1

HOW TO DESIGN RESISTIVE CIRCUITS

Chances are good that every schematic diagram you've seen contained at least a few resistors. This component is an electrical workhorse, commonly used for establishing bias voltages, programming gain, summing signals, attenuating signals, and numerous other functions. Ideal resistors dutifully follow Ohm's law, which has no frequency dependence, so it is easy to believe that designing with resistors is a simple task. This is probably the most common reason candidates are caught off guard and fail when asked to design simple resistive networks in interviews. This chapter will show you how to design and analyze practical resistive networks that solve problems you'll encounter in interviews and in the workplace.

The resistor was probably the first component you studied in school. At that time, it was the only component in your toolset, so the problems you solved were limited to finding voltages and currents in DC networks. Doing these problems taught you valuable skills such as nodal and mesh analysis, but the problems were not particularly practical and perhaps not very interesting. As a graduating engineer your knowledge of circuit elements has broadened

Ten Essential Skills for Electrical Engineers, First Edition. Barry L. Dorr.

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significantly and you have better computer tools to help with the mathematical manipulations. The examples and problems in this chapter should be much more interesting because they represent practical design problems; they should be more enjoyable because, after setting up the problems, we will rely on the computer for the brunt of the manipulations.

This chapter begins with the commonly asked interview problem of creating a voltage source with a specified Thevenin resistance. Since this is such a common problem, we derive equations so that you can easily compute the resistor values when you encounter it. Next, we design a coupling circuit with specific design requirements. Many experienced engineers design this circuit using an op-amp and numerous resistors, but you'll see that a network with only three resistors fulfills the design requirement. We then design a 50 Ω bidirectional attenuator that is commonly found in RF circuits. This is not an easy problem, but it provides a good example of converting design requirements into solvable equations and then calculating the components. Since the attenuator design is a difficult problem, we check our result using mesh analysis, and you'll see that analyzing resistive networks is simply a matter of writing the correct equations and then letting a matrix solver compute the solution.

1.1 DESIGN OF A RESISTIVE THEVENIN SOURCE

Designing circuits involves taking inventory of what is required, what is known, what is unknown, and what is available. When designing resistive networks we must often solve simultaneous nonlinear equations as shown in the following example:

Example 1.1. Given a 3.3 V power supply, design a resistive circuit that provides 1.8 V with a source resistance of 1.5 kΩ.

Solution. This is a practical problem encountered frequently when designing bias circuits. We immediately note the desired output voltage is less than the supply voltage so a resistive circuit will suffice. We also recognize that the requirement leads us to a Thevenin source.

Knowing that two design parameters (Thevenin voltage and resistance) must be simultaneously satisfied, it makes sense[1] to try the circuit with two resistors as shown in Figure 1.1b.

The design procedure is to compute R_1 and R_2 so that the Thevenin equivalent of the proposed network in Figure 1.1b is the Thevenin source of

[1]If it *doesn't* make sense don't worry. You'll pick this up with time.

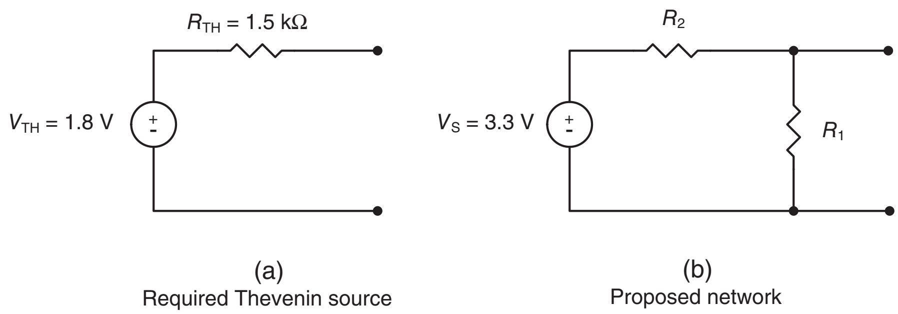

Figure 1.1. Required Thevenin source (a) and proposed resistive network (b) for Example 1.1.

