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Introduction

1.1 Background

This book grew out of several years of teaching a flight dynamics course at *The Virginia Polytechnic Institute & State University*, more commonly known as *Virginia Tech*, in Blacksburg, Virginia, USA. That course was initially based on Bernard Etkin's excellent graduate level text *Dynamics of Atmospheric Flight* (Etkin, 1972). There is a newer edition than that cited, but the author prefers his copy, as it can be relied on to fall open to the desired pages.

The author was taken on at Virginia Tech after a full career in the U.S. Navy as a fighter pilot and engineering test pilot. They taught an old dog new tricks, awarded him his PhD, and put him to work.

The author's background crept into the course presentation and Etkin's treatment became more and more modified as different approaches were taken to explaining things.

A sheaf of hand-written notes from a mentor who had actually designed flight control systems at Northrop; course material and flight experience from two different test-pilot schools; the experience of thousands of hours of flight in aircraft at the leading edge of the technology of their time; the precise and clear-minded approach to the analysis of flight dynamics problems that Fred Lutze demanded: all these things and more overlaid the tone and style of the course.

Then, one day, the author's course notes were so different from Etkin's work that it made no sense to continue using that book, and this book was born.

The course as taught at Virginia Tech was intended for first-year graduate students in aerospace engineering. The students all had previous course work in engineering mathematics. For purposes of the current treatment, multi-variable calculus, and a good understanding of ordinary differential equations and their solutions in the time domain and using Laplace transforms are needed.

The undergraduate preparation at Virginia Tech also included a sound course in aircraft performance as in, for example, Anderson's excellent text (Anderson, 1989). Our undergraduates also had an award-winning sequence of courses in aircraft design taught by Bill Mason. While that course undoubtedly gave the students a better feel for what makes airplanes fly, such background is in no way essential to the understanding of this book.

The undergraduates had also studied introductory flight stability and control, most often using another of Etkin's books (Etkin and Reid, 1995). Once again, previous exposure

to this subject matter is by no means essential to mastering the material in this book. The author has seen mechanical engineering students who had *no* previous course-work involving airplanes or *any* airplane experience stand at the top of their classes. These students often had small models of airplanes that they brought with them to class.

The chapters on automatic flight control were not part of the course as originally taught. The major thrust of the book is airplane flight dynamics, but it was felt that some discussion of control was desirable to motivate future study. It seemed unlikely that anything as comprehensive as, for example, Stevens and Lewis (1992) could be included. Therefore just a basic introduction to feedback control is presented, but with some examples that are probably not often found in flight control design.

The last chapter was motivated by the author's pride in the accomplishments of many of his past students in real-world applications of flight dynamics and control.

The method of choice in current flight control system design appears to have settled on to dynamic inversion, and the associated problem of control allocation, and so a brief introduction to these disciplines is offered. Enough material is presented that the reader will be comfortable in the midst of modern flight control system engineers, and may even know something they do not.

Finally, almost all references to MATLAB[®] will be new to previous students. The author's approach to flight dynamics and control has always been to learn the basics, then adopt the modern tools and software to implement the basics. It is not expected that any reader will often use Fedeeva's algorithm in his work, but understanding it does afford one a singular look inside the minds of the men and women who solved these problems with pencil and paper, and who later went on to develop the algorithms that underlie the simple looking MATLAB[®] commands. But the dimension of the problems kept getting bigger and bigger, so some MATLAB[®] tools are now used.

1.2 Overview

The study of aircraft flight dynamics boils down to the determination of the position and velocity of an aircraft at some arbitrary time. This determination will be developed as the *equations of motion* of the airplane. The equations of motion consist of nonlinear ordinary differential equations in which the independent variables are the *states* of the airplane—the variables that fully describe the position and velocity.

The results of the analysis of flight dynamics—the equations of motion—are important in several related studies and disciplines. Chief among these are:

- Aircraft performance. Items of flight performance typically include stall speeds, level flight performance (range, endurance, etc.), excess power and acceleration characteristics, turn performance and agility, climb performance, descent performance, and takeoff and landing performance. Each of these items is governed by the equations of motion. The equations of motion are analyzed to determine the relevant parameters of the performance item, and these parameters are used to devise flight-test techniques to measure performance, or alternatively, to modify aircraft design to improve a particular area of performance.
- Aircraft control. Aircraft control is a very broad discipline, with primary sub-disciplines of manned control, automatic control, and optimal control.

