1

Introduction to Power System Analysis

1.1 Introduction

As electricity comes out of the alternating current (AC) outlet every day, and has already been doing so for more than 100 years, it may nowadays be regarded as a commodity. It is a versatile and clean source of energy; it is fairly cheap and "always available." In the Netherlands, for instance, an average household encountered only 20 minutes' interruption to their supply in the year 2014 [1] out of a total of 8760 hours, resulting in an availability of 99.996195%!

Society's dependence on this commodity has become critical and the social impact of a failing power system is beyond imagination:

- Cars would not be refueled as gas station pumps are driven by electricity.
- The sliding doors of shops and shopping malls would not be able to open or close and people would therefore be locked out or in.
- Electrified rail systems, such as subways and trains, would come to a standstill.
- Traffic lights would not work.
- Refrigerators would stop.
- Heating/cooling installations would fail.
- Cash dispensers would be offline.
- Computers would serve us no longer.
- Water supplies would stop or run out.

Many more examples may be given, but the message is clear: electric power systems are the backbone of modern society (see Figure 1.1), and chaos would result if the electricity supply failed for an extended period.

Our society needs engineers who know how to design, build, and operate an electrical power system. So let us discover what lies beyond the AC outlet and enter the challenging world of power system analysis.

Electrical Power System Essentials, Second Edition. Pieter Schavemaker and Lou van der Sluis.

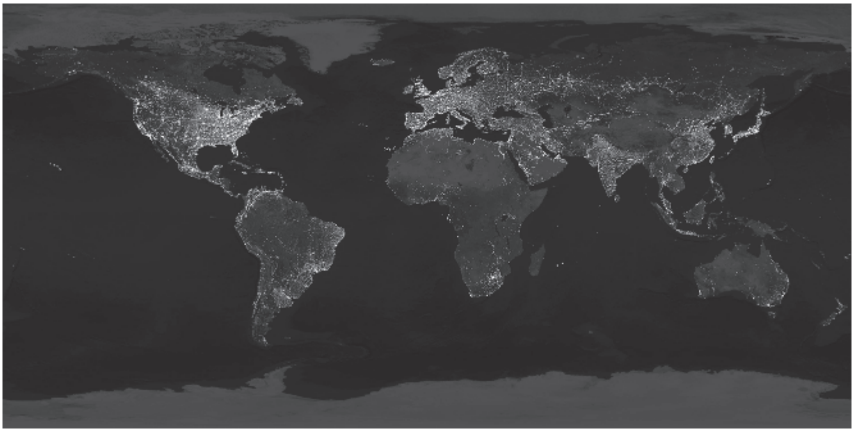


Figure 1.1 The Earth's city lights, indicating the most urbanized areas. The Visible Earth, NASA.

1.2 Scope of the Material

Power system analysis is a broad subject, too broad to cover in a single textbook. The authors confine themselves to an overview of the structure of the power system (from generation via transmission and distribution to customers) and only take into account its steady-state behavior. This means that only the power frequency (50 or 60 Hz) is considered. An interesting aspect of power systems is that the modeling of the system depends on the time scale under review. Accordingly, the models for the power system components that are used in this book have a limited validity; they are only valid in the steady-state situation and for the analysis of low-frequency phenomena. In general, the time scales we are interested in are as follows:

- Years, months, weeks, days, hours, minutes, and seconds for steady-state analysis at power frequency (50 or 60 Hz)
  This is the time scale on which this book focuses. Steady-state analysis covers a variety of topics such as planning, design, economic optimization, load flow/power flow computations, fault calculations, state estimation, protection, stability, and control.
- Milliseconds for dynamic analysis (kHz)
  Understanding the dynamic behavior of electric networks and their components is important in predicting whether the system, or a part of the system, remains in a stable state after a disturbance. The ability of a power system to maintain stability depends heavily on the controls in the system to dampen the electromechanical oscillations of the synchronous generators.

- Microseconds for transient analysis (MHz)
  Transient analysis is of importance when we want to gain insight into the effect of switching actions, for example, when connecting or disconnecting loads or switching off faulty sections, or into the effect of atmospheric disturbances, such as lightning strokes, and the accompanying overvoltages and overcurrents in the system and its components.

Although the power system itself remains unchanged when different time scales are considered, components in the power system should be modeled in accordance with the appropriate time frame. An example to illustrate this is the modeling of an overhead transmission line. For steady-state computations at power frequency, the wavelength of the sinusoidal voltages and currents is 6000 km (in the case of 50 Hz):

$$\lambda = \frac{v}{f} = \frac{3 \times 10^5}{50} = 6000\,\text{km} \tag{1.1}$$

λ the wavelength [km]
v the speed of light $\approx$ 300000 [km/s]
f the frequency [Hz = 1/s]

Thus, the transmission line is, so to speak, of "electrically small" dimensions compared to the wavelength of the voltage. The Maxwell equations can therefore be approximated by a quasi-static approach, and the transmission line can accurately be modeled by lumped elements (see also Appendix A). Kirchhoff's laws may fruitfully be used to compute the voltages and currents. When the effects of a lightning stroke have to be analyzed, frequencies of 1 MHz and higher occur and the typical wavelength of the voltage and current waves is 300 m or less. In this case the transmission line is far from being "electrically small," and it is not allowed to use the lumped-element representation anymore. The distributed nature of the transmission line has to be taken into account, and we have to calculate with traveling waves.

Despite the fact that we mainly use lumped-element models in our book, it is important to realize that the energy is mainly stored in the electromagnetic fields surrounding the conductors rather than in the conductors themselves as is shown in Figure 1.2. The Poynting vector, being the outer product of the electric field intensity vector and the magnetic field intensity vector, indicates the direction and intensity of the electromagnetic power flow [2, 3]:

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H} \tag{1.2}$$

$\boldsymbol{S}$ the Poynting vector [W/m^2]
$\boldsymbol{E}$ the electric field intensity vector [V/m]
$\boldsymbol{H}$ the magnetic field intensity vector [A/m]

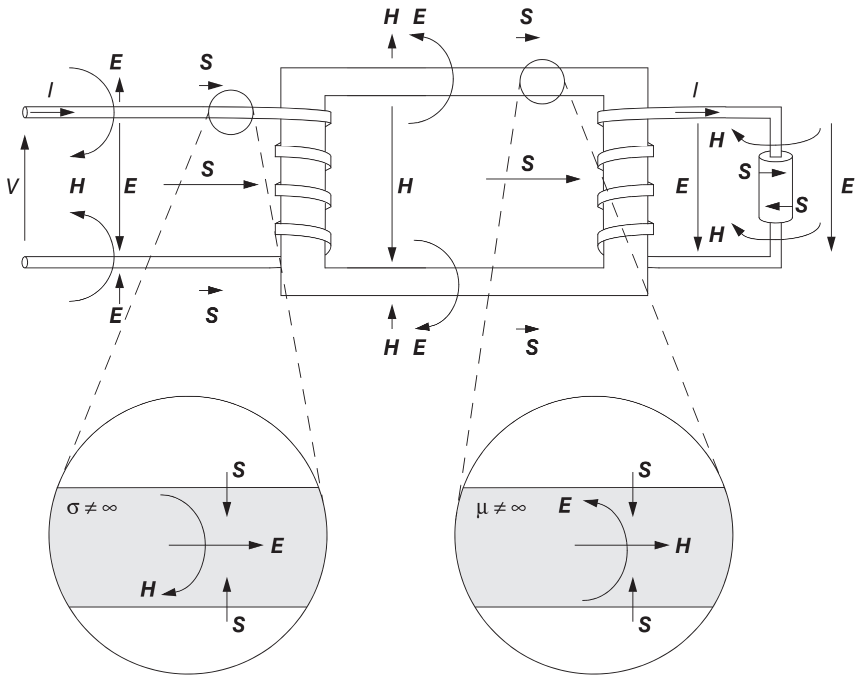


Figure 1.2 Transmission line–transformer–transmission line–load: the energy is stored in the electromagnetic field.

Due to the finite conductivity of the conductor material and the finite permeability of the transformer core material, a small electric field component is present inside the conductor and a small magnetic field component results in the transformer core:

$$\boldsymbol{E} = \frac{\boldsymbol{J}}{\sigma} \tag{1.3}$$

$\boldsymbol{J}$ the current density vector [A/m^2]
σ the conductivity [S/m]

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} \tag{1.4}$$

$\boldsymbol{B}$ the magnetic flux density vector [T = A H/m^2]
μ the permeability [H/m]

This leads to small Poynting vectors pointing toward the conductor and the transformer core: the losses in the transmission line and the transformer are fed from the electromagnetic field, as is the power consumed by the load.

1.3 General Characteristics of Power Systems

Most of the power systems are 50 or 60 Hz three-phase AC systems. The voltage levels used are quite diverse. In the following sections, we explain why these choices have been made.

1.3.1 AC versus DC Systems

The choice for AC systems over DC systems can be brought back to the "battle" between Nikola Tesla (1856–1943) and Thomas Alva Edison (1847–1931). Edison managed to let a light bulb burn for 20 hours in the year 1879. He used a 100 V DC voltage and this was one of the main drawbacks of the system. At that time a DC voltage could not be transformed to another voltage level, and the transportation of electricity at the low voltage level of 100 V over relatively short distances already requires very thick copper conductors to keep the voltage drop within limits; this makes the system rather expensive. Nevertheless, it took quite some time before AC became the standard. The reason for this was that Edison, besides being a brilliant inventor, was also a talented and cunning businessman as will become clear from the following anecdote. Edison tried to conquer the market and made many efforts to have the DC adopted as the universal standard. But behind the scenes he also tried hard to have AC adopted for a special application: the electric chair. After having accomplished this, Edison intimidated the general public into choosing DC by claiming that AC was highly dangerous, the electric chair being the proof of this! Eventually AC became the standard because transformers can quite easily transform the voltage from lower to higher voltage levels and vice versa.

Nowadays, power-electronic devices make it possible to convert AC to DC, DC to AC, and DC to DC with a high rate of efficiency, and the obstacle of altering the voltage level in DC systems has disappeared. What determines, in that case, the choice between AC and DC systems? Of course, financial investments do play an important role here. The incremental costs of DC transmission over a certain distance are less than the incremental costs of AC, because in a DC system two conductors are needed whereas three-phase AC requires three conductors. On the other hand, the power-electronic converters for the conversion of AC to DC at one side, and from DC to AC at the other side, of the DC transmission line are more expensive than the AC transmission terminals. If the transmission distance is sufficiently long, the savings on the conductors overcome the cost of the converters, as shown in Figure 1.3, and DC transmission is, from a capital investment point of view, an alternative to AC.

The following are a few of the examples of high-voltage DC (HVDC) applications.

- Long submarine crossings. For example, the Baltic cable between the Scandinavian countries and Germany and the 600 km cable connection between Norway and the Netherlands (the NorNed Cable Project).

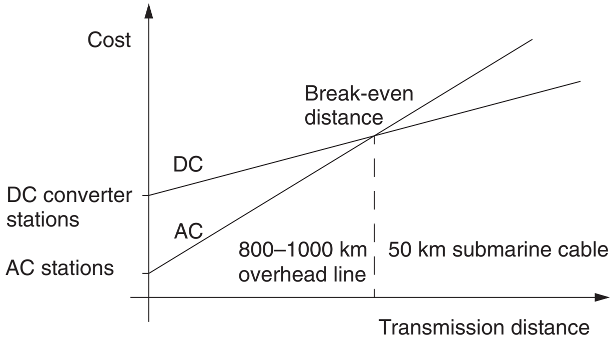


Figure 1.3 Break-even distance for HVDC [4].

- Asynchronous interconnection to interconnect networks that operate at different frequencies. For example, the HVDC intertie connection between the 50 Hz, 500 kV Argentinean system and the 60 Hz, 525 kV Brazilian system.
- Asynchronous interconnection to interconnect networks that operate at the same frequency but cannot be connected by means of AC due to stability reasons or operational differences. For example, the Scandinavian system is asynchronously connected to the western continental European system; the same applies for the US Eastern Interconnection and the US Western Interconnection.

Also in our domestic environment DC systems are present as the majority of our electronic equipment works internally with a DC voltage: personal computers, hi-fi equipment, video, DVD players, the television, and so on.

Shape of the alternating voltage

When an alternating voltage is considered, several types of alternating voltage are possible, such as sinusoidal, block, or triangular-shaped voltages, as depicted in Figure 1.4. For power systems, the sinusoidal alternating voltage is the right one to choose. By approximation, the power system can be considered to be a linear time-invariant (LTI) dynamic system. The elementary operations in such a system are multiplication with a constant number and addition and subtraction of quantities and delay in time (phase shift). When we perform these operations on a sinusoidal signal of constant frequency, another

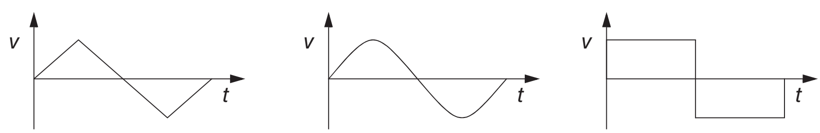


Figure 1.4 Alternating voltages: triangular, sinusoidal, and block.

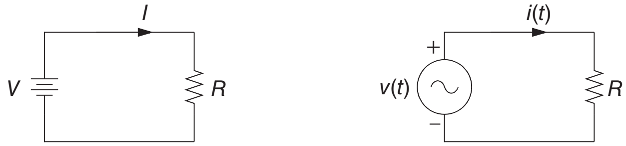


Figure 1.5 The definition of RMS values of sinusoidal quantities.

sinusoidal signal with the same frequency is the result. The same applies for differentiation and integration. The signal, after manipulation, may differ in amplitude or may be out of phase with the original signal, but the frequency and the shape of the signal have not been affected. This is not the case when the other alternating voltage shapes are used.

