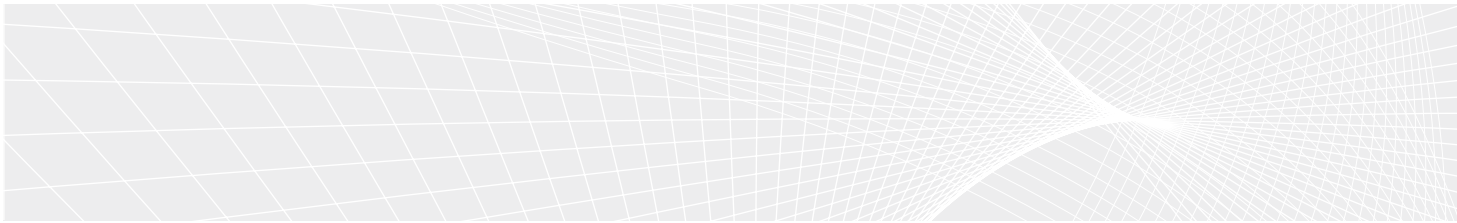




SECTION 1

# Standards and Activities for Number and Quantity

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## The Real Number System

The real number system consists of rational and irrational numbers. It is sometimes referred to as the continuum of real numbers.

Rational numbers are the set of numbers that can be expressed in the form of  $\frac{a}{b}$ , where  $a$  and  $b$  are integers,  $b \neq 0$ . Examples include integers, finite decimals, and repeating decimals.

Irrational numbers are the set of numbers that cannot be written as terminating or repeating decimals. Examples include  $\sqrt{2}$ ,  $\pi$ , and  $e$ .

### N-RN.1

“Extend the properties of exponents to rational exponents.”

1. “Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.”

#### ACTIVITY: UNDERSTANDING INTEGER AND RATIONAL EXPONENTS

Students will complete statements that show how the meaning of rational exponents follows from extending the properties of integer exponents.

##### MATERIALS

One copy of reproducible N-RN.1, “The Meaning of Rational Exponents,” for each student.

##### PROCEDURE

1. Explain that the meaning of rational exponents follows from the properties of integer exponents. Review the properties of exponents.  $a$  and  $b$  are real numbers and  $m$  and  $n$  are integers.
  - $a^m \cdot a^n = a^{m+n}$ ,  $(ab)^m = a^m b^m$ , and  $(a^m)^n = a^{mn}$ .
  - $a^0 = 1$ ,  $a^{-m} = \frac{1}{a^m}$ , and  $\frac{1}{a^{-m}} = a^m$ ,  $a \neq 0$ .





2. Explain that these properties can be extended to rational exponents as follows:  
 $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ , where  $a > 0$  and  $m$  and  $n$  are integers,  $n > 0$ .
3. Explain that some expressions that have rational exponents are rational numbers. Examples include  $16^{\frac{1}{2}} = 4$ ,  $8^{\frac{1}{3}} = 2$ , and  $9^{\frac{-1}{2}} = \frac{1}{3}$ . Other expressions, such as  $4^{\frac{1}{3}} = \sqrt[3]{4}$  and  $7^{\frac{1}{2}} = \sqrt{7}$ , are irrational numbers.
4. Explain that the reproducible, when completed, provides an explanation of how the value of  $16^{\frac{3}{4}}$  can be found using properties of exponents. Students are to complete the statements by selecting the correct expressions, which are shown at the bottom of the page. Note that the steps are sequential and each expression can be used only once.

**CLOSURE**

Ask your students to summarize how they used the properties of exponents to complete the statements.

**ANSWERS**

(1)  $\sqrt[4]{16}$  (2) 2,  $2^4$ , 16 (3)  $\left(16^{\frac{1}{4}}\right)^3$ ,  $2^3$ , 8

**N-RN.2**

**“Extend the properties of exponents to rational exponents.”**

2. “Rewrite expressions involving radicals and rational exponents using the properties of exponents.”

**ACTIVITY: FINDING THE VALUES OF EXPRESSIONS**

Working individually or in pairs, students will identify the value of expressions in simplest form.

**MATERIALS**

One copy of reproducible N-RN.2, “Equations and Their Values,” for the class; scissors for the teacher.



### PREPARATION

After making one copy of the reproducible, cut out each box (each containing an “I have” and “Who has?” statement) so that you have a total of 21 slips of paper. The slips are arranged in order on the reproducible, each providing a number that is the value of the expression written on the previous slip, except for the value of the last expression, which is written on the first slip. The original reproducible will serve as your answer key.

