# **Chapter 1**

# Awesome Operations: Math Fundamentals

### In This Chapter

- Reviewing the four arithmetic operations
- Manipulating fractions
- Using charts to convey and understand information
- Strategies to help you solve word problems

Ath has basic operations that you need to know. These operations — addition, subtraction, multiplication, and division — make all the other math in this book possible.

The good news is that you most likely learned about basics (like counting) even before you entered school, and you learned about basic arithmetic operations in elementary school. So you've been at it for a long time.

In this chapter, I review counting and the fundamentals of the four basic arithmetic operations. Other important topics I cover here are fractions, percentages, charts and graphs, and word problems. But don't worry: None of these are mysterious.

# Numbers You Can Count On

The most fundamental component of math is numbers. The first thing you do with numbers is count, and you probably started counting when you were very young. As soon as you could talk, your mother cajoled you to tell Aunt Lucy how old you were or to count from 1 to 5.

Counting was the first and most useful thing you did with math, and you still use it every day, whether you're buying oranges at the grocery store or checking the number of quarts of motor oil in a case.



Counting has been essential since people first walked the earth. In fact, the Ishango bone is a tally stick (a counting stick), and it's over 20,000 years old!

Several kinds of numbers exist. Over time, mathematicians have given them many names. The two most important kinds are whole numbers and fractions. To see a little bit about how these numbers work, use a *number line*, a simple display of numbers on a line (see Figure 1-1).



The numbers to the right of 0 are called *natural* numbers or *counting* numbers. Of course, they are the numbers you use to count. They're easy for anyone to work with because they represent how many of something someone has (for example, 6 apples or 3 oranges).

Over many centuries and in different cultures, people made up the number 0, which represents the lack of a quantity. The numbers to the left of 0 on the number line, *negative* numbers, are a harder concept to grasp. You recognize negative number in real life. For example, if your checking account is overdrawn, you have a negative balance. If someone owes you \$3.00, you have "negative cash" in your pocket.

Here are the key points to know about the number line:

- ✓ All the numbers you see in Figure 1-1 are *whole* numbers, also called *integers*. An *integer* is a number with no fraction part. The word comes from Latin, and it means "untouched," so it's the whole deal.
- ✓ The numbers to the right of zero are *positive integers*. The numbers to the left of zero are *negative integers*.



Mathematicians (and I'm not making this up) have trouble with zero. The best they can do is attach it to the positive integers and label the group *non-negative integers*.

- The number line stretches to the left and right, to infinity and beyond (as Buzz Lightyear says).
- ✓ Decimals (such as 0.75) and regular fractions (such as 3/5) are only a part of a whole number. They all have a place somewhere on the number line. They fit in between the integers. For example 2.75 "fits" between 2 and 3 on the number line, because it's greater than 2 but less than 3.

# Reviewing the Four Basic Operations

To do any sort of math, you need to know your math basics. The four basic operations — addition, subtraction, multiplication, and division — let you take care of all kinds of real life math. But what's also very important is that those same basic math operations allow you to handle fractions and percentages, which come up all the time in ordinary math tasks. Later (in Chapter 2), these operations form the basis for managing algebra equations and geometry.

The core operations are addition and subtraction. You very likely know what they are and how they work. Multiplication and division are "one step up" from addition and subtraction. The following sections give you a quick review of these four operations.

# Addition

Addition is a math operation in which you combine two or more quantities to get (usually) a larger quantity. Addition was probably the first math you ever did.

You can add numbers (called the *operands*) in any order. This property (that is, the ability to perform the operation in any order) is called *commutativity*.

21+31+41+51=144is equal to 51+41+31+21=144

No matter in what order you add the operands, the sum still equals 144.

## Subtraction

Subtraction is a math operation in which you take away the value of one number from another, resulting in (usually) a smaller quantity.

In subtraction, the order of the operands is important. You can't rearrange the numbers and get the same answer. For example, 77 - 22 (which equals 55) is not the same as 22 - 77 (which equals -55).

