
Fixed Income Markets: An Introduction

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1.1 INTRODUCTION

The last decade witnessed a profound transformation of fixed-income markets, in no small part due to the 2008–2009 financial crisis. The transformation took different forms: first, in the aftermath of the crisis, a number of countries – and the United States, in particular – had to expand their borrowing capacity in order to “stimulate” the economy. Lower tax revenues due to the economic downturn and higher government spending resulted in a large increase in government debt. As an example, Panel A of Figure 1.1 plots the outstanding U.S. debt from 1985 to 2014, compared to the mortgage-backed securities market, the corporate debt market, and the money market, which include commercial paper, bankers’ acceptance, and large time deposits. The first obvious observation is that all debt markets have been trending upward. The second observation is that while the U.S. Treasury debt market was the largest market between 1985 and the late 1990s, by year 2000, it was surpassed in sheer size by two forms of private debt, namely, the mortgage-backed securities market (the mortgage-related debt of individual households) and the corporate debt market (the debt market of private corporations). The United States has steadily becoming more in debt, but until the mid-2000s, this new debt was mainly private debt. With the advent of the financial crisis, the U.S. debt started a quick ascent to then take over both the mortgage-backed securities market and the corporate debt market, both of which instead show no growth after the crisis.

Clearly, the trends in Panel A are partly misleading, as it is true that the United States increased its nominal debt levels considerably since 1985, but the U.S. economy itself also grew substantially over the past 30 years. To take this into account, Panel B plots the same quantities as in Panel A but as a percentage of the U.S. Gross Domestic Product (GDP). The normalized U.S. debt shows a much more stable path over the sample period, as it had been indeed decreasing as a percentage of GDP between 1995 and 2008. The last 4 years, however, have seen the debt-to-GDP ratio rise from 30% to 70%, a clear concern that even led the rating agency Standard & Poor to downgrade the U.S. debt to AA+ from AAA in August 2011. In contrast to the U.S. Treasury debt, both the mortgage-backed securities market and the corporate debt market have been declining in the last few years as percentage of U.S. GDP. Finally, the size of the money markets also reached its peak in 2007, at around 20% of GDP and has been declining ever since. Smith (2015, Chapter 2 in this handbook) covers in detail the recent trends in money markets.

Besides declining in size (relative to the GDP), the mortgage-backed securities market also underwent several transformations on its own during the crisis, starting with U.S. government placing the two mortgage giants Freddie Mac and Fannie Mae under conservatorship. Duarte and McManus (2015, Chapter 4 in this handbook) discuss the evolution of the mortgage-backed

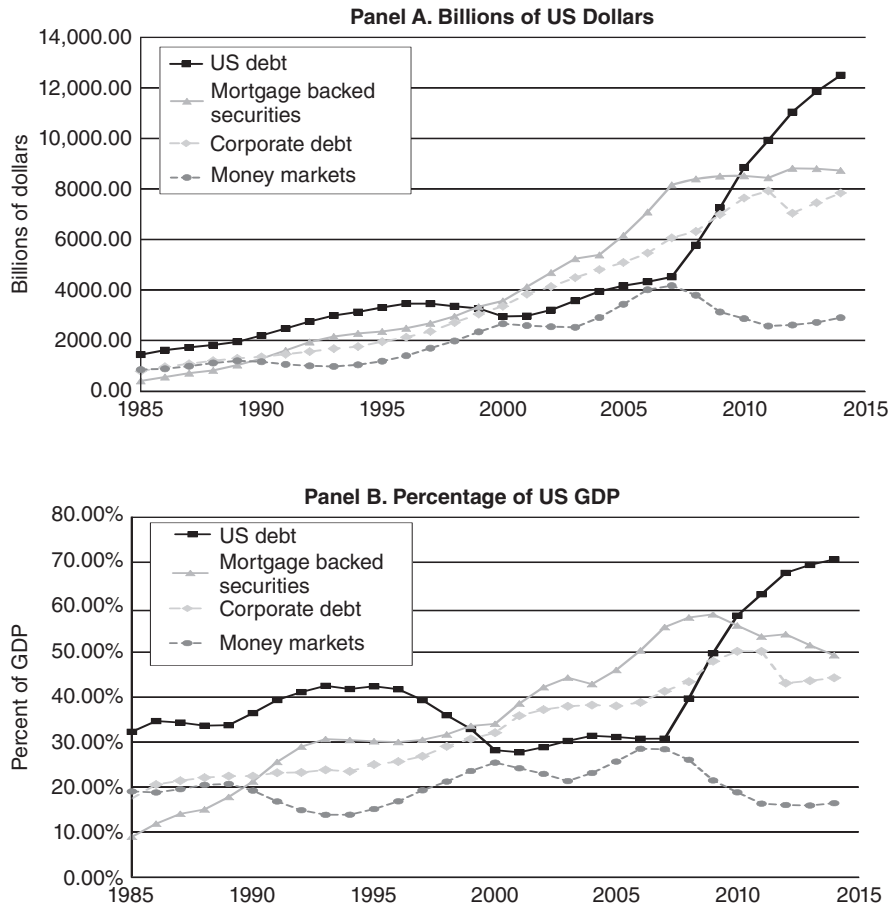


Figure 1.1 The outstanding amount of U.S. debt. Panel A shows the outstanding marketable U.S. debt, mortgage-backed securities, and corporate debt from 1985 to 2014. Panel B reports the same quantities rescaled by U.S. GDP for the same period. Source: SIFMA and FRED at Federal Reserve Bank of St Louis.

securities market after the crisis. In particular, after the crisis, large amounts of new data have become available, and Duarte and McManus discuss new methodologies to price mortgage-backed securities.

A second important change in the U.S. debt market is the unprecedented aggressive monetary policies adopted by the Federal Reserve. In 2008, the Federal Reserve slashed its main reference interest rates – the Federal funds rate – to essentially zero and then moved to the so-called “unconventional” monetary policies, which essentially entailed large purchases of U.S. government debt as well as mortgage-backed securities. Panel A of Figure 1.2 plots the Federal funds rate over time, which highlights its precipitous drop from 5.27% in July 2007, to 2% in June 2008, to 0.25% in November 2008. Panel B shows the entire term structure of interest rates over the same time period, with maturities ranging from 3 months to 30 years.¹ The short-term 3-month Treasury bill rate hit (almost) zero at about the same time as the Federal funds rate, while the long-term yields steadily decreased over time, with some ups and downs in the last few years. Several questions arise on the impact that the Federal Reserve actually has on the term structure of interest rates. Buraschi and Whelan (2015a, 2015b, Chapters 5 and 6 in this handbook) discuss the existing evidence as well as provide new evidence about the impact of monetary policy on the term structure of interest rates.

Third, in the aftermath of the 2008–2009 financial crisis, governments around the world introduced substantial regulatory changes with the aim of regulating trading in derivatives markets and curbing the risk-taking behavior of a number of financial institutions (see Culp (2015, Chapter 16 in this handbook) for a detailed discussion of the new regulatory environment). Indeed, Figure 1.3 shows that the size of the global fixed-income derivatives market has been increasing steadily over the two decades preceding the financial crisis. Panel A shows the increase in the notional amount of Over-the-Counter (OTC) derivatives from June 1998 to June 2013, starting from about \$50 trillion and increasing to over \$560 trillion over the 15-year period.²

¹Note that in Panel B of Figure 1.2, only yields up to 5 years of maturity are available until August 1971, when also the 10-year yield becomes available. The 20-year yield is available from July 1981 and the 30-year yield from November 1985.

²Data are from the Bank for International Settlements and are available at www.bis.org. The biannual survey result of June 2013 is available at http://www.bis.org/publ/otc_hy1311.pdf, accessed on January 17, 2014.

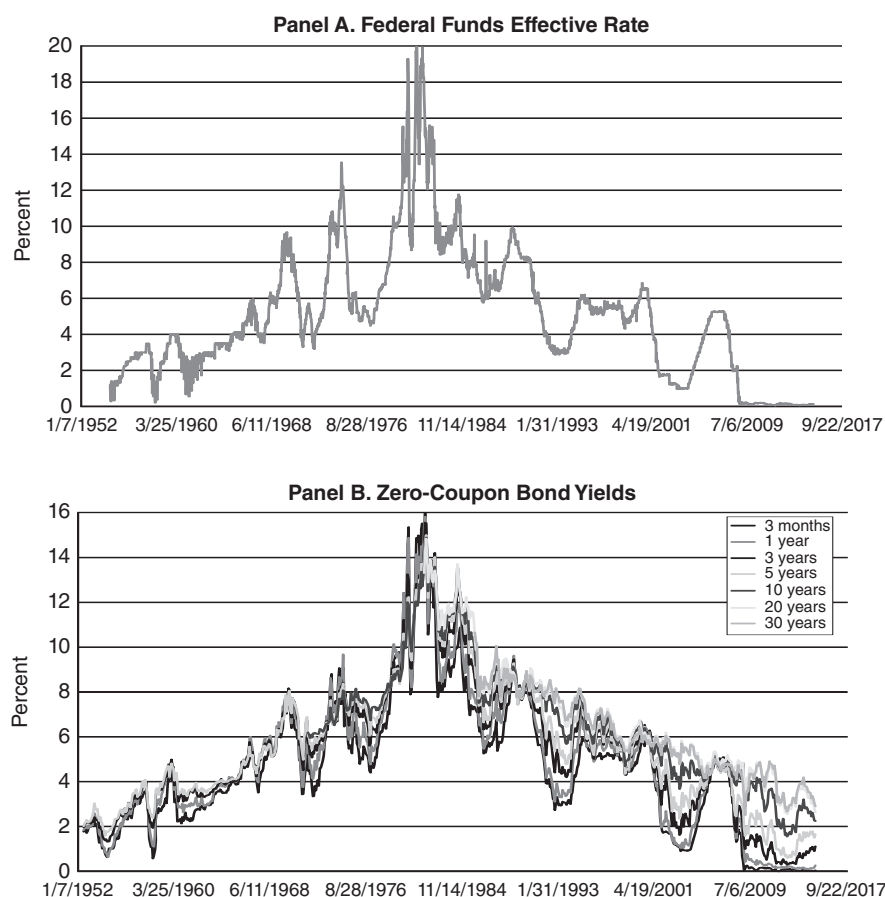


Figure 1.2 Federal funds and U.S. Treasury yields. Panel A shows the Federal funds effective rate from 1954–2014 (Source: Federal Reserve Bank of New York website). Panel B plots the zero-coupon bond yields from 1952–2014. Source: Yields up to 5 years are from the Center for Research in Security Prices (CRSP). Yields from 10 to 30 are from Gürkaynak, Sack, and Wright (2007), updated series.

The figure also shows that interest rate swaps comprise the lion’s share of the OTC fixed-income derivatives market. While the tenfold increase in the OTC market is impressive, the global economy also expanded considerably over the same period. Panel B of Figure 1.3 renormalizes the notional amount of OTC derivatives by world GDP. Even after normalization, the total notional of OTC fixed-income derivatives increased from about 1.5 times the global GDP in 1998 to 8 times the global GDP at the end of 2007 and then mildly declined to 7 times the global GDP level by the end of 2012, possibly due to the effects of the financial crisis and the new regulation on derivatives.

One additional important change that came with the new regulation, however, is that the pricing of derivative securities has become even more complex than ever before. Even relatively “simple” plain vanilla securities became challenging to price, as market participants now require (or are required to require) full collateralization of derivative positions, which entail additional costs from holding the positions open. Veronesi (2015, Chapter 18 in this handbook) and Brigo et al. (2015, Chapter 21 in this handbook) discuss a number of pricing issues that arise from the new regulation.

