Section 1

Whole Numbers and Integers: Theory and Operations

1-1 Natural Numbers ★

Natural numbers were likely the first numbers that ancient people used. They are represented by the set of numbers $\{1, 2, 3, 4, \ldots\}$. The three dots, called ellipses, mean that the numbers continue in the same pattern without ending. Natural numbers are used in virtually all aspects of life.

Problem: Write at least three common or well-known phrases in which a natural number, or numbers, is used. Example: A cat has *nine* lives. Be ready to share your answers with the class.

1-2 Natural Numbers ★★

Natural numbers, also known as the counting numbers, are represented by the set of numbers $\{1, 2, 3, 4, \ldots\}$.

Problem: Answer the questions below.

(1) The sum of the first two natural numbers = _______.

(2) The sum of the first three natural numbers = _______.

(3) The sum of the first four natural numbers = _______.

(4) The sum of the first five natural numbers = _______.

(5) The sum of the first six natural numbers = _______.

Explain the pattern of the sums.

1-3 Whole Numbers ★

Whole numbers include the natural numbers and 0 and are represented by the set of numbers $\{0, 1, 2, 3, 4, \ldots\}$.

Problem: Sometimes a pattern that uses letters is actually a numerical pattern in disguise. Find the next three letters in the pattern below. (Hint: Think of the set of whole numbers.)

Z, O, T, T, F, F

1-4 Whole Numbers ★ G

Although 0 is a part of the set of whole numbers and is extremely important, ancient people had a lot of trouble with 0. For example, the ancient Romans conquered much of the known world, but they did not have a symbol for zero in their numeral system. In fact, a symbol for 0 did not come to Europe until around the twelfth century AD, when Europeans adopted the Arabic number system.

Problem: Speculate what would happen if we did not have 0 in our number system.

1-5 Place Value with Whole Numbers ★

Jackie Robinson was the first African American baseball player to appear in a National League game in 1947. Lawrence Eugene (Larry) Doby was the first African American baseball player to appear in an American League game.

Problem: To find out the year Larry Doby first played in the American League, use these clues:

- The year was in the twentieth century.
- The sum of the digits of this year is 21.
- The tens digit is 3 less than the units digit.

1-6 Place Value with Whole Numbers ★★

At a track meet on May 6 in Oxford, England, Roger Bannister was the first person to run the mile in less than 4 minutes. His time was 3:59.4.

Problem: Use the clues below to determine the year Bannister broke the 4-minute barrier for the mile:

The sum of the thousands and hundreds digit is 10.
The sum of the tens and units digit is one less than 10.
The units digit is one less than the tens digit.

1-7 Numerical Operations ★

The word *compute* comes from the Latin word *computare*, which means a ''notched tally stick.'' Basic computation involves addition, subtraction, multiplication, and division. Computation requires an understanding of numerical facts, but it does not require creative thinking.

Problem: Explain why or why not the name *computer* is a fitting name for today's computers.

1-8 Numerical Operations ★ G

When you read a word problem in math, sometimes a key word, or words, suggests certain operations. For example, words such as *sum* and *combined* mean addition. Words like *minus* or *how much more than* mean subtraction.

Problem: Create a list of words or phrases that suggest mathematical operations.

1-9 Adding Whole Numbers ★

Although addition problems can be performed quickly with a calculator, calculators may not always be handy when numbers need to be added. Thus, the ability to add numbers remains an important skill.

Problem: Of the four problems below, which are correct? Correct those that are wrong.

(1) $\begin{array}{r} 4,659 \\ +4,979 \\ \hline 9,738 \end{array}$ **(2)** $\begin{array}{r} 1,863 \\ +5,683 \\ \hline 7,546 \end{array}$ **(3)** $\begin{array}{r} 1,359 \\ +4,055 \\ \hline 5,314 \end{array}$ **(4)** $\begin{array}{r} 58,614 \\ +38,821 \\ \hline 97,435 \end{array}$

1-10 Subtracting Whole Numbers ★

When people make mistakes in subtraction, more often than not, their mistakes are in regrouping (borrowing). Being careful to regroup reduces mistakes.

Problem: Imagine that you are explaining to a young student how to regroup. Write a step-by-step explanation of how regrouping is done. Include an example to clarify your explanation.

1-11 Subtracting Whole Numbers ★★

A solid understanding of subtraction is a basic math skill.

Problem: Find the missing digits to complete the problems below.

(1) $\begin{array}{r} \square,\square 63 \\ -3,9\square 9 \\ \hline 2,83\square \end{array}$ **(2)** $\begin{array}{r} 71,\square 4\square \\ -4,8\square 6 \\ \hline 6\square,270 \end{array}$

1-12 Multiplying Whole Numbers ★

The word *multiplication* comes from the Latin words *multi* and *plicare. Multi* means ''many,'' and *plicare* means ''fold.'' Multiplication therefore means repeated addition of numbers to obtain a solution. For example, rather than adding 318 twenty-nine times, you can simply multiply 318×29. The answer to a multiplication problem is called a *product*.

Problem: Multiply 318×29. Then write an explanation describing the steps you used to find the product.

1-13 Multiplying Whole Numbers ★★

Before multiplication was developed, people had a hard time calculating large numbers. Although calculators perform much of the multiplying we do now, a good understanding of multiplication facts is fundamental to computation.