Figure 1.1a. The Thevenin resistance of the proposed network is the resistance seen at the terminals with the voltage source shorted, so our first equation is the formula for parallel resistances:

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} \tag{1.1}$$

The Thevenin voltage is the voltage at the terminals of the circuit of Figure 1.1b when it is unloaded, so we use voltage division to write the second equation:

$$\frac{V_{TH}}{V_S} = \frac{R_1}{R_1 + R_2} \tag{1.2}$$

These two equations are nonlinear and cannot be simultaneously solved using a simple matrix. Instead, we solve Equation 1.1 for R_1 which gives

$$R_1 = \frac{R_{TH} R_2}{\left(R_2 - R_{TH}\right)} \tag{1.3}$$

Then, substitute R_1 from Equation 1.3 into Equation 1.2 and solve for R_2:

$$R_2 = \frac{R_{TH} V_S}{V_{TH}} = \frac{1500 \times 3.3}{1.8} = 2750\ \Omega \tag{1.4}$$

Finally substitute Equation 1.4 into Equation 1.3 to compute R_1:

$$R_1 = \frac{R_{TH}}{\left(1 - \frac{V_{TH}}{V_S}\right)} = \frac{1500}{\left(1 - \frac{1.8}{3.3}\right)} = 3300\ \Omega \tag{1.5}$$

To check the result, substitute these resistances into Equations 1.1 and 1.2.

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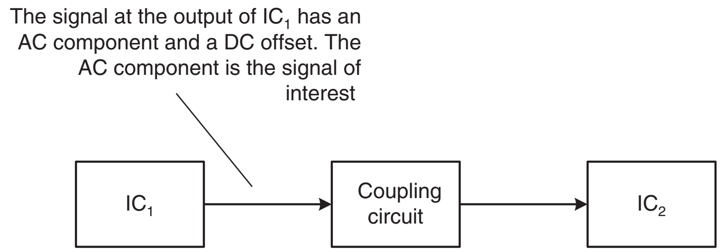

Figure 1.2. Coupling circuit.

1.2 DESIGN OF A COUPLING CIRCUIT

A common application for resistive circuits is the coupling circuit of Figure 1.2 that conditions a signal from integrated component IC_1 and feeds it to integrated component IC_2. The signal of interest, or the signal that carries useful information, is an alternating current (AC) signal with no DC component. However, DC offsets play an important role in this application because they keep the input and output signals within the working range of the amplifiers.

The required functionality of this circuit is to

1. Provide a specified load impedance for IC_1.
2. Attenuate the signal from IC_1 to IC_2 by a specified amount.
3. Provide a specified DC offset at the input of IC_2.

This functionality is provided by the circuit of Figure 1.3. Our design strategy is to first analyze the circuit and then use the analysis results to compute the component values.

The focus of this chapter is resistive circuits, so the capacitor in Figure 1.3 deserves explanation. It is called a “blocking capacitor” because its primary function is to prevent or block the DC voltage at the output of IC_1 so that it does not appear at the input of IC_2. The capacitor is selected so that it approximates a short circuit to the signal of interest. Chapter 3 shows how to select the correct value of this capacitor.

Prior to analyzing the circuit of Figure 1.3, we digress briefly to clarify the concept of “AC ground.” This powerful tool is often a source of confusion for both students and experienced engineers. The confusion arises from not fully grasping the difference between *resistance* and *impedance*. *Resistance* is the ratio of the DC voltage across a device to the DC current passing through it. *Impedance* is the frequency-dependent, complex ratio of the AC voltage to

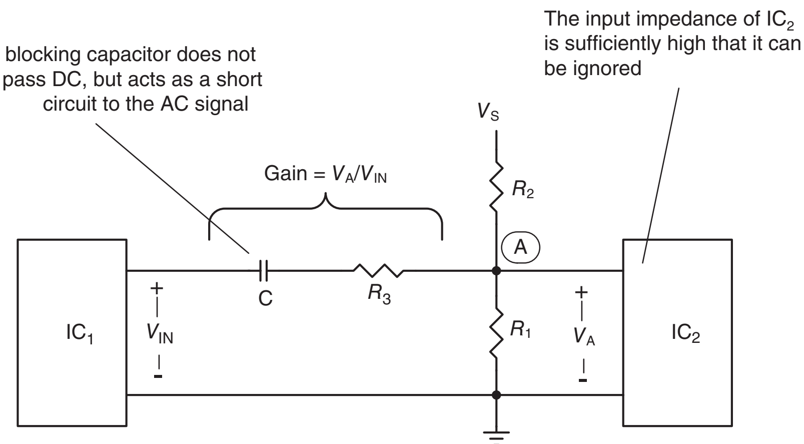

Figure 1.3. This coupling circuit provides a specified load impedance for IC_1, attenuates the signal from IC_1 to IC_2, and provides a specified DC bias for IC_2.

the AC current in a device or circuit. One way to remember this is to think of impedance as the derivative:

$$\text{Impedance} = Z = \frac{\partial V}{\partial I} \tag{1.6}$$

Note from Equation 1.6 that the impedance of a resistor is equal to its resistance.

Now consider an ideal DC power supply. Its function is to provide a fixed voltage regardless of the load current. Since its voltage does not change, Equation 1.6 shows that its impedance is zero.[2] Therefore, when computing the impedance of the circuit in Figure 1.3, we treat the power supply, V_S, as an AC ground.

