CHAPTER 1

Uncertainty

1.1 INTRODUCTION

There are some statements that you know to be true, others that you know to be false, but with the majority of statements you do not know whether they are true or false; we say that, for you, these statements are *uncertain*. This book is about understanding uncertainty in this sense, about handling it, and, above all, about helping you to live comfortably with uncertainty so that you can better cope with it in your everyday life.

There are two comments that need to be made immediately. The first arises from the fact that the set of statements that you know to be true differs from my set, for you know things that I do not. Equally, things that are uncertain for you may be known to me; but there is more to it than that, for if we take a statement about which we are both uncertain, you may have more confidence that it is true than I do; we differ in our degrees of uncertainty. The upshot of these considerations is that uncertainty is a personal matter; it is not *the* uncertainty but *your* uncertainty. Admittedly, there are some situations where almost all agree on the uncertainty but these are rare and confined to special scenarios, for example, some aspects of gambling. Statements of uncertainty are personalistic; they belong to the person making

Understanding Uncertainty, Revised Edition. Dennis V. Lindley.

them and express a relationship between that person and the real world about which a statement is being made. In particular, they are not objective in the sense that they express a property that is the same for all of us. It follows that throughout this book we will be referring to a person, conveniently called "you", whose uncertainty is being discussed; it may sometimes be appropriate for you, the reader, to interpret it as referring to yourself but generally it applies to some unidentified person, or group of persons expressing a common opinion. You are uncertain about some aspect of the world and that uncertainty does not refer solely to you, or solely to the world, but describes a relationship between you and that world.

The second comment is to note that for any of us, for any "you", the number of statements about which you are uncertain is vastly in excess of the number of statements for which their truth or falsity is known to you; thus all statements about the future are uncertain to some degree. Uncertainty is everywhere, so it is surprising that it is only in the twentieth century that the concept has been systematically studied and, as a result, better understood. Special types of uncertainty, like those arising in gambling, had been investigated earlier but the understanding of the broad notion, applicable to everyday life, is essentially a modern phenomenon. Because uncertainty is everywhere and affects everyone, a proper appreciation of it is vital for all persons, so this book is addressed to everyone who is prepared to listen to a reasoned argument about an ubiquitous concept. This book is for you, whoever you are. We begin with a collection of examples of uncertainty designed to demonstrate how varied, important, and numerous are statements where you genuinely do not know the truth.

1.2 EXAMPLES

EXAMPLE 1. IT WILL RAIN TOMORROW

For all of us who live in climates with changeable weather, this statement is uncertain. It has become almost a classic example of uncertainty because weather is of interest, even importance, to many of us; because meteorologists have seriously studied the question of how to make forecasts like this;

and because it is a statement whose uncertainty will be removed after tomorrow has passed, so that it is possible to check on the quality of the statement, a feature of which meteorologists are very conscious and which will be discussed in §5.12. Notice too, that you can change the degree of your uncertainty about rain by looking out of the window, by consulting a barometer, or by switching on the TV, and we will see in Chapter 6 just how this change may be effected.

A careful discussion here would require clarification of what is meant by "rain"; will a trace suffice, or is at least 0.01 cm in the rain gauge needed before rain can be said to have fallen? Which place is being referred to and where will the gauge be placed? What is meant by "tomorrow"—from midnight to midnight, or 24 hours from 7 A.M., as might be administratively more convenient? In this chapter we deal with illustrative examples and can be casual, but later, when more precision is introduced, these matters will assume some importance, for example, when the skills of meteorologists in predicting the weather are being assessed, or when the quality of mercy in a court of law is described. Again we return to the point in §5.12.

EXAMPLE 2. THE CAPITAL OF LIBERIA IS MONROVIA

The first example, being about the future, is uncertain for everyone living in a variable climate, but with Liberia the personal nature of uncertainty is immediately apparent, as many, but not all of us, are unsure about African politics. Your ignorance could easily be removed by consulting a reference source and, for this reason, such statements, commonly put in the form of a question, are termed almanac questions. The game of Trivial Pursuit is built around statements of this type and exploits the players' uncertainties.

