CHAPTER 1

Introduction

The foundations of what is popularly referred to as "modern portfolio theory" is attributable to the seminal work of Harry Markowitz, published more than a half a century ago.[1] Markowitz provided a framework for the selection of securities for portfolio construction to obtain an optimal portfolio. To do so, Markowitz suggested that for all assets that are candidates for inclusion in a portfolio, one should measure an asset's return by its mean return and risk by an asset's variance of returns. In the selection of assets to include in a portfolio, the Markowitz framework takes into account the co-movement of asset returns by using the covariance between all pairs of assets. The portfolio's expected return and risk as measured by the portfolio variance are then determined by the weights of each asset included in the portfolio. For this reason, the Markowitz framework is commonly referred to as *mean-variance portfolio analysis*. Markowitz argued that the optimal portfolio should be selected based on the trade-off between a portfolio's return and risk. While these concepts are considered the basis of portfolio construction these days, the development of the mean-variance model shaped how investment managers analyze portfolios and sparked an overwhelming volume of research on the theory of portfolio selection.

Once the fundamentals of modern portfolio theory were established, studies addressing the limitations of mean-variance analysis appeared, seeking to improve the effectiveness of the original model under practical situations. Some research efforts concentrated on reducing the sensitivity of portfolios formed from mean-variance analysis. Portfolio sensitivity means that the resulting portfolio constructed using mean-variance analysis and its performance is heavily dependent on the inputs of the model. Hence, if the estimated input values were even slightly different from their true values, the estimated optimal portfolio will actually be far from the best choice. This is especially a drawback when managing equity portfolios because the equity market is one of the more volatile markets, making it difficult to estimate values such as expected returns.

In equity portfolio management, there has been increased interest in the construction of portfolios that offer the potential for more robust performance even during more volatile equity market periods. One common approach for doing so is to increase the robustness of the input values of mean-variance analysis by adopting estimators that are more robust to outliers. It is also possible to achieve higher robustness by focusing on the outputs of the mean-variance model by performing simulations for collecting many possible portfolios and then finally arriving at one optimal portfolio based on all the possible ones. There are other methods that are based on the equilibrium of the equity market for gaining robustness.[2]

Although various techniques have been applied to improve the stability of portfolios, one of the approaches that has received much attention is robust portfolio optimization. Robust optimization is a method that incorporates parameter uncertainty by defining a set of possible values, referred to as an uncertainty set. The optimal solution represents the best choice when considering all possibilities from the uncertainty set. Robust optimization was developed for addressing optimization problems where the true values of the model's parameters are not known with certainty, but the bounds are assumed to be known. In 1973, Allen Soyster discussed inexact linear programming; in the 1990s, the initial approach expanded to incorporate a number of ways for defining uncertainty sets and addressing more complex optimization problems. When robust optimization is extended to portfolio selection, the inputs used in mean-variance analysis—the vector of mean returns and the covariance matrix of returns—become the uncertain parameters for finding the optimal portfolio. Since the turn of the century, there have been numerous proposals for formulating robust portfolio optimization problems. Much of the focus has been on mathematical theories behind uncertainty set construction and reformulations resulting in optimization problems that can be solved efficiently; and, as a result, there are many formulations that can be used to build robust equity portfolios.

Even though there has been considerable development on robust portfolio management, most approaches require skills far beyond perfecting mean-variance analysis. For example, it is not an easy task for a portfolio manager without extensive background knowledge in optimization and mathematics to understand robust portfolio optimization formulations. More importantly, being able to interpret robust formulations is only the first step. The second step requires solving the optimization problem to arrive at the optimal decision. Programming expertise, in addition to optimization and mathematics, is necessary in the second step because most robust formulations require complex computations. Thus, while the need and the value of robust portfolio management are apparent, only those with appropriate training will be equipped to explore the advanced methods for improving portfolio robustness.

This book is aimed at providing a step-by-step guide for using robust models for optimal portfolio construction. It is not assumed that the reader has prior knowledge in portfolio management and optimization. In this book, the basics of portfolio theory and optimization, along with programming examples, will allow the reader to gain familiarity with portfolio optimization. Once the fundamentals of portfolio management are outlined, robust approaches for managing portfolios are explained with an emphasis on robust portfolio optimization. Details on robust formulations, implementation of robust portfolio optimization, attributes of robust portfolios, and robust portfolio performance will prepare the reader to utilize robust portfolio optimization for managing portfolios. In this book, we not only review theoretical developments but provide numerous programming examples to demonstrate their use in practice. The programming examples that appear throughout the book illustrate the details of implementing various techniques including methods for constructing robust equity portfolios.

1.1 OVERVIEW OF THE CHAPTERS

The book is divided into three parts. The first part, Chapters 2 through 4, introduces the mean-variance model, discusses its shortcomings, and explains common approaches for increasing the robustness of portfolios. The second part, Chapters 5 and 6, contains an overview of optimization and details the steps involved in formulating a robust portfolio optimization problem. The third part, Chapters 7 through 10, focuses on analyzing robust portfolios constructed from robust portfolio optimization by identifying attributes and summarizing performances.

