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What is robust design?

1.1 The importance of small variation

When mass production started in the dawn of the industrial revolution, variation came in the focal point of interest. An early example that illustrates this concerns mass production of guns. The French gunsmith Honoré le Blanc realized the importance for guns to have interchangeable parts. His solution was the invention of a system for making gun parts in a standardized way. The problem that challenged le Blanc is the same as in any modern day manufacturing, as, for example, in the production of bolts and nuts. It shall be possible to pick a bolt and a nut at random that fit together. This requires that the variation in diameter, in roundness, and in thread pitch is small from bolt to bolt and from nut to nut. Unless this is the case, there will be a substantial amount of scrapping, or even worse bolts that crack or fall off while they are in use.

Before the industrial revolution, this problem was handled by good craftsmen. In the industrial era, this was not an option anymore. The importance of managing the variation became obvious. Several approaches emerged. Specifying the tolerance limits was one of them and even if the gunsmith le Blanc did not get many immediate followers in France, some Americans saw the potential of his ideas and implemented them at the armoury in Springfield. This is sometimes considered as the birth of tolerance limits (which is not quite true as tolerance limits are much older than this).

To quote Edward Deming, a forefront figure in quality engineering, ‘Variation is the enemy of quality.’ The bolt and the nut is one example. Another one is thickness variation of the plastic film on the inside of a milk package–a plastic film preventing the beverage from coming in contact with the aluminium foil that is present in most milk (and juice) packages. If this thickness varies too much, it may occasionally happen that there is a point with direct contact between the beverage and the aluminium foil. However, it is not the fact that there is a contact point that should be the centre

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of interest. The focus should rather be the size of the film thickness variation. The contact point is just a symptom of this problem.

Investigations show that a substantial part of all failures observed on products in general are caused by variation. With this in mind it is obvious that variation needs to be addressed and reduced. The issue is just how.

This book is about random variation, or more specifically how to reduce random variation in a response variable y, but not just any way to reduce this variation. It will not be about tightening tolerances, not about feedback control systems, and not sorting units outside the tolerance limits. The focus is solely on preventing variation to propagate. It is this approach to variation that is called robust design.

1.2 Variance reduction

Example 1.1 Consider a bar that is attached to a wall. There is a support to the bar and a random variation in the insertion point in the wall, as sketched in Figure 1.1. We are interested in the position (x, y) of the end point of the bar and that its variation is small.

The variation of the end point position can be reduced in two fundamentally different ways. One is to reduce the variation of the insertion point in the wall. It may be costly. Typically it can involve investment in new and better equipment. However, it might be another way to reduce the variation of the end point position, namely to move the support. In that way the design becomes less sensitive to the variation in the insertion point (Figure 1.2). This is what we mean by robust design: to make the variation of the output insensitive (robust) against incoming variation.

Example 1.2 Two metal sheets are attached to each other. There are two holes in each one and they are attached to each other using two bolts (Figure 1.3). Assume that the maximum stress σ_{max} in the metal sheets is of interest to us. A small variation in the position of the holes, or rather distance between them, affects this stress.

Suppose that this stress should be minimized. This can be achieved by increasing the precision of the positions of the holes. It can also be achieved by changing the

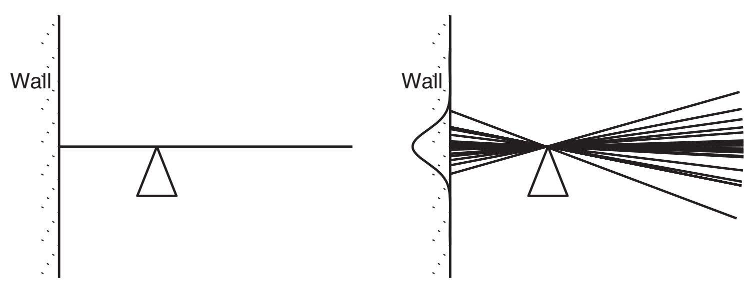

Figure 1.1 A bar is attached to a wall. There is a support to keep it in place. If there is a variation in the insertion point, there will be a variation in the end point.