### PROCEDURE

1. Mix the slips up and then distribute one slip of paper to each student or one slip to pairs of students. For a small class, you may give some students two slips. You must distribute all 21 slips.
2. Explain that each slip has a number in its simplest form on the left and a term that can be simplified on the right. Start the activity by asking a student to read the term that is written on the right side of his slip. You may find it helpful to write this term on the board. All students then should check the number they have on the left side of their slip to find the value of the term. Because of the way the slips are designed, only one slip will contain a correct match. The student who has the slip with the correct answer should say, “I have . . .,” and then provide the answer. If the student is correct, she then reads the term written on the right side of her slip. If she is incorrect, point out her error. Another student should then provide the correct answer, which is printed on the left side of his slip.
3. Continue this procedure until the student who read the first term has a number that is equal to the value of the last term.

### CLOSURE

Ask your students for examples of other expressions that can be simplified to the same number. For example,  $81 = 3^4$ , and it also equals  $9^2$ . Ask why this is so.

## N-RN.3

### “Use properties of rational and irrational numbers.”

3. “Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.”





## ACTIVITY: SUMS AND PROPERTIES OF RATIONAL AND IRRATIONAL NUMBERS

This is a two-day activity. Students will work in pairs or groups of three. On the first day, students will place in sequence the steps for proving that the sum of two rational numbers is rational. On the second day, they will place in sequence the steps for proving that the sum of a rational number and an irrational number is irrational. They will also draw conclusions about the difference between two rational numbers as well as the product of a nonzero rational number and an irrational number.

### MATERIALS

Scissors; reproducibles N-RN.3, “Proving the Sum of Two Rational Numbers Is Rational,” and N-RN.3, “Proving the Sum of Rational and Irrational Numbers Is Irrational,” for each pair or group of students.

### PROCEDURE

#### Day One

1. Distribute copies of reproducible N-RN.3, “Proving the Sum of Two Rational Numbers Is Rational,” to each pair or group of students. Explain that the table shows how to prove that the sum of two rational numbers is rational. The table has five rows, each containing a statement and an explanation. The statements and explanations are correct, but the rows are out of order.
2. Explain that students are to place the rows in the correct order. Suggest that they cut out each row, which will make it easier to arrange the rows correctly.

### CLOSURE

Discuss the proof. Ask your students what they can conclude about the difference of two rational numbers. (The difference of two rational numbers is a rational number.)

### ANSWERS

The sequence of some rows may vary; accept any sequence students can justify. One correct order of the rows follows: 2, 3, 5, 1, 4.



## PROCEDURE

### Day two

1. Hand out copies of reproducible N-RN.3, “Proving the Sum of Rational and Irrational Numbers Is Irrational,” to each pair or group of students. Explain that the table shows how to prove that the sum of a rational number and an irrational number is irrational. This is a proof by contradiction that uses the fact that the difference of two rational numbers is rational, which was discussed during the closure on day one.
2. Explain that this proof assumes that the sum of a rational number and an irrational number is rational, but a contradiction makes the assumption incorrect, leading to the conclusion that the sum of a rational number and an irrational number must be irrational.
3. Explain that the table has seven rows; each row contains a statement and an explanation. The statements and explanations are correct, but the rows are out of order. Students are to place them in the proper order. Suggest that students cut out each row to make it easier to arrange the rows correctly.

## CLOSURE

Discuss the proof. Ask your students what they think is true about the product of a nonzero rational number and an irrational number based on their understanding of the sum of a rational number and an irrational number being irrational. (The product is an irrational number.)

## ANSWERS

The sequence of some rows may vary; accept any sequence students can justify. One correct order of the rows follows: 5, 3, 1, 4, 2, 7, 6.

## Quantities

Quantities are numbers with units that involve measurement. Although in the lower grades students worked with units that addressed attributes such as length, width, height, and volume, in high school students work with other units of measurement that address a variety of problems





in many different areas. Some examples include solving problems that involve measurement and acceleration, population density, per capita income, the miles per gallon rating of a car, or the energy consumption of household appliances.

## N-Q.1

### “Reason quantitatively and use units to solve problems.”

1. “Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.”



#### ACTIVITY: A RECOMMENDATION FOR THE BOSS

This activity should be implemented over a few class periods. Students can also work outside of class. Working in groups of three to five, students are to assume that they are members of a team in a company and that their boss tasked them with recommending a vehicle the company might purchase for making deliveries. Each group will present their results to the class.

#### MATERIALS

Computers with Internet access; reproducible N-Q.1, “Guidelines for Choosing a Delivery Vehicle,” for each group of students. *Optional:* Document camera; PowerPoint; digital projector; rulers; markers; poster paper for presentations of groups’ conclusions.