In other words, a sinusoidal excitation of a linear system results in a sinusoidal response. Therefore, all the voltages and currents in the power system are sinusoidal and have the same frequency so that the components in the system can be designed for this wave shape.

Sinusoidal alternating voltage

When we talk about an alternating sinusoidal voltage (or current), we generally refer to the so-called RMS root mean square (RMS) value or effective value of the voltage (or current). This RMS or effective value of a sinusoidal alternating voltage (or current) is the equivalent value of the corresponding direct voltage (or current) that dissipates the same amount of power in a given resistor during one time period of the alternating voltage (or current). We derive this equality for the DC and AC circuit shown in Figure 1.5.

The power dissipated in the resistance in the DC circuit is

$$P = \frac{V^2}{R} = I^2 R \tag{1.5}$$

When we write the voltage and the current in the AC circuit as

$$v(t) = \sqrt{2}|V|\sin(\omega t) \quad \text{and} \quad i(t) = \sqrt{2}|I|\sin(\omega t) \tag{1.6}$$

ω the angular frequency ($\omega = 2\pi f$) [rad/s]

the instantaneous power dissipated in the AC circuit is

$$p(t) = \frac{v^2(t)}{R} = i^2(t)R \tag{1.7}$$

and for the average power this results in

$$P = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{1}{T}\int_0^T i^2 R dt \tag{1.8}$$

T the period of the sine wave ($T = 1/f = 2\pi/\omega$) [s]

When the power in the DC circuit (Equation 1.5) and the average power in the AC circuit (Equation 1.8) are demanded to be equal, we can write

$$V^2 = \frac{1}{T}\int_0^T v^2 dt \quad \text{and} \quad I^2 = \frac{1}{T}\int_0^T i^2 dt \tag{1.9}$$

and substitution of the equations for the alternating voltage and current (Equation 1.6) gives us

$$\begin{aligned} V &= \sqrt{\frac{1}{T}\int_0^T v^2 dt} = \sqrt{2}|V|\sqrt{\frac{1}{T}\int_0^T \sin^2(\omega t) dt} = \sqrt{2}|V|\sqrt{\frac{1}{2}} = |V| \\ I &= \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{2}|I|\sqrt{\frac{1}{T}\int_0^T \sin^2(\omega t) dt} = \sqrt{2}|I|\sqrt{\frac{1}{2}} = |I| \end{aligned} \tag{1.10}$$

$|V|$ the RMS or effective value of the alternating voltage
$|I|$ the RMS or effective value of the alternating current

So summing up, the RMS or effective value of a sinusoidal alternating voltage (or current) is equal to the value of the equivalent direct voltage (or current) that dissipates the same amount of power in a given resistor during one time period of the alternating voltage (or current).

The expression RMS is related to the previously derived expressions:

Root
Mean
Square

$$V = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = \sqrt{2}|V|\sqrt{\frac{1}{T}\int_0^T \sin^2(\omega t) dt} \tag{1.11}$$

The term below the square root in Equation 1.11 is in fact the mean of $\sin^2(\omega t)$, as indicated in Figure 1.6.

From the equations of the sinusoidal voltage and current (Equation 1.6), we see the relation between the RMS value and the peak value:

$$\sqrt{2}|V| = \hat{V} \quad \text{and} \quad \sqrt{2}|I| = \hat{I} \tag{1.12}$$

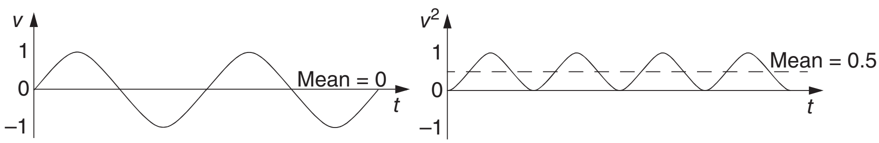


Figure 1.6 Mean value of a squared sine.

It is the RMS value of the sinusoidal voltage and current that is read by the common type of voltmeters and ammeters.

Example 1.1 ***RMS and peak value***
When we speak of a voltage of 230 V, the RMS value of the voltage amounts to 230 V, while the peak value of the sinusoidal voltage is $\sqrt{2} \cdot 230 = 325\,\text{V}$.

1.3.2 50 and 60 Hz Frequency

The choice of the frequency is not as arbitrary as one might think. Between 1885 and 1900, a diversity of frequencies was used in the United States: 140, 133⅓, 125, 83⅓, 66⅔, 60, 50, 40, 33⅓, 30, 25, and 16⅔ Hz [5–8]. Each frequency had its own field of application. The power frequency finally came out at 60 Hz in North America, Brazil, and Japan and at 50 Hz in most of the other countries. Nowadays, 16⅔ (Europe) and 25 Hz (North America) are in use for railway applications, and 400 Hz is a popular frequency on board of ships, airplanes, and oil rigs.

A too low frequency, such as 10 or 20 Hz, is useless for domestic lighting as the human eye records this as flicker. On the other hand, the frequency cannot be too high as:

- The hysteresis losses in the transformer core increase in proportion to the frequency while the eddy current losses increase in quadratic proportion to the frequency.
- The capacitive reactance of cables and transmission lines increases ($X = -1/\omega C$).
- The inductive reactance, and the related voltage drop, increases ($X = \omega L$).
- The electromagnetic interference with the radio traffic will grow.

Yet there is also an advantage in using a higher power system frequency – the power-to-weight ratio of transformers, motors, and generators is higher. In other words, the components can be smaller, while the power output is the same. The formula of Esson gives a generalized expression for the power of an electrical machine:

$$P = K \cdot D^2 \cdot l \cdot n \tag{1.13}$$

K the "output coefficient" [J/m^3], which depends on the type of machine, the type of cooling, and the magnetic material used
D the diameter of the armature [m]
l the axial length of the armature [m]
n the rotational speed of the machine [1/s]

From Equation 1.13 we see that when we increase the rotational speed, by choosing a higher system frequency, the dimension of the machine can be smaller for the same output power.

Another example is the transformer. The relation between the applied voltage and the resulting flux is given by the following equations:

$$v_1(t) = \sqrt{2}|V_1|\cos(\omega t) = N_1 \frac{d\Phi}{dt}$$
$$\Phi(t) = C + \frac{\sqrt{2}|V_1|}{\omega N_1}\sin(\omega t) = \sqrt{2}|\Phi|\sin(\omega t) \quad (1.14)$$
$$|\Phi| = |B|A$$

N_1 the number of turns of the primary transformer winding
Φ the magnetic flux [Wb = V s]
C the integration constant [Wb]; zero in steady-state conditions
B the magnetic flux density [T = Wb/m^2]
A the cross-sectional area of the iron transformer core [m^2]

We see that when the applied voltage remains the same, a higher system frequency ($\omega = 2\pi f$) results in a lower effective value of the magnetic flux ($|\Phi|$) so that we can use a smaller cross-sectional area for the iron core when we keep the magnetic flux density constant.

When there is freedom to choose the system frequency, a higher frequency can be very advantageous, especially in the case that weight and volume play a role, for example, on board of airplanes and ships.

1.3.3 Balanced Three-Phase Systems

The transmission and distribution systems are three-phase systems. In this book we restrict ourselves to balanced three-phase power systems. In the case of a balanced three-phase system, the sinusoidal voltages are of equal magnitude in all three phases and shifted in phase by 120°, as shown in Figure 1.7:

$$v_a = \sqrt{2}|V|\cos(\omega t)$$
$$v_b = \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right) \quad (1.15)$$
$$v_c = \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right)$$

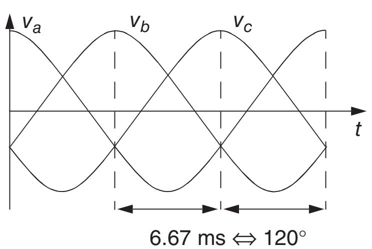


Figure 1.7 Phase voltages in a balanced three-phase power system (50 Hz).

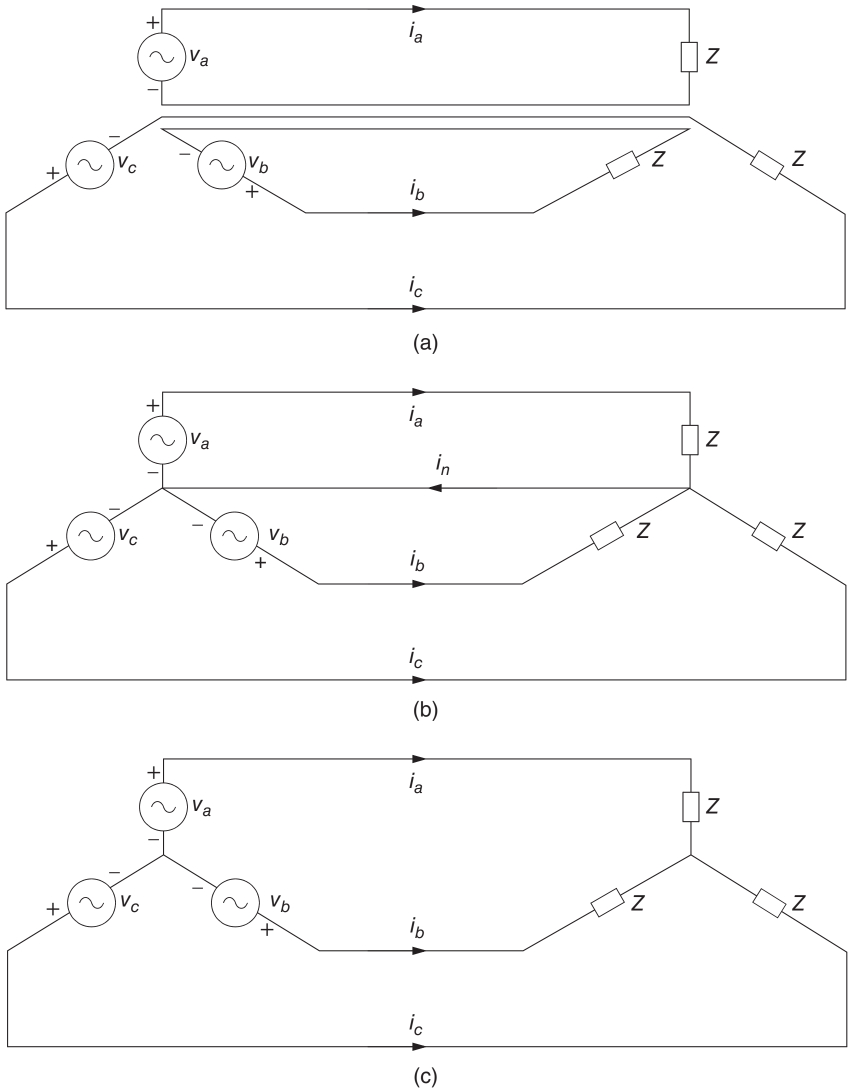


Figure 1.8 A balanced three-phase power system.

Furthermore, the network has identical impedances in each phase and the loads are identical. We can imagine the three-phase system consisting of three separate single-phase systems, as depicted in Figure 1.8 (a). Since the voltages in the three phases are equal in magnitude and 120° shifted in phase and since the impedances in the three phases are equal, the currents will also be equal in magnitude and shifted in phase by 120°:

$$\begin{aligned} i_a &= \frac{v_a}{Z} = \sqrt{2}|I|\cos(\omega t - \varphi) \\ i_b &= \frac{v_b}{Z} = \sqrt{2}|I|\cos\left(\omega t - \varphi - \frac{2\pi}{3}\right) \\ i_c &= \frac{v_c}{Z} = \sqrt{2}|I|\cos\left(\omega t - \varphi - \frac{4\pi}{3}\right) \end{aligned} \tag{1.16}$$

When the three return conductors are combined as a single return conductor, the three-phase system in Figure 1.8 (b) can be drawn. The current through the common return conductor is equal to the sum of the three individual phase currents:

$$\begin{aligned} i_n &= i_a + i_b + i_c \\ &= \sqrt{2}|I|\left[\cos(\omega t - \varphi) + \cos\left(\omega t - \varphi - \frac{2\pi}{3}\right) + \cos\left(\omega t - \varphi - \frac{4\pi}{3}\right)\right] = 0 \end{aligned} \tag{1.17}$$

Because the current in the common return conductor is zero, in the case of a balanced three-phase power system with a balanced load, this common return conductor can be removed (see Figure 1.8 (c)), and a network with only three conductors results.

Because of the fact that in the power system many single-phase loads (domestic users and also some industrial ones) are connected, the utilities try to divide the single-phase loads equally over the three phases (see also Section 4.3). The assumption that the power system is balanced is therefore a valid approximation.

The reason to build a power system as a three-phase system is twofold as explained in the following text.

Power considerations

The voltage and current of a single-phase inductive load can be written as

$$v(t) = \sqrt{2}|V|\cos(\omega t) \quad \text{and} \quad i(t) = \sqrt{2}|I|\cos(\omega t - \varphi) \tag{1.18}$$

and the instantaneous power, consumed by this load, amounts to

$$\begin{aligned} p(t) &= 2|V||I|\cos(\omega t)\cos(\omega t - \varphi) \\ &= |V||I|[\cos(\varphi) + \cos(2\omega t - \varphi)] \end{aligned} \tag{1.19}$$

From this expression we learn that the single-phase instantaneous power is not constant, but varies in time with double the power frequency ($2\omega t$). This is rather unpleasant, especially when the electrical energy is used for electrical motors and traction applications, because this results in a pulsating torque on the axis of the machine.