EXAMPLE 3. THE DEFENDANT IS GUILTY

This is uncertainty in a court of law, and "guilt" here refers to what truly happened, not to the subsequent judgment of the court. Although Example 1 referred to the future and Example 2 to the present, this refers to the past. In the two earlier examples, the truth or falsity of the statement will

ultimately be revealed; here it will usually remain forever uncertain, though the primary function of the court is, by the provision of evidence, to remove much of that uncertainty with the court's decision. The process of trial in a court of law will be discussed in §§6.6 and 10.14.

EXAMPLE 4. THE ADDITION OF SELENIUM TO YOUR DIET WILL REDUCE YOUR CHANCE OF GETTING CANCER

This is typical of many medical statements of interest today; in another example, selenium may be replaced by vitamin C and cancer by the common cold. Generally a treatment is held to affect a condition. Some medical statements you believe to be true because they are based on a large body of evidence, whereas others you may consider false and just quackery; but most are uncertain for you. They refer to topics that might come within the purview of science, where a scientist might rephrase the example in a less personal way as "selenium prevents cancer". This last statement is a scientific hypothesis, is uncertain, and could be tested in a clinical trial, where the scientist would additionally be uncertain about the number of cancers that the trial will expose. Contrary to much popular belief, science is full of uncertainty and is discussed in Chapter 11. Scientific experiments and the legal trial of Example 3 are both methods for reducing uncertainty.

EXAMPLE 5. THE PRINCES IN THE TOWER WERE MURDERED ON THE ORDERS OF RICHARD III

Richard III was the king of England and mystery surrounds the deaths of two princes in the Tower of London during his reign. Much of what happened in history is uncertain and this statement is typical in that it deals with a specific incident whose truth is not completely known. The arguments to be presented in this book are often thought to be restricted to topics like gambling (Example 7), or perhaps science (Example 4), but not relevant to cultural matters like history, art (Example 6), or the law (Example 3). In fact, they have the potential to apply wherever uncertainty

is present, which is everywhere. Admittedly historians are rarely explicit about their doubts but one historian, in accord with the thesis to be developed here, said that his probability, that the above statement about the princes was true, was 98%.

EXAMPLE 6. MANY EIGHTEENTH CENTURY PAINTERS USED LENSES AND MIRRORS

Until recently this was thought unlikely to be true but recent studies have produced evidence that strongly supports the idea. Science and art are not necessarily hostile; aside from optics and paint, they come together in the uncertainty that is present in them both.

EXAMPLE 7. A CARD DRAWN FROM A WELL-SHUFFLED PACK WILL BE AN ACE

This example is typical of those that were discussed in the first systematic studies of uncertainty in the seventeenth century, in connection with gambling, and differs from the previous ones in that the degree of uncertainty has been measured and agreed by almost everyone. Because there are four aces in a pack of 52 cards, the chance of an ace is 4 divided by 52, or 1 in 13. Alternatively expressed, since there is one ace for every 12 cards of other denominations, the odds are 12 to 1 against an ace. ("Odds" and "chance" are here being used informally; their precise meaning will be discussed in §3.8.) It is usual to refer to *the* chance but, once you accept the common value, it becomes *your* chance. Some people associate personal luck with cards, so that for them, their chance may not be 1 in 13.

EXAMPLE 8. THE HORSE, HIGH STREET, WILL WIN THE 2:30 RACE

Horse racing is an activity where the uncertainty is openly recognized and sometimes used to add to the excitement of the race by betting on the outcome. Notice that if High Street is quoted at odds of 12 to 1, so that a stake of 1 dollar

will yield 12 if High Street wins, this largely reflects the amount of money placed on the horse, not any individual's uncertainty; certainly not the bookmaker's, who expects to make a profit. Your own odds will help you decide whether or not to bet at 12 to 1. The distinction between betting odds and odds as belief is explored in §3.8. Betting is discussed in §14.5.

EXAMPLE 9. SHARES IN PHARMACEUTICAL COMPANIES WILL RISE OVER THE NEXT MONTH

The buying and selling of stocks and shares are uncertain activities because you do not know whether they will rise or fall in value. In some ways, the stock exchange is like the race course (Example 8), but there is a difference in that the odds are clearly displayed for each horse, whereas the quantitative expression of doubt for the stock can only be inferred from its price now and how it has moved in the past, together with general information about the market. Gambling in the stock market differs from that at the casino (Example 7) because the chances at the latter are generally agreed whereas the existence of buyers and sellers of the same stock at the same time testifies to lack of agreement.

