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PLL BASICS AND STANDARD STRUCTURE

This chapter provides an overview of the standard phase-locked loop (PLL) structure and illustrates that this structure is not suitable for power engineering applications. The large double-frequency ripples in the PLL cause errors beyond admissible limits of many power engineering applications. Attempts to reduce such errors by means of linear filters bring about long transient response times inadmissible for similar applications.

1.1 STANDARD PLL STRUCTURE

Structure of a standard PLL is shown in Figure 1.1 [5, 25, 31, 59]. The input signal is denoted by $u(t)$, LF stands for loop filter, and VCO stands for voltage-controlled oscillator. The input signal is multiplied with the VCO's output signal y, is passed through the LF, and the outcome is applied to the VCO.

The VCO generates a sinusoidal signal whose phase angle is proportional to the integral of the VCO's input. The VCO has a center frequency ω_n, and

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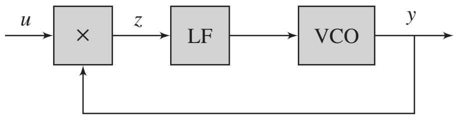

FIGURE 1.1 Structure of a standard PLL.

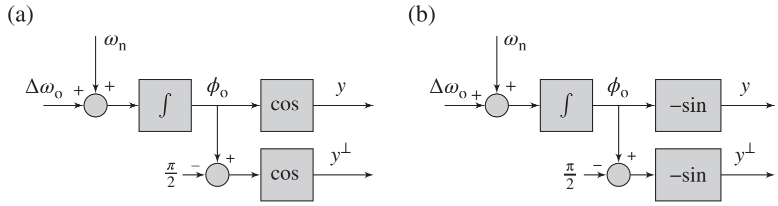

FIGURE 1.2 Two recommended structures for the VCO. (a) $y = cos\phi_o,\ y^{\perp} = cos(\phi_o - \frac{\pi}{2}) = sin\phi_o$. (b) $y = -\,sin\phi_o,\ y^{\perp} = -\,sin(\phi_o - \frac{\pi}{2}) = cos\phi_o$.

the relationship between its output signal phase angle and the input signal is given by

$$\phi_o = \int^{t} \omega_o(\tau)\mathrm{d}\tau \tag{1.1}$$

where $\omega_o = \omega_n + \Delta\omega_o$ is the estimated frequency and $\Delta\omega_o$ represents the deviation of frequency from the center frequency ω_n.[1] The VCO's operation can be modeled as an integration and a trigonometric function as shown in Figure 1.2. The center frequency for the VCO is equal to the nominal value of the input signal frequency.

Assume that the sinusoidal input signal is given by

$$u(t) = U_i \sin\phi_i \tag{1.2}$$

where U_i is its peak value and ϕ_i is its phase angle.[2] Then, using Figure 1.2a, the multiplier output signal z is equal to

$$z(t) = u(t) = U_i \sin\phi_i \cos\phi_o = \underbrace{\frac{U_i}{2}\sin(\phi_i - \phi_o)}_{\text{low frequency}} + \underbrace{\frac{U_i}{2}\sin(\phi_i + \phi_o)}_{\text{high frequency}}. \tag{1.3}$$

This signal comprises two terms: a low-frequency term and a high-frequency term. The low-frequency term is a measure of the difference between phase

[1] The subscript o denotes “output” variables, that is, those that are estimated and outputted by the PLL.
[2] The subscript i is used to denote variables of the “input” signal.

angles of input and output. Thus, the multiplier is also called the phase detector (PD) or the phase-difference detector.

If the input signal is considered as $u(t) = U_i \cos\phi_i$, then using Figure 1.2b, the multiplier output signal z is

$$z(t) = u(t) = -U_i \cos\phi_i \sin\phi_o = \underbrace{\frac{U_i}{2}\sin(\phi_i - \phi_o)}_{\text{low frequency}} - \underbrace{\frac{U_i}{2}\sin(\phi_i + \phi_o)}_{\text{high frequency}}. \quad (1.4)$$

In other words, the low-frequency term is the same in (1.3) and (1.4). The signal $y^{\perp}$ defined in Figure 1.2 is the 90° phase-delayed version of y. In steady state situation where PLL regulates $\phi_i - \phi_o$ to 0, $y^{\perp}$ is synchronous with the input signal u.

