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Introduction to Aerodynamics and Atmosphere

1.1 Motivation and Scope of Aerodynamics

Study of aerodynamics involves the ability to predict aerodynamic forces and moments acting on an airborne vehicle. However, it all began with the search for the quintessential shape that will make anything airborne in a sustained manner. Historically, the search for human flight began with lighter than air vehicles, now known variously as aerostats. While airships, blimps and/or dirigibles are still in use, the original search for vehicles heavier than air was the main attraction for human flight. In our quest for flight, we always wanted to emulate the birds, but even today this appears unattainable with present-day technologies. A bird flies in which the flapping wing performs the dual role of propulsive and aerodynamic devices. In man-made devices the propulsive device produces power or thrust for the vehicle to overcome resistance, while the aerodynamic device creates the necessary force to keep the body aloft in a dynamical equilibrium. Any device that imitates the flight of birds is known as an ornithopter and it is to the genius of Sir George Cayley (1773–1857) mankind owes a debt for the conventional aircraft shape and design. In a marked departure, he suggested that such propulsive devices did not exist and that it was more important to understand the analysis and design of aerodynamic aspect of the vehicle first. To do so, he advocated the study of powerless flight of aeronautical shapes of interest in a stable manner. This way of compartmentalizing the different aspects of flight into aerodynamics, propulsion, structures, performance, stability and control is now one essential component in the study of the discipline and was started by the need to understand the basics of flight as pioneered by Cayley. For an historic account of the development of flight and aerodynamics in particular, readers are advised to study it in Anderson (1997).

In this book, the sole motivation would be to study the aerodynamic forces and moments acting on an aircraft. Before we discuss the motivation of studying aerodynamics further, we should familiarize ourselves with different parts of an aircraft and their roles in flight. In Figure 1.1, we show the different main parts of a small aircraft as viewed externally.

Theoretical and Computational Aerodynamics, First Edition. Tapan K. Sengupta.

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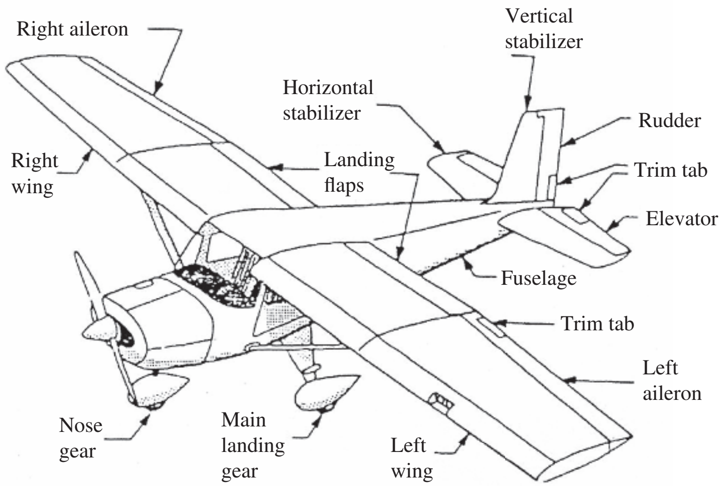

Figure 1.1 An external view of a small propeller aircraft with tricycle landing gear, identifying different parts of aerodynamic and control surfaces

The external shape of an aircraft is central to the study of aerodynamics, flight stability and controls. As an aircraft is heavier than the ambient displaced air, various surface stresses acting on different parts of the aircraft should be such that there must be a resultant force acting which will sustain the weight of the aircraft, when the aircraft is flying level and steady. This resultant force acting over the aircraft sustaining the weight is called the lift force. In a conventional aircraft, lift force is created by the flow over the wing. This is often modelled by the normal stress or the static pressure acting on the wing surface and is obtained by considering an ideal flow over the wing. When and how this is possible, will be the recurring theme of this book. Such normal stress or pressure distribution acting over the wing also creates a moment acting about a general point along the chord of the wing section and is a concomitant liability of producing the lift force. This is counteracted upon by the smaller amount of lift created on the horizontal stabilizer. One realizes that the main purpose of an aerospace vehicle is to carry payload and this is the reason for having a fuselage. Such a tubular shape of the fuselage, along with different aerodynamic and control surfaces, also creates resistance or drag for the aircraft. The required thrust to overcome drag is provided by the propeller engine located at the nose of the aircraft. The horizontal and vertical stabilizers are required for maintaining the stability of the aircraft for different flight regime.

One of the rudimentary aspects of flight operation is the cruise configuration, in which the aircraft flies level and steady at constant altitude. In Figure 1.2, we show a conventional aircraft flying level and steady in the longitudinal plane. External forces are shown to be in equilibrium, i.e. the weight of the aircraft is balanced by the vertical component of the aerodynamic forces acting on the aircraft called the lift and denoted by L. Most of it is created by the aircraft wing. The horizontal component of the aerodynamic force is termed the drag and denoted by D. Almost half the drag of an aircraft is created by the fuselage and a large portion of it is caused by the creation of lift itself. The total drag is overcome by the thrust

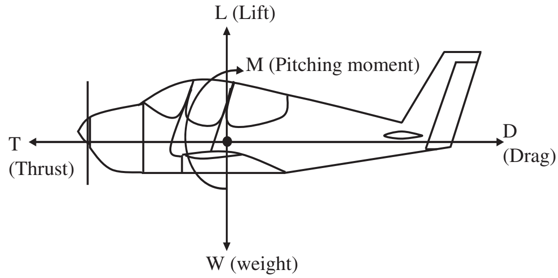

Figure 1.2 A conventional aircraft flying level and steady in the longitudinal plane. Forces and moments acting on the aircraft are shown in equilibrium; the pitching moment is balanced by the lift acting on the tailplane

(indicated by T), created by the power plant. Thrust is also a form of aerodynamic force created by internal flows through the engine where energy is added by burning fuel and ejecting a well-directed stream of fluid which by reaction creates the thrust. But this is outside the scope of this book and will not be discussed further.