Similarly the *attenuation* provided by the circuit of Figure 1.3 refers to the ratio of the AC voltage at the output of IC_1 to the AC voltage at A. When computing attenuation, the power supply, V_S, is also an AC ground.

The preceding discussion shows that for computing input impedance and gain,[3] the DC supply, V_S, is an AC ground which places R_1 in parallel with R_2 in Figure 1.3. We therefore simplify Figure 1.3 by representing R_1, R_2, and V_S as a DC Thevenin source as shown on the right side of Figure 1.4a. Since the capacitor appears as a short circuit to the desired signal and since the Thevenin voltage source is an AC ground, the circuit of Figure 1.4b can be used to compute the input impedance and gain.

[2]A large capacitor connected from a circuit node to ground results in an AC ground also.

[3]For the remainder of this discussion we use gain, V_A/V_{IN}, which is the inverse of attenuation.

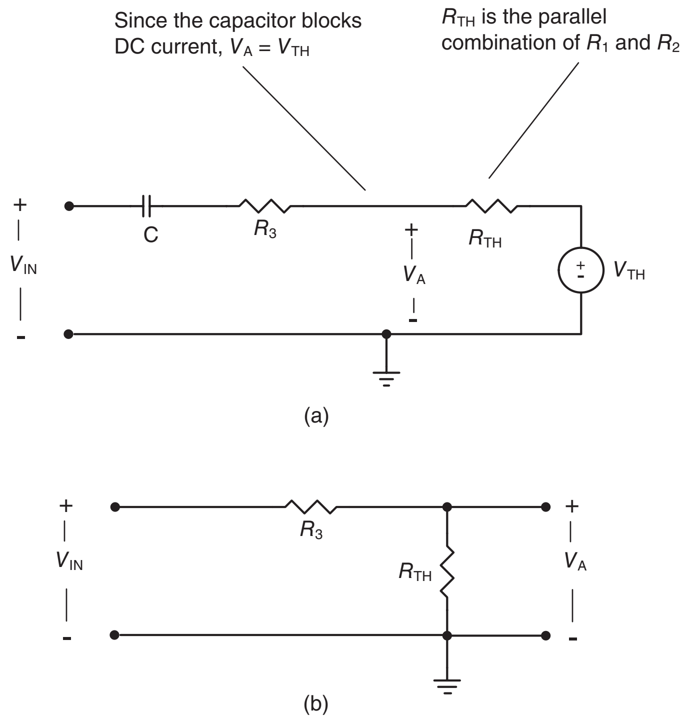

Figure 1.4. Simplifications of the circuit of Figure 1.3 are used to compute input impedance and circuit gain. A Thevenin equivalent allows R_1, R_2, and V_S to be shown as a Thevenin equivalent (a). Since the capacitor is an AC short and since voltage source V_{TH} is an AC ground, the circuit can be further simplified (b).

From Figure 1.4b the input impedance is

$$Z_{IN} = R_3 + R_{TH} \tag{1.7}$$

And the gain is computed using the voltage divider theorem

$$\text{Gain} = \frac{V_A}{V_{IN}} = \frac{R_{TH}}{R_3 + R_{TH}} \tag{1.8}$$

The analysis above can be used to design the coupler based on a design specification as shown in the following example.

Example 1.2. The circuit shown in Figure 1.5 is a coupling circuit for an AC signal. Assume that the amplifier input impedance is infinite, and the capacitor acts as a short circuit to the input signal. Pick resistors R_1, R_2, and R_3 so the DC bias at the amplifier input is 4 VDC, the signal from V_{IN} to the

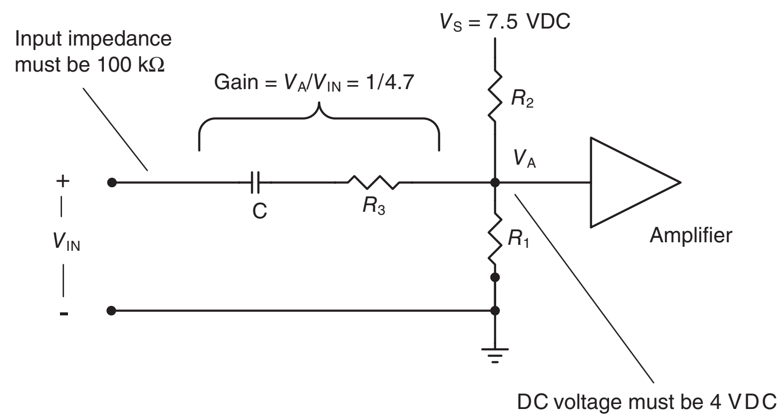

Figure 1.5. Circuit for Example 1.2.

amplifier input is attenuated by a factor of 4.7, and the impedance seen by the input signal at V_{IN} is 100 kΩ.