- Manned control refers to a human operator, manipulating control inceptors in the cockpit to drive external control effectors to modify the state and change the trajectory of the aircraft. The relative ease with which a pilot can control the aircraft is described as the aircraft's *flying qualities*. Here, the equations of motion are analyzed to determine the factors that influence the pilot's workload in controlling the aircraft. These factors drive flight-test techniques and provide design guidance to improve flying qualities.
- Automatic control ranges from stability augmentation systems, which modify the aircraft's response to manned control, to autopilots, to full fly-by-wire systems. In fly-by-wire systems, the pilot effectively flies a computer, and the computer decides how to drive the external control effectors to best satisfy what its program thinks the pilot wants to do. Most forms of automatic control are typically based on a special, linearized, form of the equations of motion. The linearized equations of motion are used to design feedback systems to achieve the desired aircraft responses.
- Optimal control is often analyzed open-loop, that is, with no human operator. Representative objectives of optimal control are to minimize (or maximize) some performance measure, such as minimum time to climb, maximum altitude, or minimum tracking error in the presence of external disturbances. The complexity of the solution techniques used to determine optimal control usually means that some simplification of the equations of motion is required. Some ways in which the equations of motion are simplified include treating the aircraft as a point-mass with no rotational dynamics, or reduction in order (reducing the number of states) of the equations of motion.
- Flight simulation. In flight simulation the full nonlinear equations of motion, as ordinary differential equations, are programmed into a computer that integrates the equations (often using high-speed computers in real-time) using numerical integration algorithms. The forces that act upon the airplane are often provided as data in tabular form. The data may be aerodynamic (from wind tunnel or flight tests), or any other description of external forces (thrust, landing gear reactions, etc.)

The information that describes an airplane's position and velocity is usually expressed as relative to some external reference frame, for instance:

- Position and velocity relative to the earth, required for navigation. A pilot navigating from city to city needs to maintain altitude, track, and airspeed within parameters dictated by air-traffic control, and aircraft performance requirements. During terminal flight phases, especially approach and landing, the position and velocity must be maintained within very close tolerances.
- Position and velocity relative to another aircraft, required for rendezvous, formation flying, in-flight refueling, air-to-air combat, and so on. Military aircraft often fly within a few feet of one another; only the relative position and speed are important to the wingman, so that position and velocity relative to the earth are relatively unimportant.
- Position and velocity relative to the atmosphere determines the aerodynamic forces and moments. Students of aircraft performance immediately recognize the relationship of an aircraft's angle-of-attack to its lift coefficient, and of lift coefficient and dynamic pressure to the lift force itself. These simple relationships are the tip of the iceberg:

angular velocities, as well as other aerodynamic relationships, are needed for the most basic understanding of the forces and moments acting on a maneuvering aircraft.

- The last example offered is less intuitive than the others. We consider the position and velocity of an aircraft relative to some reference condition. If the position tends back to the reference position, and the change in velocity tends to zero, then we have described a stable flight condition. The changes in position and velocity might be induced by some external influence, such as a gust or turbulence. An aircraft that is stable in response to such disturbances will be easier to fly. On the other hand, the changes in relative position and velocity might be created by the pilot redefining the reference condition, say by changing from straight, level flight to turning flight. In this case the response of the aircraft determines the ease with which pilot is able to control it.

The approach taken in this book is essentially an application of Newton's Second Law,

$$F = ma \quad (1.1)$$

Simplistically, we assume some knowledge of the mass m and the applied forces F , solve for the acceleration $a = F/m$, and integrate the result with respect to time twice to yield velocity and position, respectively. There are, however, several considerations that will make our analysis non-trivial:

