1

Introduction

1.1 Background and Motivation

Between two randomly selected persons in the world, roughly how many friends are there connecting them together? When searching from one webpage to another through the World Wide Web (WWW), how many clicks are needed on average? How can computer viruses propagate so fast and so wide through the Internet? How are people infected by epidemics such as AIDS, SARS, and Avian Influenza all over the world? How do rumors spread in human societies? How does a regional financial recession trigger a global economic crisis? How does an electric power blackout emerge from a small local system failure through the huge-scale power grid? How can the human brain work so efficiently while every brain cell is relatively so simple? … All these seemingly different issues have something to do with "networks" – Internet, WWW, social relationship networks, viruses and rumors propagation networks, economic trading and competition networks, power and traffic flow networks, wired and wireless communication networks, biological neural networks, ecosystem networks, and so on. Noticeably, and most important above all, these apparently different networks have a lot in common.

Since the 1990s, the rapid growth of the Internet as an icon of the high-tech era has led our life to an age of networks. The influence of various complex and dynamical networks is currently pervading all kinds of sciences, ranging from physical to biological, even to social sciences. Its impact on modern engineering and technology is prominent and will be far-reaching. There is no doubt that we are living in a networked world today. On the one hand, networks bring us convenience and benefits, improve our efficiency of work and quality of life, and create tremendous advantages and opportunities which we never had before. On the other hand, however, networks also generate harm and damage to nature and human societies, typically with epidemic spreading, computer virus propagation, and power blackouts, to name just a few. Therefore, the increasing demand for networks and networking also requires a correct view and a serious investigation of the complex properties of various networks and the dynamic mechanisms of networking. For a long time in history, studies of communication networks, power networks, biological networks, economic networks, social networks, etc., were carried out separately and independently. However, recently there has been some rethinking of the general concept and theory of complex dynamical networks towards a better understanding of the intrinsic relations, common properties and shared features of all kinds of networks in the real world, which are not isolated but actually networked together – network of systems and, more generally, network of networks. The new intention and desire of studying the fundamental properties and dynamical behaviors of most if not all complex networks, both qualitatively and quantitatively, is important and timely, although very challenging technically. The current research along this line has been considered as a "new science of networks" [1, 2], or network science and engineering, and has become overwhelming today.

Fundamentals of Complex Networks: Models, Structures and Dynamics, First Edition.
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Companion Website: www.wiley.com/go/chen/complex

Life science is perhaps the most exciting revolutionary area of scientific research in this new century. The mainstream of research in life science in the last century was reductionism-based molecular biology. The fundamental principle of reductionism is that, within different levels of the structure of a biosystem, high-level dynamical behaviors are completely determined by those at the lower levels. There was a common belief that if the individual basic ingredients of life (e.g., DNA, RNA and proteins) could be well understood, then the activities and behaviors of cells at the higher level could be comprehended, while the interactions among these basic elements even among molecules could be neglected. Yet, this traditional reductionism has been seriously challenged at the beginning of the new century due to the many significant discoveries of the importance and essence of networking interactions and interactive dynamics between different levels of life structure and among large numbers of tiny ingredients. Barabási pointed out [3]: "Reductionism, as a paradigm, is expired, and complexity, as a field, is tired. Data-based mathematical models of complex systems are offering a fresh perspective, rapidly developing into a new discipline: network science."

All these have led to a new paradigm of network science, and more recently engineering and technology as well, not just about biology but literally about almost everything.

A *network* is a diagrammatical representation of some physical system or structure. A network consists of some *nodes* (vertices) connected by some *edges* (links) in a certain topology (structure). A *graph*, on the other hand, is a mathematical notion that represents only the structure of a network without physical meanings. Throughout the textbook, however, these two terms are often used for descriptive convenience without precise distinction. Likewise, the terms of *structure* and *topology* of a network or a graph are often arbitrarily used without distinction.