1.2 APPROXIMATE LINEAR MODEL

The following set of notations is introduced:

$\omega_i = \omega_n + \Delta\omega_i$: input frequency $\quad \phi_i = \omega_n t + \Delta\phi_i$: input phase angle

$\omega_o = \omega_n + \Delta\omega_o$: output frequency $\quad \phi_o = \omega_n t + \Delta\phi_o$: output phase angle

The relationship between frequency and phase angle is[3]

$$\phi_{io} = \int^{t} \omega_{io}(\tau)\mathrm{d}\tau = \omega_n t + \int^{t} \Delta\omega_{io}(\tau)\mathrm{d}\tau.$$

The low-frequency term in (1.3) is then equal to $\frac{U_i}{2}\sin(\Delta\phi_i - \Delta\phi_o)$ and the high-frequency term is $\frac{U_i}{2}\sin(2\omega_n t + \Delta\phi_i + \Delta\phi_o)$.

Assuming that the PLL is operating such that the output frequency is close to the input frequency, the high-frequency term is around the double frequency. It will be shown that the loop has low-pass characteristics. Thus, let us now neglect the high-frequency term in the loop assuming that the low-pass loop attenuates it sufficiently. Then, (1.3) is simplified to

$$z(t) \approx \frac{U_i}{2}\sin(\Delta\phi_i - \Delta\phi_o). \quad (1.5)$$

The simplified model of the PLL based on ignoring the double-frequency term is shown in Figure 1.3. The model can further be simplified by making

[3] The subscript *io* shows that the relationship is valid for both "input" and "output" variables.

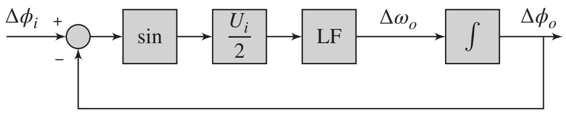

FIGURE 1.3 Simplified model of a PLL when high-frequency term is neglected.

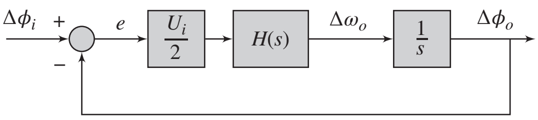

FIGURE 1.4 Linear model of the PLL.

an assumption that $\Delta\phi_i - \Delta\phi_o$ is close to 0. This means that the PD output is proportional to the difference between phase angles. In this case, the sine nonlinearity can be replaced by its linear approximation that is identity operation. The resultant transfer function diagram is shown in Figure 1.4. In this figure, $H(s)$ denotes the LF transfer function. This loop is a linear time invariant (LTI) loop if it is assumed that the input signal magnitude is constant.

Assume that the loop filter $H(s)$ is selected such that the loop is stable and the tracking error $e(t)$ tends to 0. This means that the phase angle of $y(t)$ remains in 90° phase shift from that of the input signal. The complementary output signal $y^{\perp}(t)$ as shown in Figure 1.2 will then be *in phase* (meaning that it will have the same phase angle) with the input signal. In other words, $y^{\perp}(t)$ will be a signal with unity amplitude that is in phase or synchronous with the input signal. This can serve as the synchronization signal.

The open-loop transfer function (also called the loop gain) of the system is

$$G(s) = \frac{U_i H(s)}{2s},$$

the closed-loop transfer function is given by

$$T(s) = \frac{\Delta\Phi_o(s)}{\Delta\Phi_i(s)} = \frac{G(s)}{1 + G(s)} = \frac{U_i H(s)}{U_i H(s) + 2s},$$

and the error transfer function is

$$F(s) = \frac{E(s)}{\Delta\Phi_i(s)} = \frac{1}{1 + G(s)} = \frac{2s}{2s + U_i H(s)}.$$

1.3 LOOP FILTER DESIGN

The control objectives in the PLL system are as follows:

- The output phase angle follows the changes in the input phase angle within a desirable transient time and transient behavior.
- The loop performance must be robust to noise and distortions.