Note that the drawn free body diagram in Figure 1.2, is an idealization as the forces are not collinear and also the forces and moment created by the stabilizer are not shown in this diagram. At the outset, we want to state the directions of the aerodynamic force as consisting of: the drag force, which is always along the oncoming flow direction, and lift, which is perpendicular to it. For a general motion of the aircraft, additionally another component of aerodynamic force will be experienced, which is perpendicular to both lift and drag and is termed as the side force. The moment experienced by the aircraft in the longitudinal plane has been indicated by the pitching moment. For general motion of the rigid aircraft will give rise to two more components of moment termed as the rolling (about the fuselage axis) and yawing (about the vertical axis of the aircraft) moments. In Figure 1.3, we show a sketch of all three forces and three moments relevant for a rigid aircraft. The reader is made aware of the fact that the aircraft is hardly ever a rigid body and in general the problem constitutes an

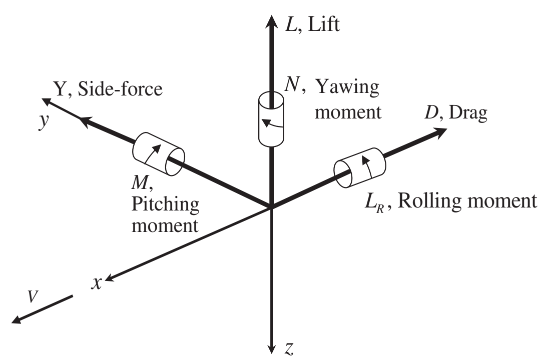

Figure 1.3 Axes and coordinate system used for any rigid aircraft. Forces and moments acting on the aircraft are shown with conventional usage

analysis for very large degrees of freedom. (The static tip deflection of Boeing 747 in cruise from the static position in ground is more than 12 feet!) This constitutes the interesting field of study called aero-elasticity where simultaneous consideration of fluid dynamics and structural dynamics are taken into account. When the design of aircraft is complicated by the fact that the stability and control of the aircraft depends and directly interacts with fluid and structural dynamics of the aircraft, then one is forced to consider aero-servo-elasticity. However creating additional complications by various forces interacting is important for many applications, but the primary goal of this book is to study the aerodynamics of simple lifting surfaces. The moot point is: How have such aerodynamic surfaces evolved to their present state of development?

1.2 Conservation Principles

We start this discussion with the innocuous question: How does an aerodynamic surface creates lift? Answer to this is related to the simple observation that if a bound vortex induces a circulatory motion in a uniform steady flow by an aerodynamic surface, the resultant superposition of the two yields the flow pattern seen around a lifting airfoil with the flow leaving the trailing edge with a small downward component of velocity. But will any bound vortex do, as in the case of a rotating cylinder in a uniform flow experiencing the Robins-Magnus effect, as explained in White (2008)? It will simply not work due to the fact that, apart from creating lift via rotation, such a body (also referred to as a bluff body) will experience large value of drag. Any aerodynamic surface, to perform efficiently, must provide not only lift but must also yield a very high value of lift to drag ratio. This ratio is also called the aerodynamic efficiency. Instead of getting into technical details of aerofoil aerodynamics, let us first understand what is expected from an aerodynamic surface from first principle of conservation laws in fluid dynamics. For low-speed applications, these would be nothing more than conservation of mass and momentum.

1.2.1 Conservation Laws and Reynolds Transport Theorem (RTT)

All conservation laws used in mechanics are nothing but interaction between system and surrounding, which are separated by real or imaginary boundaries. If the system mass is m, then for a control mass system the conservation of mass implies

$$\frac{dm}{dt} = 0. \tag{1.1}$$

If the system is moving with a translational velocity, $\vec{V}$, then the conservation of linear momentum is given by

$$\vec{F} = \frac{d}{dt}(m\vec{V}), \tag{1.2}$$

where $\vec{F}$ is the vector sum of all the applied forces acting on the system. In fluid mechanics or aerodynamics, above control mass analysis is often of limited value, as we do not track individual fluid particles. Instead we focus upon a fixed space in the flow domain and this is the rationale for control volume analysis. We will make use of the Reynolds transport theorem as given in White (2008) for an arbitrary moving and deformable control volume by noting the following. Let P be the property of the fluid (which could be mass, momentum or energy) and the corresponding intensive property be β, i.e. the property per unit mass. Considering incompressible flow with density, ρ of a control volume (CV) defined by $\mathcal{V}$, the property in the control volume will be denoted as $P_{CV} = \int_{CV} \beta\rho \, d\mathcal{V}$. The rate of change of this property of a control mass system $\left[\frac{dP_{sys}}{dt}\right]$ will be determined by, (i) a change within the control volume $\left[\frac{dP_{CV}}{dt}\right]$; (ii) outflow of β from the CV through the control surface (CS): $\int_{CS} \beta\rho\vec{V} \cos\theta \, dA_{out}$, where θ is the angle the elementary surface area dA_{out} makes with the flow velocity at the outflow and (iii) inflow of β to the CV: $\int_{CS} \beta\rho\vec{V} \cos\theta \, dA_{in}$. Written as a mathematical equation, this turns out to be

$$\frac{dP_{sys}}{dt} = \frac{dP_{CV}}{dt} + \int_{CS} \beta\rho\vec{V} \cos\theta \, dA_{out} - \int_{CS} \beta\rho\vec{V} \cos\theta \, dA_{in}. \tag{1.3}$$

The last two terms taken together constitute the net flux in the CV, which could also be written out as $\int \beta \, d\dot{m}_{out} - \int \beta \, d\dot{m}_{in}$, where $d\dot{m} = \rho(\vec{V} \cdot \hat{n}) \, dA$, with the unit vector $\hat{n}$ changes sign from inflow to outflow ports. This is the Reynolds transport theorem written in compact form as

$$\frac{dP_{sys}}{dt} = \frac{dP_{CV}}{dt} + \int_{CS} \beta\rho(\vec{V} \cdot \hat{n}) \, dA. \tag{1.4}$$

If the CV of fixed shape move at a uniform velocity $\vec{V}_s(t)$, then any observer fixed to the CV will note the velocity of fluid crossing CS as $\vec{V}_r = \vec{V} - \vec{V}_s(t)$ which will be used in calculating the flux term, so that the Reynolds transport theorem in this case will be written as

$$\frac{dP_{sys}}{dt} = \frac{dP_{CV}}{dt} + \int_{CS} \beta\rho(\vec{V}_r \cdot \hat{n}) \, dA. \tag{1.5}$$

Finally, if the CV is arbitrary deformable and move with space-time varying speed, $\vec{V}_s(r,t)$, so that $\vec{V}_r = \vec{V} - \vec{V}_s(r,t)$, then Reynolds transport equation is stated as in Equation (1.5).