Finally, besides the level of interest rates and additional regulation, the last decade also witnessed substantial changes in the behavior of bond returns themselves. For instance, the left panel of Figure 1.4 shows the quarterly series of the covariance of 5-year U.S. bond returns with the returns of the S&P 500 index. The covariance is computed from daily returns in each quarter. Quite dramatically, the covariance turned from being mostly positive until about the year 2000 to being mostly negative since then. The right panel shows that the realized “beta” of bonds with respect to the S&P 500 index (i.e., the covariance divided by the variance of the S&P 500 index) also experienced a rather dramatic fall over the same period. That is, since 2000, bonds have become an important hedge against stock market fluctuations. But why were not bonds a hedge historically? What has changed recently? The answers to these questions have obvious first-order consequences for asset allocation between two of the largest financial asset classes. David and Veronesi (2015, Chapter 15 in this handbook) review the recent literature on the movement of stock-bond covariance over time.

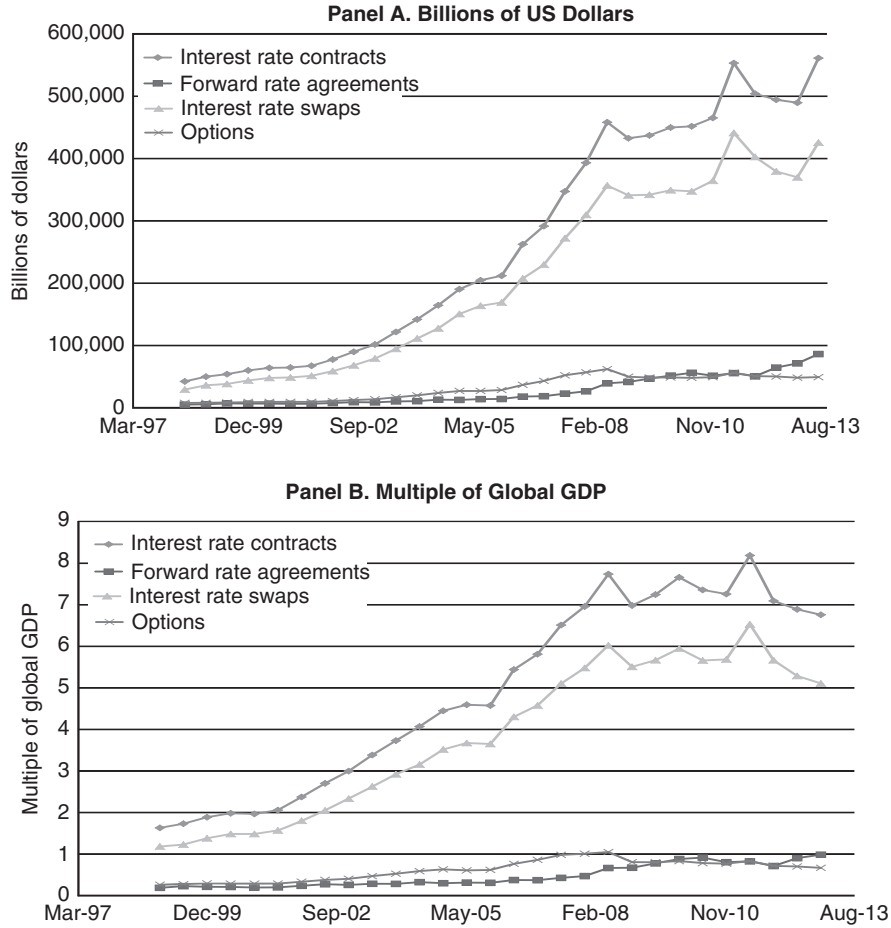


Figure 1.3 The notional amount of over-the-counter derivatives. Panel A shows the notional amount of OTC fixed-income derivatives from June 1998 to June 2013. Panel B reports the notional amount of OTC derivatives rescaled by the world global GDP from 1998 to 2012. OTC derivatives data are from the Bank for International Settlements, while global GDP data are from the World Bank.

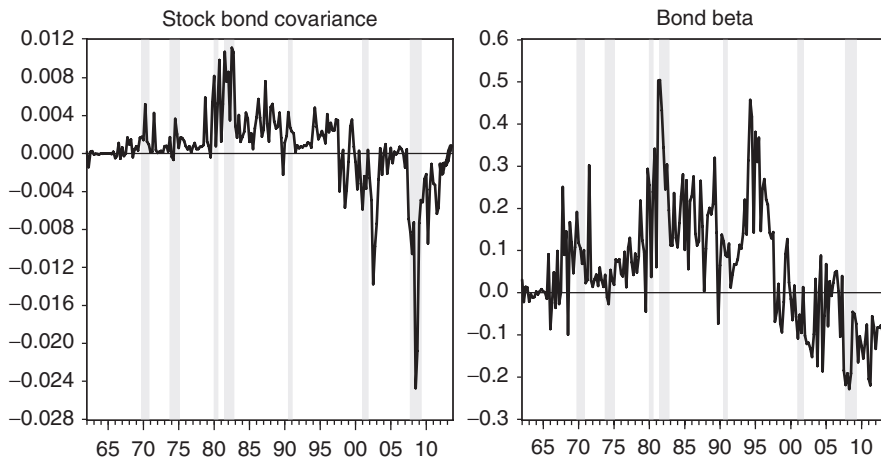


Figure 1.4 Stock-bond covariance and bond beta of 5-year treasury bonds. The left panel plots the quarterly covariance between the S&P 500 daily return and the 5-year bond return computed from daily returns. The right panel plots the quarterly beta of the 5-year bond with respect to the S&P 500 index. The vertical gray bars indicate U.S. recessions dated by the National Bureau of Economic Research. Source: Stock data are from the Center for Research in Security Prices (CRSP) while 5-year zero-coupon bond data are from Gürkaynak, Sack, and Wright (2007, updated series).

This introduction only touched upon a few of the major changes that took place in the last 10–15 years. This handbook collects recent research on these and many more topics. Indeed, in addition to sheer changes to the markets, novel methodologies and new fixed-income instruments have been introduced to fixed-income markets, and the handbook covers such recent topics as well.

In this introductory chapter, I cover some basic notions of fixed-income securities and markets. In the next section, I briefly discuss the U.S. Treasury market. Section 1.3 introduces the notions of interest rate and risk-free discounting. Section 1.4 focuses on the term structure of interest rates and on the economic forces that affect its shape. A brief discussion of the expectations hypothesis as well as forward rates as predictor of future interest rates is included in this section. Section 1.5 discusses U.S. Treasury coupon bonds and notes, as well as the methodologies to estimate the zero-coupon bond curve from coupon bonds. Section 1.6 discusses the real term structure of interest rates, as extracted from the U.S. Treasury Inflation-Protected Securities (TIPS), while Section 1.7 contains a discussion of the pricing of Floating Rate Notes (FRNs), which the U.S. Treasury started issuing in January 2014. Section 1.8 concludes.

1.2 U.S. TREASURY BILLS, NOTES, AND BONDS

A cursory look at the U.S. Treasury website immediately shows the large number of different securities that are available to investors. These securities comprise Treasury bills, Treasury notes, Treasury bonds, TIPS, and FRNs.³

The U.S. Treasury conducts regular auctions according to a well-defined calendar in order to place such securities with the public, individual investors or institutional investors. Two types of bids are available: in a competitive bid, the investor quotes the (minimum) rate that he/she will be willing to accept. In a noncompetitive bid, the investor agrees to purchase some amount of securities at the rate that is set at the auction. The Treasury allocates the amount available at the auction to noncompetitive bids and then to competitive bids (from the lowest rate to the highest rate bid) up to the amount available for sale. All investors receive the highest rate bid. Each type of security auctioned off by the U.S. Treasury is normally available with different maturities. For instance, Treasury bills are regularly auctioned off with maturities of 4, 13, 26, and 52 weeks. Treasury notes, by contrast, are regularly auctioned off with maturities of 2, 3, 5, 7, and 10 years. Treasury bonds are now only sold with a maturity of 30 years.

Table 1.1 reports the breakdown of marketable U.S. Treasury securities on March 31, 2015.⁴ As can be seen, the majority of U.S. Treasury securities held by the public (65%) are in the form of Treasury notes and thus with maturity ranging between 1 and 10 years. Short-term Treasury bills and long-term Treasury bonds are of about equal size, at 12% and 13% of the total. Securities with floating rate coupons, either tied to inflation (TIPS) or tied to interest rates (FRNs) comprise together about 10% of the total U.S. marketable debt.

TABLE 1.1 Marketable U.S. Treasury Securities (Million of Dollars)

	Debt held by the public		Intragovernmental holdings		Total
	\$	%	\$	%	
Bills	1,476,540	11.68	1,359	0.01	1,477,900
Notes	8,256,666	65.30	7,767	0.06	8,264,433
Bonds	1,607,585	12.71	62	0.00	1,607,647
Treasury Inflation-Protected Securities	1,075,141	8.50	83	0.00	1,075,224
Floating rate notes	204,991	1.62	0	0.00	204,991
Federal financing bank	0	0.00	13,612	0.11	13,612
Total	12,620,923		22,883		12,643,807

Source: U.S. Treasury website. https://www.treasurydirect.gov/govt/reports/pd/mspd/2005/2005_mar.htm.

³As of the writing of this chapter, the main web address where to find information about U.S. Treasury debt is <http://www.treasurydirect.gov/>. The products available to individual investors are listed at <http://www.treasurydirect.gov/indiv/products/products.htm>.

⁴Data are from the U.S. Treasury website https://www.treasurydirect.gov/govt/reports/pd/mspd/2015/2015_mar.htm, accessed on April 5, 2015. Data in Table 1.1 only reports the total marketable securities, that is, those available to the public. In March 2015, the U.S. government also had \$5.508 trillion in nonmarketable securities.

1.3 INTEREST RATES, YIELDS, AND DISCOUNTING

The concept of interest rates is ubiquitous to fixed-income securities. The problem is that the notion of an interest rate is not well defined without explicitly defining a compounding frequency, that is, the number of times within the year in which the interest accrues to the initial investment. For instance, 100,000 dollars invested at 10% annual rate for 10 years yields a final amount in 10 years that crucially depends on how many times per year the interest on the investment is calculated and accrued. If the interest accrues annually (*annual compounding*), then in 10 years we obtain (in thousands of dollar)

$$\$100 \times (1.1)^{10} = \$259.37$$

If instead the interest accrues twice per year (*semiannual compounding*), then we receive 5%(= 10%/2) every 6 months but 20 times (the number of 6 months in 10 years), obtaining

$$\$100 \times (1.05)^{20} = \$265.33$$

If the interest accrues quarterly, then we will receive 2.5%(= 10%/4) every 3 months 40 times, obtaining in 10 years

$$\$100 \times (1.025)^{40} = \$268.51$$

In the limit, if the interest accrues daily, we receive a rate $r = 10\%/365$ every day, 3650 times (approximately, because of leap years), obtaining in 10 years

$$\$100 \times \left(1 + \frac{0.1}{365}\right)^{10 \times 365} = \$271.79$$

In all these cases, the “quoted” interest rate on the \$100,000 investment is the same (10%), but the accrual convention makes a difference of over \$12,000 between annual compounding and daily compounding.