The voltages and currents of a balanced three-phase inductive load can be written as

$$\begin{aligned}
v_a &= \sqrt{2}|V|\cos(\omega t) & i_a &= \sqrt{2}|I|\cos(\omega t - \varphi) \\
v_b &= \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right) & i_b &= \sqrt{2}|I|\cos\left(\omega t - \varphi - \frac{2\pi}{3}\right) \\
v_c &= \sqrt{2}|V|\cos\left(\omega t - \frac{4\pi}{3}\right) & i_c &= \sqrt{2}|I|\cos\left(\omega t - \varphi - \frac{4\pi}{3}\right)
\end{aligned} \tag{1.20}$$

and the instantaneous power, consumed by this load, amounts to

$$\begin{aligned}
p(t) &= v_a i_a + v_b i_b + v_c i_c \\
&= |V||I|[\cos(\varphi) + \cos(2\omega t - \varphi)] + |V||I|\left[\cos(\varphi) + \cos\left(2\omega t - \varphi - \frac{4\pi}{3}\right)\right] \\
&\quad + |V||I|\left[\cos(\varphi) + \cos\left(2\omega t - \varphi - \frac{2\pi}{3}\right)\right] \\
&= 3|V||I|\cos(\varphi)
\end{aligned} \tag{1.21}$$

In a balanced three-phase power system, the instantaneous power is constant! This is in fact valid for every balanced power system with more than three phases. For an n-phase system, we can write the following general expression for the instantaneous power:

$$\begin{aligned}
p(t) &= n|V||I|\cos(\varphi) + |V||I|\sum_{k=1}^{n}\cos\left(2\omega t - \varphi - 2k\cdot\frac{2\pi}{n}\right) = n|V||I|\cos(\varphi) \\
&\quad + |V||I|\cos(2\omega t - \varphi)\sum_{k=1}^{n}\cos\left(2k\cdot\frac{2\pi}{n}\right) \\
&\quad + |V||I|\sin(2\omega t - \varphi)\sum_{k=1}^{n}\sin\left(2k\cdot\frac{2\pi}{n}\right)
\end{aligned} \tag{1.22}$$

A close observation of the terms behind the summation signs reveals

$$\begin{aligned}
&\sum_{k=1}^{n}\cos\left(2k\cdot\frac{2\pi}{n}\right) = 0 \quad \forall n \geq 3 \\
&\sum_{k=1}^{n}\sin\left(2k\cdot\frac{2\pi}{n}\right) = 0 \quad \forall n \geq 1
\end{aligned} \tag{1.23}$$

Thus, the instantaneous power, as given in Equation 1.22, is constant for every number of phases greater than or equal to three. Then why do we apply a three-phase system and not a four- or five-phase system? This is because each phase requires its own conductor, and the balanced three-phase system is the system with the smallest number of phase conductors capable of delivering constant instantaneous power.

The power supplied by the balanced three-phase system equals three times the average power supplied by one of the three single-phase systems of which the system is built up, while only one extra conductor is required. As the cosine term with the double frequency in Equation 1.19 has an average value of zero, the average power supplied by the single-phase system amounts to

$$P_{1\phi} = |V||I|\cos(\varphi) \tag{1.24}$$

The average power supplied by the balanced three-phase system equals the instantaneous power (Equation 1.21):

$$P_{3\phi} = p(t) = 3|V||I|\cos(\varphi) = 3P_{1\phi} \tag{1.25}$$

The three-phase system depicted in Figure 1.8 (c) transports the same amount of power as the three-phase system built of three individual single-phase systems, as shown in Figure 1.8 (a), but with only half the number of conductors!

Rotating magnetic field

A three-phase system is able to produce a rotating magnetic field, as is visualized in Figure 1.9. This is a very important property as all AC machines

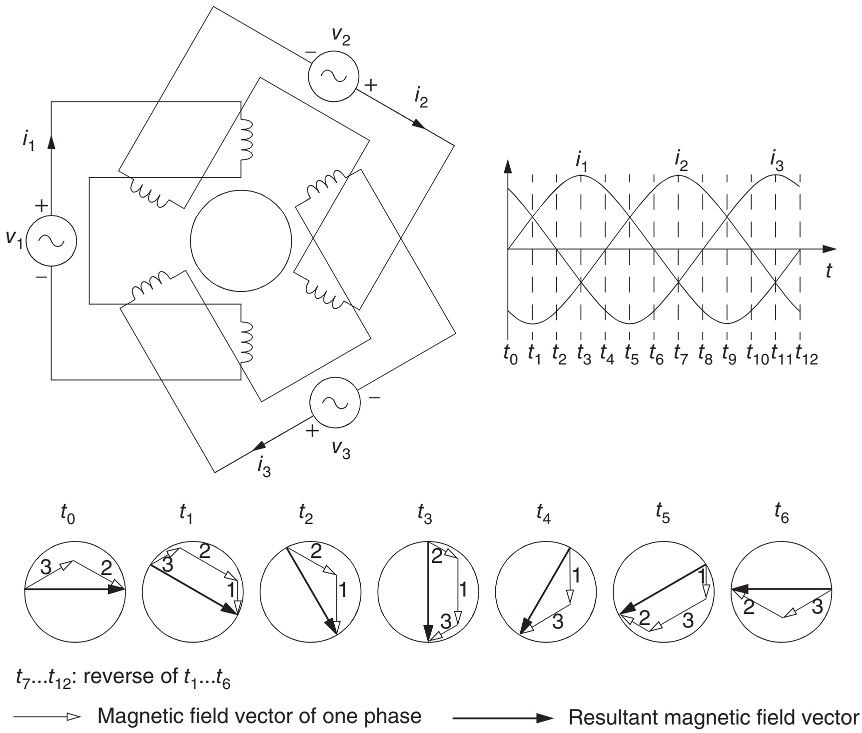


Figure 1.9 Magnetic field generated by a three-phase coil system [9].

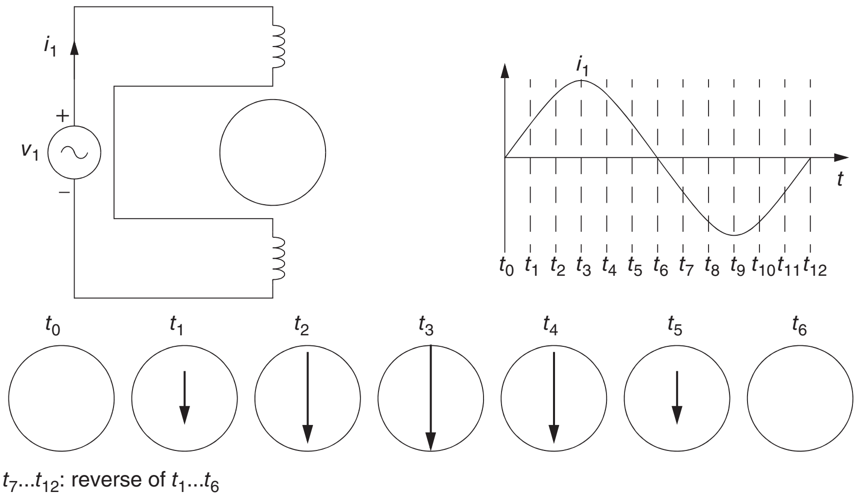


Figure 1.10 Magnetic field generated by a single-phase coil system.

operate on this principle. In a three-phase coil system, the resulting magnetic field vector rotates with a constant magnitude, while in a single-phase coil system the magnitude of the magnetic field vector varies in one direction (see Figure 1.10). The expressions for the currents in the three-phase coil system are

$$\begin{aligned} i_1 &= \sqrt{2}|I| \sin(\omega t) \\ i_2 &= \sqrt{2}|I| \sin\left(\omega t - \frac{2\pi}{3}\right) \\ i_3 &= \sqrt{2}|I| \sin\left(\omega t - \frac{4\pi}{3}\right) \end{aligned} \tag{1.26}$$

and the resulting magnetic fields can be written as

$$\begin{aligned} H_1 &= \sqrt{2}|H| \sin(\omega t) \sin(x) \\ H_2 &= \sqrt{2}|H| \sin\left(\omega t - \frac{2\pi}{3}\right) \sin\left(x - \frac{2\pi}{3}\right) \\ H_3 &= \sqrt{2}|H| \sin\left(\omega t - \frac{4\pi}{3}\right) \sin\left(x - \frac{4\pi}{3}\right) \end{aligned} \tag{1.27}$$

t the time [s]; $t = 0 \leftrightarrow H_1 = 0\ \forall x$
x the circumferential position [rad]; $x = 0 \leftrightarrow H_1 = 0\ \forall t$

Therefore, in the three-phase coil system, the resulting magnetic field amounts to

$$\begin{aligned} H_r &= H_1 + H_2 + H_3 \\ &= \frac{1}{2}\sqrt{2}|H|[\cos(\omega t - x) - \cos(\omega t + x)] \\ &\quad + \frac{1}{2}\sqrt{2}|H|\left[\cos(\omega t - x) - \cos\left(\omega t + x - \frac{4\pi}{3}\right)\right] \\ &\quad + \frac{1}{2}\sqrt{2}|H|\left[\cos(\omega t - x) - \cos\left(\omega t + x - \frac{2\pi}{3}\right)\right] \\ &= \frac{3}{2}\sqrt{2}|H|\cos(\omega t - x) \end{aligned} \tag{1.28}$$

When the time varies and the radial position is kept equal to $x = \omega t$, the resulting magnetic field vector remains constant in length $H_r = (3/2)\sqrt{2}|H|$ and rotates with a constant velocity $dx/dt = \omega$ [rad/s].

When a compass needle is positioned in the middle of the three-phase coil system, the needle keeps pace with the rotating field, which is a crude equivalent of the synchronous motor. When a copper cylinder is placed in the center of the three-phase coil system, the rotating field drags the cylinder around with it, and we have a primitive equivalent of the induction motor.

Example 1.2 ***"Two-phase" system***

It is possible to create a system with two-phase windings that delivers constant instantaneous power and that produces a rotating magnetic field. To achieve this, the voltages in the two phases should be 90° out of phase. The voltages and currents of a balanced two-phase inductive load can be written as

$$\begin{aligned} v_a &= \sqrt{2}|V|\cos(\omega t) & i_a &= \sqrt{2}|I|\cos(\omega t - \varphi) \\ v_b &= \sqrt{2}|V|\cos\left(\omega t - \frac{\pi}{2}\right) & i_b &= \sqrt{2}|I|\cos\left(\omega t - \varphi - \frac{\pi}{2}\right) \end{aligned} \tag{1.29}$$

and the instantaneous power, consumed by this load, amounts to

$$\begin{aligned} p(t) &= v_a i_a + v_b i_b \\ &= |V||I|[\cos(\varphi) + \cos(2\omega t - \varphi)] \\ &\quad + |V||I|[\cos(\varphi) + \cos(2\omega t - \varphi - \pi)] \\ &= 2|V||I|\cos(\varphi) \end{aligned} \tag{1.30}$$

The resulting magnetic field vector of the two-phase coil system rotates with a constant amplitude as visualized in Figure 1.11.

A drawback of such a two-phase system is that the current through the return conductor is not equal to zero:

$$i_n = i_a + i_b \neq 0 \tag{1.31}$$

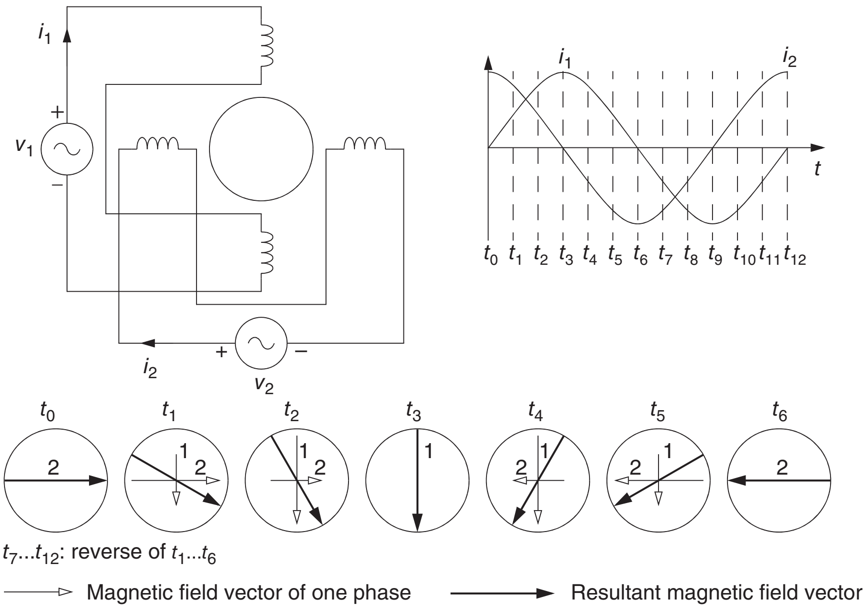


Figure 1.11 Magnetic field generated by a two-phase coil system.

Accordingly, this system requires as many conductors (three) as a balanced three-phase system, but only a power $p(t) = 2|V||I| \cos(\varphi)$ is transported (compared with $p(t) = 3|V||I| \cos(\varphi)$ in the three-phase system).

1.3.4 Voltage Levels

The range of voltage ratings finds its origin in the use of carbon arc lamps in the early days of the power system. These lamps, the source of electric lighting before the incandescent lamp was invented, worked with a DC voltage of 55 V. In those days the systems were laid out as three-wire systems, with conductors at a potential of −55, 0, and 55 V as shown in Figure 1.12 (b). In this configuration a higher voltage of 110 V was available as well. By using such a three-conductor system, one could save considerably on copper. In the system with two conductors (Figure 1.12 (a)), the losses equal $I^2 \cdot 2R$, with a resistance per conductor of R and a total copper weight of 100%. In the case of the three-conductor system, the losses equal $(I^2/4) \cdot 2r$ (note that the current in the middle conductor equals zero). In the case of equal losses in the two systems, the resistance per conductor in the three-conductor system equals $r = 4R$. Therefore, the copper weight per conductor is reduced to 25%, which brings the total required copper weight to $3/2 \cdot 25\% = 37.5\%$ of the copper needed for the two-wire layout.

