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Introduction

Until the 1960s, most research on model identification and control system design was concentrated on continuous-time (or analogue) systems represented by a set of linear differential equations. Subsequently, major developments in discrete-time model identification, coupled with the extraordinary rise in importance of the digital computer, led to an explosion of research on discrete-time, sampled data systems. In this case, a 'real-world' continuous-time system is controlled or 'regulated' using a digital computer, by sampling the continuous-time output, normally at regular sampling intervals, in order to obtain a discrete-time signal for sampled data analysis, modelling and *Direct Digital Control* (DDC). While adaptive control systems, based directly on such discrete-time models, are now relatively common, many practical control systems still rely on the ubiquitous 'two-term', *Proportional-Integral* (PI) or 'three-term', *Proportional-Integral-Derivative* (PID) controllers, with their predominantly continuous-time heritage. And when such systems, or their more complex relatives, are designed offline, rather than 'tuned' online, the design procedure is often based on traditional continuous-time concepts. The resultant control algorithm is then, rather artificially, 'digitised' into an approximate digital form prior to implementation.

But does this 'hybrid' approach to control system design really make sense? Would it not be both more intellectually satisfying and practically advantageous to evolve a unified, truly digital approach, which would allow for the full exploitation of discrete-time theory and digital implementation? In this book, we promote such a philosophy, which we term *True Digital Control* (TDC), following from our initial development of the concept in the early 1990s (e.g. Young *et al*. 1991), as well as its further development and application (e.g. Taylor *et al*. 1996a) since then. TDC encompasses the entire design process, from data collection, data-based model identification and parameter estimation, through to control system design, robustness evaluation and implementation. The TDC approach rejects the idea that a digital control system should be initially designed in continuous-time terms. Rather it suggests that the control systems analyst should consider the design from a digital, sampled-data standpoint throughout. Of course this does not mean that a continuous-time model plays no part in TDC design. We believe that an underlying and often physically meaningful continuous-time model should still play a part in the TDC system synthesis. The designer needs to be assured that the

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discrete-time model provides a relevant description of the continuous-time system dynamics and that the sampling interval is appropriate for control system design purposes. For this reason, the TDC design procedure includes the data-based identification and estimation of continuous-time models.

One of the key methodological tools for TDC system design is the idea of a *Non-Minimal State Space* (NMSS) form. Indeed, throughout this book, the NMSS concept is utilised as a unifying framework for generalised digital control system design, with the associated *Proportional-Integral-Plus* (PIP) control structure providing the basis for the implementation of the designs that emanate from NMSS models. The generic foundations of linear state space control theory that are laid down in early chapters, with NMSS design as the central worked example, are utilised subsequently to provide a wide ranging introduction to other selected topics in modern control theory.

We also consider the subject of stochastic system identification, i.e. the estimation of control models suitable for NMSS design from noisy measured input–output data. Although the coverage of both system identification and control design in this unified manner is rather unusual in a book such as this, we feel it is essential in order to fully satisfy the TDC design philosophy, as outlined later in this chapter. Furthermore, there are valuable connections between these disciplines: for example, in identifying a parametrically efficient (or parsimonious) 'dominant mode' model of the kind required for control system design; and in quantifying the uncertainty associated with the estimated model for use in closed-loop stochastic uncertainty and sensitivity analysis, based on procedures such as *Monte Carlo Simulation* (MCS) analysis.

This introductory chapter reviews some of the standard terminology and concepts in automatic control, as well as the historical context in which the TDC methodology described in the present book was developed. Naturally, subjects of particular importance to TDC design are considered in much more detail later and the main aim here is to provide the reader with a selective and necessarily brief overview of the control engineering discipline (sections 1.1 and 1.2), before introducing some of the basic concepts behind the NMSS form (section 1.3) and TDC design (section 1.4). This is followed by an outline of the book (section 1.5) and concluding remarks (section 1.6).

1.1 Control Engineering and Control Theory

Control engineering is the science of altering the dynamic behaviour of a physical process in some desired way (Franklin *et al.* 2006). The scale of the process (or system) in question may vary from a single component, such as a mass flow valve, through to an industrial plant or a power station. Modern examples include aircraft flight control systems, car engine management systems, autonomous robots and even the design of strategies to control carbon emissions into the atmosphere. The control systems shown in Figure 1.1 highlight essential terminology and will be referred to over the following few pages.