Real-work networks are generally complex, and the complexity of networks may be viewed from different perspectives:

1. *Structural complexity*: A network usually appears structurally complicated, which may even be seemingly messy and disordered (Figure 1.1). The network topology (i.e., structure) may vary in time (e.g., the WWW has new webpages to join and old websites removed everyday). Moreover, the edge connections among nodes may be directed and weighted (e.g., brain cells can be stimulated or restrained and the connections among cells can be strong or weak).
2. *Node-dynamic complexity*: A node in a network can be a dynamical system, which may have bifurcating and even chaotic behaviors (e.g., gene networks and Josephson lattices, which have dynamically evolving nodes). Moreover, a network may have different kinds of nodes (e.g., a power grid has electric generators and also has loads such as motors and machines).
3. *Mutual interactions among various complex factors*: A real-world network is typically affected by many internal and external factors (e.g., if the coupled brain cells are repeatedly excited by certain stimuli then their connections will be strengthened, which is considered the basic reason for learning and memorization). Furthermore, the close relations among networks or subnetworks make the already-complicated behaviors of each of them become much more complex and intrinsic (e.g., the blackout of a huge-scale power grid may lead to chain reactions in human lives and industrial productions, which may also slowdown the activities of other related networks such as traffic, communication and financial transactions). These are referred to as interdependent networks, which have received increasing attention recently.

In the intensive study of nonlinear science and dynamical systems, on the other hand, networked systems have been one of the focal topics for research since the mid-twentieth century. However, most such coupled dynamical systems were placed in a fixed and regularly connected network model for investigation, where the main interest was the complexity caused by the node dynamics but not that by the network topology. Typical examples of this type are coupled map lattice (CML) [4] and cellular neural (or nonlinear) networks (CNN) [5], which can generate rich spatiotemporal patterns. By assuming a network with a regular topology, one can focus on the effects of the node dynamics on the collective behaviors of the network in interest, setting aside the troublesome influence of the network structure. Moreover, the networked elements in a regular topology can be easily implemented by integrated circuits, which is a main concern in commercial applications of networked devices, systems and infrastructures.

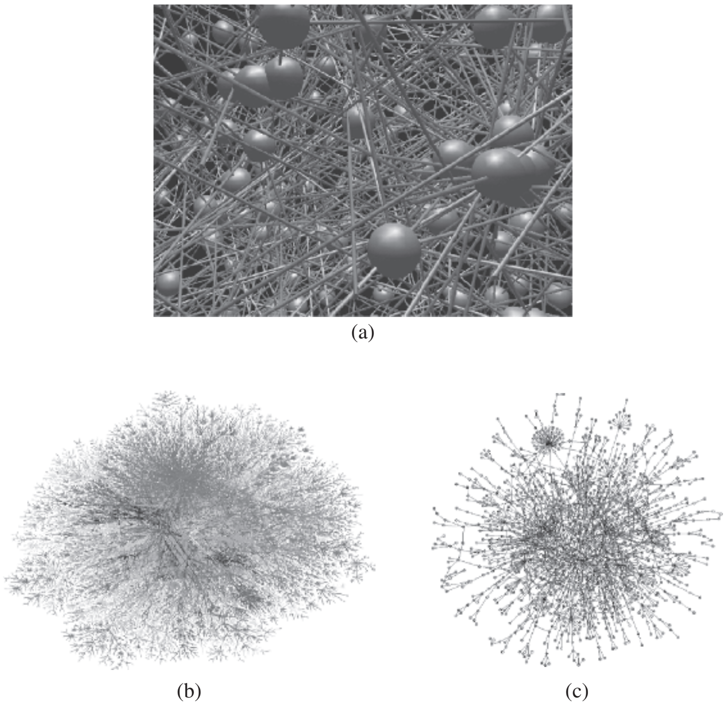

Figure 1.1 Three illustrative graphs with complex structures (from the Internet): (a) Illustrative graph of a social relationship structure in Canberra, Australia (Alden S. Klovdahl, Australian National University); (b) illustrative graph of some IP addresses on the Internet (William R. Cheswick, Lumeta Corporation, New Jersey, USA); (c) illustrative graph of interactions among proteins (Hawoong Jeong, Korea Advanced Institute of Science and Technology)

1.2 A Brief History of Complex Network Research

1.2.1 The Königsburg Seven-Bridge Problem

Complex network research has a long history. The recent study of complex networks has directed most interests to the modeling and understanding of various complex networks, especially the relations between the complexity of the network topology and the behaviors of the network dynamics.