EXAMPLE 10. INFLATION NEXT YEAR WILL BE 3.7%

Statements of this type, with their emphatic "will be", often appear in the media, or even in specialist publications, and are often called either predictions or forecasts (as with the weather, Example 1). They are surely uncertain but the confident nature of the statement tends to disguise this and makes the 3.7% appear firm, whereas everyone, were they to think about it, would realize that 3.8%, or even 4.5%, is a serious possibility. The assertion can be improved by inserting "about" before the figure, but this is still unsatisfactory because it does not indicate how much variation from 3.7% is anticipated. In general, predictions or forecasts should be avoided, because they have an air of spurious precision, and replaced by claims of the form "inflation next year will most likely be between 3.1% and 4.3%", though even here "most likely" is imprecise. Exactly how

uncertainty statements about a quantity, here an inflation index, should be made will be discussed in Chapter 9. Many people are reluctant to admit uncertainty, at least explicitly.

EXAMPLE 11. THE PROPORTION OF HIV CASES IN THE POPULATION CURRENTLY EXCEEDS 10%

At first glance this example appears similar to the previous one but notice it is not an assertion about the future but one concerning the present, the uncertainty arising partly because not every member of the population will have been tested. It improves on Example 10 by making a claim about a range of values, above 10%, rather than a single value. People are often surprised by how little we know about the present, yet at the same time, do not want the uncertainty removed because the only method of doing so involves an invasion of privacy, here the testing for HIV. Uncertainty arising from an inability to question the whole population is considered in Chapter 9.

EXAMPLE 12. IF AN ELECTION WERE TO BE HELD TOMORROW, 48% WOULD VOTE DEMOCRAT

There are two main causes for the uncertainty here, both of which are frequently commented upon and thought by many to make polls unsatisfactory. The first is the recognition that in reaching the 48% figure the pollsters only asked very few people, perhaps thousands in a population of millions; the second is caused by people either not telling the truth or changing their views between the question being posed and the action of voting. Methods for handling the first issue have been developed, and the polling firms are among the most sophisticated handlers of uncertainty in the world.

EXAMPLE 13. THERE WILL BE A SERIOUS NUCLEAR ACCIDENT IN BRITAIN NEXT YEAR

The uncertainty here is generally admitted and discussed. Two important features are the extreme seriousness of the statement if true, and the very

small chance that it will be true. The balance between these two aspects is not easy to resolve and is of very real concern in a society where people are more comfortable with small risks of moderate chance like road accidents, than with accidents of a nuclear type. Methods are developed to handle this in §5.5.

EXAMPLE 14. JESUS WAS THE SON OF GOD

For at least some Christians, this statement is not uncertain, nor is it for atheists, whereas for agnostics, it is uncertain. It is included here because some people hold that the certainty felt by believers here is different in kind from the certainty they feel about Monrovia being the capital of Liberia (Example 2), at least after the almanac has been consulted, one being based on faith, the other on facts. This is a sensible distinction, for it is unsatisfactory to equate faith with checking an almanac. Nevertheless, some of the ideas to be considered in this book may be relevant to discussions concerning faiths.

Incidentally, it was said in the first sentence of the last paragraph that the statement was "not uncertain". The double negative is deliberate because "certain" is an ambiguous word. It can mean "sure", as would be apt here, but it can also mean "particular". Uncertain does not have this ambiguity, "unsure" being a near synonym.

EXAMPLE 15. THE BRITISH SHOULD REDUCE THE AMOUNT OF SATURATED FAT IN THEIR DIET

This example is similar to that concerning selenium (Example 4) but is expressed in terms of a recommendation and comes with some authority from a government via the Ministry of Health, who also explain the reasoning, claiming it will reduce your chance of death from heart disease. Nevertheless, there is some uncertainty about it if only because people in some parts of France consume more saturated fat than some people elsewhere, yet have a lower rate of death from heart disease. Chapter 10 considers the incorporation of uncertainty into action, where statements like this one about fat can affect one's actions and where other

considerations, such as enjoyment of butter, cream, and cheese, need to be balanced against possible health effects.