Chapter 2 begins by describing how portfolio return and risk are measured, which leads to formulating the mean-variance portfolio problem. Mean-variance analysis finds the optimal portfolio from the trade-off between return and risk, and the framework also explains the benefits of diversification. Chapter 3 investigates shortcomings of the mean-variance model, which limit its use as a strategy for managing equity portfolios; improvements can be made with respect to measuring risk, estimating the input variables, and reducing the sensitivity of portfolio weights. In particular, the combination of estimation errors in the input values and high sensitivity of the resulting portfolio is a major issue with the mean-variance model. Therefore, in Chapter 4, practices for reducing the sensitivity of portfolios are demonstrated, including robust statistics, simulation methods, and stochastic programming.

Chapter 5 presents a comprehensive overview of optimization, including definitions of linear programming, quadratic programming, and conic optimization. The chapter also discusses how robust optimization transforms basic optimization problems so as to incorporate parameter uncertainty. The discussion is extended to applying robust optimization to portfolio selection in Chapter 6. While concentrating on the uncertainty caused by estimating expected returns of stocks, two robust formulations are shown with specific instructions provided as to their implementation.

Chapters 7, 8, and 9 analyze portfolio attributes that are revealed when portfolios are formed from robust portfolio optimization. In Chapter 7, we provide empirical evidence that indicates that some uncertainty sets lead to portfolios that favor skewness but penalize kurtosis. The high factor exposure of robust portfolios at the portfolio level is addressed in Chapter 8, and Chapter 9 examines portfolio weights allocated to individual stocks for comparing the composition of robust portfolios with mean-variance portfolios that assume no uncertainty. Chapter 10 illustrates the robustness of robust portfolios by observing their historical performance.

The final chapter, Chapter 11, discusses software packages that can help solve robust portfolio optimization and provides examples for finding robust portfolios.

1.2 USE OF MATLAB

Financial modeling often requires computer programs for solving complex computations. The use of powerful computing tools is inevitable in portfolio management because portfolio selection problems are mathematically expressed as optimization problems. Thus, tools that efficiently solve optimization problems give portfolio managers a great advantage; the tools are more valuable for robust portfolio management because approaches such as robust portfolio optimization involve more intense computations.

Therefore, in this book we discuss various aspects of robust portfolio management with examples on how to implement models in MATLAB, which is a programming language and interactive environment primarily for numerical computations.[3] MATLAB is widely used in academic studies as well as research in the financial industry, especially for computations that involve matrices such as portfolio optimization. The examples presented use MATLAB mainly because the language provides a straightforward approach for executing portfolio optimization. This high-level language with an extensive list of built-in functions allows beginners to easily perform various computations and visualize their results. Furthermore, the syntax for writing a script or a function is so intuitive that the reader can quickly become familiar with MATLAB even without prior experience. Hence, the MATLAB examples throughout the book will not only supplement understanding the theoretical concepts but will also let the reader apply the examples to construct optimal portfolios that reflect their investment goals.

While MATLAB features an add-on toolbox for financial computations, the examples in this book use built-in functions for solving optimization and not the functions in the financial toolbox that are customized for certain types of financial decision problems. For example, the *quadprog* function in MATLAB is used for implementing portfolio problems that are formulated as quadratic programming. This gives the reader flexibility since the examples will show how the function parameters can be modified based on different investment assumptions and portfolio constraints. Becoming familiar with the built-in optimization functions is also crucial because robust formulations are not included in the financial toolbox and therefore must be solved with the optimization functions. We also include examples that use CVX, which is a modeling system for convex optimization that runs in the MATLAB environment.[4] CVX enhances MATLAB, making it more expressive and powerful for solving optimizations like the mean-variance portfolio problems that are formulated as convex optimization problems. Many examples in this book present MATLAB codes that use the built-in functions of MATLAB as well as CVX in order to demonstrate two approaches for obtaining robust portfolios for a given problem. Since CVX is MATLAB-based, the reader will gain exposure to an additional tool without having to learn a new programming environment.

NOTES

1. Harry M. Markowitz, "Portfolio Selection," *Journal of Finance* 7, 1 (1952), pp. 77–91.
2. An example of improving the robustness of inputs is to use shrinkage estimators, introduced in Philippe Jorion, "Bayes-Stein Estimation for Portfolio Analysis," *Journal of Financial and Quantitative Analysis* 21, 3 (1986), pp. 279–292. Using simulation to gain robustness is illustrated in Richard Michaud and Robert Michaud, "Estimation Error and Portfolio Optimization: A Resampling Solution," *Journal of Investment Management* 6, 1 (2008), pp. 8–28. The Black–Litterman model is an equilibrium-based approach that incorporates an investor's views; it was proposed in Fischer Black and Robert Litterman, "Asset Allocation: Combining Investor Views with Market Equilibrium," *Goldman, Sachs & Co., Fixed Income Research* (1990). Various robust approaches including the ones mentioned here are detailed in Chapter 4.
3. MATLAB documentations and a list of functions with examples are available at http://www.mathworks.com/products/matlab/
4. A CVX user's guide and download details can be found at http://cvxr.com/cvx/