To describe the common properties and characteristic features of different types of networks, a rigorous and efficient analytic tool is needed, which has been introduced in the form of graph theory in mathematics. A network can be viewed as a *graph* consisting of *nodes* connected by *edges* according to a certain rule or form, in which the nodes and edges do not necessarily have physical meanings in the discussion of graphs.

Representing a physical problem by a graph and then solving it by mathematical analysis and computation is not a new idea. This approach can be traced back to as early as the eighteenth century when the great mathematician Leonhard Euler (1707–83) studied and solved the famous seven-bridge problem in a town named Königsburg, which is now in the territory of Russia. As shown in Figure 1.2 [6], there is a river named Pregel passing through the town Königsburg, and there are seven bridges over the river. In the old days, the residents always wondered whether someone could walk through all seven bridges and then return to the starting point without going over any bridge more than once.

In 1736, the young Euler had a good idea to describe this real-world problem by an abstract graph, using four points *A, B, C, D* to represent the four pieces of lands separated by the river in town, with lines *a, b, c, d, e, f, g* to represent the seven bridges that connected the four points together (Figure 1.3 [6]).

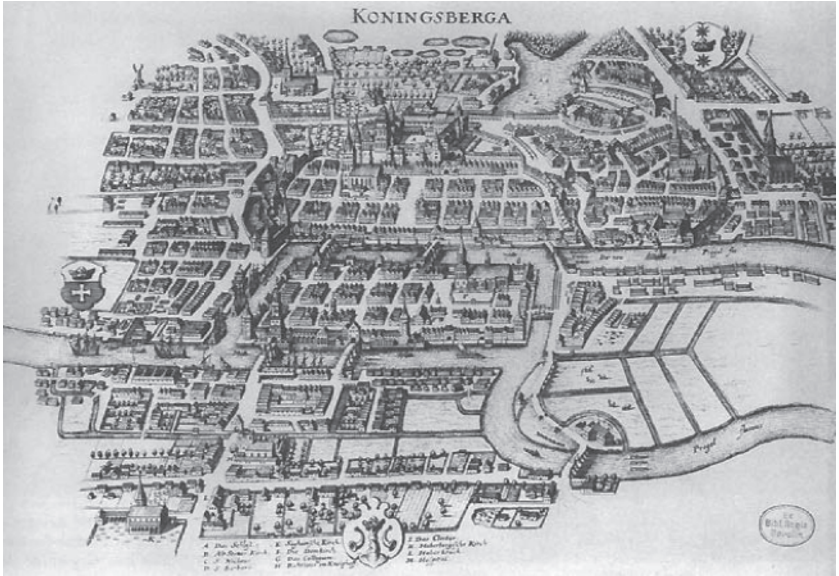

Figure 1.2 The town Königsburg and the seven bridges in 1736 [6]

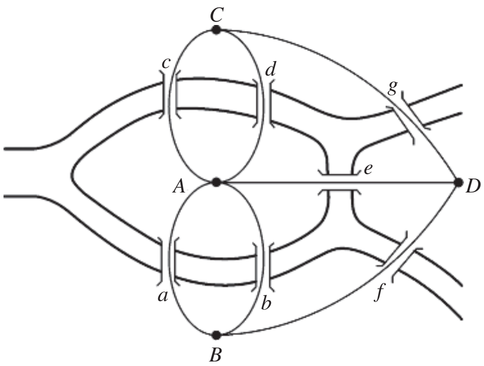

Figure 1.3 Graph of the Königsburg seven-bridge problem

Thus, Euler was able to convert the physical problem to the following mathematical problem: In the graph shown in Figure 1.3, starting from any point, is there a possible loop leading back to the starting point such that it passes all seven lines once and once only? Here and throughout, by "once and once only," it means not to miss any line and not to repeat any line.

Euler furthermore derived a necessary and sufficient condition for the existence of such a loop, thereby proving that the Königsburg seven-bridge problem has no solutions. More precisely, Euler observed that in order to have such a loop, if a point (*A, B, C,* or *D* in Figure 1.3) has one incoming edge then it should also have one outgoing edge; therefore, it is necessary for each point to have an even number of edges. However, the graph shown in Figure 1.3 does not satisfy this condition, therefore the problem has no solution. More significantly, Euler also proved that this condition is sufficient as well.