### Multiplication

Think of multiplication as repeated addition. For example, you likely know that  $3 \times 4 = 12$ , but you can also get there by adding 3 four times:

3 + 3 + 3 + 3 = 12

The technique also works for large numbers. For example,  $123 \times 7$  = 738 is equivalent to this:

123 + 123 + 123 + 123 + 123 + 123 = 738

But who wants to do all that adding?

Here's the best advice for multiplication:

- ✓ For little numbers, know your multiplication table. It's easy, up to  $10 \times 10$ .
- For big numbers, use a calculator.

As with addition, you can multiply the numbers in a list in any order. The expression  $3 \times 4$  is the same as the expression  $4 \times 3$ . Both equal 12.

### Division

Division is essentially "multiple subtraction." In a simple problem such as  $12 \div 4 = 3$ , you can get the result by subtracting 3 four times from the number 12.

 $12 \div 3 = 4$  with no remainder is equal to 12 - 3 - 3 - 3 = 0 (4 subtractions with no remainder)



In division, the order of the operands is important. You can't rearrange them and end up with the same answer.

# Finagling Fractions

Fractions take several forms, but in real life, the forms you deal with are common fractions and decimal fractions.

A *common fraction* has two parts. The *numerator* is the top number, and the *denominator* is the bottom number. You don't have to learn these words, however. Just think "top number" and "bottom number."

numerator denominator

What do you do with fractions? Arithmetic operations and conversions, that's what.



A common fraction is sometimes called a *simple fraction* or a *vulgar fraction*. The vulgar fraction isn't really rude; *vulgar* is just another word for *common* (from the Latin *vulgus*, meaning "common people").

### Getting familiar with types of fractions

Like the popular ice cream parlor, fractions come in several flavors. Not 31 flavors, however. For this book, you have to remember only a few fraction types:

- ✓ **Proper fraction:** In a *proper fraction*, the numerator is smaller than the denominator (for example,  $\frac{4}{9}$ ).
- ✓ **Improper fraction:** In an *improper fraction*, the numerator is larger than the denominator (for example,  $\frac{9}{4}$ ). Think "Honey, does this numerator make my fraction look big?"
- Mixed fraction: A mixed fraction is a combination of a whole number and a fraction. Here's an example of a mixed fraction:
  - $1\frac{3}{4}$
- Decimal fraction: A decimal fraction uses a decimal point (for example, 0.23, 1.75, or \$47.25).



Decimals are fractions, too, even though they don't look like the other types of fractions. Look at this: 0.75 is a decimal. But what does that really mean? It means 75/100.

# **Reducing fractions**

Here's fair warning: Doing fraction math often produces "clumsy" fractions. By clumsy, I mean unwieldy proper fractions (48/60, for example) and bad-looking improper fractions (37/16, for example). They are handy during the calculations but are very inconvenient as final answers.

You turn a clumsy fraction into something lovely to behold by *reducing* it.

### Reducing proper fractions

You reduce proper fractions by finding a number that the numerator and denominator share and then separating it out. This tactic is called *factoring*, and multiplication rules allow you to do it. For example, for the fraction  $\frac{48}{60}$ , you "break out" the common factor 12 in both the numerator and denominator:

$$\begin{array}{l} \frac{48}{60} = \frac{4 \times 12}{5 \times 12} \\ \frac{48}{60} = \frac{4}{5} \times \frac{12}{12} \\ \frac{48}{60} = \frac{4}{5} \times 1 \\ \frac{48}{60} = \frac{4}{5} \end{array}$$



When a fraction has the same numerator and denominator, it's equal to 1. Hence,  $\frac{12}{12}$  becomes 1.

Another way of describing this is to say, "You reduce a proper fraction by dividing the top and bottom numbers by the same number."

### Reducing improper fractions

To reduce an improper fraction, you break it into whole numbers and a remaining, smaller fraction. To do this, you divide the top number by the bottom number, and then you use the whole number and the remaining fraction to form a mixed fraction. Here's an example:

 $\begin{aligned} \frac{49}{16} &= \frac{16+16+16+1}{16} \\ \frac{49}{16} &= \frac{16}{16} + \frac{16}{16} + \frac{16}{16} + \frac{1}{16} \\ \frac{49}{16} &= 1+1+1+\frac{1}{16} \\ \frac{49}{16} &= 3\frac{1}{16} \end{aligned}$ 

### Adding, subtracting, multiplying, and dividing fractions

Fractions are just numbers. Like integers, you can add, subtract, multiply, and divide them. Before you panic, keep in mind that you perform these math calculations on fractions all the time. Don't believe me? Think about money.

