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Introduction

Nonlinear aeroelasticity is the study of the interactions between inertial, elastic and aerodynamic forces on engineering structures that are exposed to an airflow and feature non-negligible nonlinearity. There exist several good textbooks on linear aeroelasticity for aircraft (Bisplinghoff et al. 1996; Fung 1993; Hodges and Alvin Pierce 2002; Wright and Cooper 2015). Dowell (2004) even includes chapters on nonlinear aeroelasticity and stall flutter, while Paidoussis et al. (2011) discusses a number of nonlinear aeroelastic phenomena occurring in civil engineering structures. However, there is no introductory text that presents the methodologies of nonlinear dynamics and applies them to a wide range of nonlinear aeroelastic systems. The present book aims to fill this gap to a certain degree. The subject area is vast and multidisciplinary and it would be impossible to fit every aspect of it in a textbook. The main omission is high fidelity numerical simulation using Computational Fluid Dynamics and Computational Structural Dynamics solvers; these methodologies are already the subject of a dedicated text (Bazilevs et al. 2013). The aerodynamic models used in this book are analytical, empirical or based on panel methods while the structural models are either analytical or make use of series solutions.

The book is introductory but it assumes knowledge of structural dynamics, aerodynamics and some linear aeroelasticity. The main linear aeroelastic phenomena of flutter and static divergence are discussed in detail because they can affect nonlinear behaviour, but the present work is by no means a complete text on linear aeroelasticity. Unsteady aerodynamic modelling is used throughout the book and discussed in Chapters 8, 10 and in the Appendix. However, again this book is not a complete reference on unsteady aerodynamics, linear or nonlinear. On the other hand, nonlinear dynamics and bifurcation analysis are presented in great detail as they do not normally feature in most undergraduate or even graduate Aerospace and Mechanical Engineering courses. The emphasis of all discussions is on the application rather than the rigorous derivation of the theorems; there already exist several classic textbooks for the latter (Kuznetsov 1998; Guckenheimer and Holmes 1983). More application-based works on nonlinear dynamics also exist (e.g. Strogatz 1994) but they address a wide range of physical, chemical, biological, accounting models, to name a few, whereas the present book concentrates exclusively on aeroelastic phenomena.

Nonlinear aeroelasticity has become an increasingly popular research area over the last 30 years. There have been many driving forces behind this development, including faster computers, increasingly flexible structures, automatic control systems for aircraft and other engineering products, new materials, optimisation-based design methods and others. Aeroelasticians have acquired expertise from many different fields in order to address nonlinear aeroelastic problems, mainly nonlinear dynamics, bifurcation analysis, control theory, nonlinear structural analysis and Computational Fluid Dynamics. The main applications of nonlinear aeroelasticity lie in aeronautics and civil engineering but other types of structure are also concerned, such as bridges and wind turbines.

In classical linear aeroelasticity, the relationships between the states of a system and the internal forces acting on them are always assumed to be linear. Force-displacement diagrams for the structure and lift or moment curves for the aerodynamics are always assumed to be linear, while friction is neglected and damping is also linear. As an example, consider a torsional spring that provides a restoring moment M when twisted through an angle ϕ . Figure 1.1a plots experimentally measured values of ϕ and M . Clearly, the function $M(\phi)$ is not linear but, if we concentrate in the range $\phi = [-0.5^\circ \quad 2^\circ]$, the curve is nearly linear and we can curve fit it as the straight line $M = K\phi + M_0$, where K is the linear stiffness of the spring.

Figure 1.1b plots the aerodynamic lift coefficient acting on a wing placed at an angle α to a free stream of speed U , defined as

$$c_l = \frac{l}{1/2\rho U^2 c}$$

where l is the lift force per unit length, ρ is the air density and c is the chord. The curve $c_l(\alpha)$ is by no means linear but, again, if we focus in the range $\alpha = [-5^\circ \quad 10^\circ]$, we can curve fit the lift coefficient as the straight line $c_l = c_{l_\alpha}\alpha + c_{l_0}$, where c_{l_α} is the lift curve slope. An aeroelastic system featuring the spring of Figure 1.1a and the wing of Figure 1.1b will be nonlinear but, if we ensure that ϕ and α never exceed their respective linear ranges for all operating conditions, then we can treat the system as linear and use linear analysis to design it. In nonlinear aeroelasticity, the angles ϕ and α will always exceed their linear ranges and therefore we must use nonlinear analysis, both static and dynamic, in order to design the system.

Nonlinear dynamics is the field of study of nonlinear ordinary and partial differential equations, which in this book model aeroelastic systems. Unlike linear differential equations, nonlinear equations have no general analytical solutions and, in some cases, several different solutions may coexist at the same operating conditions. Furthermore, nonlinear systems can have many more types of solution than linear ones. The operating conditions of an aeroelastic system are primarily the free stream airspeed and the air density (or flight altitude), while the Reynolds number, Mach number and mean angle of attack can also be important. As these system parameters vary, the number and type of solutions of the nonlinear equations of motion can change drastically. The study of the changing nature of solutions as the system parameters are varied is known as bifurcation analysis. In this book we will use almost exclusively local bifurcation analysis, which means that we will identify individual solutions and track their nature and their intersections with other solutions for all the parameter values of interest.