Problem: Using the boxes below, one box for each of the numbers 1, 2, 3, 4, 5, write a multiplication problem that gives the largest product. You may use each number only once.

$$\begin{array}{r} \square\square\square \\ \times\square\square \\ \hline \end{array}$$

1-14 Dividing Whole Numbers ★ (6.NS.2)

Division is the mathematical operation of finding how many times a number, called the divisor, is contained in another number, the dividend. The answer to a division problem is called the quotient. Division is a basic skill in mathematics.

Problem: Which of the following quotients are correct? Correct the problems that are wrong.

(1) $\begin{array}{r} 49 \text{ R}20 \\ 37\overline{)1{,}833} \end{array}$ **(2)** $\begin{array}{r} 51 \text{ R}7 \\ 49\overline{)2{,}506} \end{array}$ **(3)** $\begin{array}{r} 88 \text{ R}4 \\ 32\overline{)2{,}820} \end{array}$ **(4)** $\begin{array}{r} 714 \text{ R}40 \\ 53\overline{)3{,}803} \end{array}$

1-15 Dividing Whole Numbers ★ (6.NS.2)

Division is much like learning to ride a bicycle. Once you master the steps, you will not forget them.

Problem: Explain the steps of long division.

1-16 Dividing Whole Numbers ★★ (6.NS.2)

Sports can be great fun for both players and spectators. Setting up teams and finding sponsors, however, may be a problem.

Problem: There are 112 students who signed up to play basketball in a recreation league. They have 14 sponsors. How many teams of 11 players each can be formed?

1-17 Whole Numbers — Multistep Problem ★★ (7.EE.3)

An old English children's rhyme goes like this:

> As I was going to St. Ives,
> I met a man with seven wives,
> Every wife had seven sacks,
> Every sack had seven cats,
> Every cat had seven kits.

Problem: How many people and animals were going to St. Ives?

1-18 Whole Numbers — Multistep Problem ★★ G (7.EE.3)

In professional football, scoring is based on 6 points for a touchdown, 1 point for an extra point after a touchdown, 2 points for a two-point conversion after a touchdown, 2 points for a safety, and 3 points for a field goal.

Problem: List any number less than 50 that is impossible for a football team's final score.

1-19 Estimation with Compatible Numbers ★ (7.EE.3)

Compatible numbers are numbers that are easy to compute mentally. They are often used to estimate answers.

Consider this example. The sophomore class made a profit of $572.70 at their annual car wash. They washed 185 cars. About how much profit did they make per car?

This answer could be easily estimated by using the compatible numbers of 600 (dollars) and 200 (cars). It is much easier to compute $600 \div 200 = 3$ than $\$572.70 \div 185$.

Problem: Make up a word problem in which compatible numbers could be used to estimate the answer. Exchange your problem with a classmate. Decide if the problems are correct. Correct them if they are incorrect.

1-20 Rounding Whole Numbers ★

Attendance at many free events is often estimated because there is no accurate way of determining the exact number of people who show up.

Problem: A magazine article reported that 16,000 people attended a free concert in the park. What number, or numbers, below *cannot* be rounded to 16,000?

(1) 15,500 **(2)** 15,400 **(3)** 16,500 **(4)** 16,499 **(5)** 15,499

1-21 Divisibility by 2, 4, and 8 ★

Understanding divisibility is useful in several mathematical applications. The divisibility tests for 2, 4, and 8 are listed below:

A natural number is divisible by:

- 2 if the last digit is even.
- 4 if the number formed by the last two digits is divisible by 4.
- 8 if the number formed by the last three digits is divisible by 8.

Problem: Using the divisibility rules above, what numbers below are divisible by 2? By 4? By 8?

(1) 1,326 **(2)** 3,174 **(3)** 7,368 **(4)** 17,208

1-22 Divisibility by 3, 6, 9, and 12 ★

Although a calculator can be used to test for divisibility, sometimes you may need to use old-fashioned divisibility tests. The divisibility tests for 3, 6, 9, and 12 are listed below.

A natural number is divisible by:

- 3 if the sum of the digits of the number is divisible by 3.
- 6 if the number is divisible by 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 12 if the number is divisible by 3 and 4.

Problem: Using the divisibility rules above, what numbers below are divisible by 3? By 6? By 9? By 12?

(1) 102 **(2)** 153 **(3)** 1,755 **(4)** 4,644

1-23 Divisibility by 5 and 10 ★

The tests for divisibility for 5 and 10 only require you to look at the last digit of the number:

- A number is divisible by 5 if it ends in 5 or 0.
- A number is divisible by 10 if it ends in 0.

Problem: Complete the sentences below with the number 5 or 10.

If a number is divisible by _______, it is always divisible by _______. If a number is divisible by _______, it may be divisible by _______.

Give an example to show that your answers are correct.

1-24 Factors ★

Although our number system is based on 10, some items are grouped by 12, such as 12 eggs in a dozen. When it comes to pricing, 12 is a more practical number than 10 because it has more factors. The factors of 12 are $1, 2, 3, 4, 6$, and 12, while the factors of 10 are $1, 2, 5$, and 10. Because of this, commercial items are often counted in multiples of 12. For example:

- 12 items = 1 dozen
- 12 dozen = 1 gross
- 12 gross = 1 great gross

Problem: How many dozens are in a great gross? How many items are in a great gross?