These ensure that the PLL will be able to supply a synchronizing signal despite variations in the input signal and despite the noise and distortions. The noise can originate from the system and/or from the measurement. The distortions are in the form of measurement noise, bias (direct current (DC) offset), harmonics, interharmonics, transient signals, and/or switching notches. The control objectives are to be achieved by proper design of the LF transfer function $H(s)$.

Loop Filter of Order Zero

The simplest LF structure is a constant: $H(s) = h_o$. The closed-loop transfer function is $T(s) = \frac{U_i h_o}{U_i h_o + 2s}$ that is stable for all $h_o > 0$ and is a first-order low-pass filter (LPF) with unity gain and cutoff frequency $\omega_c = \frac{U_i h_o}{2}$. The error transfer function is $F(s) = \frac{2s}{2s + U_i h_o}$.

The error to a unit step function in the input phase angle is

$$E(s) = \frac{2s}{2s + U_i h_o}\frac{1}{s} = \frac{2}{2s + U_i h_o}.$$

Based on the final value theorem (FVT) [24], the error signal tends to

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{2s}{2s + U_i h_o} = 0.$$

This means that the loop tracks the constant jumps in the phase angle with no steady state error.

The error to a unit ramp function in the input phase angle is

$$E(s) = \frac{2s}{2s + U_i h_o}\frac{1}{s^2} = \frac{2}{s(2s + U_i h_o)}.$$

Based on the FVT, the error signal tends to

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{2}{2s + U_i h_o} = \frac{2}{U_i h_o}.$$

This means that the loop does not track the ramp changes of the phase angle or the constant changes in the frequency.

Loop Filter of Order One

A first-order proportional-integrating (PI) structure for the LF is given by $H(s) = h_o + \frac{h_1}{s}$. The closed-loop transfer function is equal to

$$T(s) = \frac{U_i h_o + U_i \frac{h_1}{s}}{U_i h_o + U_i \frac{h_1}{s} + 2s} = \frac{\frac{U_i h_o}{2} s + \frac{U_i h_1}{2}}{s^2 + \frac{U_i h_o}{2} s + \frac{U_i h_1}{2}}. \tag{1.6}$$

The error transfer function is

$$F(s) = \frac{2s}{2s + U_i h_o + U_i \frac{h_1}{s}} = \frac{s^2}{s^2 + \frac{U_i h_o}{2} s + \frac{U_i h_1}{2}}.$$

Similar to the analysis for loop filter of order zero, the FVT can be applied to prove that step jumps and ramp variations of the phase angle (which correspond to step jumps in frequency) are tracked by the loop filter of order one with no steady state error. In other words, a PLL with a first-order LF guarantees that the “slow” variations of input frequency are tracked. This is desirable for power engineering applications.

Loop Filters of Higher Order

An LF structure with two integrators tracks ramp variations in the frequency. In addition to integrators, one may also include general transfer functions with nonzero poles and zeros. LPF and band-stop filters are of specific interest because they can attenuate the double-frequency ripples and other harmonics. For higher order filters, the design stage can be done using the classical control techniques (such as Bode diagrams, root-locus method, and Nyquist method) as well as other techniques (such as optimal control design tools including linear quadratic design methods and robust control methods) to achieve the control objectives.

1.4 REMARKS

- The transfer function (1.6) indicates that the loop modes depend on the magnitude of the input signal U_i. This means that the LF gains h_o and h_1 depend on the input signal magnitude and they must be readjusted if the magnitude experiences drastic changes.

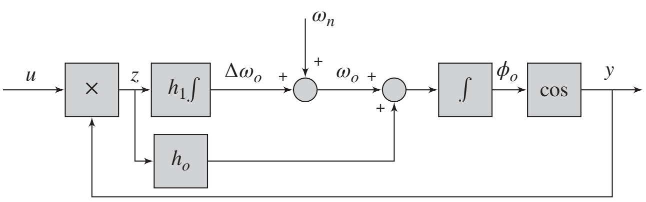

FIGURE 1.5 Tapping the frequency from the I output.