This book considers the development of digital systems that control the output variables of a system, denoted by a vector $\boldsymbol{y}$ in Figure 1.1, which are typically positions or levels, velocities, pressures, torques, temperatures, concentrations, flow rates and other measured variables. This is achieved by the design of an online control algorithm (i.e. a set of rules or mathematical equations) that updates the control input variables, denoted by a vector $\boldsymbol{u}$ in

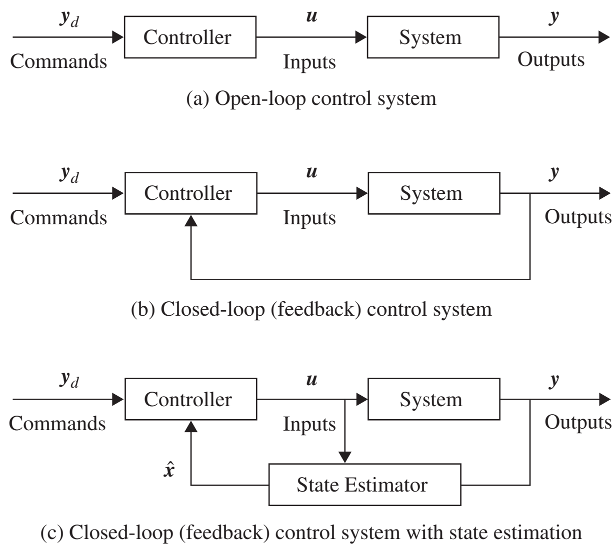

Figure 1.1 Three types of control system

Figure 1.1, automatically and without human intervention, in order to achieve some defined control objectives. These control inputs are so named because they can directly change the behaviour of the system. Indeed, for modelling purposes, the engineering system under study is *defined* by these input and output variables, and the assumed causal dynamic relationships between them. In practice, the control inputs usually represent a source of energy in the form of electric current, hydraulic fluid or pneumatic pressure, and so on. In the case of an aircraft, for example, the control inputs will lead to movement of the ailerons, elevators and fin, in order to manipulate the attitude of the aircraft during its flight mission. Finally, the command input variables, denoted by a vector $\boldsymbol{y}_d$ in Figure 1.1, define the problem dependent 'desired' behaviour of the system: namely, the nature of the short term pitch, roll and yaw of an aircraft in the local reference frame; and its longer term behaviour, such as the gradual descent of an aircraft onto the runway, represented by a time-varying altitude trajectory.

Control engineers design the 'Controller' in Figure 1.1 on the basis of control system design theory. This is normally concerned with the mathematical analysis of dynamical systems using various analytical techniques, often including some form of optimisation over time. In this latter context, there is a close connection between control theory and the mathematical discipline of optimisation. In general terms, the elements needed to define a control optimisation problem are knowledge of: (i) the dynamics of the process; (ii) the system variables that are observable at a given time; and (iii) an optimisation criterion of some type.

A well-known general approach to the optimal control of dynamic systems is 'dynamic programming' evolved by Richard Bellman (1957). The solution of the associated *Hamilton–Jacobi–Bellman* equation is often very difficult or impossible for nonlinear systems but it is feasible in the case of linear systems optimised in relation to quadratic cost functions with quadratic constraints (see later and Appendix A, section A.9), where the solution is a 'linear feedback control' law (see e.g. Bryson and Ho 1969). The best-known approaches of this type

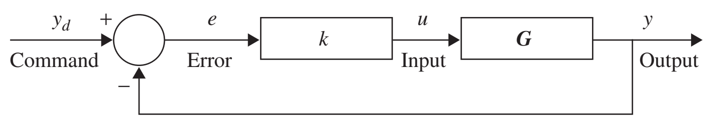

Figure 1.2 The archetypal negative feedback system

are the *Linear-Quadratic* (LQ) method for deterministic systems; and the *Linear-Quadratic-Gaussian* (LQG) method for uncertain stochastic systems affected by noise. Here the system relations are linear, the cost is quadratic and the noise affecting the system is assumed to have a Gaussian, 'normal' amplitude distribution. LQ and LQG optimal feedback control are particularly important because they have a complete and rigorous theoretical background, while at the same time introduce key concepts in control, such as 'feedback', 'controllability', 'observability' and 'stability' (see later).

Figure 1.1b and Figure 1.1c show two examples of such closed-loop feedback control. These are in contrast to the open-loop formulation of Figure 1.1a, where, given advanced knowledge of $\boldsymbol{y}_d$, a sequence of decisions $\boldsymbol{u}$ could be determined offline, i.e. there is no feedback. The potential advantages of closed-loop feedback control are revealed by Figure 1.2, where a *single-input, single-output* (SISO) system is denoted by $\boldsymbol{G}$ and k is a simple feedback control gain (that is adjusted by the control systems designer).