1-25 Factors ★★

Mr. Thomas was thrilled when he received the list of students for his first-period math class. He likes to break his class into groups with an equal number of students in each group. When he saw the number of students, he knew he could have groups of $2, 3, 4, 6, 8$, and 12.

Problem: How many students were in his class?

1-26 Greatest Common Factor ★ (6.NS.4)

The greatest common factor, GCF, of two numbers is the largest number that is a factor of both numbers. For example, the factors of 12 are $1, 2, 3, 4, 6$, and 12. The factors of 32 are $1, 2, 4, 8, 16$, and 32. The GCF of 12 and 32 is 4.

Problem: The greatest common factor of each pair of numbers below may or may not be correct. Find the incorrect greatest common factor or factors. Correct any that are wrong.

(1) 20 and 30
GCF $= 5$

(2) 15 and 18
GCF $= 3$

(3) 24 and 36
GCF $= 12$

(4) 20 and 9
GCF $= 1$

1-27 Greatest Common Factor ★★ (6.NS.4)

Over 2,000 years ago, Euclid, a Greek mathematician, developed an algorithm, or method, that he used to find the greatest common factor, GCF, of two numbers. Euclid's algorithm is still used today.

Following are the steps of Euclid's algorithm:

1. Divide the larger number by the smaller.
2. Divide the divisor in step 1 by the remainder in step 1.
3. Repeat step 2 until there is no remainder.
4. The last divisor is the GCF of the original numbers.

Problem: Using Euclid's algorithm, find the GCF of 208 and 464.

1-28 Multiples ★

A multiple of a number is the product of the number and a natural number. For example, the multiples of 4 are $4, 8, 12, 16, 20, \ldots$

Problem: Write the first five multiples of $5, 9, 12$, and 15.

1-29 Multiples ★★

The Great Wall of China, begun about 220 BC, was built over many centuries. One of the most extensive and best-preserved sections of the wall dates from the Ming dynasty (1368–1644).

Problem: Use the clues below to find the approximate length in miles of this part of the Great Wall:

The length is a four-digit number.

The tens digit and the units digit are 0.

The hundreds digit equals the thousands digit, and these digits are multiples of 5.

1-30 Least Common Multiple ★ (6.NS.4)

The least common multiple, or LCM, of two numbers is the smallest number that is a multiple of both numbers. For example:

The multiples of 6 are 6, 12, 18, 24, ...

The multiples of 8 are 8, 16, 24, 32, ...

The least common multiple of 6 and 8 is 24, because 24 is the smallest number that appears in both sets.

Problem: 12 is the least common multiple of one of the two pairs of numbers listed below. For which pair of numbers is 12 the least common multiple?

(1) 12 and 18 **(2)** 3 and 15 **(3)** 6 and 12 **(4)** 18 and 36

1-31 Least Common Multiple ★★ (6.NS.4)

A common way of finding the least common multiple of two numbers is to make lists of multiples and find the smallest number that appears in both lists. There is also a simple formula you can use:

$$\text{LCM} = \text{Product} \div \text{GCF}$$

In this equation, LCM stands for least common multiple, the Product is the result of the two numbers being multiplied together, and GCF stands for greatest common factor. Following is an example of how this equation works.

The LCM of 20 and 30 can be found by simplifying the expression $20 \times 30 \div 10$. (10 is the GCF of 20 and 30.) The LCM of 20 and 30 is 60.

Problem: Find the LCM of each pair of numbers by dividing the product of the numbers by their GCF.

(1) 15 and 20 **(2)** 10 and 50 **(3)** 21 and 63 **(4)** 17 and 29

1-32 Multiples and the Distributive Property ★★ (6.NS.4)

Mika found an amazing way to write the sum of two whole numbers as a multiple of a sum of two numbers that have no common factor other than 1.

Following is Mika's method:

1. Choose any two numbers. Example: 20 and 50
2. Find the greatest common factor, GCF, of the numbers. The GCF of 20 and 50 is 10.
3. Divide both numbers by the GCF. $20 \div 10 = 2$ and $50 \div 10 = 5$
4. Add the quotients. $2 + 5$
5. Multiply the sum of the quotients by the GCF found in step 2. $10(2 + 5) = 70$
6. This is the same as the sum of the numbers in step 1. $10(2 + 5) = 20 + 50$

Problem: Choose any two numbers, and use Mika's method to write the sum as a multiple of a sum of two numbers that have no common factor other than 1. How do you know that the sum has no common factor other than 1?

1-33 Prime Numbers ★

A prime number is a number that has only two factors, 1 and the number.

Problem: List the prime numbers between 1 and 50.

1-34 Prime Numbers ★

Because prime numbers have only two factors, 1 and the number itself, understanding factors is helpful to understanding prime numbers.

Problem: Write an explanation of why 1 is *not* a prime number.

1-35 Composite Numbers ★

A composite number is a number that has more than two factors. 6 is a composite number because its factors are 1, 2, 3, and 6.

Problem: List the composite numbers that are greater than 9 and less than 21. Then state the number of factors of each composite number you listed.

1-36 Prime and Composite Numbers ★★

Go to http://www.hbmeyer.de/eratosiv.htm. You will see a grid numbered from 1 to 400.