The general formula for compounding frequency is the following: given a quoted (annualized) interest rate r accrued n times per year for T years, the total amount at maturity from a \$1 investment is equal to

$$V = \left(1 + \frac{r}{n}\right)^{n \times T} \quad (1.1)$$

It is often convenient to work with an extremely high compounding frequency, namely, *continuous compounding*, which is the limit of Equation 1.1 as n becomes very large. Indeed, as n goes to infinity, the right-hand side of Equation 1.1 converges to

$$V = \left(1 + \frac{r}{n}\right)^{n \times T} \rightarrow e^{r \times T} \quad (1.2)$$

where $e = 2.718281828$ is the “Euler number.” The function $f(x) = e^x$ is called the “exponential function,” and it is widely used in finance and fixed-income because of its convenient mathematical properties. One can safely think of continuous compounding to be the same as daily compounding frequency and thus that the continuous compounding formula (the exponential) is nothing more than a convenient tool to approximate daily compounding. Indeed, in the previous example, if we compute the total value V using directly the number e (available on any calculator), we obtain

$$\$100 \times e^{0.1 \times 10} = \$271.82$$

which is indeed very similar to the case with daily compounding in the preceding text.

Given an interest rate (and a compounding frequency), we can invert the relations in the preceding text and obtain the discount factors $Z(T)$ to discount dollars paid at time T to dollars today. That is, how much are we willing to pay today to have \$1 at time T ? Given an interest rate r that is compounded n times per year, by inverting Equation 1.1, we obtain

$$Z(T) = \frac{1}{\left(1 + \frac{r}{n}\right)^{n \times T}}$$

That is, $Z(T)$ is the amount that we have to invest today in order to receive \$1 at T , as $Z(T) \times \left(1 + \frac{r}{n}\right)^{n \times T} = 1$. In the limit as n diverges to infinity, we obtain the discount function

$$Z(T) = \frac{1}{\left(1 + \frac{r}{n}\right)^{n \times T}} \rightarrow e^{-rT}$$

I will mostly use continuous compounding in this chapter, except when other forms of compounding are required for clarity.

1.4 THE TERM STRUCTURE OF INTEREST RATES

In general, when we discount future cash flows to the present, different discount rates apply for different maturities. It is customary to denote such discount rates by $y(T)$, where “ y ” stands for “yield.” Therefore, we denote the discount factor today for a dollar to be received at time T by

$$Z(T) = e^{-y(T)T}$$

The function $Z(T)$ (as a function of maturity T) is called zero-coupon discount function. Panel A of Figure 1.5 plots six discount functions at the end of January in 2007–2009, 2011, 2013, and 2015. The discount functions $Z(T)$ have been increasing during these years, meaning that the value of \$1 to be received in the future has been increasing over the period.

Panel B of Figure 1.5 plots the term structures of interest rate. In January 2007, the yields $y(T)$ are essentially constant across maturities. The interest rate on a 1-year risk-free investment is the same as the (annualized) interest rate of a 15-year risk-free investment or a 30-year risk-free investment. However, things are very different in the other years. In 2008, short-term yields were already much lower than long-term yields, and long-term yields themselves also decreased compared to their values in 2007. From 2009 on, short-term rates were at very low levels, while long-term rates went down and up to finally drop dramatically by January 2015.⁵

1.4.1 The Economics of the Nominal Yield Curve

What economics forces affect the term structure of interest rates? While short-term rates may be believed to be greatly affected by monetary policy (but see, e.g., Fama (2013) and the related discussion in Buraschi and Whelan (2015b, Chapter 6 in this handbook)), a number of questions arise as to what factors affect long-term yields.

A standard decomposition of long-term nominal yields is revealing:

$$\text{Nominal yield} = \text{real yield} + \text{expected inflation} + \text{risk premium} \quad (1.3)$$

Thus, a low nominal yield may be due to a low real yield, a low expected inflation, or a low risk premium (or any combination of those). We return to real yields in Section 1.6. What are the other two terms?

“Expected inflation” refers to the market expectation of average inflation over the life of the bond. Such expectations are time varying, depending on market conditions. Panel A of Figure 1.6 plots the realized inflation from 1952 to 2015 and the expected inflation from 1980 to 2015. Expected inflation is computed as the consensus inflation forecast from the Survey of Professional Forecasters available at the Federal Reserve Bank of Philadelphia. As can be seen, inflation expectations dropped substantially in the last 10 years, as inflation itself has decreased over time.

Why does expected inflation affect long-term yields? Intuitively, with the exception of TIPS (see Section 1.6), U.S. Treasury securities’ promised payments are expressed in dollars. If investors expect a high rate of inflation over the life of the bond, they are not willing to pay much money today for a security that will pay “deflated” dollars in the future. That is, the price of the U.S. Treasury security will be low today, implying a high yield. Thus, the higher the expected inflation, the higher is the (nominal) yield that investors require to buy the U.S. Treasury securities.

The last term in Equation 1.3 is a risk premium that investors require for holding nominal bonds. What types of risks does an investor in long-term bonds actually bear? While U.S. Treasuries are often referred to as “risk-free,” what is really meant is “default risk-free,” in the sense that the investor who holds the U.S. Treasury bond to maturity will receive its promised payments with an extremely high probability. However, holding long-term bonds can be quite risky for other reasons. There are two important sources of risk in particular. First, there is an inflation risk: because nominal bonds promise coupons and

⁵The yields in Figure 1.5 use data from Gürkaynak, Sack, and Wright (2007), who fit a six-parameter Nelson–Siegel model to bond data. See Section 1.5.1.

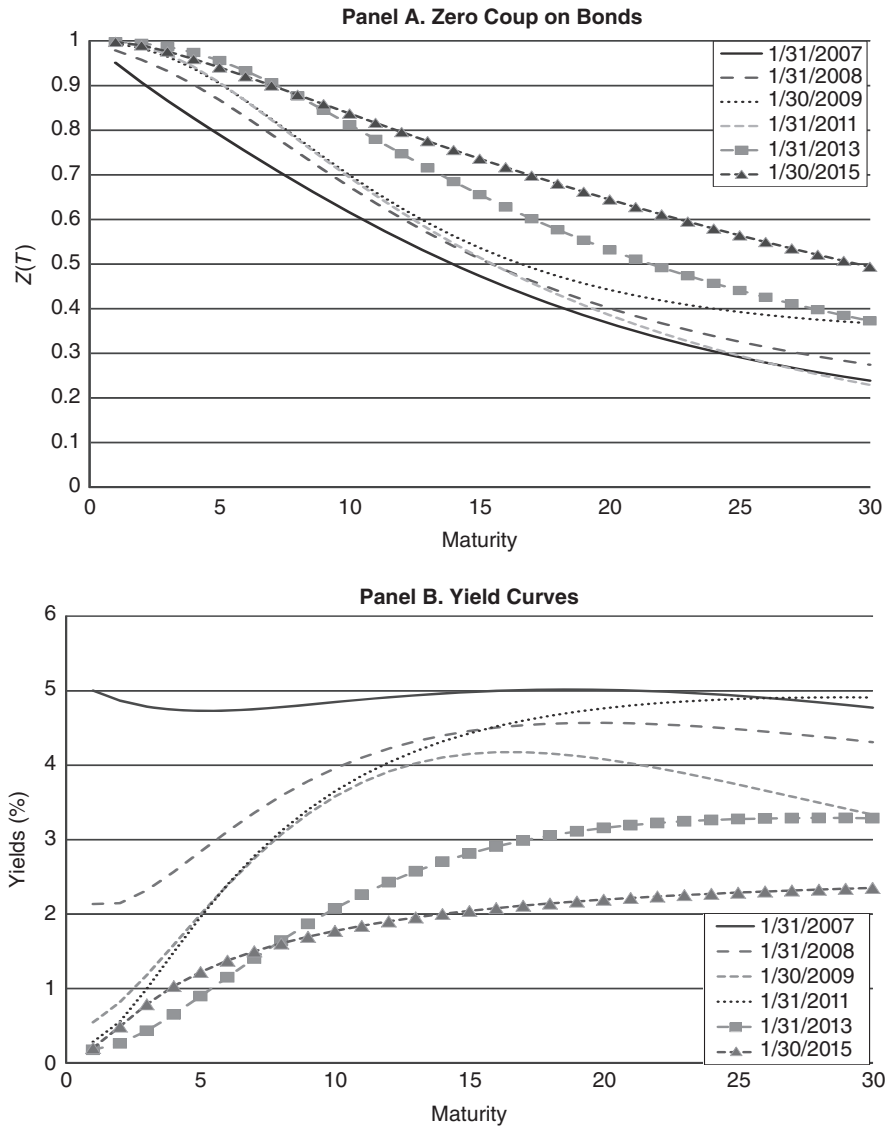


Figure 1.5 Zero-coupon bonds and yield curves. Panel A shows the zero-coupon bonds at the end of January in 2007–2009, 2011, 2013, and 2015. Panel B reports the corresponding yield curves against maturity. Yields are continuously compounded. Source: Data are from Gürkaynak, Sack, and Wright (2007), updated series.

final payoffs in dollars, if inflation unexpectedly increases over the life of the bond, the real value of these promised payments declines, and investors require a premium to hold a security with an uncertain real payoff.

The second source of risk in a long-term bond stems from interest rate risk: if the investor needs to sell the bond before maturity, he/she may suffer potentially severe capital losses if nominal interest rates increased in the meantime. To illustrate, Panel A of Figure 1.7 plots the “life cycle” of a 30-year coupon bond, issued on February 15, 1985, and maturing on February 15, 2015. The coupon rate of the bond is 11.25%. More specifically, Panel A plots the end-of-month price of the coupon bond (solid black line) together with the 1-month Treasury bill rate (dashed gray line). The price of the coupon bond shot up from its issue price of \$100 in February 1985 to \$140 by March 1986 and then dropped to \$113 by September 1987.⁶ If an investor bought the bond at \$140 in March 1986 but found it necessary to sell it in September 1987, he/she would have suffered a capital loss of 20%, which would have only partly be compensated from receiving the 11.25% coupon.

Panel B of Figure 1.7 plots the total monthly return (capital gain plus accrued interest) in excess of the 1-month T-bill rate from investing in the 30-year bond (dashed gray line). Monthly excess returns of plus/minus 5% are not uncommon for the first half of the sample. Indeed, the solid black line in the figure reports the monthly standard deviation of excess bond returns

⁶Panel A actually shows the first price in February 1985 as \$94.718 rather than \$100 as the figure reports end-of-month prices. That is, the bond dropped 5.3% in the first 2 weeks after issuance.