The contribution of Euler had gone far beyond this simple seven-bridge problem – it has opened up a new branch of mathematics: graph theory. Thereafter, Euler has been named the father of graph theory, and the picture shown in Figure 1.3 was called a Eulerian graph, an example of general Eulerian graphs to be further studied in Chapter 2. As a matter of fact, this simple graph is a foundation stone of modern mathematical graph theory, which has led to extensive studies of complex networks today.

1.2.2 Random Graph Theory

The development of graph theory had a very slow start after Euler solved the Königsburg seven-bridge problem. The first monograph on graph theory was not published until exactly 200 years later, in 1936, interestingly by a Hungarian mathematician named Dénes König [7]. Nevertheless, the theory was developed quite rapidly thereafter. The foundation of the now-famous "random graph theory" was laid by two Hungarian mathematicians, Paul Erdös (1913–96) and Alfréd Rényi (1921–70), in the late 1950s [8], which is considered the first rigorous and complete notion of modern graph theory.

Erdös and Rényi defined a random graph as N nodes connected by n edges, which are randomly chosen from the $N(N-1)/2$ possible edges.

The way to generate an ER random graph is to start with the N given nodes, from which every possible pair of nodes is being connected, once and once only, with probability p $(0 < p < 1)$. More specifically, to generate an ER random network, one may start with N isolated nodes, pick up every possible pair of nodes, once and once only, from a total of $N(N-1)/2$ pairs of nodes, and then with probability p connect the pair with an edge.

Here, "with probability p" can be performed as follows: run a pseudorandom number generator to generate a "1" with probability p or a "0" with probability $1-p$: at a step, if the generator yields a "1" then connect the pair of nodes by an edge; otherwise, the generator yields a "0," so do nothing.

Since every possible pair of nodes is picked up once and once only, there will not be multiple edges between any pair of nodes and, moreover, no node has self-connected edges.

It can easily be seen that in such an ER random graph of N nodes, the expectation value of the total number of edges will be $pN(N-1)/2$, which may not be an integer but is a random variable because p is so. Erdös and Rényi systematically studied many asymptotic properties, as $N \to \infty$, of such graphs and their relations with the edge-connectivity probability p. If a graph has a property P with probability 1 as $N \to \infty$, then they consider almost every ER random graph has property P. One of their most important and also quite surprising discoveries is that many properties of ER random graphs emerge suddenly but not gradually, in the sense that for a given edge-connectivity probability p, either almost every ER random graph has a certain property P or almost every such graph does not have property P [9].

As a historical remark, Paul Erdös is one of the most distinguished and leading mathematicians of the twentieth century, "the man who loves only numbers" [10]. Erdös was legendary; he had published more than 1600 research papers with more than 500 co-authors, and he had made very fundamental contributions in modern mathematics such as number theory, Diophantine equations, combinatory mathematics, probability theory, real and complex analysis, in addition to the random graph theory. Erdös was always excited when he met and worked with other mathematicians. When he met a colleague, he often said "My brain is open"; when he left one coworker to meet with another, he used to say "Another roof, another proof" [11]. In fact, he had devoted his entire life to the beloved mathematics. His stories and contributions will be revisited again and again throughout this textbook.

1.2.3 Small-World Experiments

1.2.3.1 Milgram's Social Experiment

A social network is a family of individuals and communities, connecting one another according to certain relationships. The human relationships in a social network can be friends, coworkers, marriages, partners, etc., or business cooperation between companies. Figure 1.1(a) illustrates one case of a social relationship in Canberra, Australia, based on real data from 5000 residents in 2005.

Take human friendship as an example. Many people have had the experience, when talking to a stranger for the first time, that both sides easily find a common friend in between, and so they yell surprisingly: "What a small world!" In general one may wonder, between two randomly selected persons in the world, how many intermediate friends are there connecting them together? In the 1960s, a psychologist at Harvard University, Stanley Milgram, did a survey in the United States and found that the average number of friends in between two randomly selected persons was only 6! This was the famous discovery of "six degree of separation."