Here is what to do:

1. Click on the number 2, and all multiples of 2 will be removed from the grid.
2. Wait until all multiples of 2 have been removed.
3. Then click on the number 3 to remove all multiples of 3 from the grid.
4. Continue this process through the number 19.

Problem: Explain why after clicking on the number 19, all composite numbers have been removed and only prime numbers remain.

1-37 Perfect Squares ★

A perfect square is a product of a number multiplied by itself. An example of a perfect square is 16, the result of 4×4. Another is 25, the result of 5×5.

Problem: List the perfect squares starting with 1 and ending with 100.

1-38 Perfect Squares and Prime Numbers ★★

We see the American flag every day at school, sporting events, and buildings.

Problem: Use the clues below to determine numbers related to the flag:

The number of red stripes is the fourth prime number.
The number of white stripes is the sixth prime number minus the number of red stripes.
The total number of stars equals the square of the number of red stripes plus 1.

1-39 Order of Operations ★

When simplifying expressions, you must perform operations in a specific order:

1. Simplify expressions within grouping symbols. If more than one grouping symbol is used, simplify the innermost group first and continue simplifying to the outermost group.
2. Multiply and divide from left to right.
3. Add and subtract from left to right.

Problem: Four expressions are listed below. Which will have the same value if you use the order of operations to simplify them? What are the values of each expression?

(1) $8 + 4 \times 3 - 1$ **(2)** $(8 + 4) \times 3 - 1$ **(3)** $8 + (4 \times 3) - 1$ **(4)** $8 + 4 \times (3 - 1)$

1-40 Order of Operations ★★ G

Numbers may always be expressed in terms of other numbers. When simplifying expressions, the order of operations must always be followed.

Problem: Using the digits 1, 2, 3, and 4 and the order of operations, write expressions that equal the numbers from 1 to 10. You must use each digit only once per expression. The first number is done for you.

$$(2 \times 1) - (4 - 3) = 1$$

1-41 Powers of Numbers ★ (6.EE.1)

An exponent is the number written in the upper-right-hand corner of a number. In the expression 2^6, 6 is the exponent and 2 is the base. 2^6 is read as "2 to the sixth power" and means $2 \times 2 \times 2 \times 2 \times 2 \times 2$, which equals 64.

Problem: 64 can be written as a base raised to a power in four different ways. 2^6 is one of these ways. 64^1 is another. What are two other ways?

1-42 Simplifying Expressions with Exponents ★ (6.EE.1)

The order of operations must be followed to simplify expressions:

1. Simplify expressions within grouping symbols first. If several grouping symbols are used, simplify the innermost group and continue simplifying to the outermost group. As you do this, follow steps 2, 3, and 4.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Problem: Place parentheses, if necessary, in each expression so that the result is 56.

(1) $7 \times 6 - 4 \times 4$ **(2)** $18 - 8 \times 5 + 6$
(3) $8^2 - 1 \times 2^3$ **(4)** $2^2 + 3 \times 9 - 1$

1-43 Simplifying Expressions with Exponents ★★ (6.EE.1)

The game of horseshoes is often played at picnics. For most players, it is a game of skill and a bit of luck. In a standard game of horseshoes, the stakes must be set a specific number of feet apart.

Problem: Simplify the expression below to find the distance in feet between stakes in a regulation game of horseshoes.

$$2 \times 3^2 + 3 \times 2^2 + 3^2 + 1^2$$

1-44 Simplifying Expressions with Exponents ★★ (6.EE.1)

Knowledge of numbers can increase your cultural literacy and provide you with insights in many areas. For this problem, knowing how to simplify expressions can even help you decipher the title of a classic book.

Problem: What is Fahrenheit $(7 + 8)^2 - (8 + 6)^2 + (9 + 10)^2 + 8^2 - 3$? What do you think the significance of this number is?

1-45 Writing Numerical Expressions ★★

A numerical expression names a number. $3+9$, 2^3+2^2, and $4(2+1)$ are examples of numerical expressions. All three of these expressions name the number 12.

Problem: Write a numerical expression for each phrase below. Find the number that each expression names.

(1) 4 squared

(2) 3 less than 2 squared

(3) The sum of 6 and 12

(4) 1 plus 5 squared

(5) 3 times 5 cubed

(6) 6 squared divided by 3 squared

1-46 Identifying Parts of a Numerical Expression ★★

The parts and value of an expression may be identified by using the correct mathematical terms. *Sum, difference, factor, product*, and *dividend* are just some of the terms that relate to expressions.

Problem: Eight numerical expressions are listed below. Match each with a correct description. Some descriptions may describe more than one expression.

(1) $2+5$

(2) 2×5

(3) $12\div 2$

(4) $3(5+2)$

(5) $3\times 2\times 6$

(6) $2-1$

(7) $12\div 3$

(8) $9(3+1)$

Descriptions

(A) The product is 36.

(B) The sum is 7.

(C) The dividend is 12.

(D) The expression has two factors.

(E) A sum is one of the factors.

(F) The difference is 1.

1-47 Integers ★ (6.NS.5)

Integers are the set of the natural numbers, 0, and the opposites of the natural numbers. The set $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ represents integers.

One of the many ways we use integers is for countdowns for space flights and satellite launchings. The negative numbers designate the seconds before blast-off, and the positive numbers represent the seconds after blast-off.