EXAMPLE 16. THE PLANTING OF GENETICALLY MODIFIED (GM) CROPS WILL DAMAGE THE ENVIRONMENT

Most people consider this statement uncertain, while others are so sure it is true that they are prepared to take action to destroy any GM crops that are planted. Indeed, some will go so far as to destroy those grown to provide information about them and thereby remove, or at least reduce, the uncertainty. Others recognize the value of GM rice in improving the diets of some people in the third world. Issues concerning genetic modification are complex because they can affect both our health and the environment and also have economic consequences. The ideas to be developed in this book are designed to fit uncertainties together and to combine them with our objectives, thus providing some assistance in balancing the many features of an issue to reach an acceptable conclusion. We have first to develop concepts appropriate for a single uncertainty, but our real emphasis has to be on combining uncertainties, and combining them with considerations necessary to implement reasonable actions in the face of uncertainty.

EXAMPLE 17. THE FLIGHT WILL ARRIVE IN LONDON TOMORROW MORNING

This is a typical, uncertain statement about transportation. Whenever we set off on a journey from one place to another, whether on foot, by bicycle, car, bus, train, boat or plane, there is uncertainty about whether we shall reach our destination without mishap and on time, so that it becomes important to compare uncertainties. It is sometimes said that travel by air is the safest form of transport, which is true if the measurement is by number of fatal accidents per thousand miles; unfortunately aviation accidents mostly occur at the start or finish of the journey, so are concentrated into relatively short periods of time. Takeoff is optional; landing is compulsory. What are needed are sensible ways of measuring and comparing uncertainties, and this is what we try to provide in

this book. People repeatedly find it hard to compare one risk with another, so that there is need for a way of assessing risks that will help us understand how the risk of car travel compares with that of planes: how the risk from Alzheimer's disease compares with that from serious indulgence in sporting activities. To achieve this it is necessary to measure uncertainty.

EXAMPLE 18. MRS. ANDERSON WAS ANASTASIA, DAUGHTER OF THE LAST TSAR OF RUSSIA

Mrs. Anderson was thought by some to be the daughter who others thought had been killed in the revolution. This historical statement was, until recently, uncertain, yet of so much interest that several books and a film were devoted to the mystery. A few years ago I made a study of the available evidence that led me to think that the statement was probably true, largely because Mrs. Anderson knew things that it was unlikely anyone but the Princess would have been expected to know. Later DNA evidence has virtually removed the uncertainty, demonstrating not merely that she was not the Princess, but establishing exactly who she was. The mystery having been destroyed, people have lost interest in Anastasia, demonstrating that uncertainty can sometimes be enjoyed.

EXAMPLE 19. THE SUN WILL RISE TOMORROW AT THE TIME STATED

Technically this statement is uncertain for you, because it is possible that some disturbance will affect our solar system; yet that possibility is so remote that it is sensible for you to act as if you knew it to be true. We shall have occasion later to return to the topic of statements that you believe to be true without totally firm evidence. A relation of mine was sure of her age but when, in her 50s, she needed a passport for the first time in her life and, as a result, needed to get her birth certificate to establish her citizenship, she was astounded to find she was a year younger than she had thought. Statements of pure logic, like $2 \times 2 = 4$, are true, but little else has the solidity of logic.

EXAMPLE 20. THE SKULL IS 7 MILLION YEARS OLD AND IS THAT OF A HOMINID

Even for palaeontologists, this is uncertain and there are different opinions that arise, not because people can be quarrelsome, but because there are understandable difficulties in fitting the pieces of fossil evidence together. In the early stages of a study, even when conducted using sound, scientific principles, there is, as discussed in Chapter 11, a lot of uncertainty. One aspect has been discussed statistically, namely the assignment of dates, so that a respectable body of evidence now exists for which the uncertainty has been, if not removed, at least lessened.

1.3 SUPPRESSION OF UNCERTAINTY

The long list of examples demonstrates how common is the phenomenon of uncertainty. Everything about the future is uncertain, as is most of the past; even the present contains a lot of uncertainty, due to your ignorance, and uncertainty is everywhere about you. Often the uncertainty does not matter and you will be able to proceed as if tomorrow will be just like today, where the sun will rise, the car will start, the food will not be poisoned, the boss will be her usual self. Without this certainty, without this assurance of continuity, life as we know it would be impossible. Nevertheless, we all encounter situations where you have to take cognizance of uncertainty and where decisions have to be made without full knowledge of the facts, as in accepting a job offer or buying a new house, or even on deciding whether to have a picnic.