It is easy to see that $y = k\,\boldsymbol{G}\,e$, where $e = y_d - y$ is the error between the desired output and the actual output, so that after a little manipulation,

$$y = \frac{k\,\boldsymbol{G}}{1 + k\,\boldsymbol{G}} y_d \tag{1.1}$$

This is a fundamental relationship in feedback control theory and we see that if $k\,\boldsymbol{G} \gg 1$ *and provided the closed-loop system remains stable*, then the ratio $k\boldsymbol{G}/(1 + k\boldsymbol{G})$ approaches unity and the control objective $y = y_d$ is achieved, *regardless of the system* $\boldsymbol{G}$. Of course, this is a very simplistic way of looking at closed-loop control and ensuring that the gain k is selected so that stability is maintained and the objective is achieved, can be a far from simple problem. Nevertheless, this kind of thinking, followed in a more rigorous fashion, shows that the main advantages of a *well designed* closed-loop control system include: improved transient response; decreased sensitivity to uncertainty in the system (such as modelling errors); decreased sensitivity to disturbances that may affect the system and tend to drive it away from the desired state; and the stabilisation of an inherently unstable system. One property that the high gain control of equation (1.1) does not achieve is the *complete* elimination of steady-state errors between y and y_d, which only occurs when the gain k is infinite, unless the system $\boldsymbol{G}$ has special properties. But we will have more to say about this later in the chapter.

The disadvantage of a closed-loop system is that it may be more difficult to design because it has to maintain a good and stable response by taking into account the potentially complex manner in which the dynamic system and its normally dynamic controller interact within the closed-loop. And it may be more difficult (and hence more expensive) to implement because it requires sensors to accurately measure the output variables, as well as the construction of either analogue or digital controller mechanisms. Control system design theory may also have to account for the uncertain or 'stochastic' aspects of the system and its environment, so that key variables may be unknown or imperfectly observed. This aspect of the problem is implied by Figure 1.1c, in which an 'observer' or 'state estimator' is used to estimate the system state $\boldsymbol{x}$ from the available, and probably noisy, measurements $\boldsymbol{y}$. However, in order to introduce the concept of the system state, we need to return to the beginnings of control theory.

1.2 Classical and Modern Control

The mathematical study of feedback dynamics can be traced back to the nineteenth century, with the analysis of the stability of control systems using differential equations. James Clerk Maxwell's famous paper *On Governors*[1], for example, appeared in 1868 and the stability criteria introduced by Routh in 1877 (Routh 1877) are still commonly taught in undergraduate control courses. With the development of long distance telephony in the 1920s, the problem of signal distortion arose because of nonlinearities in the vacuum tubes used in electronic amplifiers. With a feedback amplifier, this distortion was reduced. Communications engineers at Bell Telephone Laboratories developed graphical methods for analysing the stability of such feedback amplifiers, based on their frequency response and the mathematics of complex variables. In particular, the approaches described by Nyquist (1932) and Bode (1940) are still in common usage.

The graphical 'Root Locus' method for computing the controller parameters was introduced a few years later by Evans (1948). While working on the control and guidance of aircraft, he developed rules for plotting the changing position of the roots of the closed-loop characteristic equation as a control gain is modified. As we shall see in Chapter 2, such roots, or closed-loop poles, define the stability of the control system and, to a large extent, the transient response characteristics. This root locus approach to control system design became extremely popular. In fact, the very first control systems analysis carried out by the second author in 1960, when he was an apprentice in the aircraft industry, was to use the device called a 'spirule' to manually plot root loci: how times have changed! Not surprisingly, numerous textbooks were published over this era: typical ones that provide a good background to the methods of analysis used at this time and since are James *et al.* (1947) and Brown and Campbell (1948).

However, new developments were on the way. Influenced by the earlier work of Hall (1943), Wiener (1949) and Bode and Shannon (1950), Truxal (1955) discusses an optimal 'least squares' approach to control system design in chapters 7 and 8 of his excellent book *Control System Synthesis*; while the influential book *Analytical Design of Linear Feedback Controls,* published in 1957 by Newton *et al.*, built a bridge to what is still known as 'Modern Control', despite its origins over half a century ago.

In contrast to classical control, modern control system design methods are usually derived from precise algorithmic computations based on a 'state space' description of a dynamic system, often involving optimisation of some kind. Here, the 'minimal' linear state space model of a dynamic system described by an nth order differential equation is a set of n, linked, first order differential equations that describe the dynamic evolution of n associated 'state variables' in response to the specified inputs. And the output of the system is defined as a linear combination of these state variables. For any nth order differential equation that describes the input–output behaviour of this system, the definition of these state variables is

[1] See http://rspl.royalsocietypublishing.org/content/16/270.full.pdf.

not unique and different sets of state variables in the state vector can be defined which yield the same input–output behaviour. Moreover, the state variables for any particular realisation often have some physical interpretation: for instance, in the case of mechanical systems, the states are often defined to represent physical characteristics, such as the positions, velocities and accelerations of a moving body.