Example 1.1: To derive the conservation of mass, we note that $P = m$ and then $\beta = 1$. Hence, $\frac{dP_{sys}}{dt} = 0$ and Reynolds transport theorem provides

$$0 = \frac{d}{dt}\left(\int_{CV} \rho \, d\mathcal{V}\right) + \int_{CS} \rho \, (\vec{V}_r \cdot \hat{n}) \, dA. \tag{1.6}$$

For a fixed CV, this further simplifies to

$$0 = \frac{d}{dt}\left(\int_{CV} \rho \, d\mathcal{V}\right) + \int_{CS} \rho \, (\vec{V} \cdot \hat{n}) \, dA. \tag{1.7}$$

For a fixed CV, if the flow is steady, $\frac{\partial\rho}{\partial t} = 0$ and Equation (1.7) further simplifies to

$$\int_{CS} \rho(\vec{V} \cdot \hat{n}) \, dA = 0. \tag{1.8}$$

If one uses Gauss's divergence theorem, then the above further simplifies to $\int_{CV} \text{Div}(\rho\, \vec{V})\, d\mathcal{V} = 0$, i.e.

$$\nabla \cdot (\vec{V}) = 0. \tag{1.9}$$

1.2.2 *Application of RTT: Conservation of Linear Momentum*

In this case, $P = m\vec{V}$ and $\beta = \vec{V}$. Hence, Reynolds transport theorem for conservation of linear momentum is given by

$$\frac{d(m\vec{V})_{sys}}{dt} = \sum \vec{F} = \frac{d}{dt}\left(\int_{CV} \rho\vec{V} d\mathcal{V}\right) + \int_{CS} \rho\vec{V}(\vec{V}_r \cdot \hat{n})\, dA, \tag{1.10}$$

where $\vec{F}$ represent all body and surface forces. As $\vec{V}$ is referred to inertial frame, the x-component of Equation (1.10) can be written as

$$\sum F_x = \frac{d}{dt}\left(\int_{CV} \rho\, u\, d\mathcal{V}\right) + \int_{CS} \rho u\, (\vec{V}_r \cdot \hat{n})\, dA. \tag{1.11}$$

Now for a fixed CV, $\vec{V}_r \equiv \vec{V}$, the conservation of linear momentum equation can be written in vector notation as

$$\sum \vec{F} = \frac{d}{dt}\left(\int_{CV} \rho\vec{V} d\mathcal{V}\right) + \int_{CS} \rho\vec{V}(\vec{V} \cdot \hat{n})\, dA. \tag{1.12}$$

1.3 Origin of Aerodynamic Forces

To begin with, we lay down the principles by which an efficient aerodynamic surface generates lift, while producing significantly lower drag. Let us consider the aerodynamic device as shown in Figure 1.4 in the block diagram, which shows the input of the device indicated by a mass flow rate, $\dot{m}$ and a uniform flow, V with the direction aligned with the x-axis. The output of the device is also characterized by the same mass flow rate and speed. However, the velocity vector is indicated by a small turning angle, θ at the outflow. The aerodynamic device is characterized by the same cross-sectional area at the inflow and outflow as A.

The free body diagram, also shown in Figure 1.4, indicates that the momentum passing through the inflow to the interior of the control volume (identified by the *black-box*, termed as the aerodynamic device) is given by $\dot{m}V$, aligned along the x-axis. The momentum outflow through the control volume is also $\dot{m}V$, but the vector subtends the angle θ with the x-axis, as indicated in the free body diagram. The force acting on the aerodynamic device is indicated by the vector, $\vec{F}$, having components F_x and F_y, in x- and y-directions, respectively. Thus, the force acting on the device is given by

$$\vec{F} = \dot{m}_{out} V_{out} - \dot{m}_{in} V_{in}. \tag{1.13}$$

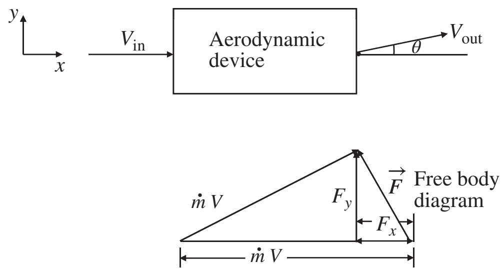

Figure 1.4 A systems approach to the analysis of aerodynamic devices. Free body diagram helps explain the force components active in flight, for $|V_{in}| = |V_{out}|$

For this case, $V_{out} = V_{in} = V$ and $\dot{m}_{out} = \dot{m}_{in} = \dot{m} = \rho AV$. Thus, the force components in the figure are given by

$$F_x = \dot{m}V(\cos\theta - 1) \tag{1.14}$$

and

$$F_y = \dot{m}V\sin\theta. \tag{1.15}$$

Thus, the net force magnitude is given by

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 2\dot{m}V\sin\theta/2. \tag{1.16}$$

And its orientation is given by

$$\phi = \pi - \tan^{-1}\frac{F_y}{F_x} = \pi/2 + \theta/2. \tag{1.17}$$

The interesting aspect of efficiency of this device is understood when the turning angle approaches a small value, i.e. $\theta \to 0$, then the magnitude and orientation of the resultant force is given by

$$|\vec{F}| = \dot{m}V\theta \quad \textit{and} \quad \phi = \pi/2, \tag{1.18}$$

This will be an ideal device, for which the resultant force is normal to the oncoming flow, i.e. only lift is produced, while the drag is vanishingly small. The efficiency of aerodynamic device is often quoted in terms of L/D which approaches a very large value in the ideal situation. It is also clearly evident that if the outflow is deflected more, then F_x also increases and it helps us identify that large flow turning indicates that the drag is lift-dependent, even when the inflow and outflow velocities are the same. In an actual flow, there will be viscous losses that will make $V_{out} < V_{in}$. This aspect of viscous losses is dealt with next, again using Reynolds transport theorem.