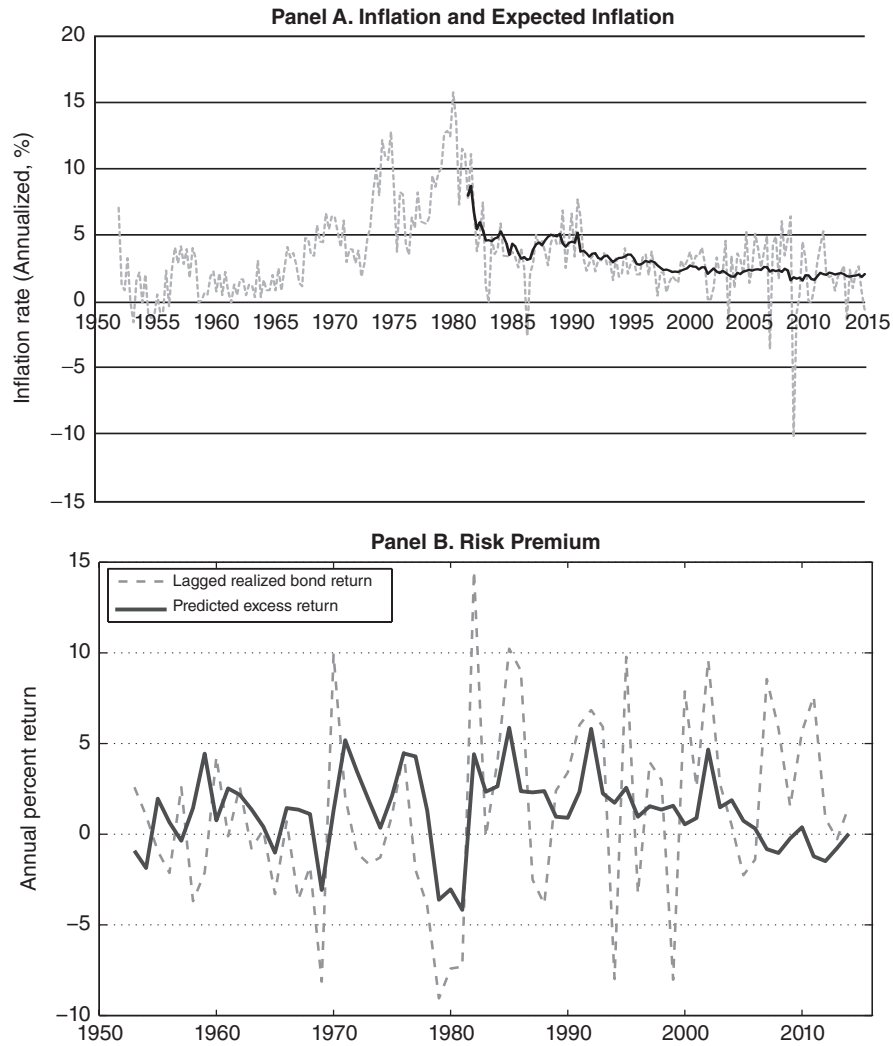


Figure 1.6 Expected inflation and risk premium. Panel A shows the quarterly realized inflation rate (Consumer Price Index (CPI), dashed gray line) and the professional forecasters' consensus forecast of future inflation with 2 years horizon (solid black line). CPI data are from the FRED database at the Federal Reserve Bank of St Louis, while consensus forecasts are from the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. CPI forecasts are only available from 1981 onward. Panel B reports the estimated expected excess return of a 5-year zero-coupon bond computed from the Cochrane and Piazzesi factor. Source: Data for Panel B are from Fama–Bliss zero-coupon bond yields available at the Center for Research in Security Prices (CRSP).

estimated over the previous 3 years, and we find that such bond return volatility was as high as 4% at the beginning of the sample (equivalent to about 13.8% annualized), a sizable variation. Panel B shows that the volatility of bond returns also declines over time as we approach maturity: indeed, long-term bonds are far more volatile than short-term bonds, due to their higher duration (see, e.g., Sydyak (2015, Chapter 7 in this handbook)). In fact, in the very last part of the sample, the price declines almost deterministically (close to zero volatility) toward \$100, which is the promised principal at maturity. The reason why the price declines so steeply is that these bonds carry a coupon of 11.25%. Because short-term interest rates were essentially zero in this period, the steep decline in the coupon bond price must compensate for the high 11.25% coupon for the total bond return to be approximately zero, that is, the same as the return on an alternative investment in Treasury securities with similar short maturity.⁷

For comparison, Panel C plots the returns of the S&P 500 index over the same sample (1985–2014). This sample includes two major drops in the stock market: October 1987 (with a negative excess return of -25%) and October 2008 (with a negative excess return of -19%). Both events are clearly visible in the figure. The S&P 500 index is more volatile than the 30-year bond

⁷For instance, the price dropped from \$112.3594 on December 31, 2013, to \$101.3008 on December 31, 2014. Thus, the total return over the year including the coupon is $(\$101.308 + \$11.25 - \$112.3594)/\$112.3594 = 0.1703\%$, which is very similar to the 1-year T-bill rate on December 31, 2013, given by 0.1691%.

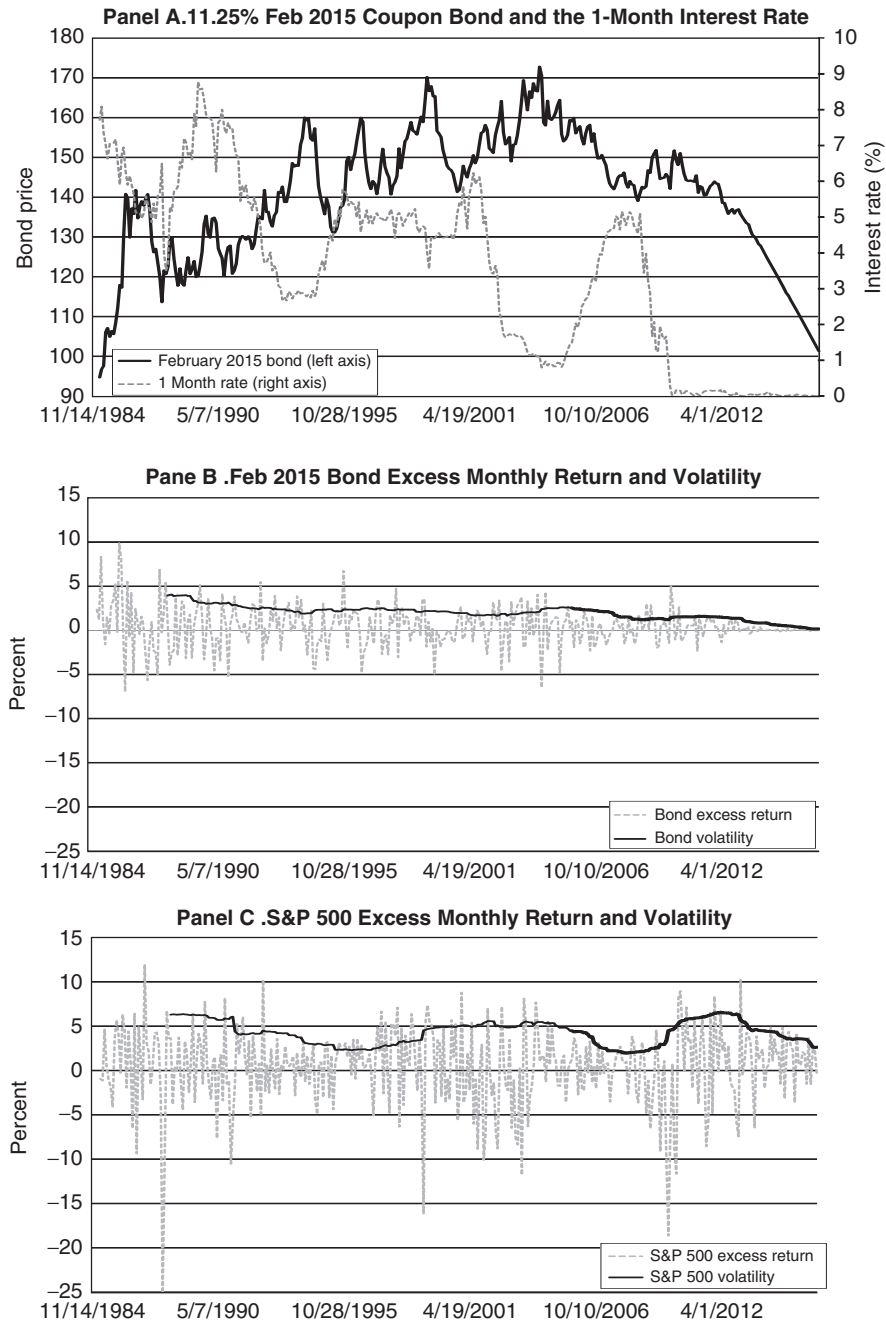


Figure 1.7 The life cycle of a 30-year coupon bond. Panel A shows the monthly (clean) price of 11.25%, 30-year U.S. Treasury coupon bond issued on February 15, 1985, and expiring on February 15, 2015, alongside the 1-month Treasury bill rate. Panel B shows the monthly excess return of the coupon bond and its monthly volatility computed as the 3-year trailing standard deviation of excess returns. Panel C shows the monthly excess return of the S&P 500 index and its monthly volatility computed as the 3-year trailing standard deviation of excess returns. Source: All data are from the Center for Research in Security Prices (CRPS).

on average, as the monthly standard deviation (the black solid line) hovers around 5% (17% annualized). However, the volatility of the S&P 500 index has been as low as 2% (7% annualized) and as high as 6.5% (23% annualized). The 4% volatility of the 30-year bond is sizable even compared to the variation of the stock market. In this sense, U.S. Treasuries represent risky investments for investors who may not hold them to maturity.

The risk premium in the last term of Equation 1.3 is therefore a compensation that investors require to hold risky long-term bonds, which could pay off little when the investors need them the most. Indeed, Panel B of Figure 1.6 plots the expected excess annual return on a 5-year zero-coupon bond computed using the Cochrane–Piazzesi factor as predictor (solid black

line).⁸ The figure also plots the lagged realized excess return from holding a 5-year zero-coupon bond. When the lines “move together,” it means that the predictor is predicting well. Assuming that the Cochrane–Piazzesi factor captures the variation of the bond risk premium well, Panel B shows quite a bit of time variation in the bond risk premium. Interestingly, the bond risk premium computed from the Cochrane–Piazzesi factor turned negative around 2005. From an economic standpoint, a negative risk premium for long-term bonds is realistic in recent years as U.S. Treasury bonds may be considered “hedging instruments” against especially negative economic scenarios (see, e.g., Campbell, Sunderam, and Viceira (2013) and David and Veronesi (2015, Chapter 15 in this handbook) for a discussion). Indeed, recall that the right panel of Figure 1.4 shows that the “beta” of the 5-year zero-coupon bond with respect to the S&P 500 index turned negative after 2000, thereby justifying the “hedge” interpretation of long-term Treasury bonds. Intuitively, bad economic news makes investors drop stocks and buy safe U.S. Treasuries, which push up the value of Treasury bonds in bad times. Treasury bonds, therefore, represent a hedge against negative economic outcomes.

To conclude, Equation 1.3 shows that nominal yields are affected by three components, all of which turned lower in the last few years: real yields declined in the aftermath of the crisis (see Section 1.6), expected inflation has been declining steadily since early 1980s (Panel A of Figure 1.6), and the risk premium has been declining too in the last 10 years, because of the hedging properties of U.S. Treasury securities (Panel B of Figure 1.6). Not surprisingly, nominal yields have been especially low in the last few years (Panel B of Figure 1.5).

1.4.2 The Expectations Hypothesis

The previous section provided a discussion of the economic forces behind the shape of the term structure of nominal interest rates that was grounded on “fundamentals,” such as real yield rates, expected inflation, and risk premium. However, there is also a more “finance-related” explanation of the yield curve which is related to investors’ expectations about future interest rates and again a risk premium:

$$\text{Long-term yield} = \text{expected future short-term rates} + \text{risk premium}$$

To illustrate, it is useful to consider the case of *perfect foresight* first. Assume investors have perfect foresight about the next 1-year interest rates. That is, they know not only the current 1-year yield $y_t(1)$ but also next year’s 1-year yield $y_{t+1}(1)$, where $y_t(\tau)$ denotes the continuously compounded yield at time t of a zero-coupon bond with time to maturity τ (and hence maturity date $t + \tau$). Given this knowledge, what is the value of a zero-coupon bond with maturity in 2 years?