#### PROCEDURE

1. Explain that students are to imagine that they work for a company that makes local deliveries to its customers. The owner of the company, their boss, has decided that the company needs a new car or van to make the deliveries. He wants to purchase the most reliable and fuel-efficient vehicle at a reasonable cost.
2. Explain that each group is to research various vehicles and then choose one that they believe will satisfy the company’s needs. They should refer to the reproducible, which contains guidelines for their research and presentation.
3. Explain that students should conduct their research on the Internet, where they can find and compare details and costs of various vehicles. They might also check print sources, especially advertisements for new vehicles. Suggest that they divide the research among the members of the group, with members researching different vehicles. They should



analyze and compare their data and then choose the vehicle they will recommend. They must support their choice with facts.

4. Explain that students should choose, interpret, and apply units consistently in any formulas they use. For example, they will need to estimate the yearly fuel costs for the vehicles they are considering.
5. Explain that students will present their results to the class, as if they were reporting their findings to their boss. They should present their information in an organized manner and include supporting materials such as graphs, tables, and/or charts. You may suggest that students share their results via a document camera, a computer, perhaps using a digital projector and a PowerPoint presentation, or simply display their data on a poster.

### CLOSURE

Have students present their results to the class. It is likely that different groups will recommend different vehicles. After all groups have made their presentations, ask students to imagine they are the boss of the company. Conduct a poll to find out which vehicle most students would choose.

## N-Q.2

“Reason quantitatively and use units to solve problems.”

2. “Define appropriate quantities for the purpose of descriptive modeling.”



### ACTIVITY: DEFINING APPROPRIATE QUANTITIES

Working in pairs or groups of three, students will identify appropriate quantities that are necessary to solve problems. Students will present their work to the class.

### MATERIALS

Computers with Internet access; reproducible N-Q.2, “Defining Appropriate Quantities for Problem Solving,” for each pair or group of students; a document camera for students to show their work. *Optional:* Instead of a document camera, you may use an overhead projector. In this case students will need transparencies and nonpermanent markers.







## PROCEDURE

1. Explain that choosing appropriate quantities, such as measures and units, is essential for solving many problems. In many cases, two or more quantities are closely related. For example, when calculating a car's fuel costs, factors such as distance (in miles), estimated miles per gallon (which will vary depending on city or highway driving), and the cost of gasoline (in gallons, which is likely to fluctuate through the year) all must be considered.
2. Explain that the reproducible contains three scenarios that present problems. In order to solve the problems, an understanding of quantities is required. Each pair or group of students is to choose one of the scenarios (or you may assign the scenarios to specific pairs or groups). Students are to determine the quantities—measures and units—necessary to solve the problem. They are to record the measures and units and also any equation or equations that could be used to solve the problem. (Because numbers are not included with the problems, students are not expected to actually solve the problems. The focus of this activity is on their identification of appropriate quantities.)
3. Suggest that, if necessary, students conduct background research on the Internet in order to better understand a scenario as well as the measures and units that would be necessary to solve the problem.

## CLOSURE

Have each pair or group of students share their work. Discuss their conclusions and note if students wrote different equations for their problems. Ask your students why different equations for the same problem might be correct.

## ANSWERS

Measures, units, and equations may vary. Possible answers follow: **(1)** First, Jeannette needs to know the volume,  $V$ , in gallons of the filled pool or she needs to know how to find the volume of the pool. For example, for a rectangular pool with an unvarying depth, she can find the volume using the formula  $V = l \times w \times h$ . Next, she needs to know the rate,  $r$ , at which water flows through the hose or pipe in gallons per minute. She must now divide the volume by the rate to find the time,  $T$ , in minutes it would take to fill the pool,  $T = V \div r$ . Finally, she would need to convert the minutes to hours. **(2)** Roberto needs to know the number of gallons of propane,  $p$ , in the tank, and the rate,  $r$ , of consumption of the generator at peak output in gallons per hour. To find how many hours,  $H$ , the generator will run at peak capacity, Roberto may divide the amount of propane by the rate,  $H = p \div r$ . **(3)** First, Rachel needs to find the area,  $A$ , in square feet of each wall, minus the area of any doors or windows. Assuming these are all rectangles, she can find the area of each wall,  $W$ , by multiplying the length,  $l$ , of the wall by its height,  $h$ ,  $A = l \times h$ . She can use the same formula



to find the area of the door and any windows, and then subtract the area of the door and windows from the area of the wall, leaving the area to be painted. Next, she must find the sum of the areas of the walls to be painted and then multiply by 2 because she intends to use two coats of paint. Assuming there are four walls, a possible formula is  $A = 2(W_1 + W_2 + W_3 + W_4)$ . She must also find the coverage area,  $c$ , of a gallon of paint. She then can find how many gallons,  $G$ , of paint she will need by dividing the total area to be covered by the coverage area of one gallon,  $G = A \div c$ .