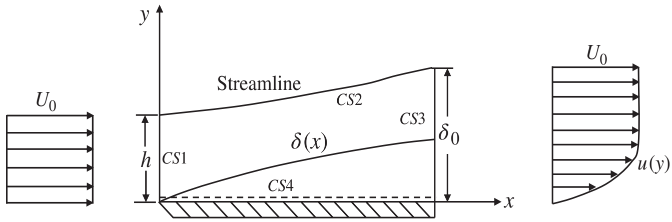

Figure 1.5 Control volume analysis of boundary layer forming over a flat plate

1.3.1 Momentum Integral Theory: Real Fluid Flow

This is the integral form of boundary layer theory developed by Von Kármán (1921). Consider the uniform flow over a flat plate with a sharp leading edge as shown in Figure 1.5, with the axis system fixed to the leading edge of the plate. This flow is over a flat plate in the absence of any pressure gradient in the free stream. Later on in Chapter 7, we will prove by boundary layer equations in differential form that the shear layer transmits the pressure through it unaltered, specifically if the flow is laminar. Hence the pressure is uniform over the depicted control volume in the figure and there is no force created by the pressure.

There are certain other features of the depicted control volume and the enclosing control surface segments. It is easy to identify the segments CS1 and CS3 through which the flow enters and leaves the control volume. While the incoming flow through CS1 is uniform, the boundary layer velocity profile is noted at CS3. In the figure, the boundary layer thickness is denoted by $\delta(x)$ and indicates the locus of the points at which the local velocity reaches a certain fraction (say, 99.9 %) of the free stream speed, indicated here by U_0. The segment CS4 of the control surface just grazes over the flat plate and a surface force caused by shear at the solid surface is transmitted through this segment on to the control volume. This contribution is denoted $-D\hat{i}$ where $\hat{i}$ is the unit vector in x-direction. This is indicative of total drag experienced by such a flat plate at zero incidence angle. Along CS4, we also note that $\vec{V} \cdot \hat{n} = 0$, as this is a no-slip wall with $\vec{V} = 0$.

The most interesting aspect of the choice of control surface is the segment CS2, which is identified to be above the boundary layer thickness at each and every x location and is considered to be along a streamline. From the definition of streamline, there cannot be any flow across it and thus, $\vec{V} \cdot \hat{n} = 0$ along this segment. This would be true, even if the flow is turbulent and then the segment CS2 would represent instantaneous streamline. We note that this is not the only choice of control surface segments, but a very convenient one. For example, if we consider a straight line as segment CS2 parallel to the flat plate, then $\vec{V} \cdot \hat{n} \neq 0$ and it would lead to additional calculations. Note specifically that there would be a net mass flow through such straight segment.

Let us consider that the flow is incompressible and steady. Hence, conservation of linear momentum equation (Equation (1.12)) becomes

$$\sum \vec{F} = \frac{d}{dt}\left(\int_{CV} \rho \vec{V} \, d\mathcal{V}\right) + \int_{CS} \rho \vec{V}(\vec{V} \cdot \hat{n}) \, dA. \tag{1.19}$$

with the first term on the right-hand side equal to zero due to steady state assumption. Considering the x-component of this equation we get

$$\sum F_x = -D = \int_{CS1} \rho u\,(\vec{V}\cdot\hat{n})\,dA + \int_{CS3} \rho u\,(\vec{V}\cdot\hat{n})\,dA$$
$$= \rho \int_0^h U_0(-U_0)\,b\,dy + \rho \int_0^{\delta_0} u^2 b\,dy, \tag{1.20}$$

where b is the breadth of the plate. Thus, the drag is estimated as

$$D = \rho U_0^2 bh - \rho b \int_0^{\delta_0} u^2 dy. \tag{1.21}$$

We also note that h and δ_0 are related to each other, as these points belong to the same streamline and the mass conservation requires

$$0 = \rho \int_0^h (-U_0)\,b\,dy + \rho \int_0^{\delta_0} u\,b\,dy,$$

which upon simplification yields

$$U_0 h = \int_0^{\delta_0} u\,dy. \tag{1.22}$$

Using Equation (1.22) in Equation (1.21) to eliminate h, one gets

$$D = \rho b \int_0^{\delta_0} u\,(U_0 - u)\,dy. \tag{1.23}$$

Note that the integrand on the right-hand side represents a measure of momentum deficit. If we represent the velocity distribution by a parabolic profile valid in the limit $0 \le y \le \delta$ (where δ is the boundary layer thickness, which is less than δ_0), then this is given by

$$\frac{u}{U_0} = \frac{2y}{\delta} - \frac{y^2}{\delta}. \tag{1.24}$$

The drag is obtained for this approximate velocity profile as

$$D = \frac{2}{15}\rho U_0^2 b\delta. \tag{1.25}$$

Note that for $\delta \le y \le \delta_0$: $u \equiv U_0$ and momentum deficit is zero. Despite such drastically simplified velocity profile being used, calculated drag for laminar flow value obtained incurs only 1% error.

1.4 Flow in Accelerating Control Volumes: Application of RTT

We are going to perform the analysis for the control volume which is fixed with the non-inertial frame (xyz axes system) in Figure 1.6, which translates at the rates given by the velocity and acceleration of its origin as $\frac{d\vec{R}}{dt}$ and $\frac{d^2\vec{R}}{dt^2}$, respectively. Additionally, the axes system performs

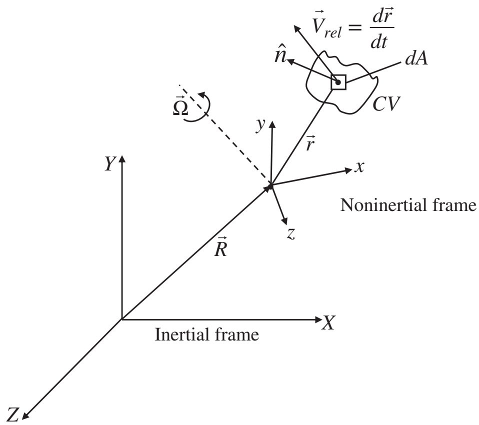

Figure 1.6 Control volume analysis of accelerating CV cases

a rotational motion given by $\vec{\Omega}$, with respect to the inertial frame of reference (XYZ axes system). Any arbitrary point inside the control volume is given by $\vec{r}$ in xyz-system.