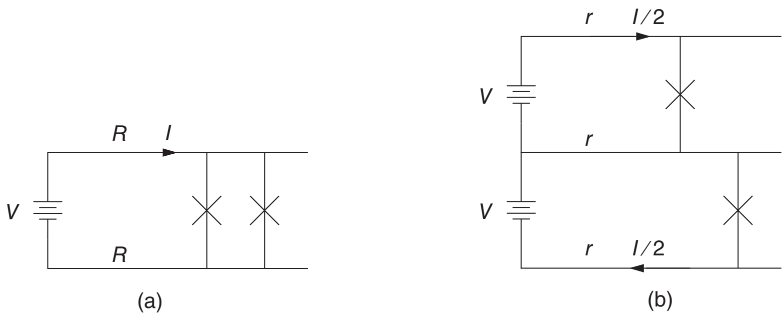


Figure 1.12 Two- (a) and three- (b) conductor system.

Figure 1.13 Reducing losses by increasing the voltage level.

The choice for a higher voltage level of 220 V (and in Europe later on for 230 V) was made in order to reduce losses, as is illustrated in Figure 1.13. Both voltage sources deliver a power of $VI = 2V \cdot I/2$. The losses in the circuit with supply voltage V equal $I^2 \cdot 2R$, while the losses in the circuit with double this supply voltage amount to $(I^2/4) \cdot 2R$, which means a loss reduction of 75%. When the choice was made for the alternating voltage system, the same voltage levels were maintained.

The voltage ratings in a power system can be divided into three levels:

- The generation level: 10–25 kV. The power is generated at a relatively low voltage level to keep the high-voltage insulation of the generator armature within limits.
- The transmission level: 110–420 kV and higher (in the former Soviet Union even the 1200 kV level is in operation).
- The distribution level: 10–72.5 kV.

The power is supplied to the customer at a variety of voltage levels; heavy-industrial consumers can be connected from 150 to 10 kV, while households are connected to the 0.4 kV voltage level. The changeover between voltage levels is made by transformers. The voltage ratings used in the Dutch system are shown in Figure 1.14.

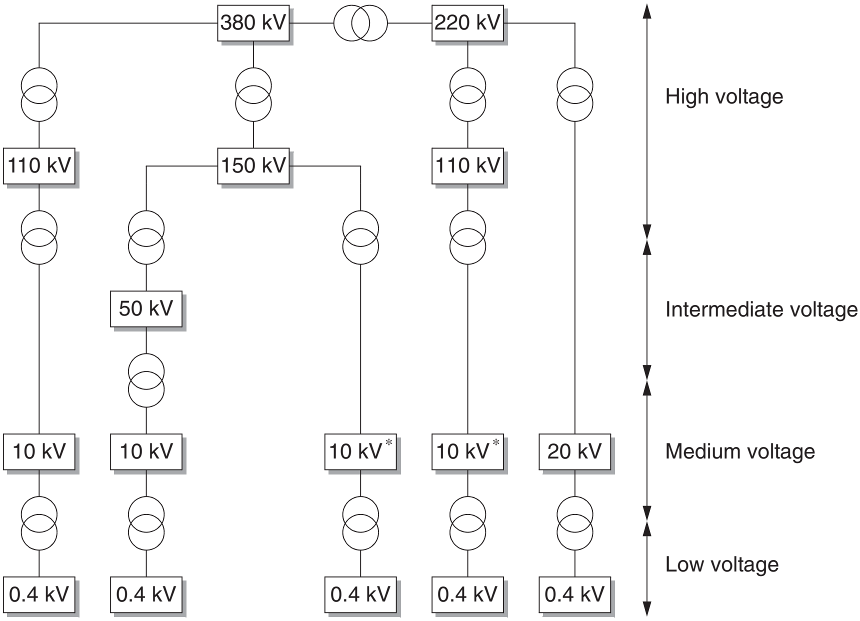


Figure 1.14 Voltage levels and transformation steps in the Dutch power system; *this voltage level can be 20 kV as well.

Line-to-line and line-to-neutral voltages

The voltage ratings of three-phase systems are usually expressed as line-to-line voltages instead of line-to-neutral voltages, as is illustrated in Figure 1.15. When the line-to-neutral voltages in the phases a and b are

$$\begin{aligned} v_{an} &= \sqrt{2}|V|\cos(\omega t) \\ v_{bn} &= \sqrt{2}|V|\cos\left(\omega t - \frac{2\pi}{3}\right), \end{aligned} \tag{1.32}$$

the line-to-line voltage between the phases a and b can be calculated as follows:

$$\begin{aligned} v_{ab} &= v_{an} - v_{bn} = \sqrt{2}|V|\left[\cos(\omega t) - \cos\left(\omega t - \frac{2\pi}{3}\right)\right] \\ &= \sqrt{2}|V|\left[2\cdot\cos\left(\frac{\pi}{6}\right)\cdot\cos\left(\omega t + \frac{\pi}{6}\right)\right] = \sqrt{3}\left[\sqrt{2}|V|\cos\left(\omega t + \frac{\pi}{6}\right)\right] \end{aligned} \tag{1.33}$$

The amplitude of the line-to-line voltage is $\sqrt{3}$ times the amplitude of the line-to-neutral voltage ($|V_{ab}| = \sqrt{3}|V_{an}|$), and the line-to-line voltage v_{ab} leads the line-to-neutral voltage v_{an} by 30 electrical degrees.

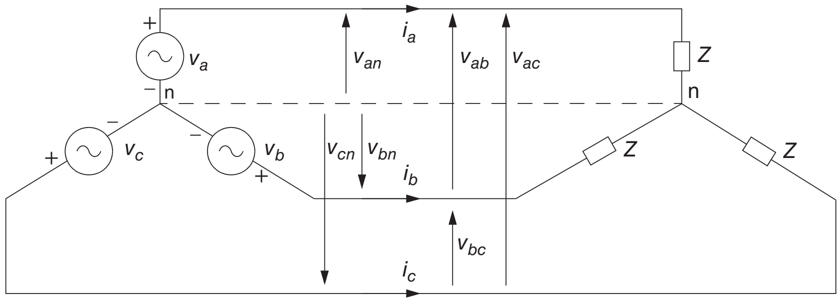


Figure 1.15 Line-to-line and line-to-neutral voltages.

In this book, a line-to-line voltage is indicated by the subscript LL, whereas a line-to-neutral voltage is indicated by the subscript LN. The relation between the line-to-line voltage and the line-to-neutral voltage is

$$|V_{\mathrm{LL}}| = \sqrt{3}|V_{\mathrm{LN}}| \tag{1.34}$$

$|V_{\mathrm{LL}}|$ the RMS value of the line-to-line voltage
$|V_{\mathrm{LN}}|$ the RMS value of the line-to-neutral voltage

---

Example 1.3 ***Line-to-line and line-to-neutral voltage***
The highest voltage level of the Dutch transmission network is 380 kV. This is the RMS value of the line-to-line voltage. The RMS value of the corresponding line-to-neutral voltage equals $380\,\mathrm{kV}/(\sqrt{3}) \approx 220\,\mathrm{kV}$. The lowest voltage used in the Netherlands is 0.4 kV (line-to-line voltage). The corresponding line-to-neutral voltage amounts to $400/(\sqrt{3}) \approx 230\,\mathrm{V}$.

1.4 Phasors

Power system calculations in the steady-state situation are considerably simplified by introducing the phasor. A phasor is an arrow in the complex plane that has a one-to-one relation with a sinusoidal signal as can be seen from Figure 1.16. When the sine/cosine in Figure 1.16 has a frequency of 50 Hz, a radius with a length $\sqrt{2}$ times the length of the phasor rotates counterclockwise in the complex plane with a frequency of 50 Hz.

Consider the following general sinusoidal voltage and current expressions:

$$v(t) = \sqrt{2}|V|\cos(\omega t) \quad \text{and} \quad i(t) = \sqrt{2}|I|\cos(\omega t - \varphi) \tag{1.35}$$

In order to express these quantities as phasors, we apply Euler's identity:

$$e^{j\varphi} = \cos(\varphi) + j\sin(\varphi) \tag{1.36}$$

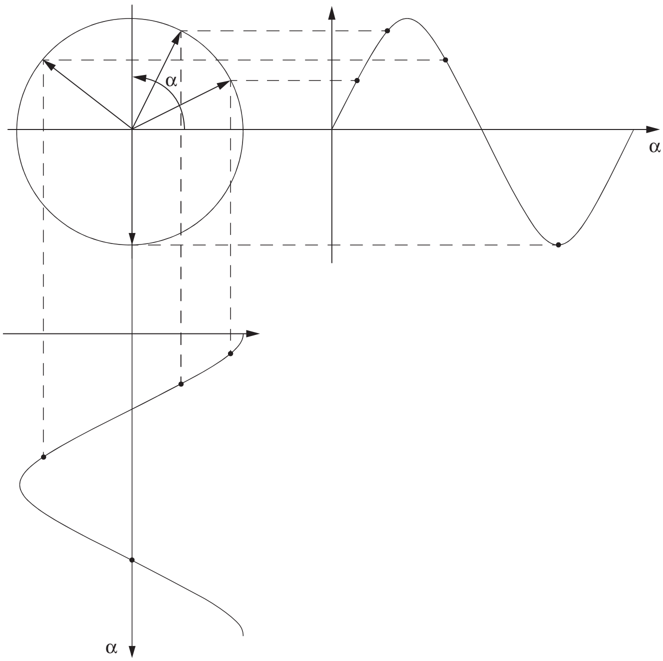


Figure 1.16 Relation between a counterclockwise rotating radius and a sinusoidal signal.

and the sinusoidal voltage and current can be written as

$$v(t) = \mathrm{Re}\{\sqrt{2}|V|e^{j\omega t}\} = \mathrm{Re}\{\sqrt{2}Ve^{j\omega t}\} \quad \text{with} \quad V = |V| = |V|\angle 0$$
$$i(t) = \mathrm{Re}\{\sqrt{2}|I|e^{j(\omega t-\varphi)}\} = \mathrm{Re}\{\sqrt{2}Ie^{j\omega t}\} \quad \text{with} \quad I = |I|e^{-j\varphi} = |I|\angle -\varphi \tag{1.37}$$

Re the operator that takes the real part of a complex quantity
V the voltage phasor
I the current phasor

The phasor represents a sinusoidal signal: the length of the phasor equals the effective or RMS value of the signal, and the angle of the phasor matches the phase shift of the signal with respect to a reference (an ideal cosine (or sine)). We see that the frequency component is absent when we use the phasor notation.

The voltages produced by the synchronous generators in the power system are 50 Hz (or 60 Hz) sinusoidal voltages. As the power system is supposed to be a linear system (see Section 1.3.1) in steady state, the voltage out of the AC outlet at home is also a sinusoidal voltage with a frequency of 50 Hz; only the amplitude of the voltage differs and a phase shift may have occurred. Therefore, in steady-state calculations, the frequency gives no extra information and can be omitted so that we can do our calculations with the phasors, fixed in the complex plane. No relevant information is lost as the information with respect to the phase angle and the amplitude is still available in this phasor.

Example 1.4 ***Phasor notation***
The phasor of the sinusoidal signal $v(t) = 141.4\cos(\omega t + \pi/6)$ is written as $V = 100\angle 30°$.

1.4.1 Network Elements in the Phasor Domain

After having introduced the phasor representation for the voltages and currents in the sinusoidal steady state, it is necessary to investigate the voltage–current relations of the resistance, inductance, and capacitance in the phasor domain. In Figure 1.17, the sinusoidal voltages and currents and also the corresponding phasors for those three elements are shown.

Consider the following sinusoidal voltage:

$$v(t) = \sqrt{2}|V|\cos(\omega t) = \mathrm{Re}\{\sqrt{2}Ve^{j\omega t}\} = \mathrm{Re}\{v'(t)\} \quad \text{with } V = |V|\angle 0° \tag{1.38}$$

The current through a resistance R, when excited with a voltage $v'(t)$ (as defined in Equation 1.38), can be calculated as follows:

$$i'(t) = \frac{v'(t)}{R} = \sqrt{2}\cdot\frac{V}{R}\cdot e^{j\omega t} = \sqrt{2}Ie^{j\omega t} \tag{1.39}$$

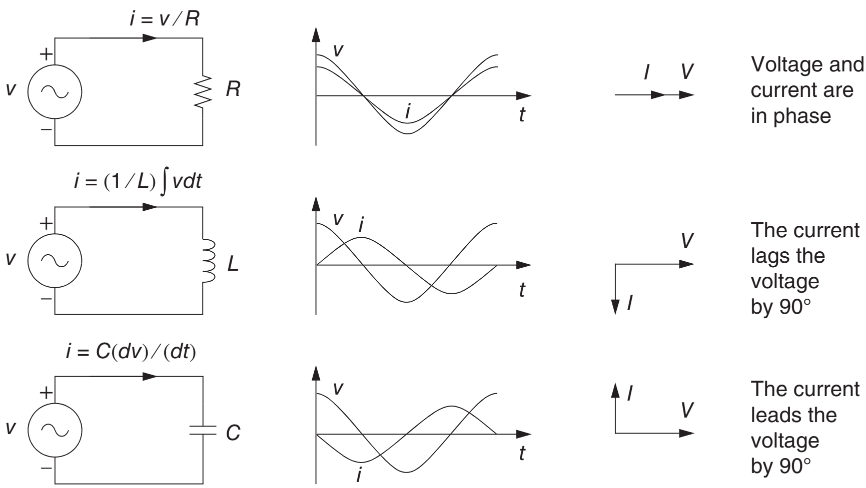


Figure 1.17 Relation between the sinusoidal voltage and current and the corresponding phasors for a resistance, inductance, and capacitance.