Milgram's social experiment was carried out as follows. He first chose two targeted persons, the wife of a graduate student in a theological college in a small town called Sharon in Massachusetts, and a stockbroker in the big city Boston, unknown to the public in the experiment. He then called for two groups of volunteers in Kansas and Nebraska, respectively. He asked the volunteers each to send a letter to a friend and then ask that friend to forward the letter to another friend, and so on, towards one of the two targeted persons respectively in Sharon and Boston. In the May 1967 issue of *Psychology Today*, [12], Milgram reported his finding – it took only five times of forwarding, on average, to reach a target. In particular, one surprising example is that a letter from Kansas was first mailed to a priest and the priest forwarded it to his friend in Sharon and then the letter reached the target at the third step! Although not all letters were forwarded in such a short path, Milgram was able to statistically reach a conclusion of "six degree of separation."

Whether or not this number 6 was accurate is actually not important, which is not the point. In fact, the Facebook Data Team announced [13], on November 21, 2011, that this number should be 4.74 according to their survey of over 721 million active Facebook users (more than 10% of the global population), with 69 billion friendships among them. In this regard, what is important is the fact that this number is an extremely small number relative to the very large populations of the region, the country, or even the whole world. This indicates the so-called "small world" property of our huge-sized human society. Milgram's idea and experiment have had a great impact on later investigations and analyses of social science and social network research (of which more will be seen below).

1.2.3.2 Bacon's Oracle

To further verify Milgram's "six degree of separation," several social experiments were carried out later, among which was the quite interesting "Oracle of Bacon," also known as Six Degrees of Kevin Bacon or Bacon's networking game. Kevin Bacon (1958–) is a Hollywood actor who has featured in more than 60 movies to date and therefore has had quite a large number of intervening partners in these movies.

The game starts with the following numbering. Bacon himself has number 0. If an actor or an actress has been in a movie with Bacon, then he or she will be given the Bacon number 1; if an actor or actress does not co-show in a movie with Bacon directly but is a partner of someone with Bacon number 1, then this person has Bacon number 2, and so on. Thus, everyone who plays in a movie will have a Bacon number, big or small, indicating that he or she is a node in the network of movie stars starting from Bacon.

There is a database of movies and movie stars established in the Computer Science Department, University of Virginia [14], containing information of near half a million movies with near 1.7 millions of actors and actresses (Table 1.1, as of January 1, 2014). As a simple example, by inputting the name of the Hong Kong movie actor Stephen Chow, one finds his Bacon number to be 2. A prominent feature of this table of statistics is that amongst the very large number of near 1.7 million actors and actresses, the average Bacon number is very small – only about 3 – a small world of movie stars indeed!

1.2.3.3 Erdös Numbers

As mentioned above, Paul Erdös published more than 1600 research papers with more than 500 coauthors. Staring from Erdös, who has a number 0, one can build a network of coauthorships: if someone (likely a mathematician) has a joint paper with Erdös, then he or she will be given an Erdös number 1; if a

Table 1.1 Bacon numbers of actors and actresses (as of January 1, 2014) [14]

Bacon number	Number of actors/actresses
0	1
1	2 799
2	313 045
3	1 078 865
4	276 680
5	22 296
6	2 361
7	251
8	24

Total number of linkable actors: 1 696 322
Average Bacon number: 3.006

person does not coauthor a paper with Erdös directly but is a coauthor of someone with an Erdös number 1, then this person has an Erdös number 2, and so on [15]. Thus, sooner or latter, every scientist who has a joint paper with someone in the fields of mathematics, physics, engineering and even social science will be a node in this coauthorship network, since mathematicians, scientists and engineers usually work in groups and frequently publish papers together.

As a simple example, Charles K Chui was a friend of Paul Erdös and they published one paper together; therefore Chui has an Erdös number 1 [16]. His former PhD student Guanrong Chen, first author of the present textbook, has an Erdös number 2 [17]. All the coauthors of Chen, such as Xiaofan Wang and Xiang Li, coauthors of this textbook, who did not have joint publications with Erdös and Chui, have a small Erdös number 3 [18] – once again, "what a small world"!