Problem: How many seconds elapse between $T - 10$ seconds and $T + 10$ seconds? On what number would blast-off occur?

1-48 Integers ★★ (6.NS.5)

Negative numbers troubled mathematicians well into the 1800s. Most mathematicians ignored them, calling them ''inadequate'' or ''absurd,'' believing that negative numbers had no practical use or meaning.

This all changed with Leonhard Euler. In his *Complete Introduction to Algebra*, Euler stated that subtracting the opposite of a number is the same as adding the number. In Euler's words: ''To cancel a debt signifies the same as giving a gift.''

Problem: Explain Euler's words. Include an example with negative numbers to support your explanation.

1-49 Opposites ★ (6.NS.5)

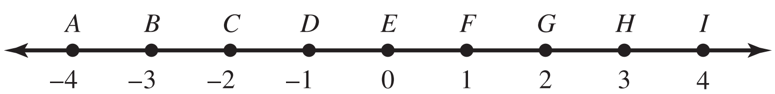
Positive numbers and negative numbers are opposites. For example, gaining 3 pounds is the opposite of losing 3 pounds. In football, gaining 10 yards is the opposite of losing 10 yards.

Problem: Consider the following statement. If $-10\,^{\circ}$F is really cold, then $10\,^{\circ}$F is really hot. Do you agree or disagree? Explain your answer.

1-50 The Number Line ★ (6.NS.6)

All numbers can be represented as points on a number line. The origin of a number line is 0. The numbers to the right of 0 are positive numbers, and the numbers to the left of 0 are negative numbers. The coordinate is the number paired with a point on the number line.

Problem: Use the number line to identify the coordinates described below.



(1) The origin

(2) The natural numbers

(3) The point halfway between *D* and *H*

(4) The point that is the same distance from the origin as *H*

1-51 The Number Line ★★ (6.NS.6)

The Smith family likes to take car trips. To help pass the time, Mrs. Smith, who does not like to rely on GPS, assumes the role of navigator and gives clues about the towns they will pass through.

Problem: Traveling south on an interstate highway, the Smiths will pass the cutoffs for three towns: Allenville, Browning, and Cooperton. The distance from Allenville to Browning is 16 miles, the distance from Browning to Cooperton is 8 miles, and the distance from Allenville to Cooperton is 24 miles. What town is between the other two? (Hint: To help you visualize the towns, sketch a number line.)

1-52 Absolute Value ★ (6.NS.7)

Absolute value is the distance of a number on the number line from 0. Note the following examples:

$$|3| = 3 \quad |7| = 7 \quad |15| = 15$$
$$|-3| = 3 \quad |-7| = 7 \quad |-15| = 15$$

Problem: From the examples above, Abby generalized that the absolute value of any number is a positive integer. Do you agree or disagree? Explain your answer.

1-53 Absolute Value ★★ G (6.NS.7)

Samantha borrowed \$25 from her sister. Samantha described her debt as $-\$25$. But her sister described the money Samantha owed her as \$25 (without the negative sign).

Problem: Is each girl's description accurate? Support your answer in terms of absolute value.

1-54 Comparing Integers ★

Every integer can be paired with a point on the number line. The larger integers are always to the right of the smaller integers.

Problem: Each group of integers below is arranged from the least to the greatest or the greatest to the least, but one integer is out of order. Identify this integer, and rewrite the set correctly.

(1) $\{-3, -2, 1, 0, 7\}$ **(2)** $\{0, -7, -6, -5, -4\}$
(3) $\{-8, -9, -7, -6, -5\}$ **(4)** $\{-4, 2, 4, 8, 6\}$

1-55 Inequality Symbols ★

Symbols are often used in mathematics and real life. Two common symbols are $<$ and $>$.

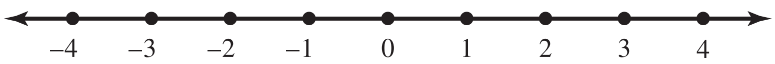
$<$ is read as "is less than."

$>$ is read as "is greater than."

Problem: Explain why the fast-forward symbol on a TV remote control resembles the greater-than symbol and the rewind resembles the less-than symbol.

1-56 Ordering Integers on a Number Line ★ (6.NS.7)

A number line is a line on which points correspond to numbers. Larger numbers are located to the right of smaller numbers. Conversely, smaller numbers are located to the left of larger numbers.



For example, because 4 is to the right of 1 on a number line, $4 > 1$. Because 1 is to the left of 4, $1 < 4$.

Problem: Write the correct inequality symbol for the numbers below. Refer to the number line above to confirm your results.

(1) -2 is to the left of 2; therefore, -2 _____ 2.

(2) -2 is to the right of -4; therefore, -2 _____ -4.

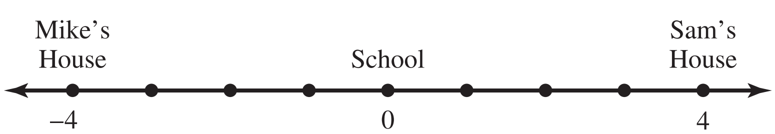
(3) 1 is to the right of 0; therefore, 1 _____ 0.

(4) -1 is to the left of 0; therefore, -1 _____ 0, or 0 _____ -1.