## N-Q.3

“Reason quantitatively and use units to solve problems.”

3. “Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.”



### ACTIVITY: DETERMINING LEVELS OF ACCURACY IN MEASUREMENT

Working in pairs or groups of three, students will measure lengths and widths to find perimeters and areas of rectangles. They will then determine the accuracy of their calculations.

#### MATERIALS

Rulers (with  $\frac{1}{16}$ -inch and millimeter scales); various items common to the classroom to measure.

#### PROCEDURE

1. Explain that levels of accuracy are limited, depending on the measurement and the item being measured. For example, a rectangle that is 10.5 centimeters long and 8.5 centimeters wide has an area of 89.25 square centimeters. However, because the measurements of the sides were accurate to tenths of a centimeter, it is inaccurate to say that the rectangle has an area accurate to hundredths of a centimeter.
2. Explain that levels of accuracy can be expressed in whole numbers, fractions, or decimals, depending on the measurement.
3. Choose rectangular objects in your classroom for your students to work with, such as desk tops, table tops, book covers, mouse pads, and sheets of paper. Having students measure a variety of objects expands the scope of the activity.

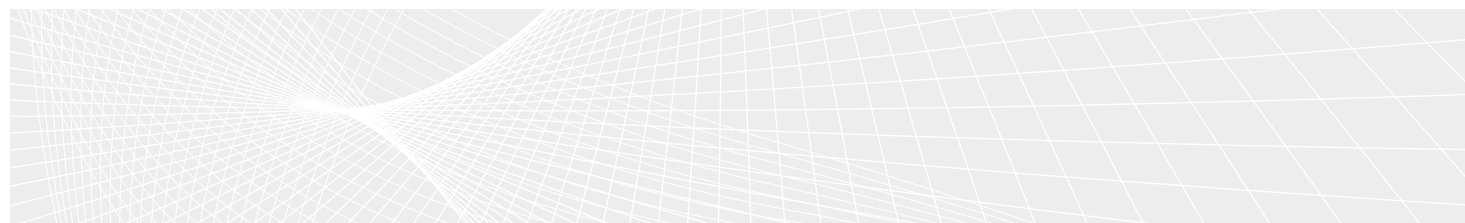




4. Explain that students are to find the perimeters and areas of at least three rectangles. They should select a ruler and an appropriate scale and measure as accurately as possible. They are to find the perimeter of each rectangle and then find its area. They are to then determine the level of accuracy of their results, stating whether the perimeter and area they found for each rectangle is accurate, according to the measures of the rectangle's length and width. They should be prepared to support their answers.

### CLOSURE

Discuss your students' results. It is likely that their results will vary, depending on the measurement tools they used and the measurements. Have students write a brief explanation of what is meant by a "level of accuracy" in measurement.



## The Complex Number System



The complex number system includes real numbers and imaginary numbers of the form  $a + bi$ , where  $a$  and  $b$  are real numbers. Complex numbers can be added, subtracted, multiplied, and divided. Some quadratic equations have solutions that are complex numbers.

### N-CN.1

**"Perform arithmetic operations with complex numbers."**

1. "Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real."

### ACTIVITY: CLASSIFYING COMPLEX NUMBERS

Students will complete a graphic organizer, classifying complex numbers. They will place given numbers in their proper place in the organizer.



**MATERIALS**

Reproducible N-CN.1, “Graphic Organizer for Complex Numbers,” for each student.

**PROCEDURE**

- Review complex numbers, real numbers, imaginary numbers, and pure imaginary numbers with your students. Explain the following:
  - To solve equations such as  $x^2 = -1$ , use the imaginary number  $i$ . Its value is  $i = \sqrt{-1}$ . Using the properties of exponents,  $i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$ .
  - Complex numbers are of the form  $a + bi$ , where  $a$  and  $b$  are real numbers. Complex numbers can be divided into two subsets: real numbers and imaginary numbers. Real numbers are of the form  $a + bi$ , where  $a$  is a real number,  $b = 0$ . Imaginary numbers are of the form  $a + bi$ , where  $a$  and  $b$  are real numbers,  $b \neq 0$ .
  - Pure imaginary numbers are a subset of the imaginary numbers. They are of the form  $a + bi$ ,  $a = 0$ , and  $b$  is a real number,  $b \neq 0$ .
- Explain that the reproducible contains four empty ovals that can be used to classify complex numbers. The ovals are numbered. Students are to classify complex numbers by writing one of the terms—“complex numbers,” “real numbers,” “imaginary numbers,” or “pure imaginary numbers”—in each oval.
- Point out that nine numbers are shown at the bottom of the sheet. Students are to place these numbers in the appropriate oval, beneath the term. Some numbers may be placed in more than one oval.