Despite uncertainty being all about us, its presence is often denied. In Britain, though not in the United States, the weather forecast will state categorically that "it will rain" (Example 1) and then sometimes look foolish when it does not. Economists will predict the rate of inflation (Example 10) and then get it wrong, though because the time scale is different from the meteorologist's, we sometimes do not notice the error. This is slightly unfair because, as mentioned in the example,

economists are mending their ways and quoting intervals, thereby recognizing the uncertainty. Newspapers can report an HIV rate (Example 11) as if it were true, or cite the numbers at a demonstration as fact even though the police and participants differ. Television executives hang desperately onto audience ratings, largely ignoring the errors present. People in the humanities rarely mention uncertainty (Example 5). Even the best historians, who are meticulous with their sources, can blur the borderline between facts and opinions. Lawyers (Example 3) do admit uncertainty and use language like "beyond reasonable doubt" or "the balance of probabilities"; nevertheless, at the end of the trial the jury has to ignore the uncertainty and pronounce the defendant "guilty" or not. Politicians are among the worst examples of people who deny any uncertainty, distorting the true scenario to make their view appear correct. There are places like the casino (Example 7) or the race course (Example 8) where the uncertainty is openly admitted and exploited to add to the excitement.

One reason for the suppression is clear: People do not like to be unsure and instead prefer to have everything sharply defined. They like to be told emphatically that the sun will shine, rather than to hear that there might be the chance shower to spoil the picnic, so they embrace the false confidence of some weather forecasts, though they are annoyed when the forecast is incorrect. But if some uncertainty is present, and we have seen that uncertainty is almost everywhere, it is usually better to face up to it and include it in your thoughts and actions, rather than suppress it. Recognition of the uncertainty in investing in stocks, or taking out a pension contract, is valuable because it helps to guard against things going wrong. Suppression of uncertainty can cause trouble, as the law has found when it claims to have removed the uncertainty by the jury announcing a verdict of guilty. To go to appeal or have a case reviewed can be difficult, partly because no one likes to admit they were wrong, but partly because the uncertainty lay unrecognized. Scientists, who are more open about uncertainty than most, still cling to their beloved theories and have trouble in accepting the maverick worker, partly because they are reluctant to entertain uncertainty. There is a clear and beautiful example of the misplaced dislike of uncertainty in the Ellsberg paradox discussed in §9.11.

Part of the thesis of this book is that, instead of neglecting or, worse still, suppressing uncertainty, it is better to recognize its presence everywhere, bringing it out into the open and discussing the concept. Previously this has not been done, partly because it is no use exposing something if, when you have done so, you do not know how to handle it, like opening a Pandora's box of misery. The past and present neglect and suppression therefore have sense behind them, but recently a change has taken place and the purpose of this book is to tell you about it. What has changed is that we now know how to handle uncertainty; we know what the rules are in Pandora's box. Beginning with the study of uncertainty in games of chance, the net has widened to the appreciation that the simple rules discovered there, and they are truly simple, just controlled addition and multiplication, apply beyond gambling to every uncertain situation, so that you can handle beliefs nearly as assuredly as facts. Early sailors had difficulty going out of the sight of land but when the rules of navigation became better understood, with the use of the stars and accurate clocks, voyages across oceans became practicable. Today we travel the seas, the air, and even space, because of our understanding of the rules; so I contend that now the rules of uncertainty have been understood, we no longer need to neglect or suppress it but can live comfortably even when we do not know.

1.4 THE REMOVAL OF UNCERTAINTY

If uncertainty is such a common feature of our lives, and yet we do not like it, the obvious thing to do is to remove it. In the case of the capital of Liberia (Example 2), this is easily done; one just goes to an almanac and checks that indeed Monrovia is the capital, though it would be as well to bear in mind that the almanac may be out of date or even wrong, or that an error can be made in consulting it, so that some uncertainty remains, but at least the uncertainty will be lessened. The removal of uncertainty is not usually as easy as it is with almanac questions. The court of law is a place where a serious attempt is made to reduce, if not remove, uncertainty. Some places use an adversarial approach, which allows both sides to present facts that they think are relevant, in the

hope that the jury will feel convinced one way or the other about the defendant's guilt. Both these examples show that the usual way to remove or reduce uncertainty is by the production of facts; these are statements that are essentially free of uncertainty, like the almanac, or are much more likely to be accepted as true than the original statement. A major task of this book is to show exactly how this reduction takes place. The legal process is considered in §10.14.

