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INTRODUCTION

The history of the fundamental general techniques that are applied in electromagnetic (EM)-solvers today is interesting. Many techniques were devised in the decades between 1960 and 1980. The first work in the EM field for the finite element (FE) technique was presented in 1965 [1]. A paper on the foundation of the numerical differential equation (DE)-based finite difference time domain (FDTD) method [2] was published in 1966.

Interestingly, a second set of techniques used in commercial solvers today had been devised in the decade from 1970 to 1980. The circuit-oriented DE-based transmission line matrix (TLM) method originated in 1971 [3]. The circuit-oriented partial element equivalent circuit (PEEC) method, which is based on an integral equation (IE) formulation, originated in 1972 [4]. Finally, the DE-based finite integration (FIT) approach was devised in 1977 [5]. Since then, many different submethods have been developed based on all the cited techniques.

We note that most of the numerical solutions are solved using the weighted residuals method (WRM) [6]. The notation is not always the same since sometimes the notation MWR is used. Importantly, the WRM applies to the majority of fundamental solution techniques that include FE, DE, or IE based, for example, Refs [6, 7].

In 1968, a key paper on the numerical implementation of integral equation-based approach was presented in Ref. [8] and it was called the method of moments (MoM). The name is used in an inconsistent way since the MoM is a subset of the WRM methods that applies to a subclass of all formulations [9]. The MoM name originated in 1932 [10] as one of the WRM methods where the two approximation functions are not the same unlike for the Galerkin method. Further, there is no relation between MoM and the moment matching method used for macromodeling [11, 12]. To avoid confusions, we do not use MoM as a name for impedance-type IE solution as is done by some researchers.

In this book, we consistently use the WRM notation. We also use it for the PEEC method since the finite, circuit-based solution of the PEEC method can be viewed as being solved using a WRM technique.

Several book chapters have been written on PEEC techniques. An introduction to the capacitance circuit elements and partial inductances is given in Ref. [13] and a book dedicated to partial inductances is [14], while sections of other books are dedicated to PEEC methods in Refs [15–17], and [18].

Making the teaching of PEEC for electromagnetic solution techniques easier and more concrete is one of the aims of the authors. Many entry-level students are discouraged with the complexity of the EM subject. Quite a number of excellent textbooks on the fundamentals of electromagnetic exist today, such as Balanis [19], Chew [20], Collin [21], Kong [22], Paul [23], Plonsey and Collin [24] and Ramo et al. [25]. Hence, the underlying physics and the derivation of Maxwell's equations are well covered in these books. Most of them do an excellent job in presenting the subject in a comprehensive way.

The aim of this book is different. The fact that the PEEC approach is circuits based should help many students and readers understand the electromagnetic concepts. Most electrical engineering students will learn basic circuit analysis before taking a course in electromagnetics. Our aim is to present the concepts in an application-oriented way. Hence, the understanding of the concepts presented in this book should be of practical as well as of theoretical interest.

The PEEC method has evolved from the initial work, for example, Ref. [26] to a multitude of works by many researchers. The technique has expanded from its original focus in the interconnect modeling area to a wider range of applications. The modeling of combined electromagnetic and circuit (EM/Ckt) problems is one of the strengths of this technique. For this reason, we also included an introduction to the necessary circuit concepts in this book.

Many new systems involve circuit aspects as well as EM parts. Therefore, it is very desirable to be able to solve combined EM/Ckt problems. Circuit-based EM approaches are usually differential equation based. This includes the transmission line method (TLM) method. Other equivalent circuit methods are derived from DEs. The IE-based PEEC method is treated in this book.

In this text, we are interested in all types of electronic systems. Unlike some other approaches, PEEC also provides a stable *dc* solution, which is important for many realistic EM/Ckt problems. Today, many applications for low-frequency problems are in the power engineering area. Quasistatic PEEC solutions lead to conventional SPICE-type circuits [27]. For higher frequencies above the quasistatic frequency range, the approach results in full-wave solutions. Unfortunately, conventional SPICE solvers cannot be employed for full-wave solutions since the resultant PEEC circuits include delays. This leads to circuit solvers with delays.

Full-wave solutions are becoming more relevant in many cases. Miniaturization and other aspects are important for the growth in electrical systems. Due to the improvements and miniaturization of the semiconductor devices, the maximum frequencies are reaching into the 1000 GHz range. At these higher frequencies and component densities, coupling among the components has become a key issue. As an extreme case, power engineering systems may include integrated circuits with currents in the microampere range in the vicinity of bars conducting hundreds of amperes. The sizes of the coupled subsystems range from micrometers to meters.

