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LEONARDO FIBONACCI

Leonardo Fibonacci, also called Leonardo Pisano or Leonard of Pisa, was the most outstanding mathematician of the European Middle Ages. Little is known about his life except for the few facts he gives in his mathematical writings. Ironically, none of his contemporaries mention him in any document that survives.

Fibonacci was born around 1170 into the Bonacci family of Pisa, a prosperous mercantile center. ("Fibonacci" is a contraction of "Filius Bonacci," son of Bonacci.) His father Guglielmo (William) was a successful merchant, who wanted his son to follow his trade.

Around 1190 when Guglielmo was appointed collector of customs in the Algerian city of Bugia (now called Bougie), he brought Leonardo there to learn the art of computation. In Bougie, Fibonacci received his early education from a Muslim schoolmaster, who introduced him to the Indian numeration system and Indian computational techniques. He also introduced Fibonacci to a book on algebra, *Hisâb al-jabr w'almuqabâlah*, written by the Persian mathematician al-Khowarizmi (ca. 825). (The word *algebra* is derived from the title of this book.)

As an adult, Fibonacci made frequent business trips to Egypt, Syria, Greece, France, and Constantinople, where he studied the various systems of arithmetic then in use, and exchanged views with native scholars. He also lived for a time at the court of the Roman Emperor, Frederick II (1194–1250), and engaged in scientific debates with the Emperor and his philosophers.

Fibonacci and Lucas Numbers with Applications, Volume One, Second Edition. Thomas Koshy.

Fibonacci*

Around 1200, at the age of 30, Fibonacci returned home to Pisa. He was convinced of the elegance and practical superiority of the Indian numeration system over the Roman system then in use in Italy. In 1202 Fibonacci published his pioneering work, *Liber Abaci* (*The Book of the Abacus*). (The word *abaci* here does not refer to the hand calculator called an abacus, but to computation in general.) *Liber Abaci* was devoted to arithmetic and elementary algebra; it introduced the Indian numeration system and arithmetic algorithms to Europe. In fact, Fibonacci demonstrated in his book the power of the Indian numeration system more vigorously than in any mathematical work up to that time. *Liber Abaci*'s 15 chapters explain the major contributions to algebra by al-Khowarizmi and Abu Kamil (ca. 900), another Persian mathematician. Six years later, Fibonacci

**Figure source*: David Eugene Smith Collection, Rare Book & Manuscript Library, Columbia University in the City of New York. Reproduced with permission of Columbia University.

revised *Liber Abaci* and dedicated the second edition to Michael Scott, the most famous philosopher and astrologer at the court of Frederick II.

After *Liber Abaci*, Fibonacci wrote three other influential books. *Practica Geometriae* (*Practice of Geometry*), published in 1220, is divided into eight chapters and is dedicated to Master Domonique, about whom little is known. This book skillfully presents geometry and trigonometry with Euclidean rigor and some originality. Fibonacci employs algebra to solve geometric problems and geometry to solve algebraic problems, a radical approach for the Europe of his day.

The next two books, the *Flos* (*Blossom* or *Flower*) and the *Liber Quadratorum* (*The Book of Square Numbers*) were published in 1225. Although both deal with number theory, *Liber Quadratorum* earned Fibonacci his modern reputation as a major number theorist, ranked with the Greek mathematician Diophantus (ca. 250 A.D.) and the French mathematician Pierre de Fermat (1601–1665). Both *Flos* and *Liber Quadratorum* exemplify Fibonacci's brilliance and originality of thought, which outshine the abilities of most scholars of his time.

In 1225, Frederick II wanted to test Fibonacci's talents, so he invited Fibonacci to his court for a mathematical tournament. The contest consisted of three problems, prepared by Johannes of Palumbo, who was on the Emperor's staff. The first was to find a rational number x such that both $x^2 - 5$ and $x^2 + 5$ are squares of rational numbers[†]. Fibonacci gave the correct answer 41/12: $(41/12)^2 - 5 = (31/12)^2$ and $(41/12)^2 + 5 = (49/12)^2$.

The second problem was to find a solution to the cubic equation $x^3 + 2x^2 + 10x - 20 = 0$. Fibonacci showed geometrically that it has no solutions of the form $\sqrt{a + \sqrt{b}}$, but gave an approximate solution, 1.3688081075, which is correct to nine decimal places. This answer appears in the *Flos* without any explanation.

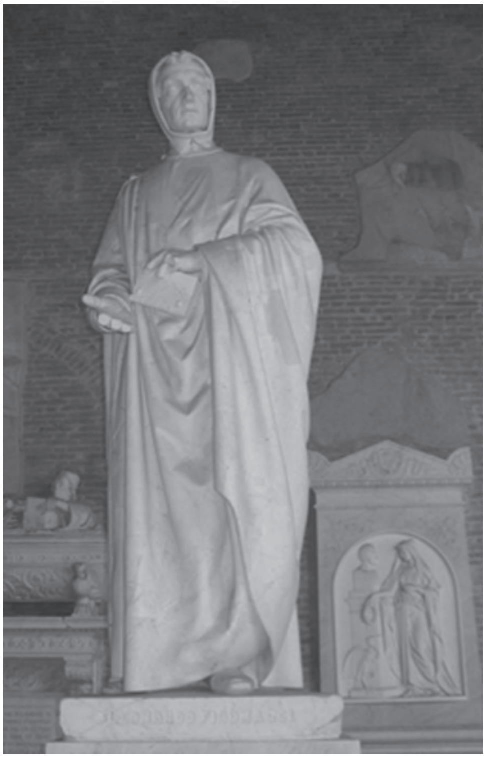
The third problem, also recorded in *Flos*, was to solve the following:

> Three people share 1/2, 1/3, and 1/6 of a pile of money. Each takes some money from the pile until nothing is left. The first person then returns one-half of what he took, the second one-third, and the third one-sixth. When the total thus returned is divided among them equally, each possesses his correct share. How much money was in the original pile? How much did each person take from the pile?

Fibonacci established that the problem is indeterminate and gave 47 as the smallest answer. None of Fibonacci's competitors in the contest could solve any of these problems.

The Emperor recognized Fibonacci's contributions to the city of Pisa, both as a teacher and as a citizen. Today, a statue of Fibonacci stands in the Camposanto Monumentale at Piazza dei Miracoli, near the Cathedral and the Leaning Tower of Pisa. Until 1990, it had been at a garden across the Arno River for some years.

[†]A solution to the problem appears in *The Mathematics Teacher*, Vol. 45 (1952), 605–606. R.A. Laird of New Orleans, Louisiana, reproduced it in *The Fibonacci Quarterly* 3 (1965), pp. 121–122. The general solution to the problem that both $x^2 - m$ and $x^2 + m$ be rational squares appears in O. Ore, *Number Theory and its History*, McGraw-Hill, New York, 1948, pp. 188–193.



Statue of Fibonacci*

Head of the Statue#

Not long after Fibonacci's death in 1240, Italian merchants began to appreciate the beauty and power of the Indian numeration system, and gradually adopted it for business transactions. By the end of the sixteenth century, most of Europe had accepted it. *Liber Abaci* remained the European standard for more than two centuries, and played a significant role in displacing the unwieldy Roman numeration system, thereby spreading the more efficient Indian number system to the rest of world.

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