- The relationship (1.6) shows a second-order transfer function with a zero. Adjusting the poles does not guarantee that the expected transient response from a standard second-order system is achieved. The zero often causes larger overshoot and longer settling time.
- The input signal to the VCO is often considered as an estimate for the frequency (or frequency deviation from the center value). However, the integrator output (within the PI unit) provides a more accurate point for this variable. In other words, the P branch can be bypassed as is shown in Figure 1.5.
- The root-locus method [22] can be used to observe the closed-loop poles and to design the LF coefficients. The loop-characteristic equation is $1+G(s)=0$, where $G(s)=\frac{U_i H(s)}{2s}$ is the loop gain and $H(s)$ is the LF transfer function. For a LF of order one, $H(s)=h_0+\frac{h_1}{s}=h_0\frac{s+z}{s}$, the characteristic equation can be written as

$$1+k\frac{(s+z)}{s^2}=0,$$

where $h_0=k\frac{2}{U_i}$ and $h_1=h_0 z$. The root locus can be drawn by selecting z and varying k. For a given z, the root locus is shown in Figure 1.6. It is observed that at $k=2z$, the poles are both at $-z\pm jz$ and at $k=4z$, the poles are at $-2z$. If $z=50$, for example, then $k=4z=200$ places both poles at -100. And for this placement, $h_0=\frac{400}{U_i}$ and $h_1=\frac{20{,}000}{U_i}$.

1.5 NUMERICAL RESULTS

Figure 1.7 shows a simulation result when the LF transfer function is $H(s)=h_o+\frac{h_1}{s}=400+\frac{20{,}000}{s}$. The system's poles are both located at -100 for this selection of h_0 and h_1, assuming a unity magnitude input signal. The input signal frequency is initially 50 Hz and jumps to 60 Hz at t = 0.1 s. The PI's output has double-frequency ripples as large as about 60 Hz peak-to-peak.

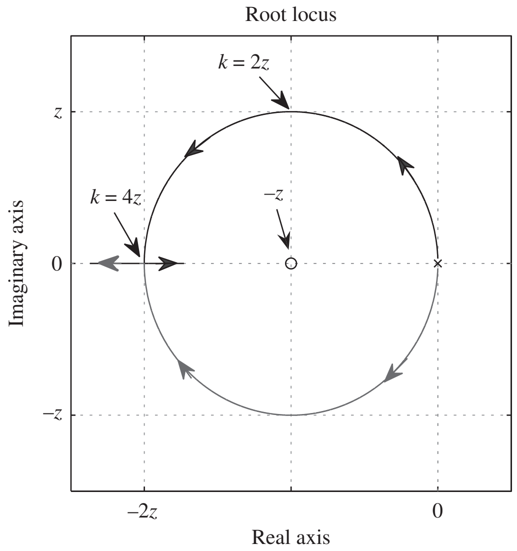

FIGURE 1.6 Root-locus of the PLL loop for $H(s) = h_0 + \frac{h_1}{s}$ where $k = \frac{U_i}{2}h_0$ and $z = \frac{h_1}{h_0}$.

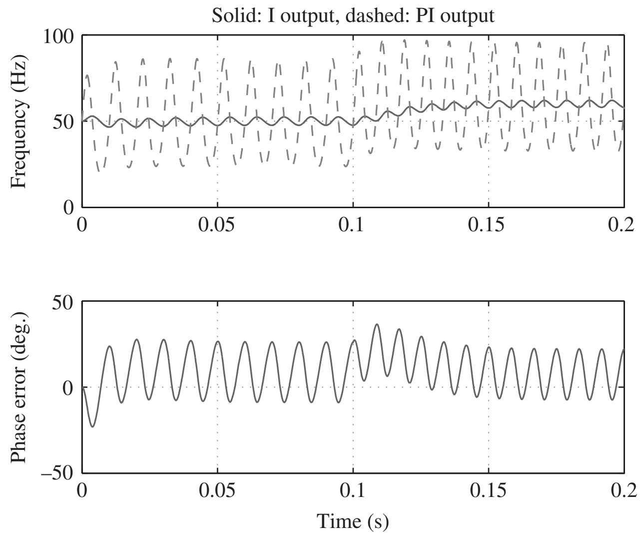

FIGURE 1.7 Performance of PLL with a LF of $H(s) = h_o + \frac{h_1}{s}$.