Therefore, the voltage–current relation of a resistance in the phasor domain is

$$I = \frac{V}{R} \tag{1.40}$$

When we take the real part of Equation 1.39, we get the expression of the time-varying sinusoidal current:

$$i(t) = \mathrm{Re}\{i'(t)\} = \sqrt{2} \cdot \frac{|V|}{R} \cdot \cos(\omega t) \tag{1.41}$$

The current through a capacitor C, when connected to a voltage source $v'(t)$ (as defined in Equation 1.38), can be obtained as follows:

$$i'(t) = C\frac{dv'}{dt} = \sqrt{2} \cdot j\omega CV \cdot e^{j\omega t} = \sqrt{2}Ie^{j\omega t} \tag{1.42}$$

Thus, the voltage–current relation of a capacitor in the phasor domain is

$$I = j\omega CV \tag{1.43}$$

Taking the real part of Equation 1.42 leads to the expression of the time-varying sinusoidal current:

$$i(t) = \mathrm{Re}\{i'(t)\} = \sqrt{2} \cdot \omega C|V| \cdot -\sin(\omega t) = \sqrt{2} \cdot \omega C|V| \cdot \cos\left(\omega t + \frac{\pi}{2}\right) \tag{1.44}$$

The current through an inductor L, driven by a voltage $v'(t)$ (as defined in Equation 1.38), is

$$i'(t) = \frac{1}{L}\int v' dt = \sqrt{2} \cdot \frac{V}{j\omega L} \cdot e^{j\omega t} = \sqrt{2}Ie^{j\omega t} \tag{1.45}$$

and the voltage–current relation of an inductor in the phasor domain is

$$I = \frac{V}{j\omega L} \tag{1.46}$$

The real part of Equation 1.45 gives us the expression for the time-varying sinusoidal current:

$$i(t) = \mathrm{Re}\{i'(t)\} = \sqrt{2} \cdot \frac{|V|}{\omega L} \cdot \sin(\omega t) = \sqrt{2} \cdot \frac{|V|}{\omega L} \cdot \cos\left(\omega t - \frac{\pi}{2}\right) \tag{1.47}$$

The voltage–current relations are summarized in Table 1.1.

Table 1.1 Voltage–current relations.

Element	Time Domain	Phasor Domain
Resistance	$v = iR$	$V = IR$
Capacitance	$i = C(dv)/(dt)$	$I = j\omega CV$
Inductance	$v = L(di)/(dt)$	$V = j\omega LI$

In the phasor domain, a general expression for the impedance can be written as

$$Z = \frac{V}{I} = R + jX \tag{1.48}$$

Z the impedance [Ω]
R the resistance [Ω]
X the reactance [Ω]

When X has a positive sign, the energy-storage element is an inductor ($jX = j\omega L$). When X has a negative sign, the energy-storage element is a capacitor ($jX = 1/(j\omega C) = -j/(\omega C)$).

When X equals zero, there is no energy storage and the impedance is purely resistive. This is the case when we have no capacitors or inductors or when the frequency is zero and we deal with DC.

In a similar way, a general expression for the admittance can be written as

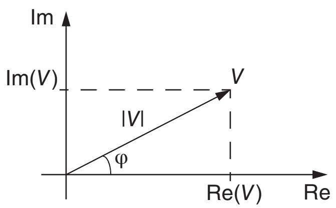
$$Y = \frac{I}{V} = G + jB \tag{1.49}$$

Y the admittance [S]
G the conductance [S]
B the susceptance [S]

1.4.2 Calculations in the Phasor Domain

When we set aside the "deeper meaning" of the phasor (being the representation of a sinusoidal signal), a phasor is nothing more than a complex number represented by a vector in the complex plane. Therefore, the mathematical rules for vector calculus can be applied for phasors.

A vector can be described by its length and its angle (polar coordinates) or represented by its real and imaginary part (rectangular or Cartesian coordinates). The relation between these two forms of notation is shown in Figure 1.18. Basically, it makes no difference whether a phasor is expressed by



$$V = |V|\angle\varphi = \mathrm{Re}(V) + j\mathrm{Im}(V)$$
$$|V| = \sqrt{\mathrm{Re}(V)^2 + \mathrm{Im}(V)^2}$$
$$\varphi = \tan^{-1}\left(\frac{\mathrm{Im}(V)}{\mathrm{Re}(V)}\right)$$
$$\mathrm{Re}(V) = |V|\cos(\varphi)$$
$$\mathrm{Im}(V) = |V|\sin(\varphi)$$

Figure 1.18 The phasor as a vector in the complex plane.

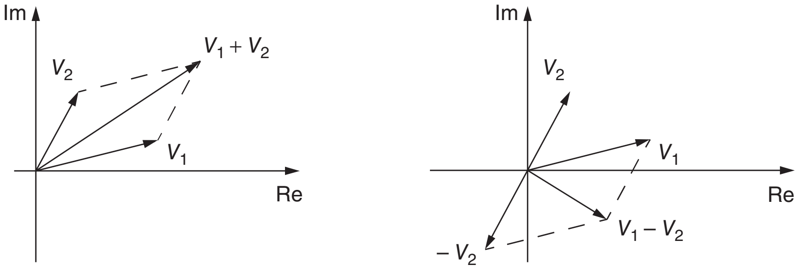


Figure 1.19 Two basic operations on vectors: addition and subtraction.

the Cartesian notation or the polar notation; however, for certain calculations it can be advantageous to use one or the other. For addition and subtraction of two phasors, the Cartesian notation is in general the easiest to apply as shown graphically in Figure 1.19:

$$V_1 = a + jb \quad \text{and} \quad V_2 = c + jd \tag{1.50}$$

$$\begin{aligned} V_1 + V_2 &= (a + c) + j(b + d) \quad && \text{addition} \\ V_1 - V_2 &= (a - c) + j(b - d) \quad && \text{subtraction} \\ V_1^* &= a - jb && \text{complex conjugate} \end{aligned} \tag{1.51}$$

For multiplication and division of phasors, the polar notation is more handy to use:

$$V = |V|\angle\alpha \quad \text{and} \quad I = |I|\angle\beta \tag{1.52}$$

$$\begin{aligned} VI &= |V||I|\angle(\alpha + \beta) \quad && \text{multiplication} \\ \frac{V}{I} &= \frac{|V|}{|I|}\angle(\alpha - \beta) && \text{division} \\ V^* &= |V|\angle - \alpha && \text{complex conjugate} \end{aligned} \tag{1.53}$$

Other complex quantities, such as the complex power (which is introduced in Section 1.6.2) or impedance or admittance, can be drawn as a vector in the complex plane too as is shown in Figure 1.20, and the same mathematical rules apply for those quantities as well. These quantities, however, cannot be interpreted as phasors, as they do not have the same mathematical background.

The familiar complex operator j (i in the mathematical literature; j is common practice in the electrotechnical world in order to avoid confusion between the current and the complex operator) is in fact a vector too:

$$j = e^{j90} = 0 + j1 = 1\angle 90^\circ \tag{1.54}$$

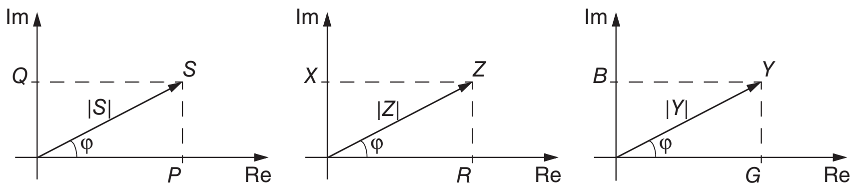


Figure 1.20 The complex power, impedance, and admittance as vectors in the complex plane.

Multiplication of a vector in the complex plane with the j operator causes a counterclockwise rotation of the vector with 90° while the length of the vector is unchanged. Another multiplication with j leads to a rotation over 180°:

$$j^2 = e^{j180} = -1 + j0 = 1\angle 180^\circ \tag{1.55}$$

In electrical power engineering another complex operator is commonly used, the so-called a operator:

$$a = e^{j120} = -0.5 + j0.866 = 1\angle 120^\circ \tag{1.56}$$

A vector multiplied in the complex plane with the a operator rotates counterclockwise by 120° while the length of the vector remains unchanged. A repeated multiplication with the a operator results in

$$\begin{aligned} a^2 &= e^{j240} = -0.5 - j0.866 = 1\angle 240^\circ \\ a^3 &= e^{j360} = 1 + j0 = 1\angle 0^\circ \end{aligned} \tag{1.57}$$

From Equations 1.56 and 1.57, it is evident that

$$1 + a + a^2 = 0 \tag{1.58}$$

This is shown graphically in Figure 1.21.

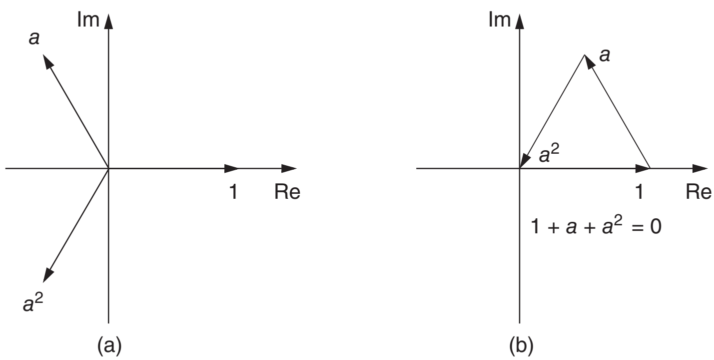


Figure 1.21 Vector diagrams of various powers of the a operator.

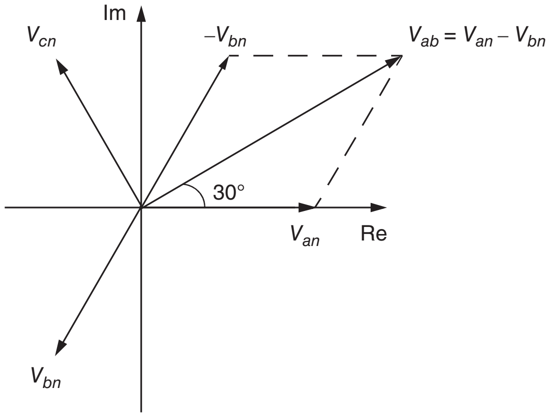


Figure 1.22 The relation between the line-to-line and the line-to-neutral voltage.

Example 1.5 ***Line-to-line and line-to-neutral voltage***

When voltage phasors are applied, the relation between the line-to-neutral and the line-to-line voltage ($|V_{LL}| = \sqrt{3}|V_{LN}|$), as found in Section 1.3.4, can be calculated as follows. The phasor representation of the line-to-neutral voltages in the phases a and b, defined earlier in Equation 1.32, are $V_{an} = |V|\angle 0°$ and $V_{bn} = |V|\angle 240°$, respectively (see Figure 1.22). The line-to-line voltage between the phases a and b is

$$\begin{aligned} V_{ab} = V_{an} - V_{bn} &= \frac{3}{2}|V| + j\frac{1}{2}\sqrt{3}|V| \\ &= \sqrt{3}|V|\angle 30° = \sqrt{3}V_{an}\angle 30° \end{aligned} \tag{1.59}$$

This is simply the phasor representation of the line-to-line voltage in the time domain as denoted in Equation 1.33. The mathematical relation can be verified graphically from Figure 1.22.

Example 1.6 ***Balanced three-phase voltage***

The voltage phasors in a balanced three-phase system can be written as (see Figure 1.22)

$$\begin{aligned} V_{an} &= |V|\angle 0° = |V| \\ V_{bn} &= |V|\angle 240° = a^2|V| \\ V_{cn} &= |V|\angle 120° = a|V| \end{aligned} \tag{1.60}$$

Therefore, the sum of these voltage phasors equals zero (see also Equation 1.58 and Figure 1.21 (b)):

$$V_{an} + V_{bn} + V_{cn} = |V|(1 + a^2 + a) = 0 \tag{1.61}$$

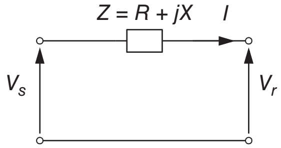


Figure 1.23 Power transport over a short single-phase transmission line.

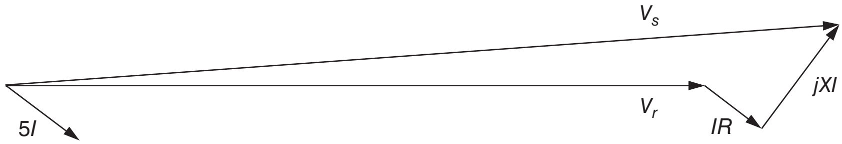


Figure 1.24 Phasor diagram.

Example 1.7 ***Phasor calculation and phasor diagram***
Given is a short single-phase transmission line, which can be modeled as a series impedance $Z = 4 + j7\ \Omega$ (see also Appendix E.4), with a (line-to-neutral) voltage at the receiving end (subscript r) of $10/\sqrt{3}\,\text{kV}$. The current amounts to $|I| = 150\,\text{A}$. At the receiving end of the line, an inductive load is present, and the current phasor lags the voltage phasor by 36.9°. This situation is depicted in Figure 1.23. We want to calculate the voltage at the sending end of the line (subscript s).