Let the fluid velocity inside the control volume be given by $\vec{V}$ in the noninertial frame for an element of mass dm. The acceleration measured in the inertial ($\vec{a}_i$) and noninertial frames ($\frac{d\vec{V}}{dt}$) for this element mass are related by

$$\vec{a}_i = \frac{d\vec{V}}{dt} + \vec{a}_{rel} \tag{1.26}$$

From Newton's second law, sum of all the applied forces are given by $\sum \vec{F} = \int \vec{a}_i \, dm$ and this could be rearranged as

$$\sum \vec{F} - \int dm \, \vec{a}_{rel} = \int dm \frac{d\vec{V}}{dt}. \tag{1.27}$$

The second term on the left hand side incorporates all noninertial effects. The velocity in inertial frame ($\vec{V}_i$) is related to the velocity in the noninertial frame ($\vec{V}$) by

$$\vec{V}_i = \vec{V} + \frac{d\vec{R}}{dt} + \vec{\Omega} \times \vec{r}. \tag{1.28}$$

Similarly, the acceleration in these two frames of references are related by

$$\vec{a}_i = \frac{d\vec{V}}{dt} + \frac{d^2\vec{R}}{dt^2} + \frac{d\vec{\Omega}}{dt} \times \vec{r} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}). \tag{1.29}$$

Note that $\frac{d^2\vec{R}}{dt^2}$ is the acceleration of the origin of the noninertial frame; $\frac{d\vec{\Omega}}{dt} \times \vec{r}$ represents the angular acceleration effect or Euler acceleration term; $2\vec{\Omega} \times \vec{V}$ represents the Coriolis acceleration term and $\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ represents the centripetal acceleration. Using the Reynolds transport theorem on the right hand side of Equation (1.27) one gets

$$\sum \vec{F} - \int dm\, \vec{a}_{rel} = \frac{d}{dt} \int_{CV} \rho \vec{V} d\mathcal{V} + \int_{CS} \rho \vec{V} (\vec{V}_r \cdot \hat{n})\, dA, \tag{1.30}$$

where $\vec{a}_{rel} = \frac{d^2\vec{R}}{dt^2} + \frac{d\vec{\Omega}}{dt} \times \vec{r} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$.

1.5 Atmosphere and Its Role in Aerodynamics

For aircraft aerodynamics, we consider its flight in atmosphere. Also the atmosphere may be regarded as an expanse of fluid substantially at rest. We can use hydrostatics and tools of reversible thermodynamics in relating pressure, temperature and other thermodynamic properties with altitude along with the assumption that the atmosphere is homogeneous.

Before we start discussing atmosphere, it is necessary to note the distinction between the height up to which the atmosphere exists to create aerodynamic forces and beyond which it does not exist. This is given by the Kármán line.

1.5.1 Von Kármán Line

The Kármán line is a fictitious dividing line used to demarcate the earths atmosphere and outer space into range of applicability of aeronautics and astronautics. This line is roughly about 100 kilometres above sea-level and named after Von Kármán who coined its first usage. The definition is used by Fédération Aéronautique Internationale (FAI), which maintains international standards and records for aeronautics and astronautics. As such there is no fixed height where Earth's atmosphere ends. On the contrary, the atmosphere gets progressively thinner with height and Von Kármán calculated that at approximately 100 kilometres from sea-level, an aircraft could maintain lift only if it flies at orbital velocity, at which a spacecraft can orbit around the Earth without losing altitude. Thus below 100 kilometres, lift can be maintained at suborbital velocities and this altitude constitute the realm of aeronautics and any other aerial activities above this altitude pertains to astronautics.

1.5.2 Structure of Atmosphere

Atmosphere consists of the following strata:

(a) The lowest region in the atmosphere is called the troposphere and extends over a range of 6 to 20 km depending upon the latitude of the place under consideration. However for aeronautical design and analysis purposes, an international standard atmosphere (ISA) is constructed in which troposphere is considered to extend up to a height of 11 km. ISA corresponds to temperate climate. Statistically speaking, the temperature is considered to drop linearly with height. This is the region where most of the aircrafts are noted to fly.

Table 1.1 Composition of atmosphere at sea level

Molecules	% by volume	Molecular weight	% by weight
N_2	78.088	28.016	75.525
O_2	20.950	32.00	23.143
Argon (A)	0.932	39.944	1.286
CO_2	0.030	44.011	0.046
+ small quantities of Neon, Helium, Krypton, Hydrogen, Xenon, Ozone and Radon.			

(b) The next region is the stratosphere where temperature remains statistically constant and is found up to a height of 50 km. The boundary between troposphere and stratosphere is called the tropopause. Stratosphere is studied with the help of the weather balloon.
(c) Above the stratosphere is the mesosphere which extends up to almost 85 km. Meteors are noted to burn in this region upon entering the atmosphere.
(d) The thermosphere extends beyond the mesosphere up to an altitude of 600 km. This is the region where the Kármán line is and the space shuttle orbit was in thermosphere. Auroras are noted in the lower reaches of thermosphere.
(e) Beyond the thermosphere, one notes the exosphere and is considered to extend up to 10 000 km.

Thus aerodynamics is related mostly to flights within the troposphere and stratosphere. In terms of composition, one notes the following variation with altitude:

1. 50% of total weight is accounted for by the first 18 000 ft.
2. 75% of total weight is accounted for by the first 36 000 ft.
3. Up to a height of 50 miles the composition of the air is more or less constant, except for variation of water vapour content.

Table 1.1 contains the composition of air at sea level, by volume and weight.

The following facts are noted furthermore for the consideration of operating aircrafts in the lower reaches of the atmosphere.

1. Up to 5–6 miles, the water vapour content is a function of ambient temperature. The higher the temperature, the more vapour can be held in a given volume.
2. At very high altitudes, the heavier gases fail to rise until around 50 miles, H_2 and He are predominant above this height.
3. Beyond 18 000 ft the human physiological limit is reached and extra O_2 must be supplied.
4. Beyond 100 000 ft sufficient O_2 is not available to support combustion of turbojets.