**CLOSURE**

Ask your students to name other numbers and discuss where they would place these numbers in the ovals.

**ANSWERS**

The oval, type of number, and numbers that belong in the oval are listed in order. **Oval 1:** Complex numbers; all nine numbers should appear in this oval; **Oval 2:** Real numbers;  $\frac{1}{2}$ ,  $\sqrt{2}$ , 36,  $\sqrt[3]{5}$ ; **Oval 3:** Imaginary numbers;  $i$ ,  $-6i$ ,  $i\sqrt{3}$ ,  $4 + 2i$ ,  $-2 - 3i$ ; **Oval 4:** Pure imaginary numbers;  $i$ ,  $-6i$ ,  $i\sqrt{3}$





## N-CN.2

### “Perform arithmetic operations with complex numbers.”

- 2.** “Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.”



### ACTIVITY: OPERATIONS WITH COMPLEX NUMBERS

Working in pairs or groups of three, students will be given 6 solution cards and 12 problem cards involving operations of complex numbers. They will match the problem cards with the solution cards.

#### MATERIALS

Scissors; reproducible N-CN.2, “Complex Number Cards—Solutions and Problems,” for each pair or group of students.

#### PROCEDURE

- 1.** Review the following operations.
  - To add or subtract complex numbers, combine the real parts and then combine the imaginary parts. Stated algebraically,  $(a + bi) + (c + di) = (a + c) + (b + d)i$ .
  - To multiply two complex numbers, think of each number as a binomial and multiply the two binomials. Then substitute  $-1$  for  $i^2$ . Stated algebraically,  $(a + bi)(c + di) = ac + adi + cbi + bdi^2 = ac - bd + (ad + cb)i$ .
- 2.** Explain that the reproducible contains 6 solution cards, numbered 1–6, and 12 problem cards, numbered 7–18. Students are to cut out the cards, keeping the two sets of cards in separate piles.
- 3.** Explain that students are to solve each problem, and then find the solution card that it matches. All solution cards can be paired with two problem cards. All of the cards will be used.

#### CLOSURE

Discuss the problems and especially any errors students might have made. Ask your students to correct any errors.

#### ANSWERS

Each solution card is listed first in parentheses, followed by its matching problem cards.

(1) 9, 17 (2) 7, 13 (3) 14, 18 (4) 8, 16 (5) 10, 12 (6) 11, 15

## N-CN.7

“Use complex numbers in polynomial identities and equations.”

7. “Solve quadratic equations with real coefficients that have complex solutions.”



### ACTIVITY: MATCHING ROOTS

Working in pairs or groups of three, students will solve quadratic equations, most of which will have complex solutions. Partial solutions are provided, and students must combine the parts of the solutions and match them to the correct quadratic equation.

### MATERIALS

Scissors; one copy of reproducible N-CN.7, “Quadratic Equations and Their Roots,” for each pair or group of students.

### PROCEDURE

1. Explain that the solutions to quadratic equations are also called *roots*. The quadratic formula states that the roots of the equation  $ax^2 + bx + c = 0$ ,  $a \neq 0$  are given by the formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . A quadratic equation may have two real roots, one real root (a double root), or two imaginary roots, depending on the value of the discriminant,  $b^2 - 4ac$ . If this value is greater than 0, there are two real roots. If this value is equal to 0, there is one real root. If this value is less than 0, there are two imaginary roots.
2. Review the quadratic formula. Emphasize that the quadratic equation must equal 0 before the quadratic formula may be used.
3. Present this example: To solve  $x^2 - 3x = -4$ , students should rewrite the equation as  $x^2 - 3x + 4 = 0$ . Then they should use the quadratic formula to find  $x = \frac{3 \pm \sqrt{9 - (4)(1)(4)}}{2} = \frac{3 \pm \sqrt{-7}}{2} = \frac{3 \pm i\sqrt{7}}{2}$ .
4. Explain that the reproducible has eight cards on the left side of the page, each with one quadratic equation. Sixteen cards, each containing a part of a solution to one of the equations, are to the right of the equation cards. Students are to solve each equation and find the cards that, when combined, will show the roots of the equation. For instance, if the previous example was an equation on the reproducible, the solutions would be formed by combining cards  $\frac{3 \pm}{2}$  and  $\frac{i\sqrt{7}}{2}$ .