The adversarial method is not the only way to obtain and process facts. Scientists collect data and perform experiments, which are assembled to infer general rules that are often deterministic and involve little uncertainty, like Newton's laws of motion. Careful measurements of the motions of the heavenly bodies led eventually to accurate calculation of their orbits so that, for example, an eclipse ceased to be uncertain but could be predicted with great accuracy. Scientific facts differ from legal facts in that they are repeatable, whereas legal evidence is not. If a scientist reports the results of an experiment, then it is an essential feature of the scientific method that other scientists be able to repeat the experiment and obtain the same result, whereas the witness's statement that he was with the defendant at the time of the crime is not capable of repetition. The repeatability aspect of science, with its consequent removal of almost all uncertainty, often leads people to think that all science is objective, as it virtually is after there has been a lot of confirmatory repetition, but active science is full of uncertainty, as healthy disagreement between scientists testifies. Science is discussed in Chapter 11.

One of our examples (Example 14) differs in style from the rest in that the agnostic's uncertainty about Jesus being the son of God is difficult to change since no further facts about Jesus are likely to be obtained. The most plausible way to change is to accept the statement as an article of faith, essentially removing the uncertainty altogether. This would ordinarily be done in connection with other features of the faith, rather than by facts. This is not to say religions do not themselves change in response to facts. The Catholic Church moved from thinking of the Earth as the center of our part of the universe, to a view that centered on the Sun; this in response to astronomical data.

Whether the ideas presented in this book, and especially the three basic rules, apply to faiths is debatable. The wisest advice is perhaps

that offered by Oliver Cromwell to the Church of Scotland, "believe it possible you may be mistaken". Acceptance of this advice would lessen tensions between different faiths. Cromwell's rule for probability is discussed in §6.8.

1.5 THE USES OF UNCERTAINTY

So far the emphasis has been on our dislike of uncertainty and methods taken to avoid the phenomenon, yet there are situations in which you actually enjoy the uncertainty and without it life would be duller. Examples are provided by mysteries where you do not know the solution, as with Mrs. Anderson in Example 18; once the mystery has been cleared up, the story loses its interest. A difference between a puzzle and, say, uncertainty about your health lies in the fact that the consequences that could flow from the removal of the uncertainty are not experienced by you in the first case, but will be in the second. Once you know she was not Anastasia, you shrug your shoulders and pass onto the next puzzle; once you are diagnosed as having cancer you have to live with the unpleasantness. So perhaps it is not that we dislike uncertainty; rather we are concerned about possible outcomes. Perhaps it is not the uncertainty about the rain (Example 1) that concerns us but rather the thought of the spoiled picnic.

Yet this cannot be the whole story, as there are uncertainties that many of us enjoy, where we do have to experience the results, some of which may, if we overindulge, be most unpleasant. The obvious ones are gambling with cards (Example 7) or betting on the horses (Example 8). Here we can, and often do, lose our money, yet we gamble because of the excitement found in the activity. Our study will reveal how this enjoyment, quite apart from monetary considerations, can be combined with the rules mentioned earlier to provide a reasoned account of gambling.

Here is a serious example of the benefits of uncertainty. In Chapter 8 we shall discuss clinical trials, that is, experiments in which patients are given a treatment or a drug to investigate whether it improves their health. In order to assess the drug's effectiveness,

it is necessary to take other, similar patients and give them a placebo, something that is outwardly like the drug but in fact contains only some innocuous material. Comparing the changes in the patients on the drug with those receiving the placebo, it is possible to measure the value of the drug. In order that the conclusions from a trial be reliable, it has to be conducted with care and one precaution is to ensure that the patients do not know whether they are receiving the drug or the placebo. To anticipate a term to be introduced in §3.2, the patients on the drug are selected at random from a pool of patients, so that every participant in the trial is uncertain about what they are taking. It is also desirable to ensure that the clinician conducting the trial is equally uncertain, as we shall see when discussing Simpson's paradox in §8.2. Many experiments today actively encourage an element of uncertainty, by selecting at random, in order to make the results more reliable than they would be were it not present.