(5) -3 is to the right of -4; therefore, -3 _____ -4, or -4 _____ -3.

1-57 Understanding Statements of Order ★★ (6.NS.7)

Sam's and Mike's math assignment was to make a map that shows the locations of their homes and their school. Sam lives 4 blocks to the east of the school, and Mike lives 4 blocks to the west. Mike said that this reminds him of a number line, where the school's location is the origin, Sam's house is paired with the number 4 on the number line, and Mike's house is paired with -4.

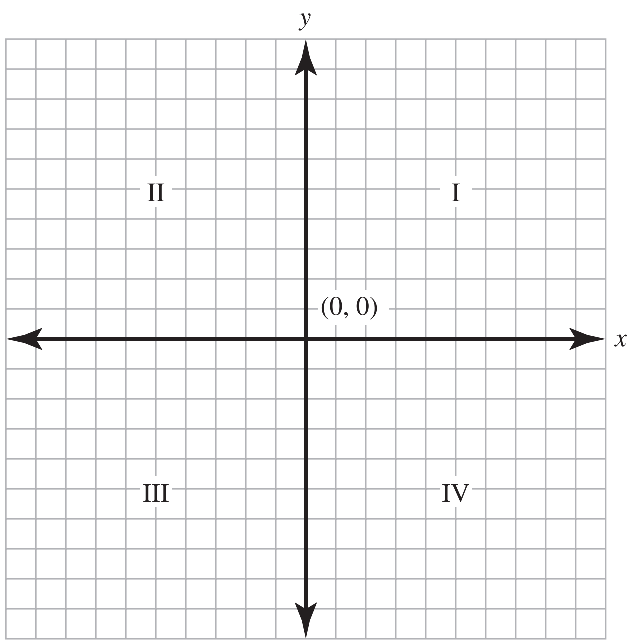


Because -4 is to the left of 0 and 0 is to the left of 4, Mike concluded that Sam's home is a greater distance from the school than his home. After all, $-4 < 0 < 4$.

Problem: Explain the error in Mike's reasoning.

1-58 The Coordinate Plane ★★ (6.NS.6)

The horizontal axis, called the *x*-axis, and the vertical axis, called the *y*-axis, divide the coordinate plane into four parts called quadrants. The point (0, 0) is called the origin and is where the *x*-axis and *y*-axis intersect.



Problem: Of the four statements below, at least three are false. Identify the false statements, and rewrite them to make them true.

(1) If the *x*-coordinate is greater than 0, the point is always in the first quadrant.

(2) If the *y*-coordinate is 0, the point is always on the *y*-axis.

(3) If the signs of the *x*- and *y*-coordinates are the same, then the point is always in the third quadrant.

(4) If the signs of the *x*- and *y*-coordinates are different, then the point could be in the fourth quadrant.

1-59 Graphing Points in the Coordinate Plane ★ (6.NS.6)

When people attend a concert, major sporting event, or theater presentation, they find their seats by checking the row and seat number on their tickets. Without this information, they would likely have trouble finding their seats.

Problem: Explain how locating a seat in a theater or arena is similar to plotting points in a coordinate plane.

1-60 Solving Problems by Graphing Points in the Coordinate Plane ★★ (6.NS.8)

The coordinate plane is used not only for graphing points; it can also be used for graphing geometric figures. Because the horizontal and vertical lines are perpendicular, graphing rectangles is especially easy.

Problem: On graph paper, draw four rectangles in the coordinate plane that have the following characteristics:

- The points $(-3, -1)$ and $(1, -1)$ are the end points of one side of each rectangle.
- The longer side of each rectangle is twice as long as its shorter side.

List the vertices of these rectangles. Find the lengths of the sides to support your answer.

1-61 Adding Integers ★ (7.NS.1)

A magic square is a square in which all of the numbers in each row, column, and diagonal add up to the same number.

Problem: In the first magic square below, find the sum of each row, column, and diagonal. In the second magic square, each row, column, and diagonal add up to 3. Find the missing numbers.

(1)

−4	−9	−2
−3	−5	−7
−8	−1	−6

(2)

	5	0
		−1
2		

1-62 Adding Integers ★★ (7.NS.1)

Tom has trouble understanding how you can add certain integers and have an answer that is less than the number you started with. For instance, $8 + (-5) = 3$.

Problem: Write an explanation of why the equation above is correct. Provide an example from everyday life that supports your explanation.

1-63 Subtracting Integers ★ (7.NS.1)

To subtract integers correctly, you must pay close attention to the signs as well as the numbers.

Problem: Complete each equation by filling in the correct number.

(1) $7 - (\_\_\_\_\_) = 12$ **(2)** $\_\_\_\_\_ - 5 = -13$
(3) $\_\_\_\_\_ - (-16) = -23$ **(4)** $9 - (\_\_\_\_\_) = 16$

1-64 Subtracting Integers ★★ (7.NS.1)

When learning how to subtract integers for the first time, many students become confused because the answer may be greater than the numbers they started with. For example:

$$20 - (-5) = 25$$

Problem: Create a word problem that requires subtraction of integers in its solution. Exchange your problem for the problem of a classmate and solve each other's problem.

1-65 Adding and Subtracting Integers ★★ (7.NS.1)

Sometimes mathematicians are not interested in actual numbers but simply whether a number is positive. A good example is the stockholder who is concerned only with whether his or her stocks go up. Another is the racer who is interested in breaking a record (going faster).