5. To make it easier to match equations and solutions, tell your students to first solve the equation and then cut out the cards. Once students find two cards that form the solutions to an equation, they should place the solution cards next to the equation card. Note that every card will be used. By placing the solution cards next to their matching equation cards, students can verify that their work is correct.

### CLOSURE

Discuss the problems and their answers. Ask your students which problems have real solutions. (Problems 4 and 5) Ask them how they know the solutions are real. (The discriminant is greater than 0.)

### ANSWERS

(1)  $2 \pm 2i$  (2)  $\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$  (3)  $-\frac{1}{2} \pm \frac{i}{2}$  (4)  $3 \pm \sqrt{6}$  (5)  $\frac{1}{8} \pm \frac{\sqrt{17}}{8}$  (6)  $-\frac{1}{3} \pm \frac{i\sqrt{2}}{3}$   
(7)  $\frac{5}{6} \pm \frac{i\sqrt{11}}{6}$  (8)  $-\frac{3}{4} \pm \frac{3i\sqrt{7}}{4}$



**N-RN.1: THE MEANING OF RATIONAL EXPONENTS**

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Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

Complete each statement with the correct expression to explain how to find  $16^{\frac{3}{4}}$ . Use the expressions at the bottom of the page. Each expression can be used only once.

1. The fourth root of 16 can be written in radical form as \_\_\_\_\_.
2. This value is \_\_\_\_\_, because \_\_\_\_\_ is equal to \_\_\_\_\_.
3.  $16^{\frac{3}{4}}$  is equal to \_\_\_\_\_, which is the same as \_\_\_\_\_ or \_\_\_\_\_.

**EXPRESSIONS**

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$2^3$       16       $(16^{\frac{1}{4}})^3$       8       $\sqrt[4]{16}$        $2^4$       2



## N-RN.2: EQUATIONS AND THEIR VALUES

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I have	Who has
10.	$5^{\frac{1}{2}} ?$

I have	Who has
$\sqrt{5}$ .	$16^{\frac{1}{2}} ?$

I have	Who has
4.	$(\sqrt{9})^2 ?$

I have	Who has
9.	$9^{\frac{1}{2}} ?$

I have	Who has
3.	$5^2 ?$

I have	Who has
25.	$4^2 ?$

I have	Who has
16.	$16^{\frac{1}{4}} ?$

I have	Who has
2.	$9^{\frac{-1}{2}} ?$

I have	Who has
$\frac{1}{3}$ .	$3^3 ?$

I have	Who has
27.	$6^0 ?$

I have	Who has
1.	$25^{\frac{1}{2}} ?$

I have	Who has
5.	$36^{\frac{1}{2}} ?$

I have	Who has
6.	$\sqrt{49} ?$

I have	Who has
7.	$10^{\frac{1}{2}} ?$

I have	Who has
$\sqrt{10}$ .	$4^{-1} ?$

I have	Who has
$\frac{1}{4}$ .	$64^{\frac{1}{2}} ?$

I have	Who has
8.	$121^{\frac{1}{2}} ?$

I have	Who has
11.	$4^{\frac{-1}{2}} ?$

I have	Who has
$\frac{1}{2}$ .	$8^{\frac{-4}{3}} ?$

I have	Who has
$\frac{1}{16}$ .	$6^{-1} ?$

I have	Who has
$\frac{1}{6}$ .	$100^{\frac{1}{2}} ?$

**N-RN.3: PROVING THE SUM OF TWO RATIONAL NUMBERS IS RATIONAL**

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Prove that the sum of two rational numbers is rational.		
1.	$ad + cb$ is an integer.	Sums of integers are integers.
2.	Let $a$ , $b$ , $c$ , and $d$ represent integers, where $b \neq 0$ , $d \neq 0$ . $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers.	Definition of rational numbers.
3.	$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$	Finding the sum of rational numbers.
4.	$\frac{ad + cb}{bd}$ is a rational number.	Definition of rational numbers.
5.	$ad$ , $cb$ , and $bd$ are integers.	Products of integers are integers.