1.2.3.4 Experiments over the Internet

From a statistical point of view, although the networks of Bacon and Erdös can easily be computed, they are nevertheless too small to be conclusive. Knowing this, Duncan J Watts and his group of researchers at the Department of Sociology at the University of Columbia, who is now a Chief Scientist at Yahoo, set up a "Small World Project" on their website in 2001 [19], trying to carry out a large-scale international experiment to verify the hypothesis of "six degree of separation." They selected some targeted people and some volunteers, with different ages, races, professions and financial statuses. Once logged in, a volunteer would be given a piece of information and asked to pass on this information by email to a friend towards an unknown targeted person, in a way similar to Milgram's experiment. They reported their findings in *Science* in 2003 [20], showing that within more than one year they had more than 60 000 volunteers from 166 countries participated in this game targeting 18 selected people in 13 countries, and 384 emails eventually arrived at some targets through 5–7 people in between on average. Although there were some uncontrollable factors such as discontinuity of forwarding emails, just like Milgram's letters game, the experimental results agree quite well with Milgram's conclusion – six degree of separation – what a small world!

The largest experiment ever carried out was the one performed by the Facebook Data Team, mentioned above [13]. On November 21, 2011, they reported that according to their survey of over 721 million active Facebook users with 69 billion friendships among them, the average degree of separation was only 4.74, revealing that our world is not only small but today has actually become even smaller!

1.2.4 Strengths of Weak Ties

How do most people find their jobs? Namely, do they rely on their close relatives, or send out letters of application at random, or try the job fairs?

In the late 1960s, Mark Granovetter, then a graduate student of Harvard University, started his research based on the simple question referred to above. He interviewed around 100 people in the greater Boston area, and sent out more than 200 questionnaires, investigating a variety of technical people who had either just been offered a new job or had just lost the old job. Surprisingly Granovetter found that in people's job hunting, usually not those close family ties but new friends or even occasionally encountered strangers linked them to the new job positions. This means that, more often than not, weak connections lead to strong interactions (here, the results of getting jobs). In the language of network science, a long-range connection may lead to a stronger interaction between two nodes than that from short connections of neighboring nodes. From a social science point of view, this can also be easily understood: your relatives and best friends typically have the same groups of common friends; therefore, they usually do not provide new connections and additional information to your own knowledge.

One can easily find many cases from the real world. Here is one typical example provided by Granovetter in his research report [21]: Edward once met a young girl in a gathering in their high school, where he got to know the boyfriend of this girl's elder sister. This man was 10 years older than Edward. Three years later, when Edward lost his job, by chance he met that man again at a party. In conversation, Edward heard that his company was looking for a graph drawer and consequently he got that new job. This example showed once again, "What a small world!" And, moreover, how powerful a weak tie could be!

As a side note, Granovetter submitted his paper "Strength of Weak Ties" to the *American Sociology Review* in August 1969, but it was rejected after four months. His paper was set aside for four years, but then was resubmitted to the *American Journal of Sociology,* consequently being accepted and published [21, 22]. Interestingly enough, this paper has turned out to be one of the few important research papers that have had the highest impact in the field of social sciences today.

1.2.5 Heterogeneity and the WWW

The small-world property of many real networks reflects the essential homogeneity of such networks, in the sense that all nodes in a network have about the same number of connections to the others, which means that every one is equally important regarding their roles in the network. On the contrary, there are also many heterogeneous networks in which a small number of nodes have very large numbers of connections while the majority of nodes have very few connections each. If the number of connection represents the degree of importance of a node, then clearly the hub nodes with large numbers of connections play more significant roles in the network.

1.2.5.1 80–20 Rule

In 1906, Italian economist Vilfredo Pareto observed that about 80% of the land in Italy was owned by about 20% of the population. He then carried out some surveys on a number of other countries and found, to his surprise, that all the distributions are quite similar. Reportedly, he further verified the universality of the 80–20 phenomenon by observing that 20% of the pea pods contained 80% of the peas in his garden [23].