Problem: Describe each situation below as positive, negative, zero, or cannot be determined.

(1) The sum of two positive integers

(2) The difference of two positive integers

(3) The sum of an integer and its opposite

(4) The sum of a negative integer and zero

(5) The difference of two negative integers

1-66 Representing Addition and Subtraction on a Number Line ★★ (7.NS.1)

Go to http://www.funbrain.com/linejump/index.html to explore how the number line can be used to model addition and subtraction of integers.

Here is what to do:

1. Click on ''Really Hard'' for the level of difficulty.
2. You will be presented with a problem. There will be a number line below the problem.
3. Click on the correct answer on the number line. You will be told if you are correct, and a number line that models the problem will be displayed.
4. Try a few more practice problems by clicking on ''Try Another One.''
5. After you have finished your practice session, complete the problem below.

Problem: Explain why $-1 - (-3) = 2$ is modeled by an arrow starting at -1 and extending 3 units to the right.

1-67 Multiplying Two Integers ★ (7.NS.2)

When multiplying positive and negative integers, you must determine whether your product is positive or negative. The rules for determining the sign of the product of two integers follow:

- If both integers have the same sign, the product is positive.
- If the integers have different signs, the product is negative.
- If one of the factors is 0, the product is 0.

Problem: Find the product of the integers below. Then arrange the products from least to greatest and explain the pattern.

(1) -8×2	**(2)** $8 \times (-1)$	**(3)** -10×0
(4) $-4 \times (-2)$	**(5)** $-8 \times (-2)$	**(6)** $-3 \times (-8)$

1-68 Multiplying More Than Two Integers ★★ (7.NS.2)

When multiplying integers, the order of the factors does not affect the product. There are several ways to multiply more than two integers. You should choose the way that is easiest for you.

- You may multiply the integers from left to right.
- You may multiply all the positive integers, multiply all the negative integers, and then find the product.
- You may look at the factors. If at least one of the factors is 0, the product is 0.
- You may count the number of negative integers. If this number is even, the product is positive. If this number is odd, the product is negative. (In either of the above cases, if one of the factors is 0, then the product is 0.) After finding whether the product will be positive or negative, multiply the absolute value of each number to find the product.

Problem: Write an explanation of which method of multiplying integers is easiest for you. Perhaps you use a combination of methods. If you do, explain which ones you use and why.

1-69 Multiplying More Than Two Integers ★★ (7.NS.2)

Multiplying integers correctly requires concentration and accuracy. Mistakes can be made in multiplication and using the correct sign with the product.

Problem: Of the four multiplication problems for each number below, one *does not* belong with the other three. Find which should be omitted, and explain your reasoning.

(1) (a) $-3 \times (-7) \times (-8)$ (b) $-8 \times 2 \times (-10) \times (-5)$
(c) $-6 \times (-3) \times (-2) \times (-7)$ (d) $-1 \times (-4) \times (-8)$

(2) (a) $-3 \times (-4) \times 0$ (b) $-6 \times (-6) \times 4$
(c) $8 \times (-1) \times (-2)$ (d) $-3 \times (-3) \times (-1)$

(3) (a) $-3 \times (-8) \times (-4)$ (b) $-1 \times 8 \times 7$
(c) $-6 \times (-8) \times 4$ (d) $-5 \times (-3) \times 9$

1-70 Dividing Two Integers ★ (7.NS.2)

The rules for dividing integers follow:

- If the signs of the divisor and the dividend are the same, the quotient is positive.
- If either the divisor or dividend is positive and the other is negative, the quotient is negative.
- If the dividend is 0, the quotient is 0.
- If the divisor is 0, the quotient is undefined because it is impossible to divide by 0.

Problem: Some of the problems below have errors. Identify which problems have errors and correct them.

(1) $15 \div (-5) = -3$ **(2)** $-21 \div (-7) = -3$ **(3)** $105 \div (-5) = -21$
(4) $-166 \div (-12) = 13$ **(5)** $-20 \div 0 = 0$ **(6)** $0 \div (-15) = 0$
(7) $-125 \div 5 = -25$ **(8)** $102 \div 17 = 6$ **(9)** $-35 \div 7 = -5$

1-71 Dividing Two Integers ★★ (7.NS.2)

A student said that if he uses the same numbers but different signs, he can make up two problems that have the same quotient. For example, $45 \div 5 = 9$ and $-45 \div (-5) = 9$.

Problem: Do you think this is true if the quotient is a negative number? Is it true if the quotient is 0? Explain your answers, and provide examples to support your reasoning.

1-72 Multiplying and Dividing Integers ★★ (7.NS.2)

Multiplication and division are inverse operations. This means that they are opposite operations. For example: $-12 \times 4 = -48$ is the inverse of $-48 \div 4 = -12$.

Problem: Fill in each blank with the correct number.

(1) $-12 \times$ _______ $\div (-4) = -42$ **(2)** _______ $\times (-15) \div 5 = -51$
(3) $-144 \div$ _______ $\times 6 = 36$ **(4)** $8 \times$ _______ $\div (-4) = 20$

1-73 Four Operations with Integers ★★ (7.NS.3)

Only when you truly understand a topic are you able to apply your knowledge of it. This is as true for mathematics as it is for any other subject.