Because investors *know* that next year’s 1-year yield will be $y_{t+1}(1)$, they also know that the zero-coupon bond price next year will be

$$Z_{t+1}(1) = e^{-y_{t+1}(1) \times 1}$$

Because under these assumptions, $Z_{t+1}(1)$ is known today, its value today is simply the discounted value using the current 1-year yield. That is, the 2-year zero-coupon bond price is

$$Z_t(2) = Z_t(1) \times Z_{t+1}(1) = e^{-(y_t(1) + y_{t+1}(1))}$$

Because the 2-year yield also satisfies $Z_t(2) = e^{-y_t(2) \times 2}$, we obtain that under perfect foresight, the 2-year yield is the average of the two 1-year yields:

$$y_t(2) = \frac{1}{2}(y_t(1) + y_{t+1}(1))$$

Indeed, a similar argument implies that if we have perfect foresight of n future 1-year yields, then the yield of a zero-coupon bond with $n + 1$ years to maturity is

$$y_t(n + 1) = \text{Average}[y_t(1), y_{t+1}(1), \dots, y_{t+n}(1)] = \frac{1}{n + 1} \sum_{i=0}^n y_{t+i}(1)$$

The **expectations hypothesis** substitutes the perfect foresight with an expectation, thereby giving the long-term yield as being equal to the market expectation of future short-term rates. Using the notation above, we have

$$y_t(n + 1) = E_t \left[\frac{1}{n + 1} \sum_{i=0}^n y_{t+i}(1) \right] \quad (1.4)$$

⁸Cochrane and Piazzesi (2005) propose a special combination of forward rates as a predictor of future excess bond returns and find that it does predict returns very well. See also Dahlquist and Hasseltoft (2015, Chapter 9 in this handbook).

Subtracting the current 1-year yield, we can also write

$$y_t(n+1) - y_t(1) = E_t \left[\frac{1}{n+1} \sum_{i=0}^n y_{t+i}(1) \right] - y_t(1) \quad (1.5)$$

The left-hand side is the term spread, the difference between the $n+1$ -year yield and the 1-year yield, while the right-hand side is the “expectation spread,” the difference between the expected future average interest rate and the current 1-year yield.

Does this relation hold in the data? Heuristically, Panel A of Figure 1.8 plots the expected future 3-month rates from the Survey of Professional Forecasters from 1981 to 2015. The forecasts are for the 3-month T-bill rate for the current quarter, denoted in the figure by $r(t)$, and for the next three quarters, denoted by $r(t+3m)$, $r(t+6m)$, and $r(t+9m)$, respectively. According to the expectations hypothesis, and assuming that the consensus forecasts from the surveys reflect market expectations, the 1-year yield $y_t(1)$ should be equal to the average forecast of future rates. Panel B shows the 1-year term spread, defined as $y_t(1y) - r(t)$,⁹ together with the 1-year “expectation spread,” which equals the expected average interest rates $\sum_{i=0}^3 E_t[r(t+i \times 3m)]/4$ minus the 3-month rate $r(t)$. The figure shows that the two series are correlated with each other (correlation of 26%), but they are quite different from each other. Though the term spread is often larger than the expectation spread, we also have cases in which the term spread is actually smaller than the expectation spread. That is, there are occasions when the expected future average rate over the next year is in fact higher than the 1-year yield.

Panels A and B of Figure 1.8 provide simple heuristic evidence to show that the expectations hypothesis is not supported in the data. More formally, using predictive regression techniques, Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), and many others (see discussion in Dahlquist and Hasseltoft (2015, Chapter 9 in this handbook) and Fontaine and Garcia (2015, Chapter 14 in this handbook)) show that there is little or no support for the expectations hypothesis in the data.

The violation of the expectations hypothesis suggests the existence of an additional term in Equation 1.4, that is, we have

$$y_t(n+1) = E_t \left[\frac{1}{n+1} \sum_{i=0}^n y_{t+i}(1) \right] + RP_t \quad (1.6)$$

where RP_t reflects a risk premium from holding the bond from t . The time variation in RP_t invalidates the expectations hypothesis. Indeed, even if expected future interest rates (the first term in Equation 1.6) does not change, the yield curve changes because RP_t changes. Therefore, movements in the yield curve do not only correspond to variations in expected future short-term yields.

To understand the source of this risk premium in this context, consider again the previous example. Let us now assume that investors do not know at t the 1-year yield at $t+1$, but they believe it has a normal distribution (for simplicity):

$$y_{t+1}(1) \sim N(\bar{y}, \sigma_y^2)$$

where $\bar{y} = E[y_{t+1}(1)]$ is the expected future 1-year yield and σ_y is the standard deviation. From the eyes of a time t investor, the 1-year zero-coupon bond at $t+1$ is risky, as $Z_{t+1}(1) = e^{-y_{t+1}(1)}$ and $y_{t+1}(1)$ is random. Therefore, it is possible that the investor at time t would like to discount the next year bond at a higher rate than the risk-free rate. That is, an intuitive formula is

$$Z_t(2) = e^{-(y_t(1)+RP) \times 1} E_t[Z_{t+1}(1)]$$

where RP is a risk premium the investor requires to hold the 2-year bond, and $E_t[Z_{t+1}(1)]$ is the current expected value of the future zero-coupon bond 1 year from now. Using the properties of the normal distribution, we then obtain

$$Z_t(2) = e^{-(y_t(1)+RP) \times 1} e^{-\bar{y} + \frac{1}{2}\sigma_y^2}$$

where $\frac{1}{2}\sigma_y^2$ is a convexity term.¹⁰ Because the 2-year yield is $Z_t(2) = e^{-y_t(2) \times 2}$ and we defined $\bar{y} = E[y_{t+1}(1)]$, we obtain

$$y_t(2) = \frac{1}{2} \{y_t(1) + E[y_{t+2}(1)]\} + \frac{1}{2} \left(RP + \frac{1}{2}\sigma_y^2 \right)$$

That is, the yield of the 2-year bond is given by the expected average 1-year yields over the 2 years, plus a term that includes a risk premium and a convexity term. If the risk premium RP (or the volatility σ_y) are time varying, then the expected future average rate may differ from the current 2-year yield in a random manner, generating patterns such as those illustrated in Panel B of Figure 1.8.

⁹Because the time step is less than 1 year, the notation here emphasizes the units used, so that 1y is 1 year, 3m is 3 months, and so on.

¹⁰The convexity term is also called a “Jensen’s” term, due to Jensen’s inequality that states that if $f(x)$ is convex, then $E[f(x)] > f(E[x])$. In the special case of a normal distribution and exponential function, we have that if $x \sim N(\mu_x, \sigma_x^2)$, then $E[e^x] = e^{\mu_x + \frac{1}{2}\sigma_x^2} > e^{\mu_x}$. Hence, $\frac{1}{2}\sigma_x^2$ quantifies the Jensen’s effect.

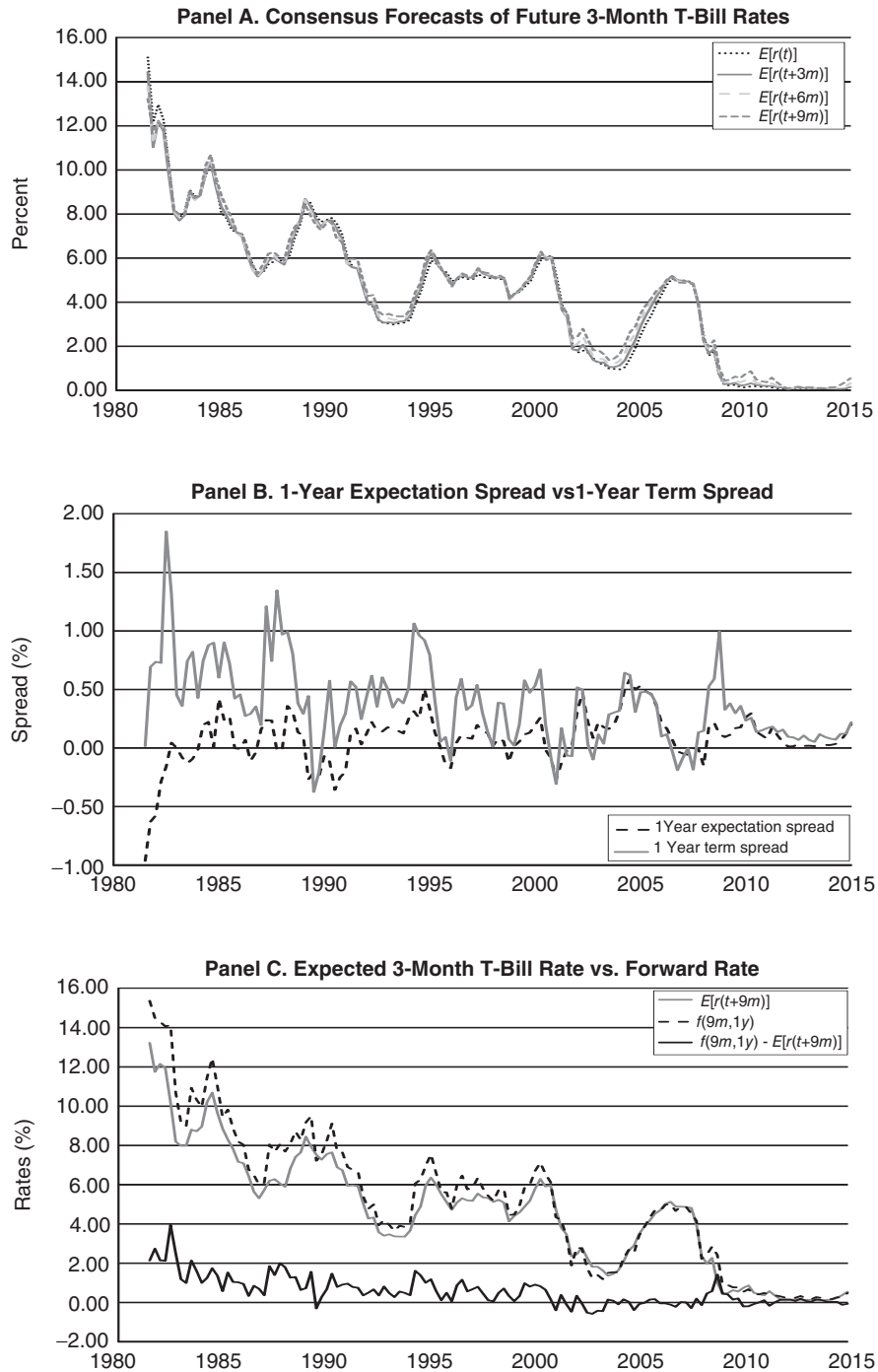


Figure 1.8 Expected future interest rates and the yield curve. Panel A plots the consensus (average) forecasts of the 3-month Treasury bill rate of the current quarter (t) and one, two, and three quarters ahead ($t + 3m$, $t + 6m$, $t + 9m$). Panel B plots the 1-year term spread, defined as the 1-year yield minus the 3-month yield, along with the 1-year “expectation spread,” defined as the average expected 1-year rate $\sum_{i=0}^3 E[r(t + i \times 3m)]/4$ minus the current 3-month yield. Panel C plots the predicted 3-month rate with 9-month horizon (gray solid line) and the corresponding 3-month forward rate $f(9m, 1y)$ (black dashed line). The bottom black line plots the difference between forward rates and expected future rates. Survey data are from the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia. Source: Yield data from the Center for Research in Security Prices (CRPS).