A wide variety of nonlinear aeroelastic phenomena will be investigated, from the galloping of cables to the buckling and flutter of panels in supersonic flow and from stall flutter to

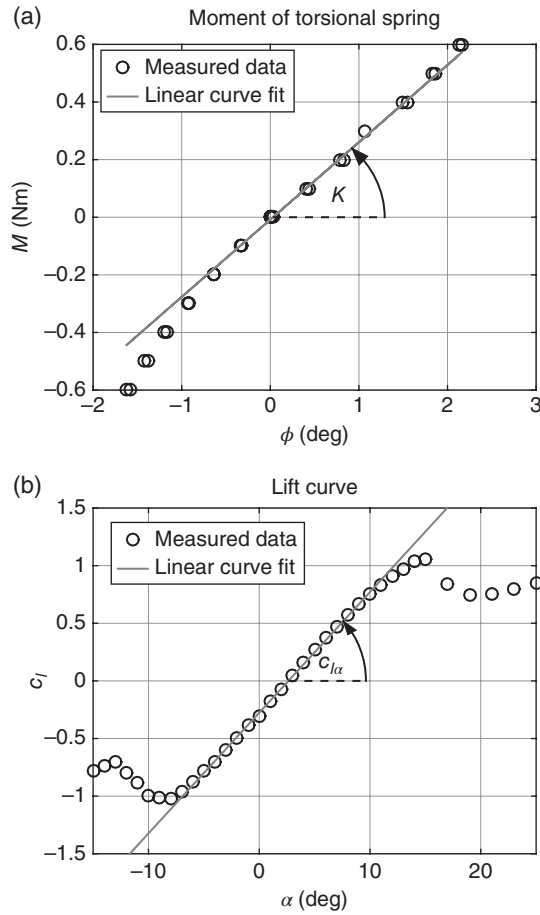


Figure 1.1 Linearised load-displacement diagrams

the limit cycle oscillations of finite wings. We will also briefly discuss transonic aeroelastic phenomena but we will not analyse them in detail because such analysis requires high fidelity computational fluid and structural mechanics and is still the subject of extensive research. The equations of motion treated in this book are exclusively ordinary differential equations; whenever we encounter partial differential equations we will first transform them to ordinary using a series solution. It is hoped that the book will contribute towards the current trend of taking nonlinear aeroelasticity out of the research lab and introducing it into the classroom and in industry.

1.1 Sources of Nonlinearity

Traditionally, a lot of effort has been devoted to designing and building engineering structures that are as linear as possible. Despite this effort, nonlinearity, weak or strong, has always

been present in engineering systems. In recent years, increasing amounts of nonlinearity have been tolerated or even purposefully included in many applications, since nonlinear analysis methods have progressed sufficiently to allow the handling of nonlinearity at the design stage. Furthermore, nonlinearity can have significant beneficial effects, for example in shock absorbers and suspension systems.

In this book we will only consider nonlinearities that are present in aeroelastic systems. Since aeroelasticity is of particular importance to the fields of aeronautics, civil engineering and energy harvesting, we will limit the discussion of nonlinearity to these application areas. The nonlinear functions that are most often encountered in these systems have three main sources:

- the structure,
- the aerodynamics and
- the control system.

The structural nonlinearities of interest occur during the normal operation of the underlying engineering system. Nonlinearities appearing in damaged, cracked, plastically deformed and, in general, off-design systems are beyond the scope of this book. The most common forms of nonlinearity appearing in structures are geometric (caused by large deformations), clearance (i.e. freeplay, contact and other non-smooth phenomena), dissipative (i.e. friction or other nonlinear damping forces) and inertial (of particular interest in rotors and turbomachinery).

Aerodynamic nonlinearities arise from the existence of either unsteady separated flow or oscillating shock waves or a combination of the two (e.g. shock-induced separation). Separation-induced nonlinearity can affect all aeroelastic systems, although bluff bodies such as bridges, towers and cables are always exposed to it. Shock-induced nonlinearity is of interest mostly to the aeronautical industry. It should be noted that aerodynamic nonlinearity is inertial, dissipative and elastic.

Engineering structures are increasingly designed to feature passive and/or active control systems. These systems can either aim to stabilise the structure (e.g. suppress or mitigate unwanted vibrations) or to control it (e.g. aircraft automatic flight control systems). Passive systems can be seen as parts of the structure and therefore included in the structural nonlinearity category (if they are nonlinear). Active systems, however, can feature a number of prescribed and incidental nonlinearities that can be turned off by running the structure in open loop mode. These nonlinear functions are in a category of their own and can take many forms, such as deflection and rate limits on actuators or nonlinear control laws. Furthermore, control actuators always feature a certain amount of freeplay, which is usually strictly limited by airworthiness regulations.