Conveniently, the state equations can be represented in the following vector-matrix form, with the state variables forming an $n \times 1$ 'state vector' $\boldsymbol{x}$, in either continuous- or discrete-time, i.e.

Continuous-time:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \qquad \left(\dot{\boldsymbol{x}}(t) = \frac{d\boldsymbol{x}(t)}{dt}\right) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \end{aligned} \tag{1.2}$$

Discrete-time:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) \end{aligned} \tag{1.3}$$

Referring to Appendix A for the vector-matrix nomenclature, $\boldsymbol{x} = [x_1\ x_2\ \ldots\ x_n]^T$ is the $n \times 1$ state vector; $\boldsymbol{y} = [y_1\ y_2\ \ldots\ y_p]^T$ is the $p \times 1$ output vector, where $p \leq n$; and $\boldsymbol{u} = [u_1\ u_2\ \ldots\ u_q]^T$ is the $q \times 1$ input vector (Figure 1.1).

Here, $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are constant matrices with appropriate dimensions. The argument t implies a continuous-time signal; whilst k indicates that the associated variable is sampled with a uniform sampling interval of Δt, so that at the kth sampling instant, the time $t = k\Delta t$. Using the state space model (1.2) or (1.3), control system synthesis is carried out based on the concept of *State Variable Feedback* (SVF). Here, the SVF control law is simply $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x}$, in which $\boldsymbol{K}$ is a $q \times n$ matrix of control gains; and the roots of the closed-loop characteristic polynomial $\det(\lambda\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})$, can be arbitrarily assigned *if and only if* the system (1.2) or (1.3) is 'controllable' (see Chapter 3 for details). In this case, it can be shown that the closed-loop system poles can be assigned to desirable locations on the complex plane. This is, of course, a very powerful result since, as pointed out previously, the poles determine the stability and transient response characteristics of the control system.

This elegant state space approach to control systems analysis and design was pioneered at the beginning of the 1960s, particularly by Rudolf Kalman who, in one year, wrote three seminal and enormously influential papers: 'Contributions to the theory of optimal control' (1960a[2]); 'A new approach to linear filtering and prediction problems' (1960b), which was concerned with discrete-time systems; and 'The theory of optimal control and the calculus of variations' (Report 1960c; published 1963). And then, in the next year, together with Richard Bucy, he published 'New results in linear filtering and prediction theory' (1961), which provided the continuous-time equivalent of the 1960b paper. The first paper deals with LQ feedback control, as mentioned above; while the second and fourth 'Kalman Filter' papers develop recursive

[2] This is not available from the source but can be accessed at: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.26.4070.

(sequentially updated) algorithms for generating the optimal estimate $\hat{\boldsymbol{x}}$ of the state vector $\boldsymbol{x}$ in equation (1.3) and equation (1.2), respectively. Here, the design of the optimal deterministic LQ control system and the optimal state estimation algorithm are carried out separately and then, invoking the 'separation' or 'certainty equivalence' principle (see e.g. Joseph and Tau 1961; Wonham 1968), they are combined to produce the optimal LQG stochastic feedback control system. In other words, the discrete or continuous-time Kalman Filter is used to estimate or 'reconstruct' the state variables from the available input and noisy output measurements; and then these estimated state variables are used in place of the unobserved state variables in the linear SVF control law: i.e. $\boldsymbol{u} = -\boldsymbol{K}\hat{\boldsymbol{x}}$ (Figure 1.1c).

Despite their undoubted power and elegance, these modern control system design methods, as initially conceived by Kalman and others, were not a panacea and there were critics, even at the theoretical level. For instance Rosenbrock and McMorran (1971) in their paper 'Good, bad or optimal?' drew attention to the excessive bandwidth and poor robustness of the LQ-type controller. And in 1972, the second author of the present book and Jan Willems, noted that the standard LQ controller was only a 'regulator': it lacked the integral action required for the kind of Type 1 'servomechanism' performance demanded by most practical control system designers, namely zero steady-state error to constant command inputs. As we shall see later, the solution to this limitation prompted the development, in the 1980s, of the NMSS-PIP design methodology discussed in this book.