At first, dollars and cents don't look like fractions because they're in decimal form. But they are fractions, for sure. To look at the details, take a gander at the following sections.

### Addition

To add two fractions, the fractions must have the same denominator (also called a *common* denominator). After the denominators are the same, you add fractions simply by adding the numerators.

When the denominators aren't the same, you need to make them the same. You can't directly add 1/2 pie to 1/4 pie to get 3/4 pie, for example. You need to convert the 1/2 pie into quarters (2/4 pie). Figure 1-2 shows what adding pieces of pie looks like.



Figure 1-2: Adding Fractions.

Getting the denominators the same is easy because you're allowed to multiply both the top number and the bottom number by the same number. In the pie example, you multiply both numerator and denominator of the fraction 1/2 by 2:

$$x = \frac{1}{2} \times \frac{2}{2}$$
$$x = \frac{1 \times 2}{2 \times 2}$$
$$x = \frac{2}{4}$$

After you have all operands in 1/4 pie units, adding 2/4 and 1/4 to get 3/4 is easy. (Remember that the denominator stays the same when you add the numerators.)

### Subtraction

To subtract two fractions, the fractions must have a common denominator (just as they must in addition); then you simply perform the operation on the numerators.

If the denominators aren't the same, you need to make them the same before you can subtract. For example, you can't directly subtract 1/4 pie from 1 whole pie (which in fraction form is 1/1) to get 3/4 pie because the denominators are different. Again, you need to convert the whole pie into quarters, and you do that by multiplying the numerator and denominator by 4 to get 4/4 pie. Then you can do the subtraction:

$$x = \frac{1}{1} \times \frac{4}{4}$$
$$x = \frac{1 \times 4}{1 \times 4}$$
$$x = \frac{4}{4}$$

After all the operands are in 1/4 pie units, subtracting 1/4 from 4/4 to get 3/4 is easy. (Remember that the denominator stays the same when you subtract the numerators.)

$$x = \frac{4}{4} - \frac{1}{4}$$
$$x = \frac{3}{4}$$

### **Multiplication**

Compared to adding and subtracting fractions, multiplying fractions is easy. Just multiply the numerators, multiply the denominators, and then reduce.

$$x = \frac{3}{5} \times \frac{6}{7}$$
$$x = \frac{3 \times 6}{5 \times 7}$$
$$x = \frac{18}{35}$$

The answer is 18/35. When possible, try to reduce the result. In this case, you can't reduce 18/35 at all.

#### Division

Here's the secret to dividing fractions: *Invert and multiply*. That is, flip the second fraction so that the numerator is on the bottom and the denominator is on the top, and then multiply as you would any other fraction.

Say you want to divide 1/4 by 2. (*Note:* The fraction form of a whole number is that number over 1.) The answer is obviously 1/8. Not so obvious, you say? Here's how you get the answer:

$$x = \frac{1}{4} \div 2$$
$$x = \frac{1}{4} \div \frac{2}{1}$$
$$x = \frac{1}{4} \times \frac{1}{2}$$
$$x = \frac{1}{8}$$

You follow the same process when you want to divide a fraction by a fraction:

$$x = \frac{1}{4} \div \frac{1}{3}$$
$$x = \frac{1}{4} \times \frac{3}{1}$$
$$x = \frac{3}{4}$$

Notice that dividing by a fraction yields a *higher* result than dividing by a whole number.



You can't divide by 0. It's mathematically impossible. The old saying is, "Never divide by zero! It's a waste of time, and it annoys the zero."

# **Converting fractions**

The handiest fraction conversions are turning common fractions into decimal fractions and turning decimal fractions into common fractions.

### A fraction is a ratio, too

In math, a *ratio* is a relationship between two numbers. I mention this because ratios come up all the time.

The size of a wide-screen DVD image is called the *aspect ratio*, usually 16:9. That amounts to 16 inches of width for every 9 inches of height, and it doesn't really matter how big your TV screen is. The ratio is always the same.