Solution. From the analysis above, we use the circuit of Figure 1.6 to compute input impedance and gain.

Substituting Z_{IN} from Equation 1.7 into Equation 1.8 and solving for R_{TH} gives

$$R_{TH} = \text{Gain} \times Z_{IN} = \frac{1}{4.7} 100\,\text{k}\Omega = 21.28\ \text{k}\Omega \tag{1.9}$$

This allows us to compute R_3 using Equation 1.7:

$$R_3 = Z_{IN} - R_{TH} = 100\,\text{k}\Omega - 21.28\,\text{k}\Omega = 78.72\ \text{k}\Omega \tag{1.10}$$

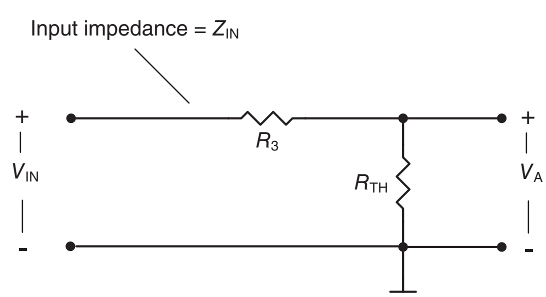

Figure 1.6. Thevenin equivalent for computing impedance and gain for Example 1.2.

Since the capacitor blocks DC current, the Thevenin voltage is the DC bias voltage of 4 VDC at the amplifier input. Knowing the Thevenin voltage, Thevenin resistance, and supply voltage, V_S, we compute R_1 and R_2 using Equations 1.5 and 1.4, respectively. From Equation 1.5:

$$R_1 = \frac{R_{TH}}{\left(1 - \frac{V_{TH}}{V_S}\right)} = \frac{21{,}280}{\left(1 - \frac{4}{7.5}\right)} = 45.60\ \text{k}\Omega \tag{1.11}$$

From Equation 1.4:

$$R_2 = \frac{R_{TH} V_S}{V_{TH}} = \frac{21{,}280 \times 7.5}{4} = 39.90\ \text{k}\Omega \tag{1.12}$$

1.3 DESIGN OF A PI ATTENUATOR

In this section we again begin with a design requirement, write descriptive equations for the proposed circuit, and then use the equations to compute the component values. We also show how mesh analysis and a matrix solver can be used to quickly and efficiently check our work by analyzing the resulting circuit.

The Pi network of Figure 1.7 is commonly found in radio frequency (RF) circuits and does the following:

1. Attenuates bidirectionally. The voltage gain, A_V, from left to right is identical to the voltage gain from right to left.
2. Provides a specified load resistance to the circuit that drives it.
3. Provides a specified source resistance to the circuit that it drives.

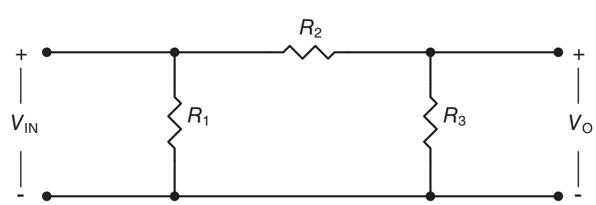

Figure 1.7. Pi network. This network is frequently used as a constant impedance RF attenuator.

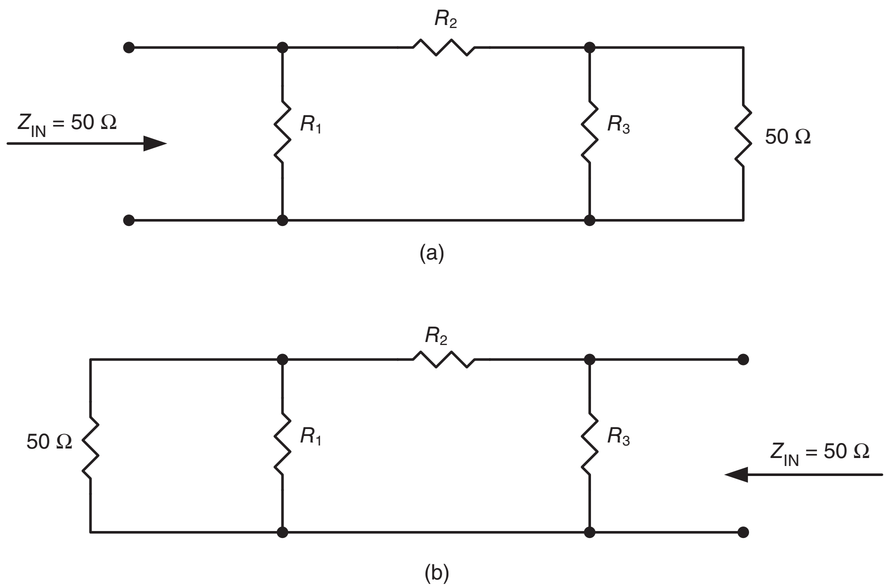

Figure 1.8. Pi network. With the output terminated in 50 Ω the input impedance is 50 Ω (a). With the input terminated in 50 Ω the output impedance is 50 Ω (b).