Thereafter, many events and phenomena support that 80–20 rule, which was then termed the Pareto principle, saying that for many events roughly 80% of the effects come from 20% of the causes. For instance, in the 1992 United Nations Development Program Report, it shows that the distribution of global income were very uneven, with the richest 20% of the world's population controlling 82.7% of the world's income. As another example, Microsoft noted that by fixing the top 20% most reported bugs, 80% of the errors and crashes would be eliminated [24]. Reportedly, in business roughly 80% of profits come from 20% of buyers, 80% of complaints come from 20% of customers, 80% of sales come from 20% of products, 80% of the sales come from 20% of the clients, and so on.

1.2.5.2 1% Rule

In Internet culture, there is a so-called 1% rule, known also as 90-9-1 rule, hypothesizing that more people will lurk than will participate in a virtual community.

The 1% rule is often used to refer to participation inequality in the context of the Internet; for example, the number of people who create content on the Internet represents approximately 1% of the people who actually view that content. For every forum posted by a user, generally about 99 other people are only viewing the forum but not posting any. The 90-9-1 version of this rule states that 1% of people create content, 9% edit or modify that content, and 90% only view the content without contributing anything to it.

It is noted, however, that according to a report in a 2012 BBC online briefing [25], about 83% of the population could properly be classified as lurkers, while 17% of the population could be classified as intense contributors of content which, instead, is closer to the 8020 rule discussed above.

1.2.5.3 World Wide Web

The World Wide Web (WWW) is a large-scale directed graph with nodes (vertices) being documents and edges (links) being URLs which point from one document to another. In 1999, Réka Albert, Hawoong Jeong and Albert-László Barabási from the University of Notre Dame, published a report in *Nature* on the connectivity topology (structure) of the WWW [26], showing that the connectivity distribution of the web is heterogeneous which follows a power law for both incoming edges and outgoing edges. This is also supported by the data analysis of Bernardo Huberman and Lada Adamic from the Xerox Palo Alto Research Center, reported in the same issue of *Nature* [27]. That means a small number of nodes have very large numbers of connections while the majority of nodes have very few connections each. Another important discovery in [26] about the WWW is that, although the web size was about 8×10^8 in that time, the average distance (number of hops) between any pair of nodes is only 18.59, namely with 19 clicks one can reach from any document to any other document on this huge web. This proves that the WWW is a small world. The web is also scale-free, since if the web were to increase 1000 percent in the same way it developed, the average distance would only increase from 19 to 21, in the sense that this and some other properties are basically independent of the scale of the network.

1.3 New Era of Complex-Network Studies

Since the 1960s, the theory of ER random graphs [8, 9] has been dominating academic research in network science, which was the only rigorous mathematical model that can describe the early small-scale telephone networks and the infant Internet the like. However, it was also aware of the fact that most real-world complex networks are not completely random, i.e., they are not generated by a completely random process. For instance, whether or not two persons are friends, whether or not two routers in the Internet are connected by optical fibers, whether or not two businessmen make a deal by signing an agreement, etc., are not totally random – these are not determined by simply tossing a coin. Besides, most natural and manmade networks are rapidly growing networks with evolutionary dynamics, very different from the fundamental framework of the ER random graphs, which is static and nongrowing, therefore more realistic network models are desirable.

The end of the twentieth century was a turning point in network science research: networks were being revisited from a physical rather than purely mathematical viewpoint by scientists, particularly applied physicists, computer scientists, engineers and biologists alike, with a new focus on the global behaviors, complex topologies, and evolving dynamics of various networks in interest.

Table 1.2 Some inferential historical markers in network science

Year	People	Event
1736	Euler	Seven-bridge problem
1959	Erdös and Rényi	Random graph theory
1967	Milgram	Small-world experiment
1973	Granovetter	Strength of weak ties
1998	Watts and Strogatz	Small-world network model
1999	Barabási and Albert	Scale-free network model

There were two groundbreaking research papers on complex networks published at the very end of the last century, opening up a new era of complex network studies. In June 1998, Duncan J. Watts, then a PhD student, and his advisor Steven H. Strogatz at Connell University, published an article in *Nature* on the so-called small-world networks [28], followed by another article in *Science* on the so-called scale-free networks by a Hungarian physicist Albert-László Barabási and his then PhD student Réka Albert from the University of Notre Dame in October 1999 [29]. These two seminal articles revealed the most fundamental characteristic of the small-world property and the defining feature of the scale-free property of various complex networks. They have stimulated a large number of new publications in diverse areas of applied physics, mathematics, computer and biological sciences, engineering and technologies, as well as social and economic sciences in the following years, on what is confronting us the most today – complex networks.