As a result of these and other limitations of LQG control, not the least being its relative complexity, it was the much simpler, often manually tuned PI and PID algorithms that became established as the standard approaches to control system design. Indeed, all of the classical methods mentioned above still have their place in control systems analysis today and involve the analysis of either continuous or discrete-time *Transfer Function* (TF) models (Chapter 2), often exploiting graphical methods of some type (see recent texts such as Franklin *et al.* 2006, pp. 230–313 and Dorf and Bishop 2008, pp. 407–492). But 'modern' control systems theory has marched on regardless. For instance, the lack of robustness of LQG designs has led to other, related approaches, as when the H_2 norm that characterises LQ and LQG optimisation is replaced by the H_∞ norm (see e.g. Zames 1981; Green and Limebeer 1995; Grimble 2006). These robust design methods can be considered in the time or frequency domains and have the virtue that classical design intuition can be used in the design process. An equivalent, 'risk sensitive' approach is 'exponential-of-quadratic' optimisation, where the cost function involves the expectation of the exponential of the standard LQ and LQG criteria (Whittle 1990). This latter approach has been considered within a NMSS context by the present authors: see Chapter 6.

The NMSS-PIP control system design methodology has been developed specifically to avoid the limitations of standard LQ and LQG design. Rather than considering robust control in analytical terms by modifying the criterion function, the minimal state space model on which the LQ and LQG design is conventionally based is modified so that state estimation, as such, is eliminated by making all of the state variables available for direct measurement. As we shall see in section 1.3, this is achieved by considering a particular NMSS model form that is linked directly with the discrete-time TF model of the system, with the state variables defined as the present and stored past variables that appear in the discrete-time equation associated with this TF model. In this manner, the SVF control law $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x}$ can be implemented directly using these measured input and output measurements as state variables, rather than indirectly using $\boldsymbol{u} = -\boldsymbol{K}\hat{\boldsymbol{x}}$, with its requirement for estimating the state variables and invoking the separation principle. As this book will demonstrate, the resulting PIP control system is not only easier to implement, it is also inherently more robust. Moreover, it can be interpreted in terms of feedback and forward path control filters that resemble those used in previous classical designs. In a very real sense, therefore, its heritage is in both classical and modern control system synthesis.

Finally, any model-based control system design requires a suitable mathematical representation of the system. Sometimes this may be in the form of a simulation model that has been based on a mechanistic analysis of the system and has been 'calibrated' in some manner. However, in the present TDC context, it seems more appropriate if the model has been obtained by statistical identification and estimation, on the basis of either experimental or monitored sampled data obtained directly from measurements made on the system. In the present book, this task is considered in a manner that can be linked with the Kalman Filter (Young 2010), but where it is the parameters in the model, rather than the state variables, that are estimated recursively. Indeed, early publications (see e.g. Young 1984 and the prior references therein) pointed out that the Kalman Filter represents a rediscovery, albeit in more sophisticated form, of the *Recursive Least Squares* (RLS) estimation algorithm developed by Karl Friedrich Gauss, sometime before 1826 (see Appendix A in Young 1984 or Young 2011, where the original Gauss analysis is interpreted in vector-matrix terms).

Chapter 8 in the present book utilises RLS as the starting point for an introduction to the optimal *Refined Instrumental Variable* (RIV) method for statistically identifying the structure of a discrete or continuous-time TF model and estimating the parameters that characterise this structure. Here, an optimal instrumental variable approach is used because it is relatively robust to the contravention of the assumptions about the noise contamination that affects any real system. In particular, while RIV estimation yields statistically consistent and efficient (minimum variance) parameter estimates if the additive noise has the required rational spectral density and a Gaussian amplitude distribution, it produces consistent, asymptotically unbiased and often relatively efficient estimates, even if these assumptions are not satisfied. This RIV method can also be used to obtain reduced order 'dominant mode' control models from large computer simulation models; and it can identify and estimate continuous-time models that provide a useful link with classical methodology and help in defining an appropriate sampling strategy for the digital control system.

1.3 The Evolution of the NMSS Model Form

The NMSS representation of the model is central to TDC system design. In more general terms, the state space formulation of control system design is the most natural and convenient approach for use with computers. It allows for a unified treatment of both SISO and multivariable systems, as well as for the implementation of the state variable feedback control designs mentioned above, which can include pole assignment, as well as optimal and robust control. Unfortunately, the standard minimal state space approach has three major difficulties. First, as pointed out in section 1.2, the required state vector is not normally available for direct measurement. Secondly, the parameterisation is not unique: for any input–output TF model, there are an infinite number of possible state-space forms, depending on the definition of the state variables. Thirdly, the number of parameters in an n-dimensional state space model is much higher than that in an equivalent TF model: e.g. a SISO system with an nth order denominator,

mth order numerator and a one sample time delay has $n + m$ parameters; while the equivalent state space model can have up to $n^2 + n$ parameters. The first problem has motivated the use of state variable estimation and reconstruction algorithms, notably the Kalman Filter (Kalman 1960b, 1961) for stochastic systems, as mentioned in section 1.2 and the Luenberger observer (Luenberger 1967, 1971) for deterministic systems. The second and third problems have led to a reconsideration of the state-space formulation to see if the uniqueness and parametric efficiency of the TF can be reproduced in some manner within a state space context.