Full-wave solutions may also be of importance due to the increasing spectrum in the noise signal frequencies and due to physical largeness of the systems. Electromagnetic

compatibility (EMC) is another area of growing interest where the ever-increasing frequencies represent new challenges. The use of semiconductor devices in power electronic systems leads to higher frequency noise. From an EM-modeling point of view, these challenges represent many new and interesting problems to be solved.

The fundamental technical idea of the PEEC approach is to convert an IE-based solution of Maxwell's equations into appropriate equivalent circuits, which can then be used in conjunction with other different linear or nonlinear circuits in a circuit solver mode. We should not assume that this will compromise the solution from an electromagnetic point of view. In many situations, the opposite is true. Solutions can very often be found in the circuit domain, which are much more difficult to obtain without circuits. Besides, we can borrow from the large number of techniques that are available from circuit theory as well as from the implementation of today's SPICE-type circuit solvers.

The recent rapid increase in performance of today's PC computers provides many new possibilities for the EM-modeling area. In addition, the processor and increased memory size have made EM-modeling affordable for everyone. The availability of a large memory is key for the solution of larger problems since the complexity of the solution increases rapidly with the problem size. The speedup of PEEC solutions for large models is one aspect that we do not cover in this text. Parallel processing is a way to enhance the compute power. Fortunately, processors also have become widely available at a low cost. All EM-modeling techniques need to be tailored to computer systems to take full advantage of these changes. However, these issues for PEEC are not included in this book.

In this book, we aim to introduce electromagnetic PEEC models in a practical useful way. The book is written such that the concepts are ready for real-life applications. We hope that this helps the understanding of the fundamental concepts. Also, it should make the book useful to industry and will help to emphasize the importance of the subject to new students in the EM field. We are also attempting to make the mathematical formulations as transparent and readable as possible. This should extend the overall readability of the book for self-study for everyone.

Besides the necessary introduction of the basic concepts in the circuit and electromagnetic theories, the techniques are presented first starting with the circuit concepts. All aspects of building PEEC models are presented in a logical way. We find it useful to implement PEEC in a SPICE circuit solver-like implementation such that both the time and frequency domain solutions can be provided without the need for the Fourier transform of the frequency domain solution. Linear equivalent circuits can lead to models that can be used equally well in both the time and frequency domains. Clearly, the SPICE solver input language represents an excellent implementation of this fact since sources can be specified such that they apply both in the time and the frequency domains. This also helps the flexibility of the overall solution.

In Chapter 2, we give an introduction to circuit analysis necessary for a PEEC solution. Our solution approach and most SPICE circuit solvers today are based on the modified nodal analysis (MNA) method. Importantly, this approach also leads to the *dc* solution for PEEC. The fundamental idea is to set up the MNA circuit matrix for all our solutions. This is accomplished with so-called matrix stamps where each circuit element is added to the circuit matrix with a *matrix stamp*. This enters the contribution to the circuit matrix in a clear, systematic way.

Stamps for different circuit elements are given in Appendix B. We need to consider the circuit solutions for both the time and frequency domains. In addition, we present a circuit-oriented approach for which frequency-dependent elements can be included also

in a time domain solution. This is done with a synthesis process. Another useful approach presented for adding macromodels to the circuit matrix is the recursive convolution approach. Section 2.11 considers circuits with delays, which is fundamental for full-wave PEEC models.

Chapter 3 introduces the underlying electromagnetic concepts necessary for the PEEC models presented in the book. Further details on the fundamentals for the EM concepts can be found in the above-mentioned texts. We have to assume that the reader of our book is knowledgeable in the solution of Maxwell's equations. The presentations are clearly oriented toward IE solutions. The necessary Green's functions that are used in this text are given. We only use scalar Green's functions for which we use the lowercase g . We also use the fundamental concepts necessary for surface IE-based PEEC equations. We end Chapter 3 with a short discussion of the numerical solution of IEs.

Chapter 4 is dedicated to the computation of capacitances – an integral part of PEEC models. The basics of IE-based solutions for capacitance problems is considered. The potential coefficients in the solutions are basic elements in the remaining chapters. For completeness, we also included computations using a differential equation approach. Further, the importance of projection meshing is considered since it may lead to errors in the results. Models for representing capacitances with retardation for PEEC equivalent circuits are presented. Finally, the computation of partial potential coefficient is given in Appendix D.

Chapter 5 considers important aspects of inductance calculations. The main topic is the development of partial inductance concepts since they are the fundamental building block of most PEEC models. The general concept of partial inductance is described and its application for general inductance computation is discussed. Concepts such as open-loop inductances are introduced and examples are given for the use of the open-loop concept.

The basic task of the computation of partial inductances is delegated to Appendix C. Other issues such as problems with the partial inductance for large conductors are shown to be of importance. Finally, the use of the PEEC concepts for the modeling of transmission lines and the efficient modeling of plane pairs is considered as a useful application of partial inductances.