This network is important to RF designers because most RF components are designed to work with 50 Ω input and output impedances.[4] Of course, there are many online calculators such as [1] available for this purpose. But the skills used in this example will help you work with any network you'll encounter in an interview or on the job.

Specific design requirements for a 3 dB Pi attenuator are as follows:

1. With the output terminated in 50 Ω, the input impedance must be 50 Ω.
2. With the input driven by a 50 Ω source and the output terminated in 50 Ω, the ratio V_O/V_{IN} must be A_V.
3. With the input driven by a 50 Ω source, the output impedance must be 50 Ω.

The toughest part of this problem is to convert the design requirements into solvable equations. First, note that the problem can be simplified using symmetry as shown in Figure 1.8. When the *output* is terminated in 50 Ω the resulting circuit is simply the reflection of the circuit when the *input* is terminated in 50 Ω. Therefore, R_1 and R_3 are identical, and the number of

[4]See Chapter 9 for examples using attenuators.

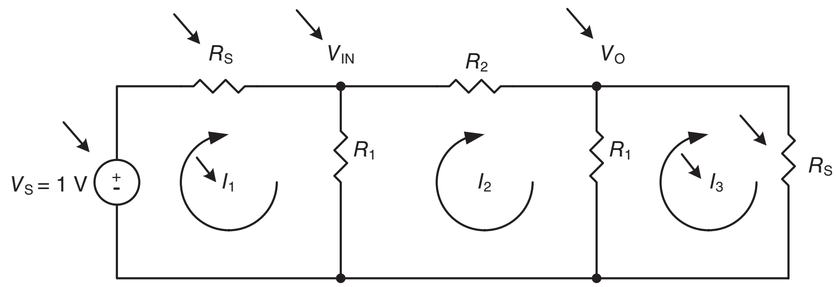

Figure 1.9. Pi network analysis model. The arrows point to quantities that are either given or can be readily computed by inspection. The unknowns in this problem are R_1, R_2, and I_2. $R_S = 50\ \Omega$.

components which must be determined is lowered from three to two. For our analysis R_3 is replaced by R_1, as shown in Figure 1.9.

The model of Figure 1.9 is used to describe the voltages and currents in the system. The calculations are simplified by setting $V_S = 1$ V. From requirement 1, the input resistance of the terminated network must be 50 Ω. Since R_S is 50 Ω voltage division gives

$$V_{IN} = \frac{V_S}{2} = \frac{1}{2} \tag{1.13}$$

Similarly, since the resistance seen by V_S is twice R_S or 100 Ω

$$I_1 = \frac{1}{2R_S} \tag{1.14}$$

Requirement 2 specifies that the ratio V_O/V_{IN} must be A_V. With voltage gain A_V known, V_O and I_3 can be computed:

$$V_O = V_{IN}A_V = \tfrac{1}{2}A_V \tag{1.15}$$

$$I_3 = \frac{V_O}{R_S} = \frac{A_V}{2R_S} \tag{1.16}$$

Returning to Figure 1.9, V_S, R_S, I_1, V_{IN}, I_3, and V_O are known and marked with arrows. R_1, R_2, and I_2 are unknown. Since there are three unknowns, three equations are needed. The first equation is written by observing that V_{IN} and I_1 are known and related to R_1 and I_2:

$$\frac{V_{IN}}{R_1} = I_1 - I_2 \tag{1.17}$$

The second equation is written by observing that V_{IN} and V_O are known and related to R_2 and I_2:

$$V_{IN} - I_2 R_2 = V_O \tag{1.18}$$

The third equation is written by observing that V_O and I_3 are known and related to R_1 and I_2:

$$\frac{V_O}{R_1} = I_2 - I_3 \tag{1.19}$$

Equations 1.17, 1.18, and 1.19 represent three nonlinear equations in three unknowns. These equations can be solved "by hand" as shown below or with a symbolic equation solver such as the one provided by Wolfram Mathematica [2].

We solve for the unknowns below using the following strategy:

1. Solve Equation 1.17 for R_1 as a function of I_2.
2. Substitute the expression for R_1 into Equation 1.19. Solve for I_2.
3. Substitute the value of I_2 into Equation 1.18. Compute R_2.
4. Substitute the value of I_2 into Equation 1.17. Compute R_1.