The first movements in this direction came in the 1960s, when it was realised that it was useful to extend the standard minimal state space form to a non-minimal form that contained additional state variables; in particular, state variables that could prove advantageous in control system design. For example, as pointed out previously, Young and Willems (1972) showed how an 'integral-of-error' state variable, defined as the integral of the error between a defined reference or command input $\boldsymbol{y}_d$ and the measured output of the system $\boldsymbol{y}$, could be appended to an otherwise standard minimal state vector. The advantage of this simple modification is that a state variable feedback control law then incorporates this additional state, so adding 'integral-of-error' feedback and ensuring inherent Type 1 servomechanism performance, i.e. zero steady-state error to constant inputs, provided only that the closed-loop system remains stable. Indeed, in the 1960s, the present second author utilised this approach in the design of an adaptive autostabilisation system for airborne vehicles (as published later in Young 1981) and showed how Type 1 performance was maintained even when the system was undergoing considerable changes in its dynamic behaviour.

In the early 1980s, the realisation that NMSS models could form the basis for SVF control system design raised questions about whether there was a NMSS form that had a more transparent link with the TF model than the 'canonical', minimal state space forms that had been suggested up to this time. More particularly, was it possible to formulate a discrete-time state space model whose state variables were the present and past sampled inputs and outputs of the system that are associated directly with the discrete-time dynamic equation on which the TF model is based? In other words, the NMSS state vector $\boldsymbol{x}(k)$ for a SISO system would be of the form:

$$\boldsymbol{x}(k) = [y(k)\ y(k-1)\ \ldots\ y(k-n+1)\ u(k-1)\ u(k-2)\ \ldots\ u(k-m+1)]^T \qquad (1.4)$$

where $y(k)$ is the output at sampling instant k; $u(k)$ is the input at the same sampling instant; n and m are integers representing the order of the TF model polynomials (see Chapter 2). Here, the order $n + m - 1$ of the associated state space model is significantly greater than the order n of a conventional *minimal* state space model (see Chapters 3 and 4 for details). Such a NMSS model would allow the control system to be implemented straightforwardly as a full state feedback controller, without resort to state reconstruction, thus simplifying the design process and making it more robust to the inevitable model uncertainty.

A NMSS model based on the state vector (1.4) was suggested by Hesketh (1982) within a pole assignment control context. This 'regulator' form of the NMSS model is discussed in Chapter 4, where the term 'regulator' is used because the model is only really appropriate to the situation where the command inputs are zero (or fixed at constant values, in which case the model describes the perturbations about these constant levels). The purpose of the control system is then to 'regulate' the system behaviour by restoring it to the desired state, as defined by the command inputs, following any disturbance. In particular, it does not include any inherent integral action that will ensure Type 1 servomechanism performance and the ability to 'track' command input changes, with zero steady-state error, when the command input remains constant for some time period greater that the settling time of the closed-loop system.

Fortunately, following the ideas in Young and Willems (1972) mentioned above, an alternative 'servomechanism' NMSS model can be defined straightforwardly by extending the state vector in (1.4) to include an integral-of-error state, i.e. again in the SISO case,

$$\boldsymbol{x}(k) = [y(k)\ y(k-1)\ \ldots\ y(k-n+1)\ u(k-1)\ u(k-2)\ \ldots\ u(k-m+1)\ z(k)]^T \quad (1.5)$$

where, in this discrete-time setting, the integral-of-error state variable is defined by the following discrete-time integrator (summer),

$$z(k) = z(k-1) + (y_d(k) - y(k)) \quad (1.6)$$

and $y_d(k)$ is the control system command input. This NMSS form, which was first introduced and used for control system design by Young *et al*. (1987) and Wang and Young (1988), is discussed in Chapter 5 and it constitutes the main NMSS description used in the present book. Indeed, as this book shows, the NMSS model provides the most 'natural' and transparent state space description of the discrete-time TF model as required for control system design (Taylor *et al*. 2000a). State variable feedback control system design based on this NMSS model yields what we have termed *Proportional-Integral-Plus* (PIP) control algorithms since they provide logical successors to the PI and PID controllers that, as mentioned previously in section 1.2, have dominated control applications for so long. Here, the 'plus' refers to the situation with systems of second and higher order, or systems with pure time delays, where the additional delayed output and input variables appearing in the non-minimal state vector (1.5), lead to additional feedback and forward path control filters that can be interpreted in various ways, including the implicit introduction of first and higher order derivative action (see Chapter 5 and Appendix D for details).