Among the above points, we note the role of human physiological limits are the limiting factors. The following line or limit is noted, which severely restricts operation of aircraft from physiological considerations.

1.5.3 Armstrong Line or Limit

This is the limiting altitude (also called the Armstrong line) where the ambient pressure (0.0618 times the sea level atmospheric pressure or 6.3 kPa) is so low that water boils at normal temperature of human body, i.e. at 37 degree Celsius or 98.6 degrees Fahrenheit (named after Harry G. Armstrong, founder of USAF's department of space medicine in 1947). This physiological limit implies a fictitious altitude which would require wearing a pressurizing suit. This is variously reported to be between 18.9 and 19.35 km (or roughly about 12 miles). At this height, bodily fluids in the exposed part will spontaneously boil due to body temperature. However, blood will not boil spontaneously!

One must remember that there would be other adverse physiological effects at significantly lower altitude which affects normal human functioning. Also, the variation of atmospheric composition also does not allow functioning of various components of aerospace vehicle. For example, at heights beyond 100 000 ft, sufficient oxygen is not available to support combustion of turbojets. This limit alerts us about the human physiological limits on design concept of aircrafts. It is just not simple optimization of engineering cost function(s)!

The human blood in a living body will not boil at Armstrong limit due to blood being always under pressure and the boiling point would be significantly higher than the normal body temperature. Blood pressure is a gauge pressure. For example, a person with a low diastolic blood pressure of 60 mm of Hg, will have absolute blood pressure which will be in addition to the ambient pressure. At the altitude of 19 km, if the ambient pressure is 47 mm of Hg, then the absolute pressure is more than double the ambient pressure and blood will not boil, while saliva, tears and liquids wetting the alveoli inside the lung will boil away.

Another major physiological issue of human sustenance and survival is related to availability of oxygen, lack of which causes what is known as hypoxia (confusion and loss of consciousness). Even a professionally well-trained pilot cannot operate on an unpressurized aircraft cabin at altitudes beyond 15 km and requires an oxygen mask. At 15 km, breathing pure oxygen through a mask has the same partial pressure with regular air at 4.7 km altitude. Commercial jetliners keep cabin pressure to an equivalent altitude of 8000 ft (2.438 km). For sustained operation for more than half an hour at altitudes above 12 500 ft or 3.81 km, a pilot must have access to supplemental oxygen supply. Passengers must also be provided with supplemental oxygen above 15 000 ft. Above this altitude, lungs expand like a balloon and tear off (condition known as pulmonary barotraumas), and this also requires pressurized suits. It is for this reason that the cabin is pressurized in commercial passenger planes.

1.5.4 International Standard Atmosphere (ISA) and Other Atmospheric Details

In Figure 1.7, the pressure, density and temperature profiles are plotted as function of altitude for ISA. Also shown are the cloud details which are essential for safe operation of aircraft. According to a general characteristic of atmosphere as released by ICAO, Nacreous and Cirrus clouds remain above the tropopause and are of less concern. However, the cumulonimbus clouds are extremely hazardous for aircraft operation. Noctilucent clouds occur near the mesopause and are caused by volcanic meteoric dust and ice crystals, the latter forming by reaction between atmospheric oxygen with outer space hydrogen.

The tropopause is not clear cut, varying from about 30 000 ft at the poles to about 54 000 ft at the equator. The high temperature noted in thermosphere is due to ionization of the atmospheric gases by cosmic radiation. Also, note the ozonosphere in Figure 1.7, which protects the atmosphere at lower heights. Maximum radiation occurs at 75 000 ft. Ozone is a toxic product of ionization and since the ozone cloud is found around 60 000 ft, the ambient air cannot be

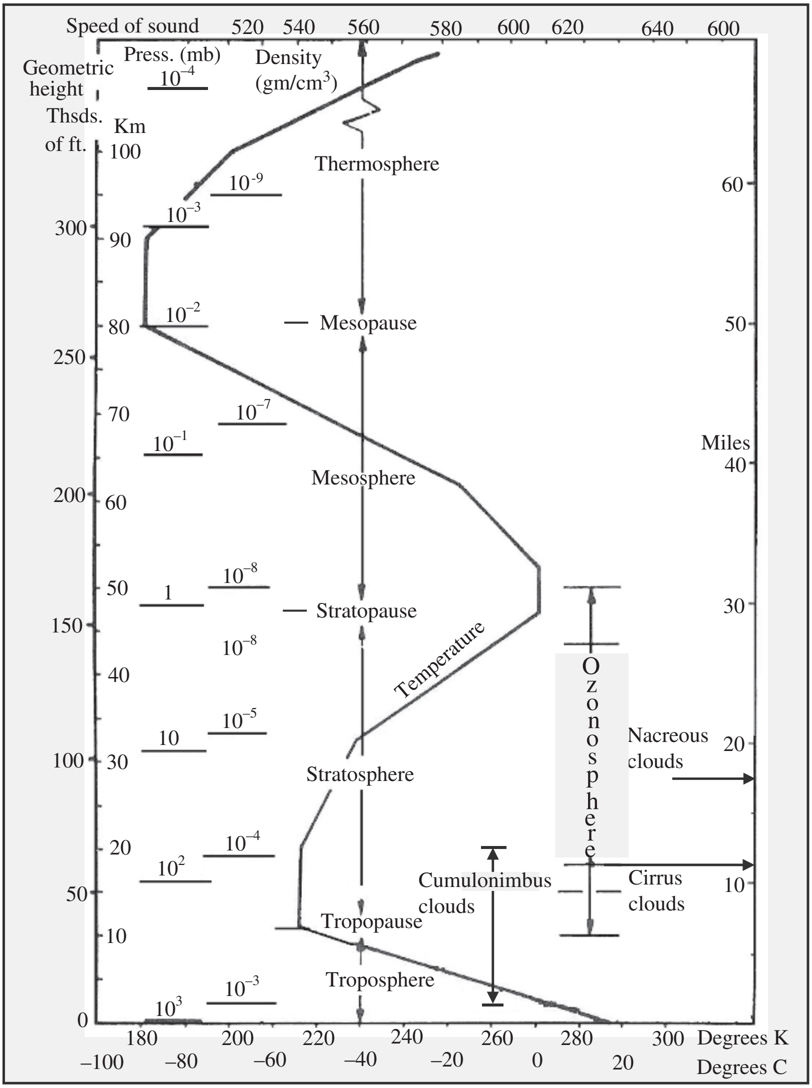

Figure 1.7 ISA atmosphere characterization and other altitude specific details useful for aeronautical operations

used for cabin pressurization. As the intensity of radiation is function of geometric height and latitude, this consideration is important for supersonic transport aircraft, which are perforce designed to fly at higher altitudes due to problems created by sonic boom.

