Figure 1.16, Figure 1.18, and Figure 1.21, the velocity of the crosshead has not been specified. This actually is an important variable. Typically, the yield stress in a test where the cross-head

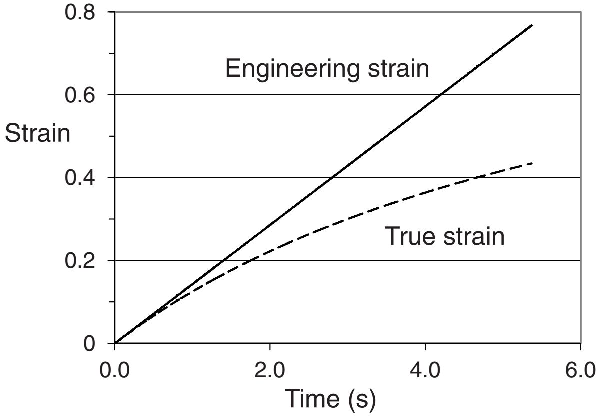


FIGURE 1.24 Engineering strain and true strain versus time for a tensile test in alloy C-22 [6].

velocity is high exceeds that in a test where the crosshead is low. The difference can be small, but it is generally measureable.

Figure 1.24 shows a plot of both the engineering strain and the true strain versus time for a tensile test in the nickel-base alloy C-22 [6]. Note that engineering strain varies very close to linearly with time. Recall that

$$\varepsilon_e = \frac{\Delta L}{L},$$

where L is the original length of the test specimen. Dividing ε_e by time gives

$$\frac{\varepsilon_e}{t} = \frac{\Delta L}{tL} = \frac{1}{L}\frac{\Delta L}{t} = \frac{1}{L}V_{CH}, \tag{1.17}$$

where V_{CH} is the cross-head velocity.

A constant value of ε_e/t (slope of the engineering strain versus time line in Figure 1.24) implies that a constant cross-head velocity has been imposed, which appears to be the case for the measurement in Figure 1.24. The quantity ε_e/t is termed the "strain rate." The unit of strain rate is s^{-1} since strain is dimensionless. Usually, it is the true strain rate rather than the engineering strain rate that is specified.* However, it is easier to operate a test machine with a constant cross-head velocity than to control the motion to yield a constant true strain rate. In the test illustrated in Figure 1.24, the true strain rate is not constant. Figure 1.25 shows how the true strain rate varies with time in this test.

* Test machines with feedback control of the cross-head velocity from a displacement measurement system can operate at a constant true strain rate.

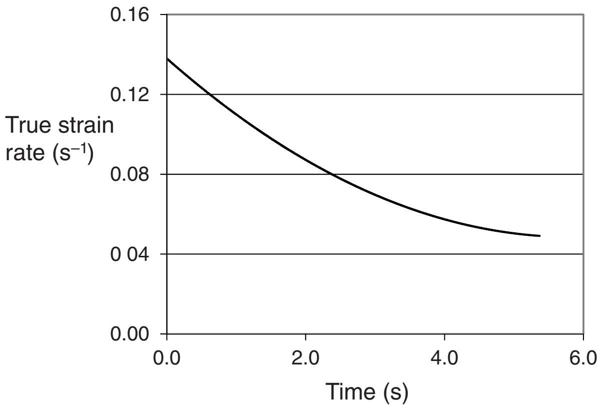


FIGURE 1.25 True strain rate versus time for the alloy C-22 tensile test (slope of the true strain versus time curve in Figure 1.24).

The true strain rate for this test started at a value nearly equal 0.14 s^{-1} at yield but decreased throughout the test. The average true strain rate was approximately equal to 0.08 s^{-1}.

1.15 MEASURING STRENGTH: SUMMARY

The mechanical test is used to measure the strength of materials. This chapter has reviewed the tensile test; introduced definitions of stress, strain, and strain rate; discussed elastic and plastic deformation; and summarized concepts such as compressibility, instability, and errors introduced by imprecise displacement measurement. Several examples of stress–strain curves—in tension as well as compression—have been presented. With this introduction, the contributions to the strength measured in a mechanical test can be presented.

EXERCISES

1.1 A copper tensile test specimen is machined with an initial diameter of 5.00 mm and an initial gage length of 40.0 mm. A load of 400 N is applied.

(a) At this load, what is the applied stress?

(b) If the specimen only deforms elastically, calculate the deflection that would be measured.