If you have a gas-powered weed eater (also known as a *string trimmer* or *weed whacker*), you've probably bought 40:1 2-cycle engine oil for it. The 40:1 ratio means that you mix 40 parts gas to 1 part oil.

### Turning a common fraction into a decimal fraction

To turn a common fraction into a decimal fraction, just divide the denominator into the numerator. A number like 4/5 easily turns into 0.80 when you divide 4 by 5.

Don't be surprised or alarmed if some division doesn't come out "even." For example, the decimal equivalent of 1/3 is 0.333333333 (and the 3s go on forever). If you see a sale item marked "33% off," it's been reduced by 33 percent or about 1/3. If the item is marked "20% off," it's been reduced by 20/100, or 1/5. (See the section "Processing Percentages" for the lowdown on how to work with percentages.)

#### Turning a decimal fraction into a common fraction

To turn a decimal fraction into a common fraction, just express the decimal as a fraction and reduce the fraction.

A decimal with one decimal place (0.6, for example) needs a fraction with 10 in the denominator. A decimal with two decimal places (0.25, for example) needs a fraction with 100 in the denominator, and so forth. Here are some examples:

$$0.6 = \frac{6}{10}$$
$$0.71 = \frac{71}{100}$$
$$0.303 = \frac{303}{1000}$$

Notice that the number of zeroes in the denominator is the same as the number of decimal places in the decimal fraction.

For example, say you want to convert 0.375 into a fraction. Here's how you'd go about it:

$$x = \frac{375}{1,000}$$
$$x = \frac{3 \times 125}{8 \times 125}$$
$$x = \frac{3}{8} \times \frac{125}{125}$$
$$x = \frac{3}{8} \times 1$$
$$x = \frac{3}{8}$$

In this example, when you "factor out" 125 from both the numerator and denominator, the result is the common fraction 3/8. See the section "Reducing proper fractions" for details on factoring.

# **Processing Percentages**

A percentage is a fraction whose denominator never changes. It's always 100. A number like 33 percent, for example, refers to 33 parts in 100, or 33/100, or 0.33. You see percentages written as "33%" and "33 percent." No matter how it's written, it's just another way of saying "thirty-three parts in one hundred."



*Percent* and *per cent* means "per centum," which is from the Latin phrase meaning "by the hundred." So a percentage always refers to a number of parts out of 100.

Percentages are especially handy for comparing two quantities. For example, if one beer contains 5.5 percent alcohol and another contains 12 percent alcohol, you can be sure that the "high octane" beer has a lot more punch.

Percentages also let you compare values to an arbitrary standard. Nutrition labels are a good example They compare items in food, such as dietary fiber, cholesterol, or vitamins and minerals, to the Dietary Reference Intake (DRI) nutrition recommendations used by the United States and Canada.



A percentage is a *dimensionless proportionality*, meaning that it doesn't have a physical unit. Fifty percent of a length is still 50 percent, whether you're talking about feet or light years.

# Converting a common fraction to a percentage

Sometimes you want to convert a fraction to a percentage. Say, for example, that you're fed up with your commute to work, because the drive requires 1 hour each way. You're at the job for 9 hours, so work consumes 11 hours of your day, 2 of those hours with you sitting in traffic. While stuck in bumper-to-bumper traffic, you wonder what percentage of your work-related time in spent commuting. The fraction is 2/11, so what's the percentage?

To convert a common fraction into a percentage, just divide the numerator by the denominator and multiply the result by 100:

```
percentage = \frac{2}{11} \times 100
percentage = 0.1818 × 100
percentage = 18.18
```

You can see that 2/11 is about 18 percent. What could be simpler than that?

### A percentage is a ratio, too

As I mention earlier, a ratio is a relationship between two numbers. A percentage can often be expressed as a ratio. For example, if a bottle of vodka contains 40 percent alcohol (which, confusingly, is called 80 proof in the United States), that means that 40 parts in 100 are alcohol. That's a ratio of 40:60, 40 parts of alcohol to 60 parts of water.

You can convert from a ratio to a percentage, too. For example, a "four to one" martini has a gin:vermouth ratio of 4:1. The vermouth is 1/5 of the cocktail, or 20 percent.