There is another merit of uncertainty that appears whenever a competitive element is present, as in sport or the conduct of war. If you are competing against an opponent, then it is to your advantage to increase their uncertainty, for example, by creating the impression that you are about to do one thing when you intend to do another. There will be little in this book about the bluffing aspect of uncertainty because we are concerned with a single person, the "you" of the language introduced in §1.7, and there are real difficulties in extending the calculus to two "yous" that are in competition. A famous, simple example of this is the prisoner's dilemma, mentioned in §5.11. We develop a calculus for "you"; there does not exist an entirely satisfactory calculus for two or more competitors and, in my view, this omission presents a serious, unsolved problem.

Notice that in the competitive situation it is not so much that you want your opponent to be uncertain, or even wrong, but that you want to have information that they do not have. You know when you are going to attack, they do not. It is your information that matters, information to be kept from them. Information is power, which is why politicians, when in power, hate the open government that they espoused when in opposition. One of our principal tasks will be to see how information can be used to your advantage. The concept of information within the calculus is treated in §6.12.

1.6 THE CALCULUS OF UNCERTAINTY

In this book uncertainty is recognized and accepted as an important part of our lives. No attempt is made to disguise or deny it; rather it is brought out into the open and we learn to handle it as confidently as we do those features about which we are sure. We learn to calculate with uncertainty, much as a card-player calculates the situations in a game of bridge. Indeed, the rules of calculation are essentially those that operate in cards or roulette.

In most circumstances that operate in cards, more than one feature is uncertain and the various uncertainties need to be combined. Similarly, a juror hearing witnesses will be uncertain about their veracity and need to meld it with the doubts concerning the defendant's guilt. A scientist performing an experiment may be uncertain about the pressure used, the purity of the material, as well as about the theory under investigation. In reacting to the offer of a job, you will be uncertain about the move involved, the nature of the work, and many other features. A doctor will need to combine appreciation of the uncertain symptoms in order to reach an overall diagnosis. In every one of these cases, many uncertainties have to be amalgamated to produce the overall judgment, so that a central task is for us to see how to put several uncertainties together.

There are things that combine very easily: numbers. Addition and multiplication are so easy that even a computer can perform them, a computer being only as wise as its programmer. One day we may have artificial intelligence but today most computers can only perform the logic they have been taught. If then, we could measure uncertainty, in the sense of attaching numbers to the statements, just as we did above with the ace drawn from the pack of cards, then the combination would present fewer difficulties and involve only the rules of arithmetic. This will be done; we will measure uncertainty, and then develop the three wonderful rules of combination. It is in the appreciation of the rules, and the ability to use them, that the strength of this book resides. We shall calculate with uncertainties and the machinery to do this is called the calculus of uncertainty.

Scientists already use statistical methods, developed from these rules, to help them interpret their data. It will be sometime before

jurors have their computer with them to assess the uncertain guilt, but the beginning of the idea can be seen in the treatment of forensic science in §6.6. One day the historian will calculate the odds against Richard III being the culprit (Example 5) rather than plucking a number out of the air as the historian quoted might have done.

It is an unfortunate fact of life that many people, especially those working in the arts or the media, have a strong dislike of numbers and are unhappy using them. Although there is likely to be genuine variation in the ease with which numbers are handled, my personal belief is that almost all can be taught to manipulate with figures and, just as important, appreciate the power that such a facility can bring. Here we shall calculate but I have tried to expound the mechanics in a simple manner. All that I ask is a willingness on the reader's part to cooperate by showing some motivation to learn, genuinely to want to understand uncertainty.

1.7 BELIEFS

We have seen that uncertainty involves a statement, whose truth is contemplated by a person. It is now convenient to introduce the standard language that is used in the calculus of uncertainty. Instead of "statement", we refer to an "event", thus the event of rain tomorrow or the event of selenium affecting cancer. Sometimes "event" will seem a strange nomenclature, as when referring to the event that Monrovia is the capital of Liberia, but it is usually apt and experience has shown that it is useful as a standard term. Thus an event is uncertain for you if you do not know whether it is true or not.

We also need to have a term for the person facing the uncertainty for, as we have seen, one person's uncertainty can be different from another's. As already mentioned, the term "you" will be used and we will talk about your uncertainty for the event. In many cases you, the reader, can think of it as a reference to yourself, while in others it may be better to think of someone else.