1.4.3 Forward Rates as Expectation of Future Interest Rates?

A subject that is closely related to the expectations hypothesis is the relation between forward rates and expected future interest rates.

First, what is a forward rate? Consider an investor who has a receivable of \$1 million in 9 months from now and wants to invest it for 3 months (i.e., from 9m to 1y from now) by investing in U.S. Treasury securities. One strategy is to wait until the receivable arrives and then invest it in the 3-month T-bill at that time. However, this strategy subjects the investor to interest rate risk: if the interest rate declines, the investor will receive a lower payment in 1 year. An alternative is to design a trading strategy to *lock in* today the future 3-month interest rate. The investor can do the following:

- **Today (time 0):** Set up a zero-cost, long/short strategy:
 1. Short 9-month U.S. Treasury bills with face value of \$1 million. The short sale will yield proceeds of $\$1 \times Z(9m)$ million, where $Z(9m)$ is the unit price of the T-bill with 9 months to maturity.
 2. Use the proceeds from the short sale to purchase M , 1-year T-bills. Given the current price $Z(1y)$ of a 1-year T-bill, the investor can purchase $M = Z(9m)/Z(1y)$ million 1-year T-bills.
- **In 9 months:** Use the receivable of \$1 million to pay off the short position in 9-month T-bills, which has become due.
- **In 1 year:** Redeem the 1-year T-bills purchased at time zero, thereby receiving $\$M = Z(9m)/Z(1y)$ million.

In frictionless markets, this strategy implies no outlays of money today, and from the perspective of the investor, it is *as if* he invests \$1 million in 9 months (the receivable) and receives $\$M = Z(9m)/Z(1y)$ million in 1 year. Thus, the implicit annualized interest rate f in this investment strategy is

$$e^{f \times 0.25} = \frac{Z(9m)/Z(1y) \text{ million}}{1 \text{ million}}$$

where 0.25 represents the investment horizon (3 months = 0.25). We thus find that the implicit interest rate f satisfies

$$f(9m, 1y) = \frac{1}{0.25} \ln \left(\frac{Z(9m)}{Z(1y)} \right)$$

where we emphasize that the interest rate f is for an investment from 9 months (9m) from now to 1 year (1y) from now. $f(9m, 1y)$ is called a **forward rate** and it is an interest rate for a future investment that is implicit in the current zero-coupon bond curve.¹¹

In general, the same argument as above but for other investment times and horizons gives the following general formula for the continuously compounded forward rate:

$$f_t(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \ln \left(\frac{Z_t(\tau_1)}{Z_t(\tau_2)} \right) \quad (1.7)$$

where t is the time when the forward rate is determined (time 0 in previous example) and τ_1 and τ_2 are the times (from t) at which the investment takes place ($\tau_1 = 9m$ and $\tau_2 = 1y$ in the previous example). Of course, $\tau_2 > \tau_1$.

Substituting $Z_t(\tau_i) = e^{-y_t(\tau_i)\tau_i}$ in Equation 1.7, we find two useful formulas:

- (i) The forward rate is related to the slope of the yield curve:

$$f_t(\tau_1, \tau_2) = y_t(\tau_1) + \tau_2 \left(\frac{y_t(\tau_2) - y_t(\tau_1)}{\tau_2 - \tau_1} \right) \quad (1.8)$$

- (ii) The long-term yield is the average forward rates

$$y_t(\tau_2) = w y_t(\tau_1) + (1 - w) f_t(\tau_1, \tau_2) \quad (1.9)$$

where

$$w = \frac{\tau_1}{\tau_2}$$

¹¹I emphasize the “investment” interpretation of the forward rate, but by reversing the trade, the same can be thought as “borrowing” in the future at the locked-in borrowing rate f .

The first expression in Equation 1.8 provides a convenient formula for forward rates: it essentially shows that the forward rate is equal to the corresponding yield $y_t(\tau_1)$ plus a term proportional to the slope of the yield curve between τ_1 and τ_2 (the term $\left(\frac{y_t(\tau_2) - y_t(\tau_1)}{\tau_2 - \tau_1}\right)$). That is, if the yield curve is increasing between τ_1 and τ_2 , then the forward rate is above the yield curve, while if the yield curve is decreasing between τ_1 and τ_2 , then the forward rate is below the yield curve. Hence, the forward rate represents the marginal increase or decrease to the yield curve, and the yield curve is an average of forward rates.

Indeed, Equation 1.9 shows that the long-term yield is an average of the short-term yield and the forward rate. Equation 1.9 can be easily generalized to a sequence of forwards. For instance, consider 1-year forward rates $f_t(1, 2), f_t(2, 3), \dots$. Then, we have that the $n + 1$ -year yield can be written as

$$y_t(n + 1) = \frac{1}{n + 1} \sum_{i=0}^{n-1} f_t(i, i + 1) \quad (1.10)$$

with the notation $y_t(1) = f_t(0, 1)$ (the first forward equals the 1-year yield to maturity).

Under the expectations hypothesis in Equation 1.4, we then have that the forward rate is equal to the expected future interest rate¹²:

$$f_t(i, i + 1) = E_t[y_{t+i}(1)]$$

A positive risk premium instead generates $f_t(i, i + 1) > E_t[y_{t+i}(1)]$. To see this, as shown in Equation 1.6, the existence of a positive risk premium implies $y_t(n + 1) > E_t\left[\frac{1}{n+1} \sum_{i=0}^n y_{t+i}(1)\right]$ which in turn implies (if $RP > 0$ for every maturity n) that $f_t(i, i + 1) > E_t[y_{t+i}(1)]$.

To check whether the forward rate is in fact equal to the expected future rate in the data, Panel C of Figure 1.8 plots the 9-month forward rate for a 3-month investment horizon ($f_t(9m, 1y)$) along with the predicted 3-month rate at the 9-month horizon ($E_t[r(t + 9m)]$). As can be seen, for most of the sample, the forward rate was above the expected future 3-month rate $f_t(9m, 1y) > E_t[r(t + 9m)]$, which is consistent with the existence of a risk premium.

1.4.4 Interpreting a Steepening of the Yield Curve

According to the expectations hypothesis, a steepening of the yield curve must be interpreted as an expected increase in future yields. However, one of the main messages of the evidence surrounding the failure of the expectations hypothesis is that essentially the opposite may be true. A steepening of the yield curve may as well signal (under some conditions) an expected *decline* in future yields. To understand the logic, intuitively, if the steepening of the yield curve is due to an increase in the risk premium RP_t in Equation 1.6, then zero-coupon bonds must have a positive expected excess return going forward. The only way a long-term zero-coupon bond may have a high return is for its price to increase, as a zero-coupon bond does not pay any coupons or dividends. For the price of a zero-coupon bond to increase, it ought to be the case that the corresponding yield has to decline. Thus, we have the paradoxical result that if a steepening of the yield curve is due to an increase in risk premium, then future rates should decline, instead of increase. Indeed, the researches carried out by Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), and many others (see Dahlquist and Hasseltoft (2015, Chapter 9 in this handbook) for a review and discussion) show that indeed the risk premium appears to be a main driver of the variation in long-term yields, providing the counterintuitive result about the slope of the yield curve and the expected future yields.

1.5 PRICING COUPON NOTES AND BONDS

As shown in Table 1.1, the vast majority of U.S. Treasury debt is made of coupon-bearing notes and bonds. From the table, about 78% of U.S. Treasuries are in fixed-coupon-bearing securities (notes and bonds), while about 12% are in (zero-coupon) Treasury bills and about 10% in securities with floating coupons, either indexed to inflation or to short-term interest rates

How do we value coupon bonds? What is their relation with discount factors introduced in the previous sections? Consider a coupon bond with maturity T_n , with semiannual coupon payments at times T_1, T_2, \dots, T_n , where $T_i = T_{i-1} + 0.5$, and coupon rate c_n . Given a set of discount factors $Z(T_1), Z(T_2), \dots, Z(T_n)$, the value of a coupon bond is provided by the no-arbitrage relation

$$P(T_n, c_n) = \sum_{i=1}^n \frac{c_n}{2} Z(T_i) + Z(T_n) \quad (1.11)$$

¹²The implication stems from observing that both Equations 1.4 and 1.10 must hold for all $n = 1, 2, \dots$.

If the price of the bond was, say, lower than the right-hand side, then an arbitrageur could buy the bond and sell a portfolio of zero-coupon bonds with principals $c_n/2$ for $i = 1, \dots, n-1$ and $(1 + c_n/2)$ for $i = n$, making a positive cash inflow today. Because at every future date the trade is perfectly hedged (as on every T_i , the arbitrageur collects the coupon – and principal at T_n – and use it to pay the corresponding zero-coupon sold), this trade represents an arbitrage opportunity.

1.5.1 Estimating the Zero-Coupon Discount Function

While in Equation 1.11 we assumed that we can price coupon bonds from a set of discounts $Z(T_i)$, in reality, we normally go in the opposite direction. In fact, computing the fair value of discount factors $Z(T_i)$ is very useful in practice, as they provide the value today of dollars in the future and thus provide an important benchmark for computing the time value of money. It is therefore common practice to use liquid U.S. Treasury notes and bonds to *extract* the discount factors $Z(T)$ from them.

There are several ways of extracting $Z(T)$ from coupon notes and notes. One possibility, called **bootstrapping**, is to invert Equation 1.11 and directly extract $Z(T)$ in a recursive manner. For instance, if we have coupon notes and bonds prices available with maturities at semiannual frequency, then inverting Equation 1.11 for $n = 1, 2, \dots$ we obtain

$$\begin{aligned} Z(T_1) &= \frac{P(T_1, c_1)}{1 + \frac{c_1}{2}} \\ Z(T_2) &= \frac{P(T_2, c_2) - \frac{c_2}{2}Z(T_1)}{1 + \frac{c_2}{2}} \\ Z(T_3) &= \frac{P(T_3, c_3) - \frac{c_3}{2}(Z(T_1) + Z(T_2))}{1 + \frac{c_3}{2}} \\ &\vdots \\ Z(T_n) &= \frac{P(T_n, c_n) - \frac{c_n}{2} \sum_{i=1}^{n-1} Z(T_i)}{1 + \frac{c_n}{2}} \end{aligned}$$

This bootstrapping procedure is widely used in practice, normally after a large filtering of the data to take into account data errors and other noises. For instance, the Fama–Bliss yields that are plotted in Figure 1.2 (those with maturities from 1 to 5 years) are indeed obtained through a bootstrapping procedure, although the methodology is more refined and elaborate than the one just illustrated.¹³ An important feature of the bootstrapping methodology is that it does not impose any parametric restriction on the structure of zero-coupon bonds $Z(T_n)$, but they are extracted in a fully nonparametric manner from traded coupon bonds. Several chapters in this handbook use the Fama–Bliss zero-coupon data.