- Newton's Second Law correctly stated is that the external forces acting on a particle of infinitesimal mass are proportional to the time rate of change of the inertial momentum of the particle. We will show that the formula $F = ma$ can be applied to the aircraft as a whole, so long as we are talking about the acceleration of the instantaneous center of mass. The problem of finding an inertial reference frame in which to measure the accelerations will be dealt with by determining a reference frame that is almost inertial, and hopefully showing that the approximation does not introduce too great an error.
- The preceding mention of Newton's Second Law dealt with linear accelerations and can be expected to yield inertial position and linear velocity. An aircraft's angular position and velocity is also of great interest and must be determined through the extension of Newton's Law that relates externally applied moments (torque) to the time rate of change of angular momentum. This will yield the angular accelerations, and twice integrating we obtain angular position and velocity. One major problem with this formulation is that the various reference frames of interest generally rotate with respect to one another, which will require consideration of the various rotational accelerations.
- The mass of what we consider to be the aircraft may become redistributed and change, for example as fuel is burned or external stores are released. The redistribution of mass within the aircraft can arise from many sources and may be easy to consider (rotating machinery) or very difficult to formulate (aeroelastic flutter). It is impractical and undesirable to keep track of the various particles of mass that are moving around or are no longer attached to the aircraft, so we will look for reasonable approximations that allow us to neglect them.
- The various motions, forces, and moments we will consider all have coordinate systems in which they are most naturally characterized. For example, in a suitably defined coordinate system gravity always points 'down' or in the z -direction. For another, our

study of airfoils yields characterizations in which lift is perpendicular to the mean airflow and drag is parallel to it, suggesting a coordinate system in which some axis points in the direction of the relative wind. We will therefore define several coordinate systems that typically are rotating relative to one another. The problem then becomes one of describing the orientation of one system with respect to another, and determining how the orientation varies with time.

- The externally applied forces and moments may be hard to calculate accurately. Only for the force of gravity do we have a reasonably accurate approximation. Propulsive forces depend on a variety of factors that are difficult to predict, such as propeller efficiency or duct losses. Aerodynamic forces and moments in particular create difficulties because they are dependent in complex ways upon the various quantities we are trying to determine. Our understanding of those dependencies relies on empirical and analytical studies, the most extensive of which capture only the most salient relationships. Our study of flight dynamics will typically ignore the uncertainties in aerodynamic and propulsive data, marking these uncertainties for future consideration.
- Assuming we can find tools and reasonable approximations to deal with the aforementioned difficulties and can formulate $a = F/m$ (and its rotational counterpart) then we have the problem of solving these equations for positions and velocities. While it is easy to state that the velocity is the first integral of acceleration, and that position is its second, it is quite another thing to actually solve these equations. The complex relationships among the variables involved guarantee that we will have to find solutions to coupled, nonlinear ordinary differential equations for which analytical solutions are generally unavailable. We will overcome this difficulty by first considering unaccelerated motion of the aircraft, and then asking the question: how does the aircraft behave following small disturbances from this motion, or in response to control inputs? The result of this analysis will be systems of coupled, linear ordinary differential equations for which solutions are available.
- The solutions to the systems of linear ordinary differential equations (equations of motion) will tell us a good deal about how the aircraft behaves as a dynamical system. Unanswered is the question of what the behavior should be in order for the aircraft to be a ‘good’ aircraft. The problem is that most aircraft are piloted, and the human pilot does not want to spend all the time making corrections to keep the aircraft pointing in the right direction. The response and behavior of a piloted aircraft is called flying qualities (sometimes, handling qualities). The difficulty here is that how a pilot would like an aircraft to handle varies from pilot to pilot, and depends on what the pilot is trying to get the aircraft to do. Our study of flying qualities will be based on statistical analyses of a range of pilots’ opinions that give rise to criteria and guidance for the design of aircraft.
- In the event an aircraft does not have inherently good flying qualities, it may be necessary to modify certain dynamical response characteristics. Structural modifications to an aircraft late in the design or in its operational phase are costly. It is relatively inexpensive to incorporate electronic control systems that can be tuned to provide the desired characteristics. The approach we will first investigate utilizes classical feedback control theory. Then we will briefly examine a recent development in automatic control called *dynamic inversion*, and examine its application to interesting control problems.

1.3 Customs and Conventions

We will attempt to proceed from the general to the specific in addressing the various aspects of flight dynamics and control. We will define certain generic quantities and operations needed to address the problem. In applying these results to the study of aircraft, custom and convention dictate that certain variables get their own names. The definitions used here are largely those of Etkin (1972), generally held in the United States to be authoritative. One should always be careful when encountering any terminology to be sure the exact definitions are understood.

References

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