As this book explains, the servomechanism NMSS model provides a very flexible basis for control system design. For example, the state vector is readily extended to account for the availability of additional measured or estimated information, and these additional states can be utilised to develop more sophisticated NMSS model structures and related control algorithms (see Chapter 6 and Chapter 7). Also, the PIP algorithm is quite general in form and resembles various other digital control systems developed previously, as well as various novel forms, including: feedback and forward path structures, incremental forms and the Smith Predictor for time-delay systems (Taylor *et al*. 1998a); stochastic optimal and risk sensitive control (Taylor *et al*. 1996b); feed-forward control (Young *et al*. 1994); generalised predictive control (Taylor *et al*. 1994, 2000a); multivariable control (Taylor *et al*. 2000b); and state-dependent parameter, nonlinear control (see e.g. Taylor *et al*. 2009, 2011 and the references therein). The multivariable extensions are particularly valuable because they allow for the design of full multi-input, multi-output control structures, where control system requirements, such as channel decoupling and multi-objective optimisation, can be realised.

Of course, a control system is intended for practical application and this book will only succeed if it persuades the reader to utilise PIP control systems in such applications. In this regard, its application record and potential is high: in the 25 years that have passed since the seminal papers on the servomechanism NMSS model and PIP control were published, the methodology has been extended and applied to numerous systems from the nutrient-film (hydroponic) control of plant growth (e.g. Behzadi *et al*. 1985); through various other agricultural (e.g. Lees *et al*. 1996, 1998; Taylor *et al*. 2000c, 2004a, 2004b; Stables and Taylor 2006), benchmark simulation, laboratory and industrial applications (e.g. Chotai *et al*. 1991; Fletcher *et al*. 1994; Taylor *et al*. 1996a, 1998b, 2004c, 2007; Seward *et al*. 1997; Ghavipanjeh *et al*. 2001; Quanten *et al*. 2003; Gu *et al*. 2004; Taylor and Shaban 2006; Taylor and Seward 2010; Shaban *et al.* 2008); and even to the design of emission strategies for the control of atmospheric carbon dioxide in connection with climate change (Young 2006; Jarvis *et al*. 2008, 2009). And all such applications have been guided by the underlying TDC design philosophy outlined in the next subsection.

1.4 True Digital Control

In brief, the TDC design procedure consists of the following three steps:

1. Identification and recursive estimation of discrete and continuous-time models based on the analysis of either planned or monitored experimental data, or via model reduction from data generated by a physically based (mechanistic) simulation model.
2. Offline TDC system design and initial evaluation based on the models from step 1, using an iterative application of an appropriate discrete-time design methodology, coupled with closed-loop sensitivity analysis based on MCS.
3. Implementation, testing and evaluation of the control system on the real process: in the case of self-tuning or self-adaptive control, employing online versions of the recursive estimation algorithms from step 1.

Step 1 above is concerned with stochastic system identification and estimation, i.e. the identification of the control model structure and estimation of the parameters that characterise this structure from the measured input–output data. In the present book, this will normally involve initial identification and estimation of TF models on which the NMSS model form, as required for PIP control system design, is based. It may also involve the identification and estimation of continuous-time models that allow for the direct evaluation of the model in physically meaningful terms, as well as the evaluation of different sampling strategies. Here, the continuous-time model, whose parameter estimates are not dependent on the sampling interval unless this is very coarse, can be converted easily (for example using the c2d conversion routine available in MATLAB®[3]) to discrete-time models defined at different sampling intervals, which can then be evaluated in control system terms in order to define the most suitable sampling frequency.

In step 2, the stochastic models also provide a useful means of assessing the robustness of the TDC designs to the uncertainty in the model parameters, as estimated in step 1 (see e.g. Taylor *et al*. 2001). However, if experimental data are unavailable, the models are instead obtained from a conventional, usually continuous-time and physically based, simulation model of the system (assuming that such a model is available or can be built). In this case, the identification

[3] MATLAB®, The MathWorks Inc., Natick, MA, USA.

step addresses the combined problem of model reduction and linearisation, and forms the connection between the TDC approach and classical mechanistic control engineering methods.

Step 3 is, of course, very problem dependent and it is not possible to generalise the approach that will be used. Implementation will depend on the prevailing conditions and technical aspects associated with specific case studies. We hope, therefore, that the various examples discussed in later chapters, together with other examples available in the cited references, will be sufficient to provide some insight into the practical implementation of TDC systems.

1.5 Book Outline

Starting with the ubiquitous PI control structure as a worked example, and briefly introducing essential concepts such as the backward shift operator z^{-1}, we try to make as few prerequisite assumptions about the reader as possible. Over the first few chapters, generic concepts of state variable feedback are introduced, in what we believe is a particularly intuitive manner (although clearly the reader will be the judge of this), largely based on block diagram analysis and straightforward algebraic manipulation. Conventional minimal state space models, based on selected canonical forms, are considered first, before the text moves onto the non-minimal approach.

