Part I

THE DETERMINISTIC LIFE CONTINGENCIES MODEL

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Introduction and motivation

1.1 Risk and insurance

In this book we deal with certain mathematical models. This opening chapter, however, is a nontechnical introduction, designed to provide background and motivation. In particular, we are concerned with models used by actuaries, so we might first try to describe exactly what it is that actuaries do. This can be difficult, because a typical actuary is concerned with many issues, but we can identify two major themes dealt with by this profession.

The first is *risk*, a word that itself can be defined in different ways. A commonly accepted definition in our context is that risk is the possibility that *something bad* happens. Of course, many bad things can happen, but in particular we are interested in occurrences that result in *financial loss*. A person dies, depriving family of earned income or business partners of expertise. Someone becomes ill, necessitating large medical expenses. A home is destroyed by fire or an automobile is damaged in an accident. No matter what precautions you take, you cannot rid yourself completely of the possibility of such unfortunate events, but what you can do is take steps to mitigate the financial loss involved. One of the most commonly used measures is to purchase insurance.

Insurance involves a sharing or pooling of risks among a large group of people. The origins go back many years and can be traced to members of a community helping out others who suffered loss in some form or other. For example, people would help out neighbours who had suffered a death or illness in the family. While such aid was in many cases no doubt due to altruistic feelings, there was also a motivation of self-interest. You should be prepared to help out a neighbour who suffered some calamity, since you or your family could similarly be aided by others when you required such assistance. This eventually became more formalized, giving rise to the insurance companies we know today.

With the institution of insurance companies, sharing is no longer confined to the scope of neighbours or community members one knows, but it could be among all those who chose to purchase insurance from a particular company. Although there are many different types

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of insurance, the basic principle is similar. A company known as the *insurer* agrees to pay out money, which we will refer to as *benefits*, at specified times, upon the occurrence of specified events causing financial loss. In return, the person purchasing insurance, known as the *insured*, agrees to make payments of prescribed amounts to the company. These payments are typically known as *premiums*. The contract between the insurer and the insured is often referred to as the *insurance policy*.

The risk is thereby transferred from the individuals facing the loss to the insurer. The insurer in turn reduces its risk by insuring a sufficiently large number of individuals, so that the losses can be accurately predicted. Consider the following example, which is admittedly vastly oversimplified but designed to illustrate the basic idea.

Suppose that a certain type of event is unlikely to occur but if so, causes a financial loss of 100 000. The insurer estimates that about 1 out of every 100 individuals who face the possibility of such loss will actually experience it. If it insures 1000 people, it can then expect 10 losses. Based on this model, the insurer would charge each person a premium of 1000. (We are ignoring certain factors such as expenses and profits.) It would collect a total of 1 000 000 and have precisely enough to cover the 100 000 loss for each of the 10 individuals who experience this. Each individual has eliminated his or her risk, and in so far as the estimate of 10 losses is correct, the insurer has likewise eliminated its own risk. (We comment further on this statement in the next section.)

We conclude this section with a few words on the connection between insurance and gambling. Many people believe that insurance is really a form of the latter, but in fact it is exactly the opposite. Gambling trades certainty for uncertainty. The amount of money you have in your pocket is there with certainty if you do not gamble, but it is subject to uncertainty if you decide to place a bet. On the other hand, insurance trades uncertainty for certainty. The uncertain drain on your wealth, due to the possibility of a financial loss, is converted to the certainty of the much smaller drain of the premium payments if you insure against the loss.

1.2 Deterministic versus stochastic models

The example in Section 1.1 illustrates what is known as a *deterministic* model. The insurer in effect pretends it will know exactly how much it will pay out in benefits and then charges premiums to match this amount. Of course, the insurer knows that it cannot really predict these amounts precisely. By selling a large number of policies they hope to benefit from the diversification effect. They are really relying on the statistical concept known as the ‘law of large numbers’, which in this context intuitively says that if a sufficiently large number of individuals are insured, then the total number of losses will likely be close to the predicted figure.

To look at this idea in more detail, it may help to give an analogy with flipping coins. If we flip 100 fair coins, we cannot predict exactly the number of them that will come up heads, but we expect that most of the time this number should be close to 50. But ‘most of the time’ does not mean always. It is possible for example, that we may get only 37 heads, or as many as 63, or even more extreme outcomes. In the example given in the last section, the number of losses may well turn out to be more than the expected number of 10. We would like to know just how unlikely these rare events are. In other words, we would like to quantify more precisely just what the words ‘most of the time’ mean. To achieve this greater sophistication a stochastic model for insurance claims is needed, which will assign probabilities to the occurrence of various numbers of losses. This will allow adjustment of premiums in order to allow for the risk that the actual number of losses will deviate from that expected. We will however begin the study of actuarial mathematics by first developing a deterministic approach, as this seems to be the best way of learning the basic concepts. After mastering this, it is not difficult to turn to the more realistic stochastic setting.