Problem: Create four problems that use integers: one for addition, one for subtraction, one for multiplication, and one for division. When you are done, exchange your problems with the problems of a partner and solve each other's integer problems. Remember to create an answer key for your problems.

1-74 Four Operations with Integers ★★ (7.NS.3)

Mrs. Wilson often gives quizzes with 10 problems. To discourage students from filling in just any answer, she devised this scoring method: students receive 1 point for each correct answer, no points for an unanswered question, and −1 point for an incorrect answer.

Alberto had 7 problems correct on his quiz, he left 1 problem unanswered, and he had 2 wrong answers. His score was 5, computed as $7 \times 1 + 0 + 2 \times (-1) = 5$.

Problem: Use Mrs. Wilson's grading system to answer these questions.

(1) What is the highest possible score? What is the lowest?

(2) How many ways could a student score an 8?

(3) Annie's score was −5, and Jason's score was twice Annie's. What was Jason's score?

(4) The scores of five students were $-5, -4, 3, 1$, and 0. Find the average of the scores.

1-75 Four Operations with Integers ★★★ (7.NS.3)

It is nice to have help for finding the solutions to some problems. This is one of those problems.

Problem: Use the numbers $20, 10, -4, -2$, and -1 and the signs $+, -, \times$, and $\div$ to obtain the highest and lowest numbers possible. All numbers and operations must be used once for each problem, and the answers must be integers. You may not use grouping symbols or exponents.

1-76 Using Positive Exponents with Integers ★

In the expression 2^3, 3 is the exponent and 2 is the base. The expression means that 2 is a factor three times, or $2 \times 2 \times 2 = 8$.

Whenever a number or group of numbers is written in parentheses with an exponent, the number or group of numbers is the base. For example, $(-3)^2 = -3 \times -3 = 9$, and $-3^2 = -(3 \times 3) = -9$. Here is another example: $(8+2)^3$ means 10^3 or $10 \times 10 \times 10 = 1{,}000$. This is not the same as $8 + 2^3$ which means $8 + 2 \times 2 \times 2 = 16$.

Problem: Simplify each expression.

(1) $(-5)^2$ **(2)** -5^2 **(3)** $5 + 2^4$
(4) $(5+2)^4$ **(5)** -5×3^3 **(6)** $(-5 \times 3)^3$

1-77 Using Scientific Notation to Express Large Numbers ★★ (8.EE.3)

A number is written in scientific notation when it is expressed as a product of a number greater than or equal to 1 but less than 10 with an integral power of 10. For example:

$$1{,}800{,}000 = 1.8 \times 10^6 \qquad 15{,}600 = 1.56 \times 10^4$$

Problem: A googol is the numeral 1 with 100 zeroes after it. A googolplex is the numeral 1 with a googol of zeroes after it. Write a googol and a googolplex in scientific notation.

1-78 Computing with Numbers Written in Scientific Notation ★★ (8.EE.3)

Scientific notation is often used to express very large numbers. We can use scientific notation to write the average distances of the planets from the sun:

Mercury: 5.79×10^7 km
Venus: 1.08×10^8 km
Earth: 1.50×10^8 km
Mars: 2.28×10^8 km
Jupiter: 7.78×10^8 km
Saturn: 1.43×10^9 km
Uranus: 2.87×10^9 km
Neptune: 4.50×10^9 km

Problem: Answer the following questions.

(1) Which planet is about 5 times the distance from the sun as Earth?

(2) Which planet is less than 2 times as far from the sun as Earth?

(3) Which planet is over 30 times as far from the sun as Earth?

(4) Which planet is about 13 times as far from the sun as Mars?

1-79 Changing Numbers in Scientific Notation to Standard Form ★★ (8.EE.4)

Any number written in scientific notation can be written in standard form. Since whole numbers and integers written in scientific notation will always be expressed as a positive power of 10, move the decimal point to the right the same number of places as the exponent. Following are two examples:

- Our solar system is about 4.6×10^9 years old. To write this number in standard form, move the decimal point 9 places to the right. 4.6×10^9 years = 4,600,000,000 years.
- The product of 1,600,000 and 6,000,000 will be displayed on a calculator as 9.6 E12. This is the same as 9.6×10^{12}, or 9,600,000,000,000. The decimal point is moved 12 places to the right.

Problem: Express each number in standard form.

(1) The diameter of the sun: 1.392×10^6 miles

(2) The distance of a light-year: 9.461×10^{12} km

(3) The calculator display: 5.46 E11

(4) The calculator display: 4.326 E15

1-80 A Quotation Applicable to Mathematics ★

Hypathia of Alexandria (370–415) was the first woman mathematician about whom we know many details. Along with her pursuits in mathematics, Hypathia was also a writer, teacher, scientist, and astronomer at a period in history when it was unusual for women to receive an education. By all counts, she was a remarkable person and a true trendsetter of her time.

Hypathia received much of her passion for learning from her father, Theon, a noted writer and thinker. Although at the time Christians believed that math and science were heresy, Theon is credited with helping to keep mathematical and scientific thought alive through tireless inquiry and discussion.

Problem: Explain these words that Theon said to Hypathia: ‘‘Reserve your right to think, for even to think wrongly is better than not to think at all.’’ How might they be applied to mathematics?