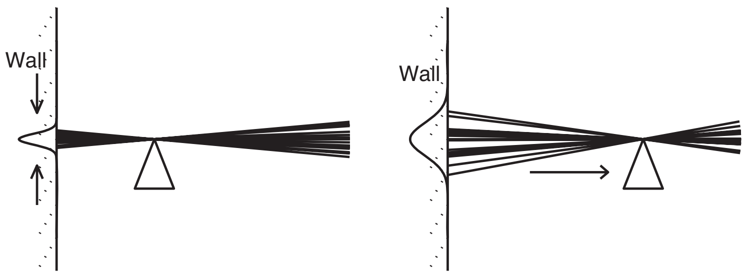

Figure 1.2 *A bar is attached to a wall. The variation in the end point can be reduced in two fundamentally different ways, namely by reducing the incoming variation or by making the design robust against this variation.*

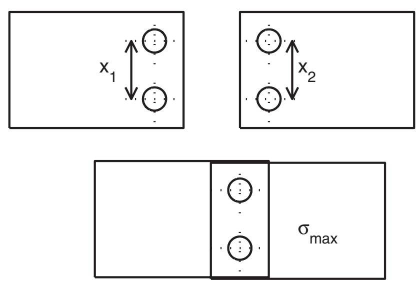

Figure 1.3 *Two metal sheets are mounted together. Since there is a variation in the attachment position, there will be a variation in the maximum stress.*

design so that one hole is exchanged for a slot (Figure 1.4). In that way, the variation in the response, the maximum stress, is decreased without reducing the variation in the sources of variation, the hole positions. The stress is robust to the hole position.

We have seen two examples of robust design, the end point of the bar and the stress of the metal sheets. For both of them, ways to reduce the variation of the output without reducing or removing the original source of variation were pointed out.

In robust design, the original source of variation is called noise. This noise is typically sources of variation that the engineer cannot remove or even reduce, or something that can be reduced but at a considerable cost.

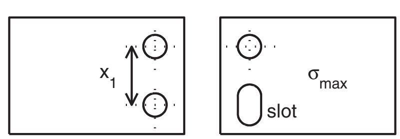

Figure 1.4 *Exchanging one hole for a slot will make the stress robust against the variation in the hole position.*

1.3 Variation propagation

The essence of robust design is to make use of nonlinearities in the transfer function $y = g(x, z)$ in such a way that variation in the noise z is prevented from propagating. The key is in the derivative of the transfer function. We will study how this can be expressed mathematically.

Assume that Z is a random variable with mean μ_z and standard deviation σ_z and that x is a nonrandom variable. Further, assume that Y is a function of Z and x,

$$Y = g(x, Z).$$

For example, Z can be the distance from the nominal attachment point and x the distance from the wall of the support in Example 1.1. Since Y is a function of the random variable Z, it is also a random variable. Taylor's formula gives

$$g(x, z) = g(x, \mu_z) + g'(x, \mu_z)(z - \mu_z) + \frac{g''(x, \mu_z)}{2!}(z - \mu_z)^2 + \cdots$$

where all the derivatives are taken with respect to z. This will be applied to study how the mean and variance of Z propagate to Y. For the mean of Y we obtain

$$E[Y] = E[g(x, Z)] \approx g(x, \mu_z) + g'(x, \mu_z)E[Z - \mu_z] = g(x, \mu_z)$$

and for the variance

$$\begin{aligned} Var[Y] &= Var[g(x, Z)] \approx Var\left[g'(x, \mu_z)(Z - \mu_z)\right] \\ &= \left(g'(x, \mu_z)\right)^2 \sigma_z^2. \end{aligned} \tag{1.1}$$

These are called Gauss' approximation formulas.

Let us apply this to Example 1.1. Define

$$\begin{aligned} Y &= \text{height of end point} \\ Z &= \text{insertion point in wall} \\ x &= \text{position of support, distance from wall} \end{aligned}$$

and

$$L = \text{length of bar.}$$

Note that x has a different meaning here than in Example 1.1. We obtain

$$Y = g(x, Z) = Z + L\sin\left(\arctan\left(-\frac{Z}{x}\right)\right)$$

and

$$Var[Y] \approx \left(g'(\mu_z)\right)^2 \sigma_z^2 = \left(1 - L\left(\frac{x\cos\left(\arctan\left(-\frac{\mu_z}{x}\right)\right)}{x^2 + \mu_z^2}\right)\right)^2 \sigma_z^2. \qquad (1.2)$$

In robust design, Z is called a noise factor or variable that the engineer cannot fully control and x a control factor since it can be controlled.

The main difference between classical ways to approach variation and robust design is which factor in Gauss' approximation formula (Equation 1.1) to focus on.

Gauss' approximation formulas:

$$\mu_y \approx g(\mu_z)$$
$$\sigma_y^2 \approx \left(g'(\mu_z)\right)^2 \sigma_z^2$$

It is how the second one of these, $\sigma_y^2 \approx (g'(\mu_z))^2\sigma_z^2$ (see Figure 1.5), is used that makes robust design different from traditional ways to reduce variation. In traditional engineering, the variance of the variation source, σ_z^2, is reduced in order to reduce the variation of the response, σ_y^2. In robust design the objective is still to reduce σ_y^2, but it is done by reducing $g'(\mu_z)$.