# Converting a percentage to a fraction

Sometimes a fraction may be more convenient than a percentage. Perhaps you want to know what fraction of your salary goes to taxes. Or maybe you're less inclined to eat a whole 8-ounce bag of chips when you think in terms of it having 1/2 rather than 50 percent of your daily recommended amount of sodium.

To convert a percentage into a common fraction, just divide the percentage by 100 and reduce the result. For example, say to want to convert 80 percent into a common fraction:

80 percent = ???  
80 percent = 
$$\frac{80}{100}$$
  
80 percent =  $\frac{4 \times 20}{5 \times 20}$   
80 percent =  $\frac{4 \times 20}{5 \times 20}$   
80 percent =  $\frac{4}{5}$ 

The value 80 percent means 80/100. Form a fraction and reduce it. As you can see, 80 percent is 4/5.

# Grasping Charts and Graphs

A chart or graph is a *visual representation* of numbers. Charts and graphs come in many forms, but for day-to-day math, you need to know about only three kinds — the line chart (or graph), the pie chart, and the bar chart.

The key point is that a chart is visual, and people usually find a visual display to be more understandable than a list of numbers. Expect to encounter charts when you read about the economy or when you compare consumer products. Also, the best thing is that you can make your own charts, which you may want to do, for example, to get a better picture about your personal finances.

### Looking at line charts

A line chart (sometimes called a *line graph*) displays information as data points connected by a line. With this chart, you can easily see how an item is trending. Figure 1-3 shows typical temperatures over a week. What can you glean from this data? That the weekend was hot!



A line chart can easily show you how the economy is doing. Think unemployment figures. Also, you can make a chart that shows how one (or all) of your investments is doing.

# Gobbling up pie charts

A pie chart looks like, er, a pie, which is divided into "slices" that show the relative proportion of various elements. This type of chart lets you see both the relationship between elements and the relationship of individual elements to the whole pie.

Pie charts are great when you have to compare only a few elements. When you must compare many elements, the slices get too thin and they're harder to understand.

Figure 1-4 shows a typical monthly budget. After paying the rent, making the car payment, and buying food, you can see that not much is left for everything else. Note that it doesn't matter whether you make \$1,000 a month or \$10,000 a month. The pie chart shows *relative* proportions.





A pie chart is great for making comparisons of government expenses relative to each other. And seeing where your tax money goes is always fun. Visit the Center on Budget and Policy Priories: http://www.cbpp.org/cms/index.cfm?fa=view&id=1258.

## Bellying up to bar charts

A bar chart has rectangular bars that can be either horizontal or vertical. The size of the bars represents bigger or smaller values.

Bar charts are great for showing anything over time, including variable income, variable expenses, and even the number of burgers sold at the local drive-in. Figure 1-5 is a bar chart that shows what my Visa bill was for seven months. Can you tell when I went on vacation?



Figure 1-5: A bar chart.

# Working Wicked Word Problems

Do you remember word problems (sometimes called "story problems") from school? A few people loved them, but many people hated them. What's peculiar is that most of life's math problems start as word problems, a fact that's understandable because we speak in words, not numbers. So if you say, "The boss gave me a 10 percent raise," figuring out your new salary starts as a word problem.

At first glance, some word problems appear to be baffling. But that's just at first glance. You simply need to know a few tricks that can make all word problems easy to solve. The basic process for solving word problems is to first do some analysis and then do the math.

# Doing the analysis

Two parts are involved in solving a story problem. The first part is to study the problem a little. That makes the second part (doing the math) easy.

For example, a shed has a roof that's 6 feet by 10 feet on each side. The barn's roof is twice as long and twice as deep as the shed's roof. Both buildings are red. If it takes 120 shingles to cover the shed's roof, how many shingles does it take to cover the barn's roof?

When you analyze a story problem, you go through the problem to get the info you need to eventually solve it. Follow these steps:

#### 1. Read the problem and list the facts.

Always read word problems more than once. Facts are hiding in the question. From the question, you know the dimensions of the shed roof *on each side*. You get a sense of the dimensions of the barn's roof, and you know how many shingles are needed to cover the shed. Good!