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### N-RN.3: PROVING THE SUM OF RATIONAL AND IRRATIONAL NUMBERS IS IRRATIONAL

Prove that the sum of a rational number and an irrational number is an irrational number.		
1.	$\frac{a}{b} + x = \frac{c}{d}$	Assume the sum of a rational number and an irrational number is rational.
2.	$\frac{c}{d} - \frac{a}{b}$ is a rational number.	The difference of two rational numbers is rational, as previously proven.
3.	Let $a$ , $b$ , $c$ , and $d$ represent integers, where $b \neq 0$ , $d \neq 0$ . $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers.	Definition of rational numbers.
4.	$x = \frac{c}{d} - \frac{a}{b}$	Subtraction property of equality.
5.	Let $x$ represent an irrational number.	Beginning of proof by contradiction.
6.	$\frac{a}{b} + x$ must be an irrational number.	The sum of a rational number and an irrational number is irrational.
7.	Therefore, $x$ is a rational number.	This contradicts the assumption that $x$ is an irrational number.

## N-Q.1: GUIDELINES FOR CHOOSING A DELIVERY VEHICLE

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Your boss wants your group to research and recommend a vehicle that the company will use to make deliveries to local customers. The deliveries will be small boxes and packages. You may recommend a car or van, keeping in mind that your boss wants the most reliable, fuel-efficient, and least costly vehicle you can find. Use the following guidelines to determine the best choice for a delivery vehicle.

### The Vehicle

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1. The vehicle should be relatively small. It will be in service five days per week in all but the most extreme weather conditions. (You might want to consider an all-wheel or four-wheel drive vehicle, but remember this will affect gas mileage.)
2. Because the vehicle is expected to travel at least 30,000 miles per year, it should get good mileage. (The vehicle may run on gasoline, electricity, or a combination of both.)
3. The vehicle should be reliable and have reasonable costs for maintenance.
4. The purchase price of the vehicle should be reasonable.
5. Your boss will want an estimate of the total yearly cost for operating and maintaining the vehicle.

Terms such as “extreme weather conditions,” “good mileage,” “reliable,” “fuel-efficient,” and “reasonable costs for maintenance” are to be defined by the group.

### Tips for Your Research and Presentation

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1. Gather your data. Consider several vehicles. Use the Internet as well as print sources for research.
2. Analyze your data. Narrow down your potential choices to one recommendation.
3. Choose, interpret, and apply the appropriate units and formulas for determining fuel costs. You will need to consider the approximate distance the vehicle will travel each year, the cost of fuel (gas or electricity or both), and the miles the vehicle is expected to travel on a unit of fuel.
4. In determining the estimated total yearly costs for operating and maintaining the vehicle, be sure to include all typical costs such as fuel, routine service such as oil changes, and possible repairs.
5. Organize your data.
6. Prepare a presentation of your conclusions, highlighting data in graphs, charts, or tables.

## N-Q.2: DEFINING APPROPRIATE QUANTITIES FOR PROBLEM SOLVING

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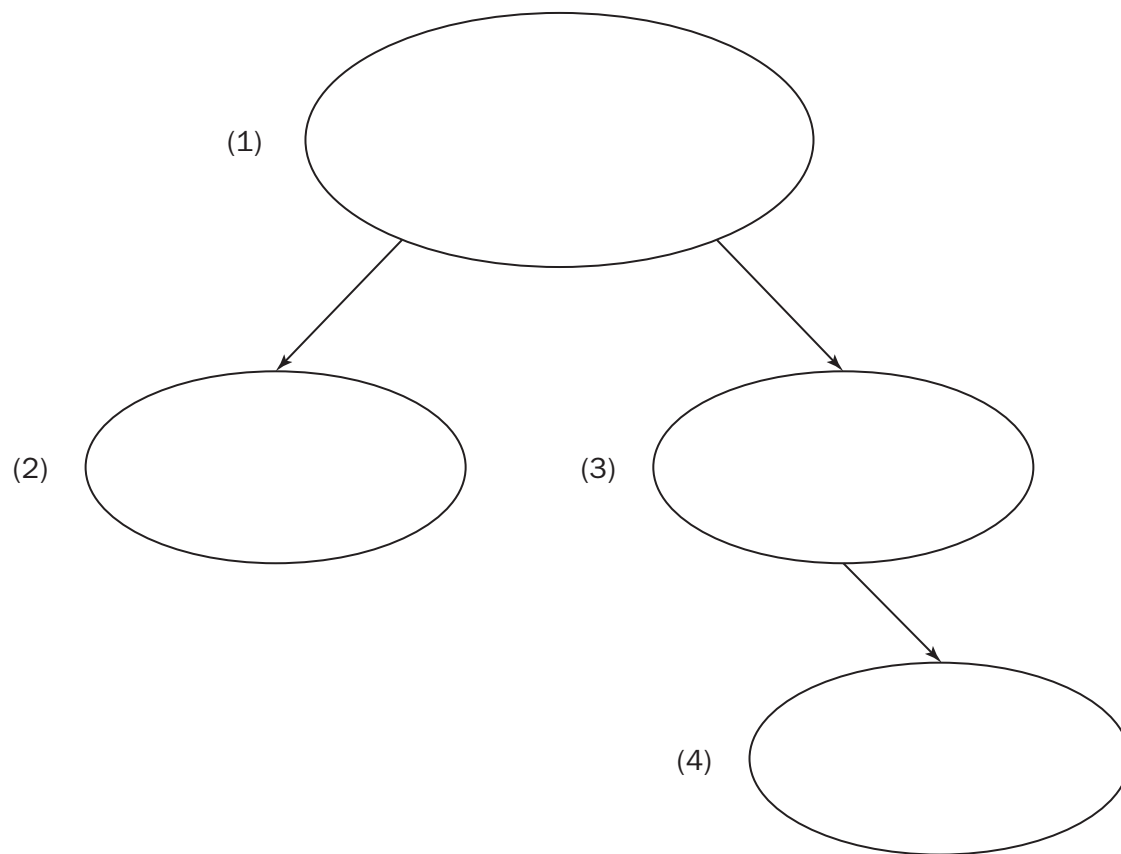
Choose one of the following scenarios. Write the measures and units that are necessary to find a solution to the problem. Then provide an equation or equations that might be used to solve the problem.