In Chapter 6, we present the building details for PEEC models that utilize partial inductances as well as partial coefficients of potential and resistances. The presentation of these concepts is done systematically so that the derivations in the previous chapters can be used. Importantly, the conductor cells are described in detail and the PEEC equivalent circuits are given. Other aspects such as the implementation of the continuity equation as Kirchhoff's current law is derived in terms of the physical geometry. More details for the capacitance models used in PEEC are given. Finally, the circuit equations for the model are presented, which includes models with delays or retardation.

Chapter 7 gives a detailed development of the equations for the nonorthogonal case, which is the extension of the PEEC approach to nonorthogonal conductors and dielectrics. This necessary step will facilitate the computation problems with nonrectangular-shaped objects. Mainly, we found that a purely rectangular or Manhattan representation of the geometry is insufficient for some problems.

Manhattan modeling can do an excellent job for many problems, especially for on-chip problems. The additional complexity introduced by nonrectangular coordinates certainly leads to increases in compute time and additional challenges in the algorithms. Hence, we do address both the rectangular and the nonrectangular cases in a consistent way. Advances

have also been made for both orthogonal and nonorthogonal geometries. We present formulations for the evaluation of the nonorthogonal partial elements in the appendices.

Chapter 8 is dedicated to the important topic of the geometrical description of the bodies and their subdivision or meshing. This subdivides the problem into the appropriate geometrical cells for which the partial elements are computed. Two fundamentally different approaches exist today. In many IE approaches, the geometry is subdivided into small triangular cells that can be computed at a relatively small cost. This approach is well suited, for example, in solving problems such as the scattering from airplanes.

For electronic systems, efficient subdivision may consist of cells with very large aspect ratios. This may result in a reduction in the number cells and ultimately unknowns. Large differences in cell sizes are allowed. However, a larger effort is needed for the meshing as well as for the computation of the partial elements. In this chapter, we discuss details for the implementation for this type of meshing.

Chapter 9 covers the inclusion of skin-effect in PEEC models. The issue is fundamentally very challenging since it can add considerably to the compute time needed to solve a problem. Fortunately, the PEEC method is inherently suitable for 3D skin-effect models. Several different models are included depending on the current flow and other geometrical features. The modeling of skin-effect problems in terms of circuits is a natural, convenient solution. This is shown to be the case in this chapter where several circuit-oriented models are presented. Also, a surface IE skin-effect model is presented. Examples are given where we also compare the integral equation results with the volume skin-effect models.

Chapter 10 considers PEEC with dielectric models. These models also include loss models for the dielectrics. It is clear that the inclusion of the dielectrics in PEEC is important today. Approaches such as the method-of-images are presented for the computation of the Green's functions for layered dielectric structures. In the gigahertz range, the dielectric losses can dominate skin-effect losses. Hence, the higher frequency made the inclusion of dielectric losses relevant.

The challenge is to provide loss models that operate equally well in both the time and the frequency domains. It is evident from this chapter that it is easy to include other circuit models in the implementation of PEEC circuits. The last topic in this chapter is inclusion of dielectrics in the time domain using recursive convolution. This allows the inclusion of relatively complex structures without excessive compute times.

In Chapter 11, we consider techniques for the modeling of problems that include magnetic materials. First, we consider the conventional magnetic reluctance concepts since they lead to circuit models, although they are not directly connected to PEEC. Concepts such as magnetic potentials and magnetic charge are considered next for the IE solution.

The key aspect of this part of the chapter is concerned with PEEC formulations that also includes magnetic bodies. The derivations are parallel to Chapter 6, where we are building conventional PEEC models. Finally, surface IE approaches are also suitable for the inclusion of magnetic materials. For this reason, we include a PEEC surface formulation in this chapter.

Chapter 12 includes incident and radiated fields, which represent an important part of EM models, especially for electromagnetic interference (EMI) and antenna problems. First, the incident field computation is considered, which results in additional independent voltage sources. Then a PEEC-specific, sensor-based approach is used to compute both the electric and magnetic fields. Finally, more conventional direct far field computations are included.

The concepts of stability and passivity are considered in Chapter 13. We introduce the fundamental circuit concepts needed for considering this complicated topic. Both the time

and frequency domain solutions are included. Conventional PEEC models without delays can be treated in a relatively straightforward way. While the theory is needed, we also consider the more practical side where possible.

Interesting results have been obtained from running conventional circuits. However, PEEC circuits with delays lead to very challenging *descriptor* systems. This chapter also considers techniques to improve the stability and passivity for realistic problems. Examples are given to show the effectiveness and the implementation of some of these concepts.

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