1.4 Discussion

There have always been engineers that have made their designs insensitive to factors that are outside their own control. However, the modern development started in the 1950s, when Genichi Taguchi divided the factors in designed experiments into two different categories, noise and control factors. For some decades, it was primarily in Japan that these ideas attracted any attention.

Around 1980 the ideas of Taguchi reached North America and Europe. The basic principles, like dividing the factors into two different categories, were highly appreciated but some other ideas of Taguchi were more controversial. These controversies will be touched upon at several occasions in this book.

Even if robust design has been around for quite a long time, it is only since around the year 2000 that the application of it in industry has started to grow. One reason is that many companies have started programmes for Design for Six Sigma, DFSS, where robust design plays a central role. Another reason for the growth is the availability of software for CAE (computer aided engineering) based robust design.

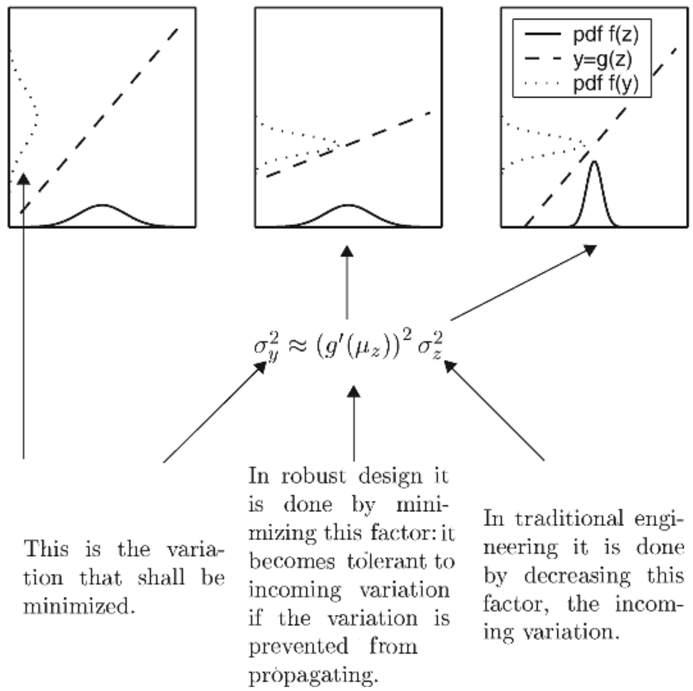

Figure 1.5 Robust design is one way to reduce variation.

1.4.1 Limitations

Since robust design is a broad field, we need to limit ourselves in this book. Two of these limitations are worth mentioning. When Taguchi puts robust design in a framework, he mentions three steps:

(i) system design;

(ii) parameter design;

(iii) tolerance design.

The first one of them, system design, deals with the development of the concept. It is about developing a concept that in itself is robust. The second one, parameter design, is a particular kind of design of experiments (DOE). It deals with the question of how the factor levels (parameter levels) shall be selected given a certain concept. The third one, tolerance design, is left aside in this discussion.

The focus of this book is on the second step, parameter design. It might be somewhat unfortunate, since it is often in the development of the concept or the selection of the concept that robustness is born. However, system design is often difficult and hard to grasp. There is no data available at that point of time and limited possibilities for data collection, so the work has a tendency to become more of a speculation than data driven analysis. There are of course exceptions, often based on a solid understanding of the mathematics and physics of the concepts and intensive usage of the variation propagation formulas of Section 1.3. Therefore, robust design in the concept development phase may look beneficial from a theoretical point of view, but when it comes to the practice it mostly turns out that the benefit is limited. Therefore, the focus of this book is on the step that Taguchi calls parameter design.

Even if we limit our discussion to the DOE related part of robust design, parameter design, there are still different ways to perform the experiments. They may be virtual or physical. The main stream in the book is physical experimentation, but one chapter is devoted to the specific kind of issues arising in CAE based robust design.

1.4.2 The outline of this book

The order in which the topics are introduced in this book is not chronological. It does not start off by deciding what to make robust, followed by a discussion on what it should be robust against. It will rather start with a fairly late step, namely the design of experiments and the analysis of the results (Chapter 2). Our reason is to quickly get into a stage where we can do calculations and from this provide an understanding for the preparations needed before these steps are reached. Starting with DOE will make it possible to quickly come to practical exercises.