The ripples at the I's output are almost 15 times smaller (about 4 Hz peak-to-peak). The extent of ripples on the phase angle is about 25°.

It is possible to decrease the ripples by modifying the LF. For example, as shown in Figure 1.8, a LPF with a cutoff frequency of 50 Hz decreases the ripples to almost about 50% of the original values, see Figure 1.9.

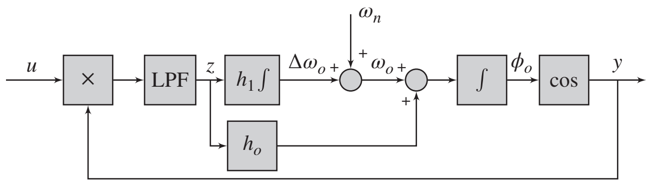

FIGURE 1.8 Structure of a standard PLL with modified LF.

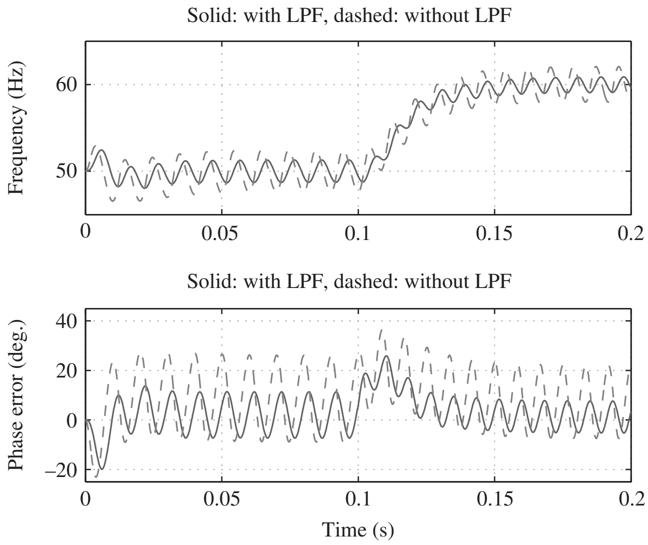

FIGURE 1.9 Performance of PLL with a LF of $H(s) = \frac{1}{\tau s+1}\left(h_o + \frac{h_1}{s}\right)$.

It is clearly observed from Figure 1.7 and Figure 1.9 that the phase angle error not only has a double-frequency ripple, it also has some offset. In other words, there is a bias error in the estimated phase angle. This phenomenon cannot be explained by the linear model of the PLL because the linear model (which includes a PI filter) states that there must be no bias in the estimated angle. This error has its origin in the nonlinearity of PLL.

1.6 SUMMARY AND CONCLUSION

The conventional PLL structure is discussed and its performance is studied for a power system situation. It was observed that for a pure sinusoidal signal with no distortion or noise, the error in phase locking is in the order of several degrees when a first order integrating plus a LPF is used for the loop filter.

This level of error is not admissible for most power engineering applications let alone that harmonics and noise would contribute to increase this error in a realistic scenario. Adding more low-pass filtering in the loop can decrease the ripples, but, by the time the ripples are small enough, the transient time of the loop is beyond the admissible range for power system applications. This is why this standard PLL structure has not been of much interest in power engineering.

PROBLEMS

1.1 Let $u = U_i \sin(\phi_i(t))$ be the input signal to the PLL of Figure 1.1. Show that ϕ_o will be equal to ϕ_i if the VCO realization of Figure 1.2a is used. However, $\phi_o = \phi_i - \frac{\pi}{2}$ if the VCO realization of Figure 1.2b is used.