We will not get into all the complications that can arise. In actual coin flipping it seems clear that the results of each toss are independent of the others. The fact that one coin comes up heads, is not going to affect the outcomes of the others. It is this independence which is behind the law of large numbers, and which results in outcomes that are usually close to what is expected. There are some risks, often referred to as *systematic* or *non-diversifiable,* where the independence assumption fails, and which can adversely affect all or a large number of members of a group at the same time. For example, a spreading epidemic could cause life or health insurers to pay more in claims than they expected. Selling more policies in order to diversify would not help their financial situation. It could in fact make it worse, if the premiums were not sufficient to cover the extra losses. Severe climatic disturbances causing storms could impact property insurance in the same way. In 2008, falling real estate prices in the United States affected mortgage lenders and those who insured mortgage lenders against bad debts, to the extent that this helped trigger a global financial crisis. A detailed discussion of these matters is not within the scope of this work, and for the most part, the stochastic model we present will confine attention to the usual insurance model where the risks are considered as independent. It should be kept in mind however that the detection and avoidance of systematic risk are matters that the actuary must always be aware of.

1.3 Finance and investments

The second theme involved in an actuary's work is finance and investments. In most of the types of insurance that we focus on in this book, an additional complicating factor is the long-term nature of the contracts. Benefits may not be paid until several years after premiums are collected. This is certainly true in life insurance, where the loss is occasioned by the death of an individual. Premiums received are invested and the resulting earnings can be used to help provide the benefits. Consider the simple example given above, and suppose further that the benefits do not have to be paid until 1 year after the premiums are collected. If the insurer can invest the money at, say, 5% interest for the year, then it does not need to charge the full 1000 in premium, but can collect only 1000/1.05 from each person. When invested, this amount will provide the necessary 1000 to cover the losses. Again, this example is oversimplified and there are many more complications. We will, in the next chapter, consider a mathematical model that deals with the consequences of the payments of money at various times. A much more elaborate treatment of financial matters, incorporating randomness, is presented in Chapter 20.

1.4 Adequacy and equity

We can now give a general description of the responsibilities of an actuary. The overriding task is to ensure that the premiums, together with investment earnings, are *adequate* to provide for the payment of the benefits. If this is not true, then it will not be possible for the insurer to meet its obligations and some of the insureds will necessarily not receive compensation for their losses. The challenge in meeting this goal arises from the several areas of uncertainty. The amount and timing of the benefits that will have to be paid, as well as the investment earnings, are unknown and subject to random fluctuations. The actuary makes substantial use of probabilistic methods to handle this uncertainty.

Another goal is to achieve *equity* in setting premiums. If an insurer is to attract purchasers, it must charge rates that are perceived as being fair. Here also, the randomness means that it is not obvious how to define equity in this context. It cannot mean that two individuals who are charged the same amount in premiums will receive exactly the same back in benefits, for that would negate the sharing arrangement inherent in the insurance idea. While there are different possible viewpoints, equity in insurance is generally expected to mean that the mathematical expectation of these two individuals should be the same.

1.5 Reassessment

Actuaries design insurance contracts and must initially calculate premiums that will fulfill the goals of adequacy and equity, but this is not the end of the story. No matter how carefully one makes an initial assessment of risks, there are too many variables to be able to achieve complete accuracy. Such assessments must be continually re-evaluated, and herein lies the real expertise of the actuary. This work may be compared to sailing a ship in a stormy sea. It is impossible to avoid being blown off course occasionally. The skill is to detect when this occurs and to take the necessary steps to continue in the right direction. This continual monitoring and reassessing is an important part of the actuary's work. A large part of this involves calculating quantities known as *reserves*. We introduce this concept in Chapter 2 and then develop it more fully in Chapter 6.

1.6 Conclusion

We can now summarize the material found in the subsequent chapters of the book. We will describe the mathematical models used by the actuary to ensure that an insurer will be able to meet its promised benefits payments and that the respective purchasers of its contracts are treated equitably. In Part I, we deal with a strictly deterministic model. This enables us to focus on the main principles while keeping the required mathematics reasonably simple. In Part II, we look at the stochastic model for an individual insurance contract. In Part III, we look at more advanced stochastic models and introduce the mathematics of financial markets. In Part IV, we consider models that encompass an entire portfolio of insurance contracts.