The impedance of the transmission line can be written in polar coordinates $Z = 8\angle 60^\circ\ \Omega$. When the voltage at the receiving end of the line is used as a reference, the corresponding phasor is $V_r = 5773.5\angle 0^\circ$ V, and the current phasor is $I = 150\angle -36.9^\circ$ A. The voltage at the sending end of the transmission line can be calculated in the following way:

$$\begin{aligned} V_s &= V_r + ZI = 5773.5\angle 0^\circ + 8\angle 60^\circ \cdot 150\angle -36.9^\circ \\ &= 5773.5 + 1200\angle 23.1^\circ = 6877.3 + j470.8 \\ &= 6.9\angle 3.9^\circ\,\text{kV} \end{aligned} \tag{1.62}$$

When the voltage and current phasors are drawn in a single diagram, the phasor diagram in Figure 1.24 results. The voltage drop across the transmission line ($V_s = V_r + ZI$) is now visualized. For a better visibility, the current has been drawn at a five times larger scale: $5I$ instead of I. The voltage phasor IR is the product of the current phasor I and the resistance R: only the length of the current vector changes, whereas the angle remains the same. The voltage phasor jXI is the product of the current phasor I, the reactance value X, and the complex operator j. The length of the current vector alters (XI) and the vector rotates counterclockwise by 90° (jXI).

1.5 Equivalent Line-to-neutral Diagrams

When solving balanced three-phase systems, one can work with a single-phase equivalent of the three-phase system. In fact, the consecutive steps made

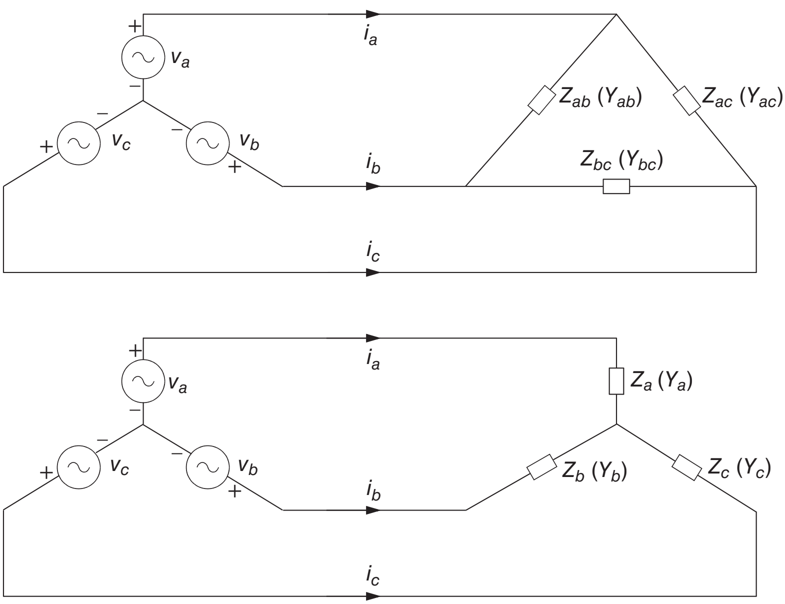


Figure 1.25 Conversion of a delta-connected load to a wye-connected load.

to arrive at the balanced three-phase system consisting of three conductors in Figure 1.8 are reversed: the three-phase system is split up into three single-phase networks, of which only one needs to be analyzed. When the voltages and currents are known in this single phase, one can simply obtain the expressions for the voltages and currents in the other two phases by rotating the corresponding phasors with 120° and 240°.

When the three-phase network contains delta-connected elements, they have to be converted to their equivalent wye connections first, as shown in Figure 1.25. The delta–wye transformation formulas for both impedances and admittances are given in Table 1.2.

Table 1.2 Delta–wye transformation.

Impedance	Admittance
$Z_a = \dfrac{Z_{ab}Z_{ac}}{Z_{ab}+Z_{ac}+Z_{bc}}$	$Y_a = \dfrac{Y_{ab}Y_{ac}+Y_{ab}Y_{bc}+Y_{ac}Y_{bc}}{Y_{bc}}$
$Z_b = \dfrac{Z_{ab}Z_{bc}}{Z_{ab}+Z_{ac}+Z_{bc}}$	$Y_b = \dfrac{Y_{ab}Y_{ac}+Y_{ab}Y_{bc}+Y_{ac}Y_{bc}}{Y_{ac}}$
$Z_c = \dfrac{Z_{ac}Z_{bc}}{Z_{ab}+Z_{ac}+Z_{bc}}$	$Y_c = \dfrac{Y_{ab}Y_{ac}+Y_{ab}Y_{bc}+Y_{ac}Y_{bc}}{Y_{ab}}$

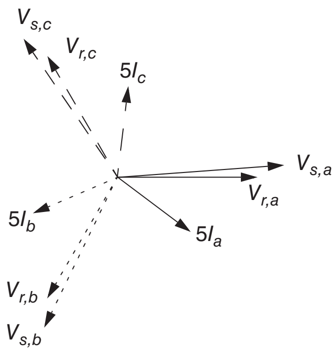


Figure 1.26 Phasors in Figure 1.24, obtained from single-phase computations (solid), are rotated counterclockwise with 120 (dashed) and 240 (dotted) degrees.

As we assume the system to be balanced (i.e., $Z_{ab} = Z_{ac} = Z_{bc}$ and $Y_{ab} = Y_{ac} = Y_{bc}$), the two following delta–wye transformation formulas can be derived:

$$Z_Y = \frac{Z_\Delta}{3} \tag{1.63}$$

$$Y_Y = 3Y_\Delta \tag{1.64}$$

Example 1.8 ***Equivalent line-to-neutral diagram and single-phase computation***

The loaded single-phase short transmission line of Example 1.7 (p. 28), with a (line-to-neutral) voltage at the receiving end of $10/\sqrt{3}$ kV, can be interpreted as an equivalent line-to-neutral diagram of a balanced three-phase short transmission line with a line-to-line voltage at the receiving end of 10 kV, which supplies a balanced wye-connected inductive load. As the voltages and currents are known in one phase (e.g., phase a), the voltages and currents in the other two phases (phases c and b) can be obtained by rotating the corresponding phasors with 120° and 240° counterclockwise as shown in Figure 1.26.

1.6 Power in Single-phase Circuits

In Section 1.3.3, we learned that the single-phase instantaneous power is a function of time and therefore not constant. In this section, we examine the power concept of a single-phase circuit more thoroughly, and we determine the relations between the voltage and current phasors and the power.

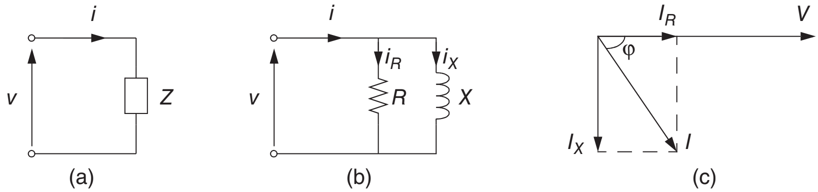


Figure 1.27 An inductive load split up into a resistor in parallel with an inductor.

1.6.1 Active and Reactive Power

The sinusoidal expressions for voltage and current of a general single-phase load, as shown in Figure 1.27 (a), are

$$v(t) = \sqrt{2}|V|\cos(\omega t) \quad \text{and} \quad i(t) = \sqrt{2}|I|\cos(\omega t - \varphi) \tag{1.65}$$

φ the phase shift between the voltage and the current: φ is positive for a current lagging the voltage and negative for a leading current

The instantaneous power consumed by the impedance Z amounts to

$$\begin{aligned} p(t) &= 2|V||I|\cos(\omega t)\cos(\omega t - \varphi) \\ &= |V||I|\cos(\varphi)[1 + \cos(2\omega t)] + |V||I|\sin(\varphi)\sin(2\omega t) \\ &= P[1 + \cos(2\omega t)] + Q\sin(2\omega t) \end{aligned} \tag{1.66}$$

The first term in Equation 1.66 ($P[1 + \cos(2\omega t)]$) describes an unidirectional component of the instantaneous power with an average value P, which is called the average power and is also addressed as real or active power. So

$$P = |V||I|\cos(\varphi) \tag{1.67}$$

P the active/real/average power [W]

$\cos(\varphi)$ the power factor (the cosine of the phase shift between the voltage and current (or, in other words, the cosine of the phase angle between the voltage and current phasor (see Figure 1.27 (c))))

The second term in Equation 1.66 ($Q\sin(2\omega t)$) is alternately positive and negative and has an average value of zero. This term describes a bidirectional, that is, oscillating, component of the instantaneous power. When this term has a positive sign, the power flow is toward the load; when the sign is negative, the power flows from the load back to the source of supply. Because the average value of this oscillating power component equals zero, it gives on average no transfer of energy toward the load. The amplitude of this oscillating power is called imaginary or reactive power Q and is defined as

$$Q = |V||I|\sin(\varphi) \tag{1.68}$$

Q the reactive/imaginary power [var; reactive volt-amperes]

A better understanding of these aspects of electrical power is obtained when an inductive load is modeled as a resistor in parallel with an inductor as visualized in Figure 1.27 (b). The current can be resolved into two components: one that is in phase with the voltage (the current through the resistor) and one that is 90° out of phase (the current through the inductor). As the length of the current phasor in phase with the voltage equals $|I_R| = |I|\cos(\varphi)$ (see the phasor diagram in Figure 1.27 (c)), the instantaneous current component in phase with the voltage can be written as

$$i_R = \sqrt{2}|I_R|\cos(\omega t) = \sqrt{2}|I|\cos(\varphi)\cos(\omega t) \tag{1.69}$$

Similarly, the length of the current phasor lagging the voltage with 90° equals $|I_X| = |I|\sin(\varphi)$, and the instantaneous current component lagging the voltage with 90° can be written as

$$i_X = \sqrt{2}|I_X|\sin(\omega t) = \sqrt{2}|I|\sin(\varphi)\sin(\omega t) \tag{1.70}$$

The instantaneous power consumed by the resistor (see Figure 1.28 (b)) equals

$$\begin{aligned} vi_R &= 2|V||I|\cos(\varphi)\cos^2(\omega t) \\ &= |V||I|\cos(\varphi)[1 + \cos(2\omega t)] \\ &= P[1 + \cos(2\omega t)] \end{aligned} \tag{1.71}$$

The instantaneous power toward the inductor (see Figure 1.28 (c)) equals

$$\begin{aligned} vi_X &= 2|V||I|\sin(\varphi)\sin(\omega t)\cos(\omega t) \\ &= |V||I|\sin(\varphi)\sin(2\omega t) \\ &= Q\sin(2\omega t) \end{aligned} \tag{1.72}$$

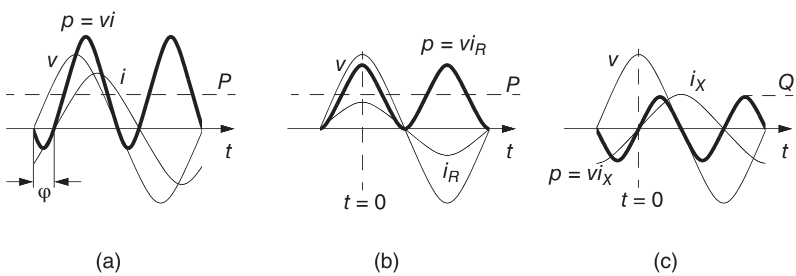
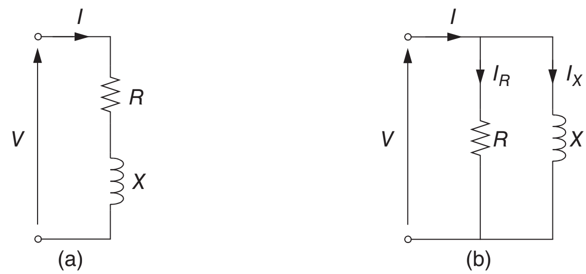


Figure 1.28 Voltage, current, and instantaneous power of an inductive load.

Figure 1.29 A simple series (a) and parallel (b) circuit.



When we add both instantaneous power components (see Figure 1.28 (a)), we recognize the same expression as found earlier in Equation 1.66:

$$p = vi = vi_R + vi_X \tag{1.73}$$

Therefore, of the instantaneous power p, consumed by an element at any instant, a part (vi_R) is utilized for permanent consumption, such as conversion into heat. This part always has a positive value, that is, it cannot be returned to the rest of the circuit. The remainder (vi_X) is used to establish either a magnetic or electric field, that is, it is taken up and returned to the circuit with the rhythm of double the power frequency.

In a simple series circuit, as shown in Figure 1.29 (a), easier expressions for the active and reactive power can be derived. When the series impedance is written as $Z = R + jX$ and $|V| = |Z||I|$ is substituted into the equations for the active and reactive power, the following equations are the result:

$$P = |V||I|\cos(\varphi) = |I|^2|Z|\cos(\varphi) \tag{1.74}$$

$$Q = |V||I|\sin(\varphi) = |I|^2|Z|\sin(\varphi) \tag{1.75}$$

When we recognize that $R = |Z|\cos(\varphi)$ and $X = |Z|\sin(\varphi)$ (see Figure 1.20), the active and reactive power consumed by a series impedance can be written as

$$P = |I|^2 R \quad \text{and} \quad Q = |I|^2 X \tag{1.76}$$

In a simple parallel circuit, as shown in Figure 1.29 (b), easier expressions for the active and reactive power can also be derived. Deconstructing the current phasor into two components with a length $|I_R| = |I|\cos(\varphi)$ and $|I_X| = |I|\sin(\varphi)$ (see the phasor diagram in Figure 1.27 (c)) results in the following equations for the active and reactive power:

$$P = |V||I|\cos(\varphi) = |V||I_R| \tag{1.77}$$

$$Q = |V||I|\sin(\varphi) = |V||I_X| \tag{1.78}$$

When we realize that $|V| = |I_R|R = |I_X|X$, the active and reactive power consumed by a parallel impedance can be written as

$$P = \frac{|V|^2}{R} \quad \text{and} \quad Q = \frac{|V|^2}{X} \tag{1.79}$$

Example 1.9 ***Single-phase power***

In Example 1.7 (p. 28), the single-phase inductive load consumes both active and reactive power (an explanation why the word "consumes" has been used here is given in Section 1.6.2):

$$P = |V_r||I|\cos(\varphi) = \frac{10\times10^3}{\sqrt{3}} \cdot 150 \cdot \cos(36.9) = 692.5\,\text{kW} \tag{1.80}$$

$$Q = |V_r||I|\sin(\varphi) = \frac{10\times10^3}{\sqrt{3}} \cdot 150 \cdot \sin(36.9) = 520\,\text{kvar} \tag{1.81}$$

The inductive load can be represented as a resistance in parallel with an inductance. The component values can be computed from the following relations (the frequency is 50 Hz):

$$Q = \frac{|V_r|^2}{X} \rightarrow 520\times10^3 = \left(\frac{10\times10^3}{\sqrt{3}}\right)^2 /(2\pi f L) \tag{1.82}$$

$$P = \frac{|V_r|^2}{R} \rightarrow 692.5\times10^3 = \left(\frac{10\times10^3}{\sqrt{3}}\right)^2 /R \tag{1.83}$$

This results in the following values: $L = 0.2$ H and $R = 48.1\ \Omega$.