1.2 Let $u = U_i \cos(\phi_i(t))$ be the input signal to the PLL of Figure 1.1. Show that ϕ_o will be equal to ϕ_i if the VCO realization of Figure 1.2b is used. However, $\phi_o = \phi_i + \frac{\pi}{2}$ if the VCO realization of Figure 1.2b is used.

1.3 Consider the standard PLL structure with a pure sinusoidal input signal $u(t) = U_i \sin(\omega_i t)$. The LF is given by the general transfer function $H(s)$.

a. Show that the steady state peak-to-peak value of the ripples in the estimated phase angle is equal to $\frac{U_i}{2\omega_i}|H(j2\omega_i)|$ rad.

b. Calculate the steady state peak-to-peak value of the ripples in the estimated frequency.

c. Express the results in (a) and (b) when $H(s) = h_0 + \frac{h_1}{s}$.

d. Repeat (b) when Figure 1.5 is used.

e. Verify the results in part (c) and (d) by simulations for the case where $U_i = 1$, $\omega_i = 120\pi$, $h_0 = 400$, and $h_1 = 20{,}000$.

1.4 Consider the standard PLL structure with a pure sinusoidal input signal $u(t) = U_i \sin(\omega_i t)$. The LF is a combination of a PI and a LPF given by $H(s) = (h_0 + \frac{h_1}{s})(\frac{\omega_c}{s+\omega_c})$ as shown in Figure 1.8. Choose $U_i = 1$, $\omega_i = 120\pi$, $h_0 = 400$, and $h_1 = 20{,}000$. Assume that a maximum peak-to-peak ripple of 1° is allowed in the phase angle.

a. Find the value of ω_c analytically.

b. Verify your answer by simulation.

c. Based on the simulation, how long is the transient time of the system with this value of ω_c?

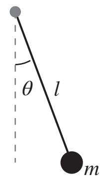

FIGURE 1.10 Pendulum system.

1.5 Consider the second-order LF of the form $H(s) = h_0 + \frac{h_1}{s} + \frac{h_2}{s^2}$.

a. Prove that a PLL with this LF can track the ramp variations of frequency with no steady state error.

b. Write down the closed-loop characteristic equation of the system. Set value of h_0, h_1, and h_2 so that the closed-loop poles are all at -50 for a unity magnitude input signal.

c. Simulate the system and observe its responses when the input signal frequency changes in step and ramp formats.

d. Write the LF in the form $H(s) = h_0 \frac{(s+z_1)(s+z_2)}{s^2}$, assume that z_1 and z_2 are known, and draw the root locus of the system to find a suitable value for h_0. Consider three different cases:

i. $z_1 = z_2 = -100$,

ii. $z_1 = -100,\ z_2 = -50$,

iii. $z_{1,2} = -100 \pm j100$.

1.6 Consider the pendulum system shown in Figure 1.10. A mass m is suspended from a pivot through a massless rod with length ℓ and can swing freely. The angle of the rod with the vertical axis is θ.

a. Show that

$$I\ddot{\theta} = -mg\ell \sin\theta - k\dot{\theta}$$

describes the pendulum movement where $I = m\ell^2$ is the moment of inertia, g is the gravitational constant, and k is the friction constant.

b. Define the state variables $x_1 = \theta$ and $x_2 = \dot{\theta}$ and derive a state-space representation for this system.

c. Show that $P_k = (k\pi, 0)$ for all integer k are equilibrium points of the system.

d. Physically, the possible equilibrium points are $P_0 = (0, 0)$ and $P_1 = (\pi, 0)$. Intuitively, it is clear that P_1 is globally stable and P_2 is unstable. Show this fact analytically.

1.7 Consider the PLL system with the LF of $H(s) = \frac{1}{\tau s+1}$. Write down the equations of the PLL, ignore the double-frequency terms, and show that it is mathematically equivalent to the pendulum system introduced in problem 1.6. You may use the block diagram of Figure 1.3.

1.8 Consider the PLL system with a general LF described by the transfer function $H(s)$. Let (A, B, C, D) be a minimal state-space representation for $H(s)$.

a. Write down a state-space representation for the PLL system with this LF.

b. Show that $P_k = (k\pi, 0)$ for all integer k are equilibrium points of the system.