Equation 1.17 is solved for R_1:

$$R_1 = \frac{V_{IN}}{I_1 - I_2} \tag{1.20}$$

Equation 1.20 is substituted for R_1 in Equation 1.19 and then Equation 1.19 is solved for I_2:

$$I_2 = \frac{\frac{V_O}{V_{IN}} I_1 + I_3}{1 + \frac{V_O}{V_{IN}}} \tag{1.21}$$

Substitution of A_V for V_O/V_{IN}, Equation 1.14 for I_1, and Equation 1.16 for I_3 into Equation 1.21 gives

$$I_2 = \frac{A_V}{R_S\left(1 + A_V\right)} \tag{1.22}$$

Equation 1.22 is substituted into Equation 1.18 and Equation 1.18 is solved for R_2:

$$R_2 = \frac{\left(V_{\text{IN}} - V_{\text{O}}\right)\left(A_{\text{V}} + 1\right) R_{\text{S}}}{A_{\text{V}}} \tag{1.23}$$

Substituting Equation 1.15 for V_{O} and Equation 1.13 for V_{IN} into Equation 1.23 gives

$$R_2 = \frac{\left(1 - A_{\text{V}}^2\right)}{2A_{\text{V}}} R_{\text{S}} \tag{1.24}$$

Finally, substitution of Equation 1.13 for V_{IN}, Equation 1.14 for I_1, and Equation 1.22 for I_2 into Equation 1.17 gives

$$R_1 = \left[\frac{1 + A_{\text{V}}}{1 - A_{\text{V}}}\right] R_{\text{S}} \tag{1.25}$$

Example 1.3. Design a 3 dB, 50 Ω, Pi attenuator. Verify the design gives the specified impedance and attenuation.

Solution. The attenuation is specified as 3 dB. This is converted to voltage gain A_{V} using the definition of the decibel[5]

$$\text{dB} = 20 \log_{10}(A_{\text{V}}) \tag{1.26}$$

Solving for voltage gain A_{V} gives[6]

$$A_{\text{V}} = 10^{-\text{dB}/20} = 10^{-3/20} = 0.708 \tag{1.27}$$

Knowing A_{V} and R_{S}, we use Equations 1.24 and 1.25 to compute the resistors as shown below:

$$R_2 = \frac{\left(1 - A_{\text{V}}^2\right)}{2A_{\text{V}}} R_{\text{S}} = \frac{\left(1 - 0.708^2\right)}{2 \times 0.708} 50 = 17.6\ \Omega \tag{1.28}$$

[5]Note that the attenuation is specified as 3 dB, not 3 dB *voltage* or *power* gain. Students frequently get confused using dB on interviews. A given value of dB specifies *different* values for the voltage and power gains.

[6]The negative sign in front of dB in Equation 1.27 is used because attenuation has been specified as a positive quantity.

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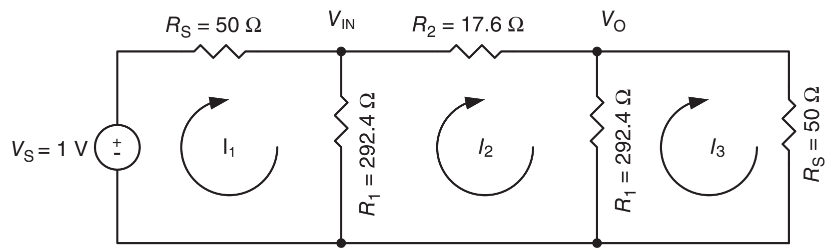

Figure 1.10. Resistor values for the Pi attenuator are checked by writing and solving mesh equations. Next, the input impedance and attenuation are compared to the specified values.

$$R_1 = \left[\frac{1 + A_V}{1 - A_V}\right] R_S = \left[\frac{1 + 0.708}{1 - 0.708}\right] 50 = 292.4\ \Omega \tag{1.29}$$

To check this result, the circuit of Figure 1.10 is analyzed by writing three equations in the three unknown mesh currents, and then solving them using $V_S = 1$ V.[7] We then verify that the input impedance is 50 Ω and the attenuation is 3 dB.