The active power loss in the transmission line can be computed as follows:

$$P = |I|^2 R = (150)^2 \cdot 4 = 90\,\text{kW} \tag{1.84}$$

The reactive power consumed by the transmission line amounts to

$$Q = |I|^2 X = (150)^2 \cdot 7 = 157.5\,\text{kvar} \tag{1.85}$$

1.6.2 Complex Power

We now examine whether the expressions that we derived for the active and reactive power can be obtained from some kind of multiplication of the voltage phasor ($V = |V|\angle\alpha$) and the current phasor ($I = |I|\angle\beta$) as shown in Figure 1.30. The angle between the voltage and the current phasor is defined as $\varphi = \alpha - \beta$. Direct multiplication of both phasors gives the following expression:

$$\begin{aligned} VI &= |V||I|\angle(\alpha+\beta) \\ &= |V||I|[\cos(\alpha+\beta) + j\sin(\alpha+\beta)] \end{aligned} \tag{1.86}$$

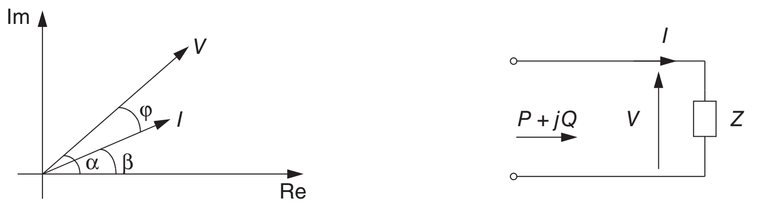


Figure 1.30 Phasor diagram of a single-phase load.

It is obvious that the equations of the active and reactive power do not result. Two other voltage and current phasor multiplications are shown in the following:

$$\begin{aligned} V^{*}I &= |V||I|\angle(-\alpha+\beta) \\ &= |V||I|[\cos(-\alpha+\beta)+j\sin(-\alpha+\beta)] \\ &= |V||I|[\cos(\varphi)-j\sin(\varphi)] \\ &= P-jQ \end{aligned} \tag{1.87}$$

$$\begin{aligned} VI^{*} &= |V||I|\angle(\alpha-\beta) \\ &= |V||I|[\cos(\alpha-\beta)+j\sin(\alpha-\beta)] \\ &= |V||I|[\cos(\varphi)+j\sin(\varphi)] \\ &= P+jQ \end{aligned} \tag{1.88}$$

In both equations, an active and reactive power component is present, but how about the sign of the reactive power?

We adopt the sign convention recommended by the International Electrotechnical Commission (IEC). A capacitor supplies reactive power, whereas an inductor consumes reactive power. Or in other words, the reactive power absorbed by an inductive load has a positive sign, and the reactive power absorbed by a capacitive load a negative sign. In the case that $\alpha > \beta$ (see Figure 1.30), the current lags the voltage. Therefore, the load is inductive and, in line with the IEC convention, consumes reactive power. To obtain the proper sign for the reactive power, it is necessary to calculate VI^{*}.

The mode of operation of an element, in terms of active and reactive power, can be seen from a quadrant diagram as shown in Figure 1.31. The non-reference phasor points to the quadrant of operation. As an example, let us consider the situation shown in the quadrant diagram in Figure 1.31. The current lags behind the voltage. Therefore, the load is inductive and consumes both active (+P) and reactive power (+Q). The load can be represented by a resistance in series with an inductance as shown in quadrant 1. Note that the quadrants 2 and 3 require an active element, that is, an underexcited generator and an overexcited generator (see also Section 2.5).

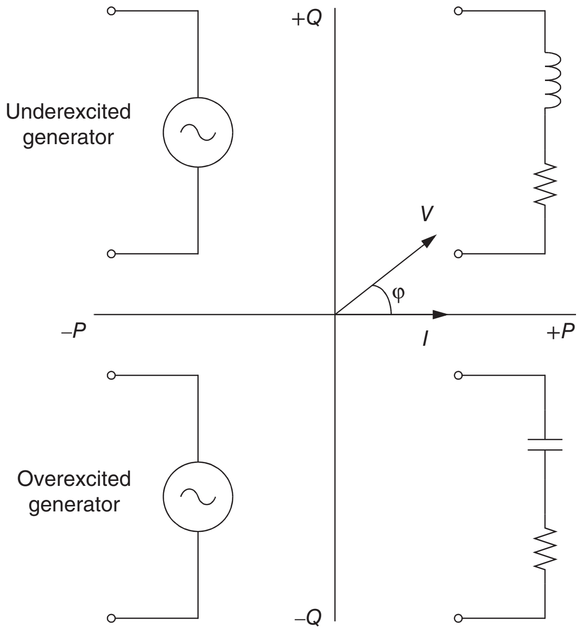


Figure 1.31 Quadrant diagram.

The quantity, obtained after the multiplication of the voltage phasor and the complex conjugated of the current phasor, is called the complex power S:

$$S = VI^* = P + jQ \tag{1.89}$$

S the complex power [VA]

The complex power S is a complex quantity and can be expressed in both polar coordinates and rectangular or Cartesian coordinates, as shown in Figure 1.32:

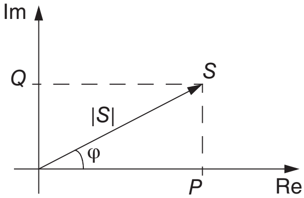
$$\begin{aligned} S &= VI^* \\ &= |V||I|\angle\varphi \\ &= |S|\angle\varphi \quad \text{(polar)} \\ &= P + jQ \quad \text{(Cartesian)} \end{aligned} \tag{1.90}$$

φ the phase shift between the voltage and the current [rad]
$|S|$ the apparent power [VA]

The apparent power $|S|$ is defined as (see also Figure 1.32)

$$|S| = |V||I| = \sqrt{P^2 + Q^2} \tag{1.91}$$

The apparent power is a useful practical quantity for specifying the rating of electrical apparatus when the maximum voltage and maximum current are fixed, and the phase angle is not considered.



$$S = |S|\angle\varphi = P + jQ$$
$$|S| = \sqrt{P^2 + Q^2}$$
$$\varphi = \tan^{-1}\left(\frac{Q}{P}\right)$$
$$P = |S|\cos(\varphi)$$
$$Q = |S|\sin(\varphi)$$

Figure 1.32 The complex power (consumed by an inductive load).

Table 1.3 Power definitions.

Symbols	Terminology	Units
p	Instantaneous power	W
S	Complex power	VA
$\|S\|$	Apparent power	VA
P	Active power	W
	Real power	
	Average power	
Q	Reactive power	var
	Imaginary power	

The different symbols that we use to address electrical power are summarized in Table 1.3. There is no “deeper meaning” behind the different units (W, VA, var) that are used to express the different types of power values. (It is in fact no different from the impedance terms $Z = R + jX$; we do not use three different kinds of ohm!) But it is convenient that we are able to read from the unit which type of power is addressed.

Example 1.10 ***Complex power***

Consider Example 1.7 (p. 28). In Example 1.9 (p. 34), the active and reactive power consumed by the single-phase inductive load were calculated:

$$P = |V_r||I|\cos(\varphi) = \frac{10 \times 10^3}{\sqrt{3}} \cdot 150 \cdot \cos(36.9) = 692.5\,\text{kW} \tag{1.92}$$

$$Q = |V_r||I|\sin(\varphi) = \frac{10 \times 10^3}{\sqrt{3}} \cdot 150 \cdot \sin(36.9) = 520\,\text{kvar} \tag{1.93}$$

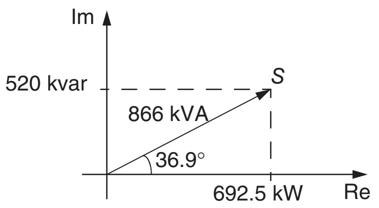


Figure 1.33 The complex power consumed by the inductive load.

Therefore, the complex power consumed by the load amounts to

$$\begin{aligned} S &= P + jQ = 692.5 + j520\,\text{kVA} \\ &= |S|\angle\varphi = 866\angle 36.9^\circ\,\text{kVA} \end{aligned} \tag{1.94}$$

This relation is displayed in Figure 1.33. The complex power can also be calculated in a different way:

$$S = V_r I^* = 5773.5\angle 0^\circ \cdot 150\angle 36.9^\circ = 866\angle 36.9^\circ\,\text{kVA} \tag{1.95}$$

Taking the real and imaginary parts of this complex power results in the values for the active and reactive power that we found earlier.

1.6.3 Power Factor

In previous sections, the active power has been defined as

$$P = |V||I|\cos(\varphi) = |S|\cos(\varphi) \tag{1.96}$$

The term $\cos(\varphi)$ is called the power factor, being the cosine of the phase shift between the voltage and current, that is, the cosine of the phase angle between the voltage and current phasor. In fact, the power factor is that part of the apparent power that is related to the mean energy flow, like mechanical energy in the case of a machine or heat in the case of a resistor.

The power factor can be computed by using several (equivalent) formulas. They can be obtained easily by inspection of the power diagram in Figure 1.32:

$$\begin{aligned} \cos(\varphi) &= \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}} \\ &= \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right) \end{aligned} \tag{1.97}$$

The power factor gives the information to calculate the active power from the apparent power. When we want to calculate the reactive power that is consumed, extra information is needed in order to specify whether the consumed reactive power is positive or negative. This information is expressed in words. When the current lags the voltage, the power factor is said to be lagging. When the current leads the voltage, the power factor is said to be leading.

It is evident from Equation 1.96 that a value of the power factor ($\cos(\varphi)$) close to 1 is beneficial. When a certain amount of active power is consumed at a fixed voltage, a power factor lower than 1 leads to increased currents and higher ohmic losses in the power system. From the utility point of view, it is therefore desirable that the large (industrial) loads have a power factor close to 1. Unfortunately, most of the heavy-industrial loads are inductive (electrical drives, for instance) and have rather low power factors. As a consequence, power-factor corrections should be made by these larger industries to improve their power factor. The power factors for households and small commercial users are usually closer to 1.

Example 1.11 ***Power factor***

Consider Example 1.7 (p. 28). The power factor is defined as the cosine of the phase angle between the voltage and current phasor. Thus, the power factor of the inductive load is

$$\cos(\varphi) = \cos(36.9) = 0.8 \tag{1.98}$$

Because the current lags the voltage, the power factor is said to be 0.8 lagging. This value can be calculated from the consumed power as well (see also Example 1.10 (p. 37)):

$$\cos(\varphi) = \frac{P}{|S|} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{692.5 \times 10^3}{866 \times 10^3} = 0.8 \tag{1.99}$$

$$\cos(\varphi) = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right) = \cos\left(\tan^{-1}\left(\frac{520 \times 10^3}{692.5 \times 10^3}\right)\right) = 0.8 \tag{1.100}$$

Example 1.12 ***Power-factor improvement***

Consider Example 1.7 (p. 28) and the previous example. The power factor of the inductive load amounts to 0.8 and is to be increased to the value of 1. This can be done by connecting a capacitor in parallel to the load. The value of the capacitance is such that the capacitor supplies the amount of reactive power that is consumed by the inductive load. When we assume that the voltage at the load terminals remains fixed, the required capacitance can be calculated as follows:

$$Q = \frac{|V_r|^2}{-X} \rightarrow 520 \times 10^3 = \left(\frac{10 \times 10^3}{\sqrt{3}}\right)^2 (2\pi f C) \tag{1.101}$$

The capacitance value amounts to $C = 50\,\mu\text{F}$.

After the power-factor correction, $\cos(\varphi) = 1$, the current equals

$$|I| = \frac{P}{|V_r|\cos(\varphi)} = \frac{692.5 \times 10^3}{(10 \times 10^3/(\sqrt{3})) \cdot 1} = 120\,\text{A} \tag{1.102}$$

Now, the power loss in the short transmission line is

$$P_{\text{loss}} = |I|^2 R = (120)^2 \cdot 4 = 57.6\,\text{kW} \tag{1.103}$$

which means that considerable savings, by a loss reduction from 90 kW at a $\cos(\varphi) = 0.8$ to 57.6 kW at a $\cos(\varphi) = 1$, have been established.