A second possibility is to postulate a parametric functional form for $Z(T_i)$ and then *estimate* the parameters by minimizing the squared difference between the observed bonds and the model bonds. For instance, a popular methodology, called the **extended Nelson–Siegel model**, is to assume that the continuously compounded yield with maturity T_i is given by

$$y(T_i) = \beta_0 + (\beta_1 + \beta_2) \frac{e^{-\frac{T_i}{\lambda_1}} - e^{-\frac{T_i}{\lambda_2}}}{\frac{T_i}{\lambda_1} - \frac{T_i}{\lambda_2}} - \beta_2 e^{-\frac{T_i}{\lambda_1}} + \beta_3 \left(\frac{1 - e^{-\frac{T_i}{\lambda_2}}}{\frac{T_i}{\lambda_2}} - e^{-\frac{T_i}{\lambda_2}} \right) \quad (1.12)$$

For given parameters $(\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$, we can compute

$$Z(T_i) = e^{-y(T_i)T_i} \quad (1.13)$$

¹³The Fama–Bliss methodology extracts forward rates from Treasury bills and notes by iteratively extending the maturity of the zero-coupon curve. The methodology essentially uses Equation 1.10, once it is generalized to any intermediate maturity.

where $y(T_i)$ is given by Equation 1.12. Thus, for given parameters, we can compute a “model price” for each available bond $P^{\text{model}}(T_n, c_n)$ by using Equation 1.11, with $Z(T_i)$ given by 1.13. We can finally estimate the parameters of the model by minimizing

$$\min_{(\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)} \sum_{n=1}^N (P^{\text{model}}(T_n, c_n) - P^{\text{data}}(T_n, c_n))^2$$

where $P^{\text{data}}(T_n, c_n)$ denotes the observable data. Indeed, the zero-coupon bonds $Z(T_n)$ and the yield curves $y(T_n)$ in Panels A and B, respectively, of Figure 1.5 are estimates of the extended Nelson–Siegel model in Equation 1.12 (see updated series of Gürkaynak, Sack, and Wright (2007)). Several chapters in this handbook use these data.

1.5.2 Data and Bond Illiquidity

Clearly, the type of data used in the computation of discount factors $Z(T_i)$ affects the coupon bond curves. For instance, to illustrate, Table 1.2 contains some U.S. Treasury note and bond data on November 15, 2007, and November 15, 2008. I selected only the notes and bonds that have maturity on consecutive February 15 or August 15, so that we can easily extract the zero-coupon bond curve using the bootstrap methodology.¹⁴ It is evident from Table 1.2 that there are multiple bonds maturing exactly on the same date. Hence, which bonds we use may affect the zero-coupon bonds $Z(T)$ and thus the yields $y(T)$.

To illustrate, Figure 1.9 shows the discount functions (top panels) and the yield curves (bottom panels) on November 15, 2007 (left panels), and on November 15, 2008 (right panels), the last date chosen as the financial crisis was in its full swing. Each panel reports two lines: the line with squares uses the oldest notes or bonds available at each maturity. So, for instance, for the first maturity (2/15/2008) on November 15, 2007, it uses the note issued on 2/15/1998 (see Table 1.2). The line with circles, instead, uses the most recently issued bonds or notes available at each maturity. So, for instance, again for the first maturity on November 15, 2007, it uses the note issued on 2/15/2005, which is the latest issued with that maturity, and so on.

Looking at the top-left panel of Figure 1.9, we see that there is little difference in the two zero-coupon bond curves on November 15, 2007, independently on whether the oldest or the most recent notes and bonds were used in the computation. The bottom-left panel shows that some little difference is visible in yields between the two curves, but the difference is small.

Different is the situation displayed in the top-right panel of Figure 1.9, when on November 15, 2008, the two zero-coupon discount curves show some significant price differences at maturities around 6 through 8 years. In fact, the bottom-right panel shows that on this date, the yield curves displayed a difference as high as 1% at these maturities. What is the source of the discrepancy between old bonds and new bonds? By examining the data in Table 1.2, we see that at these maturities the oldest Treasuries were bonds issued back in the 1980s. Such bonds tend to be less liquid on average compared to other U.S. Treasury notes with the same maturities but issued more recently (some differences in yield curves are in fact visible also on November 2007 on the left-bottom panel). During the financial crisis, the illiquidity of such bonds turned out to be very significant, generating a large spread in the zero-coupon bond curves issued at this time. Fontaine and Garcia (2015, Chapter 14 in this handbook) contains a discussion of the literature that highlights these and other similar events.

1.6 INFLATION-PROTECTED SECURITIES

The U.S. Treasury, as well as the treasury departments of many other countries such as the United Kingdom and Italy, issues inflation-protected bonds, that is, bonds that do not pay a fixed amount but a quantity proportional to the cumulative inflation during the life of the bond (see Fleckenstein, Longstaff, and Lustig (2015, Chapter 3 in this handbook) and Pflueger and Viceira (2015, Chapter 10 in this handbook) for details and discussions about inflation-protected bonds). Figure 1.10 plots the real yields obtained from U.S. TIPS on the same six dates as in Figure 1.5. The variation is quite interesting, as the real yields were all positive in 2007 and then moved to negative for short-term maturities, while they remained mostly positive for long-term maturities. Indeed, negative real rates persisted since the beginning of the 2008–2009 financial crisis up until today. In January 2013, moreover, the short-term real rate was extremely negative, at -2% .

What economic forces determine the real yields, such as the ones plotted in Figure 1.10? A standard decomposition of the real yield is as follows:

$$\begin{aligned} \text{real yield} &= a \times \text{expected real economic growth} \\ &\quad - b \times \text{macroeconomic risk} + \text{impatience} \end{aligned} \quad (1.14)$$

where a and b are two positive proportionality factors.

¹⁴Note that the price data reported are “clean” prices, that is, without accrued interest. In order to extract the zero-coupon bond curve, we need to use “invoice prices” which equal “clean prices” plus “accrued interests.” Accrued interest equals the semiannual coupon payment times the fraction of time since the last coupon payment. For instance, on November 15, 2007, the accrued interest of the 3% coupon note expiring on 2/15/2008 is $\frac{3}{2} \times 0.5 = 3 \times 0.25 = \0.75 .

TABLE 1.2 U.S. Treasury Note and Bond Data on Two Dates with Maturity at Semiannual Frequency

November 15, 2007		November 15, 2008						
Issue Date	Maturity date	Type	Coupon rate	Bid	Ask	Price	Bid	Ask
2/18/03	2/15/08	Note	3	99.92188	99.95313		100.6172	100.6484
2/15/05	2/15/08	Note	3.375	100.0078	100.0391		100.9922	101.0234
2/15/98	2/15/08	Note	5.5	100.5313	100.5625		101.7656	101.7969
8/15/03	8/15/08	Note	3.25	99.85156	99.88281		102.75	102.7813
8/15/05	8/15/08	Note	4.125	100.4531	100.4844		103.5938	103.625
2/17/04	2/15/09	Note	3	99.52344	99.55469		103.0078	103.0391
2/15/06	2/15/09	Note	4.5	101.3359	101.3672		104.4297	104.4609
8/16/04	8/15/09	Note	3.5	100.2734	100.3047		106.5	106.5313
8/15/06	8/15/09	Note	4.875	102.5469	102.5781		105.2344	105.2656
8/16/99	8/15/09	Note	6	104.4766	104.5078		108.1484	108.1797
2/15/05	2/15/10	Note	3.5	100.4219	100.4531		108.6094	108.6406
2/15/07	2/15/10	Note	4.75	103.0781	103.1094		110.4844	110.5156
2/15/00	2/15/10	Note	6.5	106.8828	106.9141		110.8438	110.875
8/15/05	8/15/10	Note	4.125	102.0781	102.1094		110.5313	110.5625
8/15/00	8/15/10	Note	5.75	106.2656	106.2969		108.0781	108.1094
2/15/01	2/15/11	Note	5	104.9219	104.9531		109.3281	109.3594
8/15/01	8/15/11	Note	5	105.2969	105.3281		108.2656	108.2969
2/15/02	2/15/12	Note	4.875	105.0156	105.0469		109.9063	109.9375
8/15/02	8/15/12	Note	4.375	103.3125	103.3438		145.0625	145.0938
2/18/03	2/15/13	Note	3.875	100.9219	100.9531		107.2813	107.3125
8/15/03	8/15/13	Note	4.25	102.625	102.6563		143.2344	143.2656
2/17/04	2/15/14	Note	4	100.9688	101		107.5156	107.5469
8/16/04	8/15/14	Note	4.25	102.1094	102.1406		135.7813	135.8125
2/15/85	2/15/15	Bond	11.25	145.3594	145.3906		108.7656	108.7969
2/15/05	2/15/15	Note	4	100.2969	100.3281		109.9063	109.9375
8/15/85	8/15/15	Bond	10.625	143.7188	143.75		107.6719	107.7031
8/15/05	8/15/15	Note	4.25	101.5625	101.5938		136.4844	136.5156
2/18/86	2/15/16	Bond	9.25	136.0469	136.0781		108.2344	108.2656
2/15/06	2/15/16	Note	4.5	103.125	103.1563		99.64063	99.67188
8/15/06	8/15/16	Note	4.875	105.5938	105.625		102.8594	102.8906
2/15/07	2/15/17	Note	4.625	103.7813	103.8125			
8/17/87	8/15/17	Bond	8.875	137	137.0313			

Source: CRSP

This table reports data on U.S. Treasury notes and bonds on two dates, November 15, 2007, and November 15, 2008. The data were selected to have maturities at a semiannual frequency (February and August of each year) for as many years as possible without gaps. All bonds and notes available on that maturity calendar are reported.

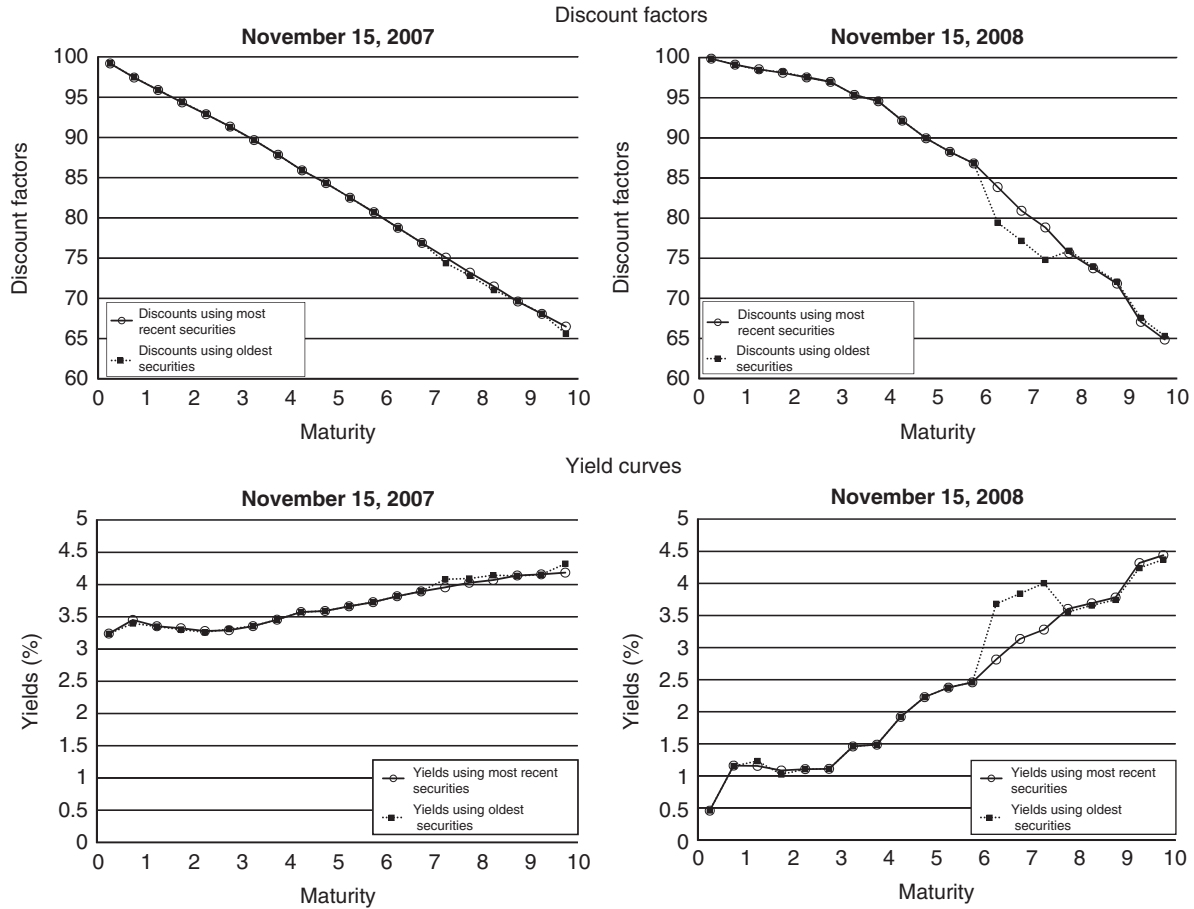


Figure 1.9 Discounts and yields on two dates. The top two panels plot the discount functions $Z(T)$ obtained from bootstrapping on November 15, 2007, and November 17, 2008, while the bottom two panels plot the corresponding yield curves. For each panel, the lines with squares are computed using the oldest securities available, while the lines with circles use the most recent securities available. Source: Data are from the Center for Research on Security Prices (CRSP).