1.2 Measurements of load versus deflection in a metallic tensile test specimen with an initial diameter of 6.00 mm and an initial gage length of 50.0 mm are given in the following table.

(a) Does this specimen appear to be deforming elastically?

(b) From the elastic constants in Table E1.2, determine the metal.

TABLE E1.2

ΔL (μm)	Force (N)
5.4	572
16.2	1662
26.9	2807
37.6	3921
48.4	5089
59.2	6189
69.9	7326
80.7	8457

1.3 An Al_2O_3 solid cylinder with an initial length of 1.50 cm and an initial diameter of 1.00 cm is loaded in compression. An optical extensometer is used that continuously measures the diameter of the test specimen. Table E1.3 lists the measured force versus diameter. Assuming that Poisson's ratio for this material is 0.25, compare the elastic modulus with the value listed in Table 1.2. (Note that the forces are listed as negative to reflect the compressive loading.)

TABLE E1.3

ΔD (μm)	Force (N)
0.36	−3,960
1.81	−19,600
3.26	−35,300
4.71	−51,000
6.16	−66,800
7.61	−82,400
9.06	−98,200
10.51	−113,800

1.4 Compare the forces required to perform the tensile test in Exercise 1.2 with the forces required to perform the compression test in Exercise 1.3. Instron® and MTS® are two companies that supply mechanical testing machines. Go to either the Instron or MTS website and research which testing machine would be required to perform these mechanical tests.

1.5 A copper tensile test specimen is machined with an initial diameter of 5.00 mm and an initial gage length of 40.0 mm. The measured load (N) versus displacement (μm) data is listed in Table E1.5.

(a) Calculate and plot stress versus strain for this test.

(b) Has the yield stress been exceeded in this test?

TABLE E1.5

ΔL (μm)	Force (N)
4.1	269
7.9	532
12	813
16	1068
19.2	1231
24	1560
28.1	1817
31.9	1973
36.1	2108
40.4	2195
48.1	2407
56.1	2583
64.1	2740
72.2	2903
80	3104
88.3	3182
95.9	3368
112.2	3659
128.2	3845
144.3	4141
160.3	4289
176.5	4507

1.6 For the test described in Exercise 1.5,

(a) estimate the proportional limit and

(b) estimate the 0.002 offset yield stress.

1.7 The test described in Exercise 1.5 was actually strained to a higher total elongation. Table E1.7 lists the measured elongation versus force data. Add these data to the data listed in Exercise 1.5.

(a) Plot the engineering stress–strain curve.

(b) Calculate and plot the true stress–strain curve. (Remember that the conversion from engineering strain and stress to true strain and stress needs to be performed only for points beyond the yield stress.)

TABLE E1.7

ΔL (μm)	Force (N)	ΔL (μm)	Force (N)
192.5	4,694	1,103.0	11,074
208.5	4,880	1,168.7	11,311
224.5	5,142	1,234.6	11,655
240.7	5,303	1,300.7	11,861
256.9	5,462	1,366.9	12,204
272.9	5,608	1,433.1	12,466
289.0	5,793	1,557.5	12,968
321.2	6,122	1,682.5	13,432
353.5	6,416	1,807.7	13,830
385.9	6,619	1,933.3	14,208
418.3	6,937	2,059.2	14,599
450.5	7,164	2,185.6	15,017
483.0	7,392	2,312.4	15,433
515.3	7,651	2,439.4	15,730
547.6	7,937	2,567.1	16,081
580.2	8,102	2,694.9	16,474
612.7	8,373	2,823.2	16,732
645.0	8,574	2,951.8	17,095
677.6	8,755	3,080.8	17,408
710.2	8,988	3,210.2	17,724
742.8	9,138	3,340.1	17,912
775.5	9,296	3,470.3	18,274
808.0	9,489	3,600.9	18,531
840.7	9,742	3,732.0	18,727
906.0	10,033	3,863.3	19,078
971.5	10,416	3,995.1	19,283
1037.2	10,741	4,127.4	19,494

1.8 The highest elongation and force recorded in the test described in Exercise 1.7 were 4127.4 mm and 19,494 N, respectively. If at this point the load were reduced to zero, what would be the final length of the tensile specimen?

1.9 Could the tensile test described in Exercise 1.7 deform to the maximum elongation listed without forming a neck?

1.10 Table E1.10 lists displacement versus force for a tantalum cylinder deformed in compression. (Note that the displacement and force values are negative to reflect the compression loading conditions.) The original sample diameter was 8.00 mm and the original sample length was 12.00 mm.