One more source of nonlinearity can be external stores on aircraft that carry them (mainly military aircraft). Stores such as external fuel tanks, bombs and missiles can cause store-induced oscillations, particularly at transonic flight conditions. However, the mechanisms behind these oscillations are still not fully understood and the relevant analyses usually involve computational fluid-structure interaction. Consequently, these phenomena will not be discussed further in this book. Human operator-related nonlinearities (pilot, driver, rider etc.) will not be considered either.

1.2 Origins of Nonlinear Aeroelasticity

Some of the first investigations of nonlinear aeroelasticity concerned stall flutter and started just after WWII. For example, Victory (1943) reported that the airspeed at which wings undergo flutter decreases at high incidence angles, while Mendelson (1948) attempted to model this phenomenon. Rainey (1956) carried out a range of wind tunnel experiments of aeroelastic models of wings and noted the parameters that affect their stall flutter behaviour. It was quickly recognised that, in order to analyse stall flutter, the phenomenon of unsteady flow separation known as dynamic stall needed to be isolated and studied in detail. Bratt and Wight (1945) and Halfman et al. (1951) carried out two of the first experimental studies of the unsteady aerodynamic loads acting on 2D airfoils oscillating at high angles of attack. They were to be followed by a significant number of increasingly sophisticated experiments, covering a wide range of airfoil geometries, Reynolds numbers, Mach numbers and oscillation amplitudes and frequencies. The phenomena of dynamic stall and stall flutter are discussed in Chapter 8.

The effects of structural nonlinearity were first investigated by Woolston et al. (1955, 1957) and Shen (1959). They both set up aeroelastic systems with structural nonlinearity and solved them using analog computers. The systems included 2D airfoils with nonlinear springs, wings with control surfaces and buckled panels in supersonic flow. Such systems have been explored ever since, using increasingly sophisticated mathematical and experimental methods. They are in fact the basis of nonlinear aeroelasticity and will be discussed in detail in the present book. Two-dimensional airfoils with nonlinear springs will be analysed in Chapters 2 to 7, panels in supersonic flow will be presented in Chapter 9 and 3D wings in Chapter 10.

Wind tunnel experiments on nonlinear aeroelastic systems with nonlinear springs have been carried out since the 1980s, notably by McIntosh Jr. et al. (1981); Yang and Zhao (1988); Conner et al. (1997). These works provided both valuable insights into the phenomena that can be encountered in nonlinear aeroelasticity and a basis for the validation of various modelling and analysis methods. The focus of the present book is the application of nonlinear dynamic analysis to nonlinear aeroelasticity. Modelling will be discussed in the last three chapters, as well as in the Appendix.

Shen (1959) was one of the first works to apply the Harmonic Balance method to nonlinear aeroelasticity. This method was first presented in the West by Kryloff and Bogoliuboff (1947) and has since become one of the primary analysis tools for nonlinear dynamic systems undergoing periodic oscillations. We will use several different versions of the Harmonic Balance technique throughout this book.

One of the first studies to apply elements of bifurcation theory to nonlinear aeroelastic systems was carried out by Price et al. (1994). They used stability boundaries, Poincaré sections and bifurcation diagrams to analyse the behaviour of a simple 2D mathematical nonlinear aeroelastic system with structural nonlinearity. Aside from the Hopf bifurcation, they also observed period-doubling bifurcations and chaotic responses. Bifurcation analysis is used throughout the present book but most of the bifurcations typically encountered in nonlinear aeroelasticity are discussed in detail in Chapter 5.

Alighanbari and Price (1996) were the first to use numerical continuation in nonlinear aeroelasticity. Numerical continuation (Allgower and Georg 1990) is a set of mathematical methods for solving nonlinear problems that have static or periodic dynamic solutions. Continuation methods are strongly linked to bifurcation analysis, as they very often start

evaluating solutions at bifurcation points. Such methods will be presented in detail in Chapter 7 and used in all subsequent chapters.

Towards the end of the 1990s, Friedmann (1999) identified nonlinear aeroelasticity as a major research direction in his paper on the future of aeroelasticity. Lee et al. (1999) published a lengthy and authoritative review of past and current nonlinear aeroelastic research, describing all major advances in both understanding and methodologies. A few years later, the nonlinear aeroelasticity chapter by Dowell (2004) provided an extensive description of nonlinear aeroelastic phenomena encountered in flight and in benchmark aeroelastic wind tunnel models and summarised the state of the art.

Thirteen years later, there has been a significant increase in the research and application of nonlinear aeroelasticity. Transonic aeroelastic phenomena, the highly flexible structures of High Altitude Long Endurance aircraft, aeroelastic tailoring, gust loads acting on nonlinear aircraft, wind turbine aeroelasticity and high-fidelity fluid structure interaction have all become major areas of research. Major national and international research projects have addressed such issues and the results are slowly starting to be applied in industry. Given this wealth of activity in the field, it was felt that an introductory text in nonlinear aeroelasticity is missing from the literature. It is hoped that the present book will come to fill this gap, providing a basis for understanding nonlinear aeroelastic phenomena and methodologies on relatively simple systems and preparing the reader for more advanced work in state-of-the-art applications.

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