More specifically, the book is organised as follows:

- In **Chapter 2**, we introduce the general discrete-time TF model, define the poles of the system and consider its stability properties. Here, as in Chapters 3–6, the analysis is based on this SISO model. Some useful rules of block diagram analysis are reviewed and these are the utilised to develop three basic, discrete-time control algorithms. The limitations of these simple control structures are then discussed, thereby providing motivation for subsequent chapters.
- **Chapter 3** considers *minimal* state space representations of the TF model and shows how these may be employed in the design of state variable feedback control systems. Two particularly well-known representations of this minimal type are considered, namely the controllable canonical form and the observable canonical form. These are then used to illustrate the important concepts of controllability and observability.
- We start **Chapter 4** by defining the regulator NMSS form, i.e. for a control system in which the command input $y_d = 0$ (Figure 1.1). Once the controllability conditions have been established, the non-minimal controller can be implemented straightforwardly. The final sections of the chapter elaborate on the relationship between non-minimal and minimal state variable feedback, while the theoretical and practical advantages of the non-minimal approach are illustrated by worked examples.
- **Chapter 5** develops the complete version of the NMSS-PIP control algorithm for SISO systems. Most importantly and in contrast to Chapter 4, an integral-of-error state variable (1.6) is introduced into the NMSS form to ensure Type 1 servomechanism performance, i.e. if the closed-loop system is stable, the output will converge asymptotically to a constant scalar command input y_d specified by the user. Two main design approaches are considered: pole assignment and optimal LQ control.
- In **Chapter 6**, we extend the NMSS vector in various ways to develop generalised linear PIP controllers. The robustness and disturbance response characteristics of the main control

structures that emerge from this analysis are considered, including incremental forms for practical implementation, the Smith Predictor for time-delay systems, stochastic optimal design, feed-forward control and predictive control.
- **Chapter 7** is important in practical terms because it considers the full NMSS-PIP control system design approach for more complex multivariable systems of the kind that are likely to be encountered in practical applications. Here, the system is characterised by multiple command and control inputs that affect the state and output variables in a potentially complicated and cross-coupled manner. Two design approaches are discussed: optimal LQ with multi-objective optimisation; and a combined multivariable decoupling and pole assignment algorithm. These can be contrasted with the classical 'multichannel' approach to design in which each channel of the system, between a command input and its associated output, is often designed separately and 'tuned' to achieve reasonable multivariable control and cross-coupling.
- **Chapter 8** provides a review of data-based modelling methods and illustrates, by means of simulation and practical examples, how the optimal RIV methods of statistical identification and estimation are able to provide the TF models utilised in previous chapters (including both SISO and multivariable). We also demonstrate how these stochastic models provide a useful means of assessing the robustness of the NMSS designs to uncertainty in the model parameters. In order to help connect with classical methods for modelling in engineering and also allow for the appropriate selection of the sampling interval Δt, both discrete-time and continuous-time estimation algorithms are considered.
- **Chapter 9** considers several additional topics that are not central to TDC design but could have increased relevance in future design studies. These include control system design for rapidly sampled systems using δ-operator models (Middleton and Goodwin 1990); and nonlinear NMSS control system design using *Time-Variable* (TVP) and *State-Dependent* (SDP) parameter models.
- Finally, **Appendix A** revises matrices and the essentials of matrix algebra, as used at various points in the text. The other appendices cover supplementary topics, such as selected theorem proofs, and are cited in the main text where appropriate.

These chapters blend together, in a systematic and we hope readable manner, the various theoretical and applied research contributions made by the authors and others into all aspects of TDC system design over the past half century. This allows for greater integration of the methodology, as well as providing substantially more background detail and examples than the associated academic articles have been able to do.

1.6 Concluding Remarks

The present book, taken as a whole, aims to provide a generalised introduction to TDC methods, including both NMSS design of PIP control systems, and procedures for the data-based identification and estimation of the dynamic models required to define the NMSS form. In this initial chapter, we have both introduced the TDC design philosophy, drawing a distinction between it and *Direct Digital Control*, and reviewed the historical context in which TDC and its associated methodological underpinning have been developed. In TDC design, all aspects of the control system design procedure are overtly digital by nature, with continuous-time concepts introduced only where they are essential for the purposes of describing, simulating and understanding the process in physically meaningful terms; or for making decisions about digitally related aspects, such as the most appropriate sampling strategies. We believe that the approach we have used for the development of the NMSS form and the associated PIP class of control systems provides a relatively gentle learning curve for the reader, from which potentially difficult topics, such as stochastic and multivariable control, can be introduced and assimilated in an interesting and straightforward manner.

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