It is followed by a chapter about noise factors, and not until thereafter will the issue of what to make robust be reached. The attribute to make robust can be a single value like the mechanical tension of Example 1.2 or a relation between a third type of factor, called a signal, and a response. It is in turn followed by a deeper discussion about factors. It is not always obvious whether a factor is a noise factor (that the engineer cannot fully control) or a control factor.

Already in the discussion about response variables, some of the specific ideas of Taguchi are introduced. In Chapter 5, more will come. It covers a special type of DOE, inner and outer arrays, that originates from Taguchi. The chapter includes a discussion about the pros and cons of such designs compared to the ones covered earlier on in the book.

Once the analysis has been performed, a mathematical model is built and the design that is the most robust is pointed out. However, in order to have confidence in this, it must be checked whether the mathematical model is trustworthy and the design pointed out as the most robust one really is good enough.

Up to Chapter 5, the target value is supposed to be of the type ‘nominally the best’. Chapter 6 covers other types of target values, such as ‘the larger the better’ and ‘the smaller the better’.

In most texts on robust design, it is stated that noise factors must be possible to control in the experimentation despite the fact that they are not controllable in real life. Unless they are, they cannot be included in the designed experiment and it will be impossible to make the design robust against this noise. This is not quite true. Even if noise factors cannot be controlled in the experimentation, it may still be possible to make the design robust against them. The remedy is regression analysis. However, this requires that noise factors are possible to observe and measure. Regression is the topic of Chapter 7.

Then, a number of chapters will follow where various topics of robust design are covered. These chapters may not be of interest to each and every reader, but will be invaluable for those who need a deeper and wider knowledge in the field. The first one of them (Chapter 8) goes deeper into a concept introduced in Chapter 7, namely various types of optimality criteria for robustness. Chapter 8 will also introduce concepts that pave the way to Chapter 9, which is about the special types of experimental methodology that are useful in virtual experimentation and simulation based robust design.

With mass production, there is no or limited possibility to fine-tune the geometry of each and every produced unit (e.g. by shimming). The geometry of the final product must be robust against the geometry variation of its components and of the manufacturing operations. It turns out that Monte Carlo methods are very useful for robust design applied on variations in geometry. This possibility has caught some attention in the industry, especially in the automotive industry, where the usage is fairly wide spread. These Monte Carlo methods are the topic of Chapter 10.

In the last chapter in the book, the history of robust design, or rather Taguchi methods, is presented and the ideas of Taguchi are discussed.

Exercises

1.1 When two objects of material are attached together with glue, the glue can be oxidized in the moment just before the attachment to increase adhesion. It is done by blowing gas on to the glue. However, the dosage (amount) of this gas cannot always be fully controlled. There is a variation around the nominal value as marked with the Gaussian curves of Figure 1.6. In some applications, there is a saturation point for the gas dosage, as in Figure 1.6. Which nominal setting of the dosage is best from a robustness point of view if the adhesion should be maximized, x_0 or x_1?

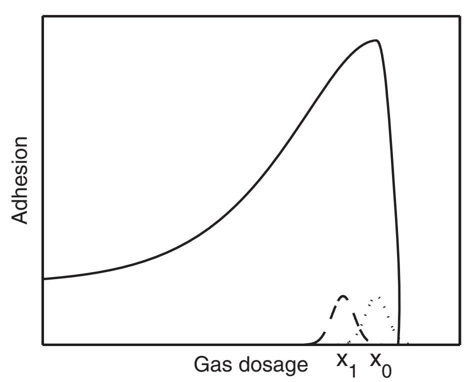

Figure 1.6 The adhesion between the two objects may depend on the gas dosage.

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1.2 Consider Example 1.1 where a bar is attached to a wall. Denote the end point position (y_x, y_y), the length of the bar L, the attachment point in the wall Z, and the position of the support (distance from wall) x. For simplicity, assume that $\mu_z = 0$ and that the height of the support always takes value zero.

(a) Sketch the variance of Y_y as a function of the support position x by using Equation (1.2). Let x be in the range $0.2L$ to $0.95L$, where L is the length of the bar. What happens when $x \to L$?

(b) Use trigonometry to prove Equation (1.2) and to derive the function $Y_x = g(Z, x)$ for the abscissa coordinate of the end point position.

(c) Use Gauss' approximation formulas to obtain the mean and variance of Y_x and reflect on the result. Does the result make sense? If not, explain why!

(d) Use a higher order Taylor expansion to and sketch the variance of Y_x as a function of the support position.