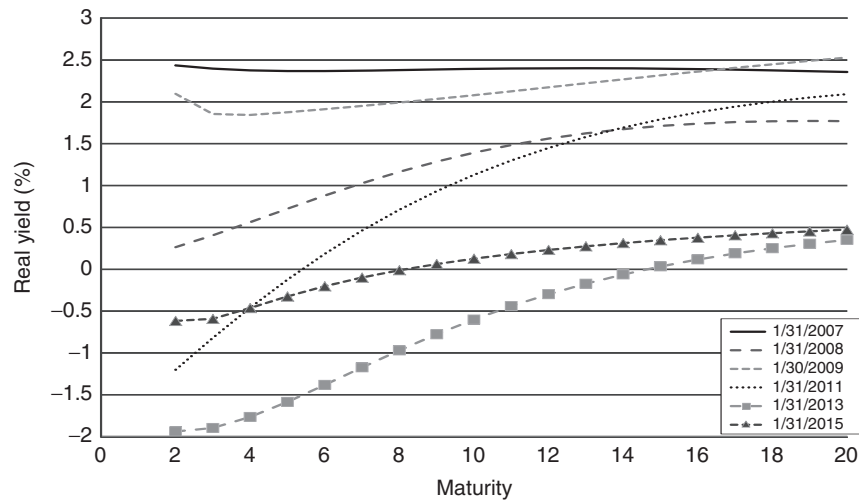


Figure 1.10 TIPS-implied real yield curves. This figure reports the real yield curves extracted from TIPS at the end of January in 2007–2009, 2011, 2013, and 2015. Yields are continuously compounded. Source: Data are from Gürkaynak, Sack, and Wright (2010), updated series.

Intuitively, let us consider an investor who is thinking about how much to borrow or save today. Let the investment or borrowing horizon be denoted by T . Assume there is no inflation, so that the only reason to save or borrow is to transfer “real consumption” from today to time T (if he saves) or vice versa (if he borrows). The logic is as follows: If the investor expects to be very rich at time T , then he knows he will be able to buy a lot of goods in the future, such as a home or a better car, and so on. Thus, it is likely that this investor would like to borrow today to consume more today in expectation of being able to pay back the loan in the future (time T). For instance, many households borrow money to purchase their homes or their cars in the expectation that their income will increase (or at least persist) over time and thus will be able to repay the mortgage or the auto loans. That is, they do not wait to save enough to buy their homes or cars, but rather they buy them today through borrowing. If many households want to borrow today, the strong demand for funds push up the (real) interest rate today, as the higher demand for scarce financial resources induces banks to charge more for borrowing. Extending this argument to the whole economy, we can expect that the higher the expected real growth rate of the economy, the more firms and individuals would like to borrow today and thus the higher the real rate of interest should be.

On the other hand, even if an investor expects to be rich at time T but there is a lot of risk – for instance, he could be fired in the meantime – then he would like to save some money today to cover against the rainy days, so to speak. If a lot of investors want to save because of too much economic uncertainty, they will buy a lot of (inflation-protected) Treasury securities. Such high demand for U.S. Treasuries in turn decreases the real interest rate. Thus, higher macroeconomic risk reduces the risk-free real interest rate. Finally, investors’ natural impatience has also an impact on real rates. That is, the more people prefer to consume today versus the future (i.e., the more they are impatient to consume, such as buy new cars or bigger homes), the more they need to borrow today, which in turn pushes up the real rate of interest, as discussed.

These arguments imply that observing low or negative real rates may indicate that investors expect a low or negative real growth of the economy or perceive high macroeconomic risk. Figure 1.10 is consistent with this interpretation: in 2007, before the crisis, macroeconomic risk was low and expectation of economic growth was strong. However, the 2008 financial crisis has slashed agents’ expectations of real economic growth and dramatically increased macroeconomic risk. Both channels moved to reduced dramatically the real rate of interest, as agents stop borrowing for the future. Of course, other forces are at play as well, such as an aggressive monetary policy that slashed nominal interest rates in 2008 and kept them close to zero ever since. With low expected inflation, real rates are mechanically negative from Equation 1.3. Still, why was the monetary policy so aggressive that led to a massive decrease in the reference short-term rate to start with? Likely, because the Federal Reserve’s own expectations were that real economic growth was predicted to be low and that macroeconomic risk was large. That is, the decomposition 1.14 is a useful reference to interpret the data and the market (or Federal Reserve) expectations about the future of the real economy.

1.7 FLOATING RATE NOTES

The U.S. Treasury started issuing FRNs in January 2014. As of March 2015, 1.6% of U.S. marketable securities are in FRN. FRNs are securities whose coupon is not fixed, but it depends on realized short-term rates. That is, if the short-term interest rate increases, the periodic coupon of the FRN increases, and vice versa. For instance, the new U.S. Treasury FRNs pay coupons at the quarterly frequency using the 13-week U.S. Treasury bill rate as a reference rate, plus a spread determined at the auction of the FRN.

Let T_1, T_2, \dots, T_n denote the coupon payment dates of the FRN, and let $T_{i+1} = T_i + 0.25$. The standard coupon-adjustment formula is that the (annualized) coupon rate at time T_i is given by

$$c(T_i) = r(T_{i-1}) + \text{spread} \quad (1.15)$$

where $r(T_{i-1})$ is the quarterly compounded, (annualized) 3-month rate. While the price of FRNs seems very hard to obtain, because its cash flows (coupons) depend on the realization of future interest rates, which are unknown today, it turns out the FRN pricing is relatively simple although conceptually a little complicated.

Indeed, we can obtain the price from a recursive argument. We consider first the case in which the “spread” in Equation 1.15 is zero. Suppose the FRN has maturity date T_n , where n is the total number of coupon payments. Suppose that today is actually T_{n-1} and thus the FRN has only 3 months to maturity. Today we know the reference 3-month rate $r(T_{n-1})$ and therefore we know the full cash flow at time T_n . This is given by the principal \$1 plus the 1/4 of the coupon rate $c(T_n) = r(T_{n-1})$,

$$\text{Cash flow at } T_n = 1 + \frac{1}{4}c(T_n) = 1 + \frac{1}{4}r(T_{n-1})$$

Therefore, denoting by $P^{FRN}(t)$ the value of the FRN at t , the FRN value at time T_{n-1} ought to be equal to the discounted value of its cash flow at T_n , that is

$$P^{FRN}(T_{n-1}) = \frac{\text{Cash flow at } T_n}{1 + \frac{1}{4}r(T_{n-1})} \quad (1.16)$$

$$= \frac{1 + \frac{1}{4}c(T_n)}{1 + \frac{1}{4}r(T_{n-1})} \quad (1.17)$$

$$= \frac{1 + \frac{1}{4}r(T_{n-1})}{1 + \frac{1}{4}r(T_{n-1})} \quad (1.18)$$

$$= 1 \quad (1.19)$$

That is, no matter what the interest rate is at T_{n-1} , the value of the FRN 3 months before maturity is always 1. The intuition is that the interest rate at T_{n-1} affects both the future cash flow *and* the discount rate used for discounting the cash flow next period back to T_{n-1} . A high interest rate implies not only a higher future cash flow but also a higher discount rate, and vice versa. The two effects offset each other, and the value of the security 3 months before maturity is always 1.

Consider now the value of the security 6 months before maturity, that is, at T_{n-2} . At this time, the investor knows that by purchasing an FRN he gets a security that pays the (annualized) coupon $c(T_{n-1}) = r(T_{n-2})$ next period (i.e., T_{n-1}), when the price of the security will revert back to unity $P^{FRN}(T_{n-1}) = 1$. Thus, the FRN value at T_{n-2} ought to be

$$P^{FRN}(T_{n-2}) = \frac{P^{FRN}(T_{n-1}) + \frac{1}{4}c(T_{n-1})}{1 + \frac{1}{4}r(T_{n-2})} \quad (1.20)$$

$$= \frac{1 + \frac{1}{4}r(T_{n-2})}{1 + \frac{1}{4}r(T_{n-2})} \quad (1.21)$$

$$= 1 \quad (1.22)$$

where we used Equations 1.15 and 1.19 to move from the first to the second line. Once again, the FRN is equal to one also at time T_{n-2} .

Using the same logic and using backward induction, we obtain the rule that an FRN is always equal to unity at every reset time T_i :

$$P^{FRN}(T_i) = 1$$

The FRN is going to be slightly different from one outside reset times, however. Consider a time t such that $T_{i-1} < t < T_i$. At this time, the next cash flow at T_i has already been set according to Equation 1.15. However, at t the new interest rate $r_t(T_i)$ with maturity T_i may have changed. Thus, the value at t is the discounted value of $(1 + c(T_i)/4) = (1 + r(T_{i-1})/4)$ using the discount at t with time to maturity $T_i - t$, $Z_t(T_i - t)$:

$$P^{FRN}(t) = Z_t(T_i - t)(1 + r(T_{i-1})/4) \quad (1.23)$$

The discussion so far assumed that the spread in Equation 1.15 is zero. Adding a spread to the coupon payments is simple because the spread is constant over time, and therefore its present value can be computed using the existing zero-coupon discount curve as $\frac{\text{spread}}{4} \times \sum_{i=1}^n Z(T_i)$, a quantity that needs to be added to $P^{FRN}(0) = 1$. The spread in U.S. Treasury FRN has been extremely small since their first issuance in January 2014, ranging between 0.045% and 0.84%.¹⁵ Because the default risk of U.S. Treasury FRN is the same as the one of short-term Treasury bills and notes, the additional spread over T-bills is likely due to a compensation for the likely less liquidity of FRN compared to the other short-term U.S. Treasury securities.

¹⁵Data from treasurydirect.gov accessed on May 4, 2015.

1.8 CONCLUSION

This chapter contains an introductory overview of fixed-income markets. I highlighted a number of recent changes in fixed-income markets as well as some basic methodologies for their analysis, such as zero-coupon discount functions, forward rates, and the various market forces affecting the term structure of interest rates. This handbook covers many of these subjects, and many more, in far more detail.

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