- 1.** A swimming pool was installed in Jeannette's yard. What measures and units would she need to find how many hours it will take to fill the pool with water? What equation or equations could she write to solve the problem?
- 2.** Roberto's father purchased a generator that will provide emergency electrical power to their home in the event of a power outage. The generator uses propane gas for fuel, which is stored in a tank. What measures and units would Roberto need to determine how many hours the generator will run at peak capacity on a full tank of propane? What equation or equations can he write to solve the problem?
- 3.** Rachel has decided to paint the walls of her room. She will not paint the ceiling. To make sure that she covers all of the old paint completely, she plans to use two coats of paint on the walls. What measures and units would Rachel need to determine how much paint to buy? What equation or equations could she write to solve the problem?

### N-CN.1: GRAPHIC ORGANIZER FOR COMPLEX NUMBERS

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

Classify complex numbers by completing the organizer. Write “complex numbers,” “real numbers,” “imaginary numbers,” and “pure imaginary numbers” in the appropriate oval. Then write the numbers at the bottom of the page in the appropriate oval. Some numbers will be used more than once.



$i$     $-6i$     $i\sqrt{3}$     $\frac{1}{2}$     $\sqrt{2}$     $4 + 2i$     $36$     $\sqrt[3]{5}$     $-2 - 3i$





## N-CN.2: COMPLEX NUMBER CARDS—SOLUTIONS AND PROBLEMS

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### Solution Cards

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1) $5$	2) $-4$	3) $12 + 2i$
4) $7 + i$	5) $3 + 4i$	6) $5 + i$

### Problem Cards

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7) $(8-4i)-(12-4i)$	8) $(3-i)(2+i)$	9) $(7+4i)-(2+4i)$
10) $(2+i)^2$	11) $(3+4i)-(-2+3i)$	12) $i(4-3i)$
13) $3+7i^2$	14) $(9-2i)+(3+4i)$	15) $(3-2i)(1+i)$
16) $(12-7i)-(5-8i)$	17) $(2-i)(2+i)$	18) $2i(-6i+1)$

### N-CN.7: QUADRATIC EQUATIONS AND THEIR ROOTS

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Equations	Parts of Solutions	
1) $x^2 - 4x + 8 = 0$	$\frac{1}{8} \pm$	$\frac{i}{2}$
2) $x^2 - x = -1$	$3 \pm$	$\frac{3i\sqrt{7}}{4}$
3) $2x^2 + 2x = -1$	$\frac{1}{2} \pm$	$\frac{i\sqrt{2}}{3}$
4) $x^2 - 6x + 3 = 0$	$2 \pm$	$\frac{i\sqrt{11}}{6}$
5) $4x^2 - x = 1$	$-\frac{3}{4} \pm$	$\sqrt{6}$
6) $3x^2 + 2x + 1 = 0$	$-\frac{1}{3} \pm$	$2i$
7) $3x^2 + 3 = 5x$	$-\frac{1}{2} \pm$	$\frac{\sqrt{17}}{8}$
8) $2x^2 + 3x = -9$	$\frac{5}{6} \pm$	$\frac{i\sqrt{3}}{2}$

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