(a) Estimate the proportional limit.

(b) Estimate the (–) 0.002 offset strain yield stress.

TABLE E1.10

ΔL (μm)	Force (N)	ΔL (μm)	Force (N)
−2.4	−1,886	−185.7	−23,958
−4.9	−3,725	−209.3	−24,651
−7.2	−5,571	−233.0	−25,244
−9.6	−7,523	−256.4	−25,792
−11.9	−9,392	−279.9	−26,247
−14.4	−11,233	−315.0	−27,013
−16.9	−13,040	−350.0	−27,685
−19.3	−14,626	−385.0	−28,310
−21.6	−15,283	−419.8	−28,889
−24.0	−15,654	−454.4	−29,441
−28.8	−16,323	−523.5	−30,429
−33.6	−16,760	−592.1	−31,414
−38.3	−17,215	−660.3	−32,375
−43.2	−17,695	−728.2	−33,187
−52.6	−18,395	−840.3	−34,510
−62.2	−19,108	−951.3	−35,781
−71.8	−19,660	−1,061.2	−37,051
−81.3	−20,097	−1,170.2	−38,105
−90.8	−20,598	−1,277.9	−39,256
−109.8	−21,408	−1,384.6	−40,335
−128.9	−22,185	−1,490.3	−41,339
−147.8	−22,885	−1,594.7	−42,412
−166.8	−23,489	−1,698.2	−43,444

1.11 Plot the engineering stress–strain curve and the true stress–strain curve for the entire data set listed in Exercise 1.10.

1.12 If this material were tested in tension rather than in compression, would the tension specimen be able to elongate to the maximum strain listed without undergoing a tensile instability (i.e., form a neck)?

1.13 Table E1.13 lists the measured displacement versus applied force for a tungsten compression test. The original specimen diameter was 8.00 mm and the original specimen length was 12.0 mm.

(a) Plot engineering stress versus engineering strain and determine the apparent elastic modulus.

(b) How does the apparent modulus compare with the value listed in Table 1.2 for tungsten?

(c) Can you suggest a rationale for the difference?

TABLE E1.13

ΔL (μm)	Force (N)	ΔL (μm)	Force (N)
–23.8	–3,916	–528.9	–75,442
–47.7	–8,170	–575.4	–76,089
–71.6	–12,216	–624.1	–76,656
–95.4	–15,890	–672.1	–77,415
–119.9	–20,134	–719.5	–77,703
–145.0	–23,989	–767.3	–78,266
–167.5	–28,082	–815.3	–78,835
–192.2	–32,012	–864.5	–79,343
–216.8	–36,201	–912.4	–79,574
–239.3	–40,076	–959.6	–80,102
–264.7	–44,121	–1,007.5	–80,259
–287.5	–48,382	–1,055.1	–80,934
–312.9	–52,415	–1,104.8	–80,983
–335.4	–56,339	–1,152.7	–81,591
–360.1	–60,182	–1,200.1	–81,797
–384.7	–64,281	–1,248.9	–82,345
–407.8	–68,243	–1,295.7	–82,626
–432.2	–71,999	–1,343.1	–82,794
–480.5	–73,694	–1,391.6	–83,012

1.14 Assuming that the true elastic modulus is 411 MPa, perform a correction to remove the artificial elastic strain. Compare the corrected and uncorrected engineering stress–strain curves.

1.15 Table E1.15 lists the measured displacement versus time for a tensile test on a specimen with an initial length of 25.0 mm.

(a) Plot the engineering strain versus time and the true strain versus time.

(b) Was this test run at a uniform engineering strain rate or a uniform true strain rate?

(c) What is this strain rate?

TABLE E1.15

Time (s)	ΔL (μm)	Time (s)	ΔL (μm)
2.93	0.070	104.79	2.758
8.83	0.216	111.17	2.942
14.99	0.372	116.71	3.098
20.96	0.527	123.12	3.281
27.2	0.674	129.07	3.438
32.79	0.846	134.75	3.610
39.09	0.987	141.21	3.785
45.1	1.161	146.88	3.961
50.87	1.298	153.27	4.129
57.19	1.457	159.04	4.298
62.85	1.619	165.08	4.495
68.88	1.786	171.22	4.662
74.93	1.943	177.03	4.844
80.88	2.105	182.97	5.026
86.8	2.265	189.16	5.195
92.76	2.444	195.1	5.378
98.71	2.604	201.08	5.557

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