Even when we assume the atmosphere to be static, the temperature variations within the atmosphere cause air mass to convect, resulting in wind, downdraft, wind shear and turbulence as a consequence of instabilities. Turbulence is the major hazard affecting the safe operation of aircraft. Thus, in this chapter we familiarize readers with static stability of atmosphere as

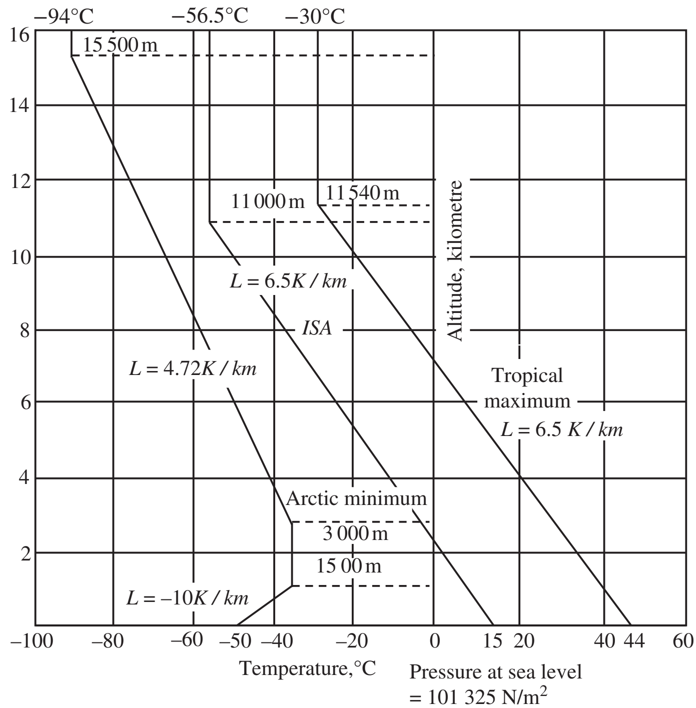

Figure 1.8 ISA atmosphere compared with that seen at arctic minimum and tropical maximum latitudes

a motivation to learn more about dynamic instability of atmosphere, which can be found in Sengupta (2012).

In Figure 1.8, the atmosphere as depicted by ISA condition is compared with those at the tropics and at the arctic.

In designing a new aircraft, it is necessary to note the extremes over which the vehicle is exposed in its lifetime. For example, operating in tropical latitudes the maximum sea level temperature endured is 44°C and in contrast, for minimum arctic climate the other extreme sea level temperature is about −50°C. For ISA, the sea level temperature is 15°C and the temperature falls at the rate of 6.5 K per km. The same lapse rate is noted for the tropical maximum temperature distribution. As the dynamic range of temperature is more at the tropics (from 44°C at sea level to −30°C at tropopause), than at temperate latitudes (from 15°C at sea level to −56.5°C at tropopause) – all low temperature flight trials are carried out in the tropics.

1.5.5 Property Variations in Troposphere and Stratosphere

In the following, the property variation for the ISA is obtained in troposphere and stratosphere by considering static atmosphere, so that conditions of hydrostatics apply. Consider a small cylindrical control volume of height dh and area of cross section A in the still atmosphere.

The temperature variation of the ambient air also causes the static pressure applied on A on two ends of a cylindrical control volume to differ by dp, so that from hydrostatic equilibrium one gets $dp = -\rho g\ dh$. If we treat the surrounding air as perfect gas, then $\frac{p}{\rho} = RT$ where R is the ideal gas constant equal to 287.26 J/(kg K). As noted in Figures 1.7 and 1.8, for the stratosphere the temperature remain constant as $T = T_s$. Using hydrostatic equilibrium and equation of state in the stratosphere

$$\frac{dp}{dh} = -\rho g = -\frac{gp}{RT} = -\frac{pg}{RT_s}.$$

Therefore by separation of variables one gets

$$\frac{dp}{p} = -\frac{g}{RT_s}dh.$$

Which upon integration between two heights h_1 and h_2 yields the relation between pressure at these two heights as

$$\mathrm{Ln}\frac{p_2}{p_1} = -\frac{g}{RT_s}(h_2 - h_1).$$

Thus, the pressure and density ratios are obtained as

$$\frac{p_2}{p_1} = Exp\left[\frac{g}{RT_s}(h_1 - h_2)\right] = \frac{\rho_2}{\rho_1} \tag{1.31}$$

In the troposphere, the temperature variation with altitude is given as

$$T = T_0 - Lh,$$

where L is the Lapse rate in $K/metre$. As before,

$$\frac{dp}{p} = -\frac{g}{R}\frac{dh}{T} = -\frac{g}{R}\frac{dh}{(T_0 - Lh)},$$

which allows us to integrate between two states indicated by subscripts '1' and '2' in the following

$$\mathrm{Ln}\frac{p_2}{p_1} = \frac{g}{LR}\left[\mathrm{Ln}(T_0 - Lh)\right]_{h_1}^{h_2}.$$

This can be alternately stated also as

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{g}{LR}}, \tag{1.32}$$

where $T_i = T_0 - Lh_i$

Since $p/(\rho T)$ is constant, therefore using Equation (1.32) one gets

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2} = \left(\frac{T_2}{T_1}\right)^{\frac{g}{LR}-1}.$$

Also, one can relate pressure and density as

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{g}{g-LR}}.$$

So in general within troposphere, $p = \text{const}\ \rho^{\frac{g}{(g-LR)}}$.