The aforementioned three network models, namely the ER random-graph network model, the WS small-world network model and the BA scale-free network model, will be studied in detail in the following chapters of this textbook (for more related information, see [30]).

To this end, a short list of inferential markers in the network science history is summarized in Table 1.2.

Scientific research on complex networks had very significant progress during the crossing time of the two centuries, attributing to basic natural science and several high-tech developments in engineering: (i) the tremendous supercomputing power and the broad-spanned Internet embedded with a vast volume of databases, enabling researchers to collect and process huge amount of real data of different kinds; (ii) fast and wide overlapping of even seemingly unrelated fields of research, leading to many new findings in a broad spectrum of interdisciplinary areas; (iii) recent breakthroughs in the study of complexity, from reductionism to global and structural understanding, spurs a renewal of interest and a rethinking of the network science, leading the contemporary research focus to move from local to global, from lower level to higher level, and from steady to dynamic.

Due to the restriction of large-scale computational ability and the nonavailability of sufficiently large amount of real data, the network study in the past was typically limited to a few hundred or even just a few dozens of nodes in a physical network model. However, we are facing with networks having millions or even billions of nodes and edges such as the Internet, power grids, cash dollars in world economics, human populations in worldwide social studies, biological neural networks at the cell level, biosystems at the gene level, crystals in nanoscale, and so on. Many traditional theories, methodologies and techniques, meaningful and efficient for small-scale computation and analysis, are no longer applicable to such giant networks. The interactions among nodes and between different levels of the complex structure of a huge network generate many unexpected or unpredictable behaviors, such as emergence and chaos, going much beyond the traditional thinking of networks as simple and steady graphs. Consequently, the traditional approaches to network research have to be drastically modified and advanced, or even completely changed. New viewpoints, new theories, new models, and new methods for investigating complex networks are all needed. We have, indeed, already entered a new era of complex network studies.

At present, studies of complex networks may be roughly categorized as follows:

1. *Discovering*: To reveal the global statistical properties of a network and to develop measures for these properties.

2. *Modeling*: To establish a mathematical model of a given network, enabling better understanding of the network statistical properties and the causes for their appearance.
3. *Analysis*: To find out the basic characteristics and essential features of nodes, edges, and the whole network, connected in a certain topology, to develop fundamental mathematical theories that can describe and predict network dynamical behaviors.
4. *Control*: To develop effective methods and techniques that can be used to modify and improve network properties and performances, suggesting new and possibly optimal network designs and utilizations, particularly in regard to network stability, synchronizability, and data-traffic management.
5. *Applications*: To apply and utilize some special and fundamental properties and characteristics of complex networks to facilitate the design and applications of network-related problems, such as data-flow congestion control on the Internet and traffic control for city transportations, optimal integrated circuit design for chip fabrication, better decision-making of policy and strategy for commercial trading and financial management, etc.

It is clear that, accomplishment notwithstanding, complex dynamical networks as a promising and profound research subject is merely at the beginning of a foreseeable far-reaching as well as long-sustainable research endeavor. The scientific research along this line has been considered as "network science", and "network science and engineering" more recently. New discoveries, developments, enhancements and improvements are still yet to come.

Exercises

1.1 In the Königsburg seven-bridge problem shown in Figure 1.3, if the requirement is to go through every bridge once and once only, but not necessarily return to the starting point, is the problem solvable? If so, show one solution; if not, explain why.

1.2 Name and briefly describe a few real-world examples of random-graph networks, small-world networks, and scale-free networks.

1.3 How many people will connect you to Paul Erdös, Kevin Bacon, Albert-László Barabási, and Barack Obama?

1.4 Give some real-world examples of strong interactions generated by weak connections, namely, a long-range connection leads to a stronger interaction between two nodes than those short connections of neighboring nodes.

1.5 Show some examples where the minority rules the majority (both in terms of numbers) in their social activities, cooperation, competition, business, etc. Also, show some other examples where majority rules the game.

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