Equations for the three meshes are

$$\begin{aligned} -V_S + I_1 R_S + \left(I_1 - I_2\right) R_1 &= 0 \\ \left(R_S + R_1\right) I_1 - R_1 I_2 &= V_S \end{aligned} \tag{1.30}$$

$$\begin{aligned} \left(I_2 - I_1\right) R_1 + I_2 R_2 + \left(I_2 - I_3\right) R_1 &= 0 \\ -R_1 I_1 + \left(2R_1 + R_2\right) I_2 - R_1 I_3 &= 0 \end{aligned} \tag{1.31}$$

$$\begin{aligned} \left(I_3 - I_2\right) R_1 + I_3 R_S &= 0 \\ -R_1 I_2 + \left(R_1 + R_S\right) I_3 &= 0 \end{aligned} \tag{1.32}$$

In matrix notation this is

$$[\boldsymbol{A}]\,[\boldsymbol{I}] = [\boldsymbol{V}] \tag{1.33}$$

[7]Chapter 3 shows that this technique is also used for AC circuits.

where

$$[\boldsymbol{A}] = \begin{bmatrix} R_S + R_1 & -R_1 & 0 \\ -R_1 & 2R_1 + R_2 & -R_1 \\ 0 & -R_1 & R_1 + R_S \end{bmatrix} \tag{1.34}$$

$$[\boldsymbol{I}] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.35}$$

$$[\boldsymbol{V}] = \begin{bmatrix} V_S \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{1.36}$$

To solve Equation 1.33 premultiply both sides by the inverse of $[\boldsymbol{A}]$ or $[\boldsymbol{A}]^{-1}$

$$[\boldsymbol{A}]^{-1}\,[\boldsymbol{A}]\,[\boldsymbol{I}] = [\boldsymbol{A}]^{-1}\,[\boldsymbol{V}] \tag{1.37}$$

Since a matrix multiplied by its inverse gives the identity matrix, and multiplying a matrix by the identity matrix gives the original matrix,

$$[\boldsymbol{I}] = [\boldsymbol{A}]^{-1}\,[\boldsymbol{V}] \tag{1.38}$$

Using an matrix tool such as [3] or Matlab gives $I_1 = 10$ mA, $I_2 = 8.29$ mA, and $I_3 = 7.08$ mA.

We check for 50 Ω input impedance by verifying that voltage V_{IN} is half the input voltage, V_S, or 0.5 V:

$$V_{IN} = 1 - I_1 \times 50 = 0.5 \text{ V} \tag{1.39}$$

We check for 3 dB attenuation by verifying that the output voltage, V_{OUT} is 0.7071 times the input voltage, V_{IN}.

$$V_O = I_3 R_S = 7.08 \times 10^{-3} \times 50 = 0.7071 \times 0.5 \tag{1.40}$$

PROBLEMS

1.1 Given a 5 V supply, design a circuit with $V_{TH} = 3.3$ V and $R_{TH} = 1000\ \Omega$.

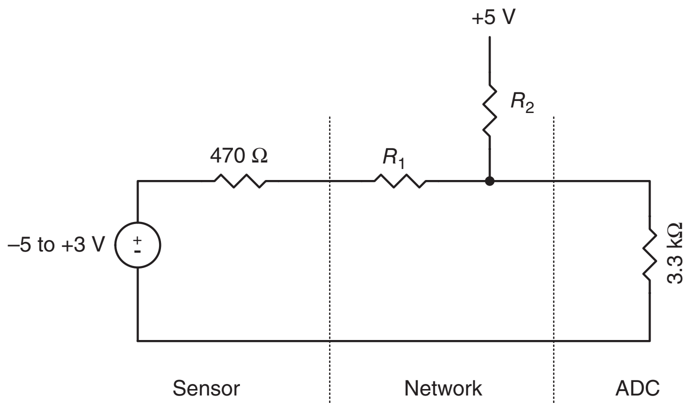

Figure 1.11. Circuit for Problem 1.2.

1.2 A sensor produces a voltage between –5 and +3 V. The output resistance of the sensor is 470 Ω. The sensor must be interfaced to an ADC with an input range of 0 to +3.3 V and an input resistance of 3300 Ω. Determine the values of R_1 and R_2 for the interface network shown in Figure 1.11. Check your work by analyzing the resulting network.

1.3 Components IC_1 and IC_2 are connected using the coupling network shown in Figure 1.12. IC_1 has a DC offset of 15 VDC and must be loaded with 41 kΩ. The AC signal from IC_1 is 7 Vpp. IC_2 requires a DC bias of 4.2 VDC and a input signal level of 3 Vpp. The input of IC_2 has a 47 kΩ resistor connected directly to ground. Compute R_1, R_2, and R_3 for this network.

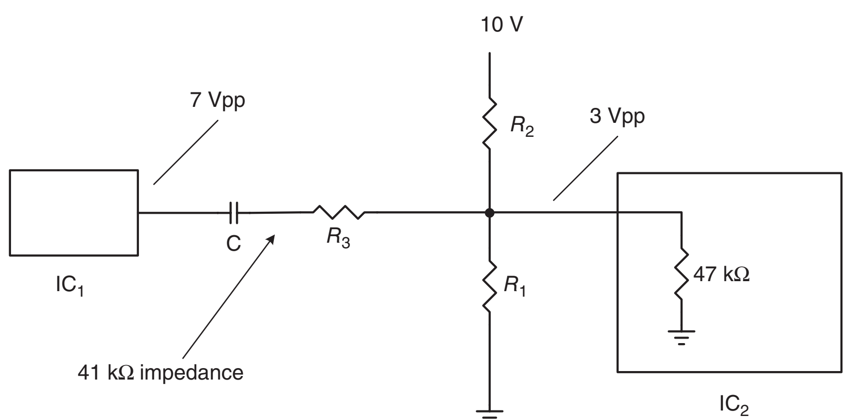

Figure 1.12. Circuit for Problem 1.3.