#### 2. Figure out exactly what the problem is asking for.

In every word problem, you run the risk of solving — correctly — for the *wrong* thing. So make sure you know what the question asks for. In the example, you know that the answer is "number of shingles to cover the barn." The question could have been about calculating the number of shingles to cover both the shed and the barn, but it's not.

#### 3. Eliminate excess information.

Both real life and school story problems tend to have extraneous facts. Ignore them. For example, the fact that both buildings are red is interesting but not important.

#### 4. See what information is missing.

Sometimes a major fact is missing. What's more likely, however, is that the information is *hiding*. For example, the info that the barn's roof is twice as long and twice as deep as the shed's gives you a clue about calculating the area of the barn's roof.

#### 5. Find the keywords.

Be on the lookout for key words and phrases, such as "how much more," "how much less," and "total." Those words and phrases usually indicate what kind of math operations are involved.

### Applying the math

Almost every story problem uses a simple algebra formula that's "hiding" in it. When you develop the formula, you then insert the numbers to solve the problem. Math instructors often call this last step "plug and chug."

To apply the math, take the info you gleaned from your analysis and do the following:

#### 1. Convert information supplied into information needed.

First, use the given dimensions of the shed roof to calculate how many square feet are covered by 120 shingles.

area (shed) = length  $\times$  depth  $\times 2$ area (shed) =  $6 \times 10 \times 2$ area (shed) = 120

The answer is 120 square feet. (*Note:* You multiply by 2 to take into account *both* sides of the shed's roof.)

Then use the given dimensions of the shed roof to calculate the area of the barn's roof. The barn's roof is twice as long and twice as deep as the shed's roof.

area (barn) = 2(shed length)  $\times$  2(shed depth)  $\times$  2

area (barn) =  $2(6) \times 2(10) \times 2$ area (barn) =  $12 \times 20 \times 2$ area (barn) = 480

The answer is 480 square feet.

2. Apply a formula.

There's a technique called *ratio-proportion*. Don't worry about the details now (I explain it fully in Chapter 3). Here, you apply the technique:

known quantity (shed area) _	known quantity (barn area)
known quantity (shed shingles)	desired quantity (barn shingles)
$\frac{120}{120} =$	$\frac{480}{r}$
$120 \times 3$ 120x = 57,600	
<i>x</i> =	480

You cross multiply and solve. The answer is 480 shingles.

Pay attention to units and phrase the answer in the units asked for. In the example, you must express the answer in shingles, not square feet.

#### 3. Check for reasonableness.

Always make sure the answer is reasonable. Because the barn is bigger than the shed, the barn should take more shingles than the shed. The 120 versus 480 is one reasonableness check. In the example, if you get an answer of 48 shingles or 48,000 shingles, something is wrong.



If you crave a shortcut, consider this: The fact that the barn's roof is twice as long and twice as deep as the shed's roof means that the barn's roof has four times the area. With that info at hand, the calculation is easy: Simply multiply the shed's 120 shingles by 4, giving you 480 shingles.

### Other story problem tricks

If you find yourself totally stuck on a word problem, a few tricks may help you out:

- ✓ Draw a diagram. Sometimes, drawing a picture using the facts in the problem can be a help. This tactic works when you need to find the area of a garden, the board feet you need for a deck, or how old your brothers and sisters will be when you reach a certain age.
- ✓ Find a formula. When you encounter a problem about interest on your savings account or the amount of mortgage payments, chances are excellent that someone has already developed a formula to solve it. Chances are also very good that you can find an online calculator or an embedded function in a spreadsheet application to help you out.



- ✓ Develop a formula. Sometimes you can make your own simple formula instantly. For example, as soon as you know that a hamburger has 21 grams of protein and that dietary guidelines recommend 56 grams of protein a day, a little quick math (divide 56 by 21) shows that about three burgers at that backyard barbecue will give you a full day's worth of protein. And a formula works all the time, after you develop it.
- ✓ Consult a reference. Using a reference isn't just desirable; it's also sometimes necessary. For example, if you're painting a room, calculating the area to be covered isn't hard (see Chapter 8), but it's essential to consult the paint manufacturer's information to learn how much area a gallon of paint will cover.