A term is needed to describe what it is that you feel about the event. The phrase usually employed is "degree of belief"; and we will talk about your degree of belief in the truth of the event, so that you

have the highest belief when you think it is true, and least when false. Belief is a useful word because it does emphasize that the uncertainty we are talking about is a relationship between you, on the one hand, and an event, on the other. Belief does not reside entirely with you because it refers to the world external to you. Belief is not a property of that world because your degree of belief may reasonably be different from mine. Rather belief expresses a relationship between you and the world, in particular between you and an event in that world. The word that will be used to measure the strength of your belief is probability, so that we talk about your probability that an event is true, or more succinctly, your probability for the event. One of the greatest experts on probability, having written a two-volume work on the topic, calling it simply *Theory of Probability*, wanted an aphorism to include in his preface that would encapsulate the basic concept expressed therein. He chose:

Probability does not exist.

It was intended to shock, for having written 675 pages on a topic, it did not seem sensible to say the topic did not exist. But having brought it to your attention by the shock, its meaning becomes apparent; probability does not exist as a property of the world in the way that distance does, for distance between two points, properly measured, is the same for all of us, it is objective, whereas probability depends on the person looking at the world, on you, as well as on the event, that aspect of the world under consideration. Throughout this book we will refer to *your* probability, though the use of *the* probability is so common in the literature that I may have slipped into the false usage unintentionally.

Our task in this book is to measure beliefs through probability, to see how they combine and how they change with new information. This book is therefore about your beliefs in events. It is not about what those beliefs should be, instead it is solely about how those beliefs should be organized; how they need to relate, one to another. An analogy will prove useful, provided it is recognized that it is only an analogy and cannot prove anything but is merely suggestive. Suppose that this was a book about geometry, then it would contain results about the shapes of figures, for example, that the angles of a

plane triangle add to 180 degrees, but it would not tell you what the angles have to be. In fact they can be anything, provided they are positive and add to 180 degrees. It is the same with the beliefs described here, where there will be results, analogous to the sum of the angles of a triangle being 180 degrees, that provide rules that beliefs must obey. We shall say little about what the individual beliefs might be, just as little is said about the individual angles. If you have high belief that the Earth is flat, then there is nothing in our rules to say you are wrong, merely that you are unusual, just as a triangle with one angle only a fraction of a degree is unusual. We claim that the rules provided are universal and should not be broken, but that they can incorporate a wide range of disparate opinions.

Before writing these words, I had heard an argument on the radio between a representative of a multinational corporation and another from an environmental organization. The arguments presented in this book have little to say about who is correct but they have a lot to say about whether either of the participants had organized their beliefs sensibly. It is my hope that correct organization, combined with additional information, will help in bringing the speakers together.

1.8 DECISION ANALYSIS

We all have beliefs and in this book we try to show how they should be organized, but not what they should be. There is, however, a basic question that we need to answer:

What is the point of having beliefs and why should we organize our opinions?

The answer is that we have beliefs in order to use them to improve the way in which we run our lives. If you believe that it will rain tomorrow, you will act on this and not go on with the picnic, but go for an indoor entertainment instead. Action is not essential for beliefs and most of us will not be influenced in our actions by our beliefs concerning the Princes in the Tower (Example 5), but if action is contemplated, as with the picnic, then our beliefs should be capable of being used to decide what the action should be.

This attitude toward beliefs is pragmatic in the sense that it assesses them by how they perform as a guide to action, and it leads from the sole consideration of your attitude toward an uncertain world, to how you are to behave in that world. Some hold that belief is inseparable from action, while we prefer to develop the calculus of belief first, and then extend it to embrace action. The relationship here is asymmetric: actions require beliefs, but beliefs do not necessitate action.

The topic that deals with the use of beliefs in action is called "decision analysis", and it analyzes how you might decide between different courses of action, without saying what the decisions should be, only how they should be organized. The passage from belief to action will introduce a new concept that needs to be blended with the beliefs in order to produce a recommended action. Example 13 supplies an illustration, where the seriousness of the nuclear accident needs to be blended with the small belief that it will happen, in order to decide whether to build more nuclear power plants. The subject is covered in Chapter 10.

In summary, this book is about your approach to uncertainty, how your beliefs should be organized, and how they need to be used in deciding what to do. Before we embark on the program, it is necessary to comment on the method used to tackle these problems. These commentaries form the content of the next chapter and only in Chapter 3 will the development proper begin.