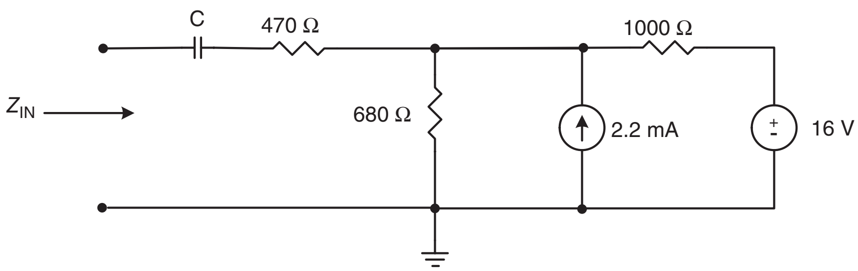

Figure 1.13. Circuit for Problem 1.5.

1.4 Using an argument similar to the one used for a DC voltage source in Section 1.2, determine the impedance of a DC current source. How should a DC current source be represented when analyzing an AC circuit? *Hint:* Use Equation 1.6.

1.5 Compute the input impedance of the circuit of Figure 1.13 for AC signals. Assume the frequency is sufficiently high so the capacitor can be considered a short circuit.

1.6 A coupling network connects components IC_1 and IC_2 as shown in Figure 1.14. Assume the output impedance of IC_1 is zero, the input impedance of IC_2 is infinite, and the capacitor appears as a short circuit to the signal of interest.
(a) Determine the attenuation provided by the network.
(b) Determine the DC bias at the input of IC_2.

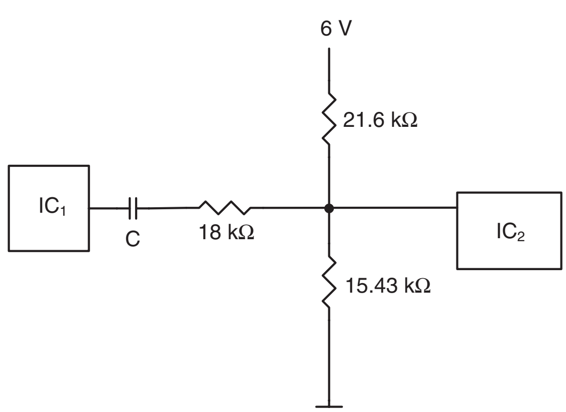

Figure 1.14. Circuit for Problem 1.6.

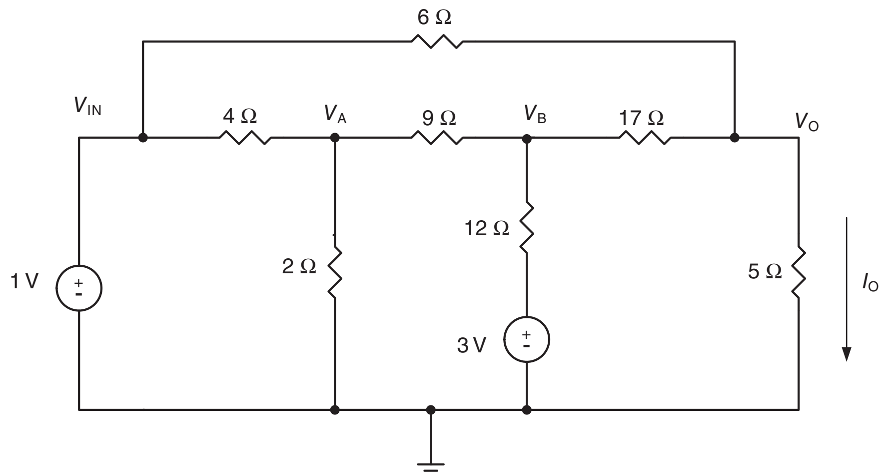

Figure 1.15. Circuit for Problems 1.7 and 1.8.

(c) Determine the load impedance seen by IC_1.

(d) Determine the source impedance seen by IC_2. *Hint:* It is the impedance looking from IC_2 back to the network.

1.7 Determine current I_O in the circuit of Figure 1.15 using mesh analysis.

1.8 Determine total power dissipation in the circuit of Figure 1.15.

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