In SI unit: L = 0.0065 K/m and R = 287.26 J/(kg K) and one has the following relations

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{5.256}; \quad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{4.256} \quad \text{and} \quad p = k\rho^{1.235}. \tag{1.33}$$

where k is constant.

Thus, movement in static air from one height to another is a polytropic process with polytropic constant $n = 1.235$.

A quantity often useful in aerodynamics is the relative density and is defined as

$$\sigma = \frac{(\rho)_{\text{alt}}}{(\rho)_{\text{SL and ISA}}},$$

where

$$(\rho)_{\text{SL and ISA}} = \frac{p_0}{RT_0} = \frac{101.325}{287.26 \times 288} \simeq 1.2256\ kg/m^3.$$

Thermodynamic properties as given in Equations (1.31)–(1.33) define the atmosphere, which is considered to be static. It is important to study the stability of this atmosphere for aerodynamic applications. For example, if the atmosphere is unstable, then there would be thermals which can be used for gliding, as the soaring birds do. If static instability is present in the local atmosphere, then that will ensure a necessary condition for updraft. It does not ensure the sufficient condition for instability, yet it provides indication for a need to study dynamic instability. Also, instability can cause adverse effects, as in the creation of turbulence and downdraft. In performing the study of static stability, we consider the atmosphere as an expanse of fluid that is at rest.

1.6 Static Stability of Atmosphere

In studying static stability of the atmosphere in the troposphere, we note that the static temperature decreases with altitude and is given by

$$T = T_0 - Lz$$

where T_0 is the sea-level temperature in K and L is the lapse rate in $K/metre$. To study static stability, consider a small element of atmosphere to be suddenly displaced a short distance upward, as shown in Figure 1.9. One is then interested in finding the tendency of the displaced element. Three possibilities exist:

(i) If the displaced mass takes up the new position, then the atmosphere is neutrally stable.
(ii) If the displaced parcel tends to return to its original undisturbed position, then the atmosphere is considered to be statically stable.

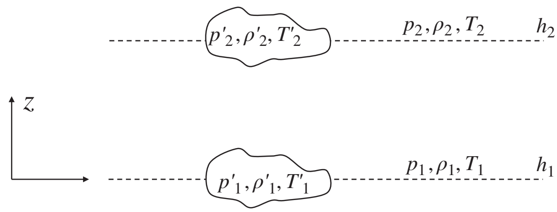

Figure 1.9 An element displaced from h_1 and h_2 with its conditions indicated by primed quantities and ambient condition indicated by unprimed quantities

(iii) If the displaced element tends to continue upward further, then the original equilibrium atmosphere is considered statically unstable.

Let a parcel of air be disturbed from the altitude h_1 to h_2, where the prevailing conditions at that altitude are denoted by unprimed quantities, while the primed quantities refer to the condition of the air-parcel, as shown in Figure 1.9.

For upward migration in the ambient air, we have shown that the pressure and density are related by

$$\left.\begin{aligned} p &= k\rho^{1.235} \text{ in troposphere of ISA} \\ p &= k\rho \text{ for stratosphere in ISA} \end{aligned}\right\}.$$

Or in general

$$p = k\rho^n. \tag{1.34}$$

Hence for the ambient air

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \left(\frac{p_2}{p_1}\right)\left(\frac{p_1}{p_2}\right)^{1/n},$$

which can be simplified as

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}. \tag{1.35}$$

If the upward migration of the air packet is very quick, then the process of moving the parcel can be considered adiabatic to be given by

$$p' = c\ \rho'^{\gamma}.$$

The temperature ratio of the air-parcel at h_1 and h_2 is related to the static pressure at these two heights by

$$\frac{T_2'}{T_1'} = \left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}} \tag{1.36}$$

Note that initially before the air-parcel is moved upward, it is in thermodynamic equilibrium with its surrounding. For mechanical, thermal and chemical equilibrium, one must have, $p_1' = p_1$, $T_1' = T_1$ and $\rho_1' = \rho_1$. Since we are studying the statics of the problem, then even after moving the parcel, it must be in mechanical equilibrium with its surrounding, i.e. the force balance demands $p_2' = p_2$, otherwise there will be expansion work done, along with acceleration of the parcel.

Thus from Equations (1.35) and (1.36) one gets

$$\frac{T_2'}{T_2} = \frac{T_2'}{T_1'}\left(\frac{T_1'}{T_1}\right)\frac{T_1}{T_2}.$$

As $T_1' = T_1$, this simplifies to

$$\frac{T_2'}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-n}{\gamma n}}. \tag{1.37}$$

Also from the equation of state and dynamic equilibrium at the displaced condition

$$\frac{\rho_2'}{\rho_2} = \frac{p_2'}{T_2'}\frac{T_2}{p_2} = \frac{T_2}{T_2'} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-n}{\gamma n}}. \tag{1.38}$$

Consider the following cases:

(1) For $n < \gamma$, one has $\frac{\gamma-n}{\gamma n} > 0$ and which leads to the following observation:

$$\frac{\rho_2'}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\alpha^2} > 1 \tag{1.39}$$

where α^2 is a positive quantity.

Thus, the displaced parcel is heavier than the ambient fluid which has been displaced and the parcel will sink back, indicating static stability.

(2) For $n = \gamma$, one obtains $\frac{\rho_2'}{\rho_2} = 1$ and this indicates neutral stability.

(3) For $n > \gamma$, then one obtains $\frac{\rho_2'}{\rho_2} < 1$ and as a consequence the parcel will continue in its motion upward, indicating static instability.

As $n = \frac{g}{g-LR}$, the condition for neutral stability $n = \gamma$ implies a lapse rate given by

$$(L)_{ns} = \left(-\frac{g}{\gamma} + g\right)\frac{1}{R} = \frac{g}{R}\left(\frac{\gamma-1}{\gamma}\right) \simeq \frac{9.807 \times 0.4}{1.4 \times 287.26} = 9.75K/km.$$

$(L)_{ns}$ is called the adiabatic lapse rate.

It is possible to study the dynamic stability of an air-parcel displaced from its equilibrium position. However, that is outside the scope of the present book. Interested readers can consult Sengupta (2012).

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