1.7 Power in Three-phase Circuits

The power (consumed by a load or produced by a generator) in a three-phase network can be found easily by adding up the power for each of the three phases. For balanced three-phase systems, the three-phase complex power is three times the complex power of the single-phase equivalent network. Therefore, the following equations hold for the complex, apparent, active, and reactive power:

$$S_{3\phi} = 3S_{1\phi} = 3V_{\text{LN}}I^* \tag{1.104}$$

$$|S_{3\phi}| = 3|S_{1\phi}| = 3|V_{\text{LN}}||I| = \sqrt{3}|V_{\text{LL}}||I| \tag{1.105}$$

$$P_{3\phi} = 3P_{1\phi} = 3|V_{\text{LN}}||I|\cos(\varphi) = \sqrt{3}|V_{\text{LL}}||I|\cos(\varphi) \tag{1.106}$$

$$Q_{3\phi} = 3Q_{1\phi} = 3|V_{\text{LN}}||I|\sin(\varphi) = \sqrt{3}|V_{\text{LL}}||I|\sin(\varphi) \tag{1.107}$$

V_{LL} the line-to-line voltage phasor
V_{LN} the line-to-neutral voltage phasor
I the current phasor

Example 1.13 ***Three-phase power***

The loaded single-phase short transmission line in Example 1.7 (p. 28), with a (line-to-neutral) voltage at the receiving end of $|V_{r,\text{LN}}| = 10/\sqrt{3}\,\text{kV}$, can be interpreted as an equivalent line-to-neutral diagram of a balanced three-phase short transmission line with a line-to-line voltage at the receiving end of $|V_{r,\text{LL}}| = 10\,\text{kV}$, which is loaded by a balanced wye-connected inductive load. The three-phase active power consumed by the load can be calculated in the following (equivalent) ways (see also Example 1.9 (p. 34)):

$$P_{3\phi} = 3P_{1\phi} = 3 \cdot 692.5 \times 10^3 = 2077.5\text{ kW} \tag{1.108}$$

$$\begin{aligned} P_{3\phi} &= 3|V_{r,\text{LN}}||I|\cos(\varphi) \\ &= 3 \cdot \frac{10 \times 10^3}{\sqrt{3}} \cdot 150 \cdot \cos(36.9) = 2077.5\text{ kW} \end{aligned} \tag{1.109}$$

$$P_{3\phi} = \sqrt{3}|V_{r,\mathrm{LL}}||I|\cos(\varphi) = \sqrt{3}\cdot 10\times 10^3\cdot 150\cdot\cos(36.9) = 2077.5\ \mathrm{kW} \tag{1.110}$$

1.8 Per-unit Normalization

In power engineering a normalization of numerical values is common practice. This so-called per-unit (pu) normalization is similar to calculating with percentages. In the percentage system, the product of two quantities expressed as percentages must be divided by 100 to obtain the result in percentage. This drawback is circumvented in the pu system. In the pu system, 100% corresponds to 1 pu. The pu value of a certain quantity is defined as

$$\text{Per-unit value} = \frac{\text{actual value}}{\text{base value}} \tag{1.111}$$

In power system analysis, the following quantities are of interest and frequently normalized: voltages, currents, impedances, and powers. These quantities are interrelated and this means that a selection of two base values fixes immediately the base values for the other two. Therefore, selecting a base for the pu calculation requires a selection of two base values only, and the other two base values can be calculated. Taking into account the constant voltage nature of the power system and the fact that the equipment is rated in volt-amperes, the base voltage and (apparent) power are usually the quantities to specify the base values, as shown in Table 1.4.

Example 1.14 ***Base quantities and per-unit calculation***
Consider Example 1.7 (p. 28). The calculation of the voltage at the sending end of the short transmission line is repeated here in per-unit quantities. The base

Table 1.4 Base quantities in the per-unit system.

Base Quantity	Single Phase (Line-to-Neutral Voltage, Single-Phase Power)	Three Phase (Line-to-Line Voltage, Three-Phase Power)
Voltage [V]	$\|V_b\|$ (selected)	$\|V_b\|$ (selected)
(Apparent) Power [VA]	$\|S_b\|$ (selected)	$\|S_b\|$ (selected)
Current [A]	$\|I_b\| = \frac{\|S_b\|}{\|V_b\|}$	$\|I_b\| = \frac{\|S_b\|}{\sqrt{3}\|V_b\|}$
Impedance [Ω]	$\|Z_b\| = \frac{\|V_b\|}{\|I_b\|} = \frac{\|V_b\|^2}{\|S_b\|}$	$\|Z_b\| = \frac{\|V_b\|/\sqrt{3}}{\|I_b\|} = \frac{\|V_b\|^2}{\|S_b\|}$

voltage and base (apparent) power are selected: $|V_b| = |V_{r,\mathrm{LN}}| = 10/\sqrt{3}\,\mathrm{kV}$ (line-to-neutral) and $|S_b| = 1\,\mathrm{MVA}$ (single-phase power). Now the base current and base impedance can be calculated:

$$|I_b| = \frac{|S_b|}{|V_b|} = \frac{1 \times 10^6}{10 \times 10^3/\sqrt{3}} = 173.2\,\mathrm{A} \tag{1.112}$$

$$|Z_b| = \frac{|V_b|^2}{|S_b|} = \frac{(10 \times 10^3/\sqrt{3})^2}{1 \times 10^6} = 33.3\,\Omega \tag{1.113}$$

The network quantities expressed in pu are obtained by dividing the actual value by the corresponding pu value: $|V_{r,\mathrm{LN}}| = 1\,\mathrm{pu}$, $I = 0.87\angle{-36.9^\circ}\,\mathrm{pu}$, $Z = 0.24\angle 60^\circ\,\mathrm{pu}$. Now, the voltage in pu at the sending end of the transmission line can be computed:

$$\begin{aligned} V_{s,\mathrm{LN}} &= V_{r,\mathrm{LN}} + ZI = 1\angle 0^\circ + 0.24\angle 60^\circ \cdot 0.87\angle{-36.9^\circ} \\ &= 1 + 0.21\angle 23.1^\circ \\ &= 1.19 + j0.08 = 1.19\angle 3.9^\circ\,\mathrm{pu} \end{aligned} \tag{1.114}$$

The value in volts is found by multiplication of the pu value with the base voltage value:

$$V_{s,\mathrm{LN}} = \frac{10 \times 10^3}{\sqrt{3}} \cdot 1.19\angle 3.9^\circ = 6.9\angle 3.9^\circ\,\mathrm{kV} \tag{1.115}$$

This is the same value as we found earlier in Example 1.7 (p. 28).

There are several reasons why the pu normalization is used. First of all, it is more comfortable to work with a voltage value of, for example, 1.00895 pu than with a value of 151343 V (in this case, 1 pu corresponds to a base value of 150 kV): the actual deviation in the voltage value can be more easily observed. Another advantage is that a pu quantity contains "more information." Consider, for instance, a voltage drop of 1500 V along a transmission line. This information is rather insignificant since we have no information about the nominal voltage rating of the transmission line. A voltage drop of 1500 V on a 10 kV transmission line is extraordinary, while on a 150 kV transmission line it is an acceptable value. When we have a pu value, for example, a voltage drop of 0.015 pu, all relevant information for a useful interpretation is present: the voltage drop amounts to 1.5% of the base voltage level. Another advantage of the pu calculation appears when we deal with systems with transformers. When the base quantities are selected properly, the transformers disappear from our line diagrams if we suppose them to be ideal. In Figure 1.34, an ideal transformer is shown, and the relations between voltage and current at the primary side

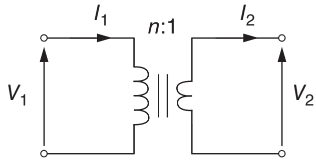


Figure 1.34 An ideal (single-phase) transformer.

(subscript 1) and secondary side (subscript 2) are (see also Appendix B.2)

$$|V_2| = \frac{|V_1|}{n} \tag{1.116}$$

$$|I_2| = n|I_1| \tag{1.117}$$

n the turns ratio: the number of windings at the primary side divided by the number of windings at the secondary side ($n = N_1/N_2$)

To achieve the previously mentioned advantage, the base (apparent) power ($|S_b|$) is selected and must be the same at both the primary and the secondary sides of the transformer.

Furthermore, the base voltages at the primary and secondary sides of the transformer should be selected such that they have the same ratio as the turns ratio of the transformer windings:

$$|V_b|_2 = \frac{|V_b|_1}{n} \tag{1.118}$$

In that case, the base currents at the primary and secondary sides are related as follows:

$$|I_b|_2 = \frac{|S_b|}{|V_b|_2} = n \cdot \frac{|S_b|}{|V_b|_1} = n|I_b|_1 \tag{1.119}$$

Dividing the voltage relation of the ideal transformer (Equation 1.116) by the base voltage $|V_b|_2$ and substituting the relation $|V_b|_2 = |V_b|_1/n$ results in

$$\frac{|V_2|}{|V_b|_2} = \frac{|V_1|}{n|V_b|_2} = \frac{|V_1|}{|V_b|_1} \tag{1.120}$$

In other words, the pu voltage at the secondary side of the ideal transformer equals the pu voltage at the primary side. The same procedure can be followed for the current. Dividing the current relation of the ideal transformer (Equation 1.117) by the base current $|I_b|_2$ and substituting the relation $|I_b|_2 = n|I_b|_1$ results in

$$\frac{|I_2|}{|I_b|_2} = n \cdot \frac{|I_1|}{|I_b|_2} = \frac{|I_1|}{|I_b|_1} \tag{1.121}$$

In other words, the pu current at the secondary side of the ideal transformer equals the pu current at the primary side. As both the pu voltage and the pu current are equal at the primary and secondary sides of the ideal transformer, the electrotechnical symbol of the ideal transformer does not serve any purpose in the circuit diagram and can be left out.

Example 1.15 ***Per-unit calculation and ideal transformers***

Consider the single-phase power system of Figure 1.35. In the system are two ideal transformers with ratings:

$$A - B\text{: } 10\,\text{MVA}, 13.8\,\text{kV}/138\,\text{kV}$$
$$B - C\text{: } 10\,\text{MVA}, 138\,\text{kV}/69\,\text{kV}$$

As base power the value for the apparent power is chosen: $|S_b| = 10\,\text{MVA}$. The base voltage in circuit B is chosen to be equal to $|V_b|_B = 138\,\text{kV}$. The base voltages in the circuits A and C are derived from the base voltage in circuit B and the turns ratio of the transformers:

$$\begin{aligned} |V_b|_A &= \frac{1}{10} \cdot |V_b|_B = 13.8\,\text{kV} \\ |V_b|_C &= \frac{1}{2} \cdot |V_b|_B = 69\,\text{kV} \end{aligned} \tag{1.122}$$

Now, the base impedances in the three circuits can be calculated as

$$\begin{aligned} |Z_b|_A &= \frac{|V_b|_A^2}{|S_b|} = 19\,\Omega \\ |Z_b|_B &= \frac{|V_b|_B^2}{|S_b|} = 1904\,\Omega \\ |Z_b|_C &= \frac{|V_b|_C^2}{|S_b|} = 476\,\Omega \end{aligned} \tag{1.123}$$

The resistive load in circuit C, expressed in pu, is

$$R_C = \frac{100}{|Z_b|_C} = \frac{100}{476} = 0.21\,\text{pu} \tag{1.124}$$

When the resistance is referred to circuit B (see also Appendix B.2) and expressed in pu, the following value is the result:

$$R_B = 2^2 \cdot 100 = 400\,\Omega \quad R_B = \frac{400}{|Z_b|_B} = \frac{400}{1904} = 0.21\,\text{pu} \tag{1.125}$$

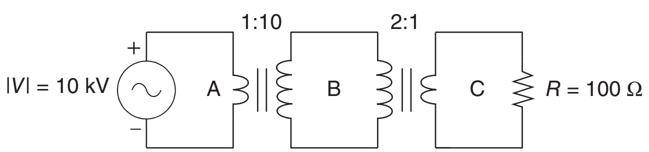


Figure 1.35 Single-phase circuit with ideal transformers.

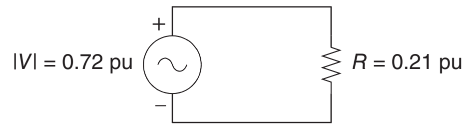


Figure 1.36 Per-unit diagram of the single-phase circuit in Figure 1.35.

When the resistance is referred to circuit A and expressed in pu, the following value results:

$$R_A = 0.1^2 \cdot 2^2 \cdot 100 = 4\ \Omega \quad R_A = \frac{4}{|Z_b|_A} = \frac{4}{19} = 0.21\,\text{pu} \tag{1.126}$$

Because of the well-chosen base quantities, the pu resistance of the load, when referred to another part of the system, is the same as the pu resistance of the load in its original position. It is evident that the single-phase circuit of Figure 1.35, when expressed in pu, turns into the simplified circuit, which is shown in Figure 1.36, in which all the ideal transformers have disappeared.

1.9 Power System Structure

The graphical layout of the three-phase power system is often displayed as a single-line equivalent or as a one-line diagram. Such a one-line diagram gives only an overview of the topology of the power system. The components are identified by means of standardized symbols and not by models built up from lumped-circuit elements (as is the case with the equivalent line-to-neutral diagram as introduced in Section 1.5). The symbols used in such a one-line diagram are shown in the list of symbols. Sometimes lumped-circuit elements appear in a "one-line diagram." An example of such a "one-line diagram" is shown in Figure 5.28 (Section 5.5.3): it shows a device used in the power system for sophisticated control actions. The lumped elements in the diagram show what is actually inside the device.

A one-line diagram that schematically displays part of the structure of a power system is shown in Figure 1.37. The one-line diagram shows a clear vertical structure: a relatively small number of large power stations supply the transmission network (380 and 150 kV). Besides some large industrial consumers that are connected to the higher voltage levels (150 kV), most of the power is transported and distributed to the consumption centers located at the lowest voltage levels (10 and 0.4 kV). Nowadays, more and more decentralized generation, that is, small-scale generators connected to the lower voltage levels (like the generator connected to the 10 kV bus shown in Figure 1.37), is being integrated in the system (see also Section 8.3). Examples of such decentralized generators are windmills, solar panels, and combined heat-power units (producing steam for industrial processes and electricity as a by-product).

Our book closely follows the vertical structure of the power system. In Chapter 2 the generation of electric energy is examined. In Chapter 3 the

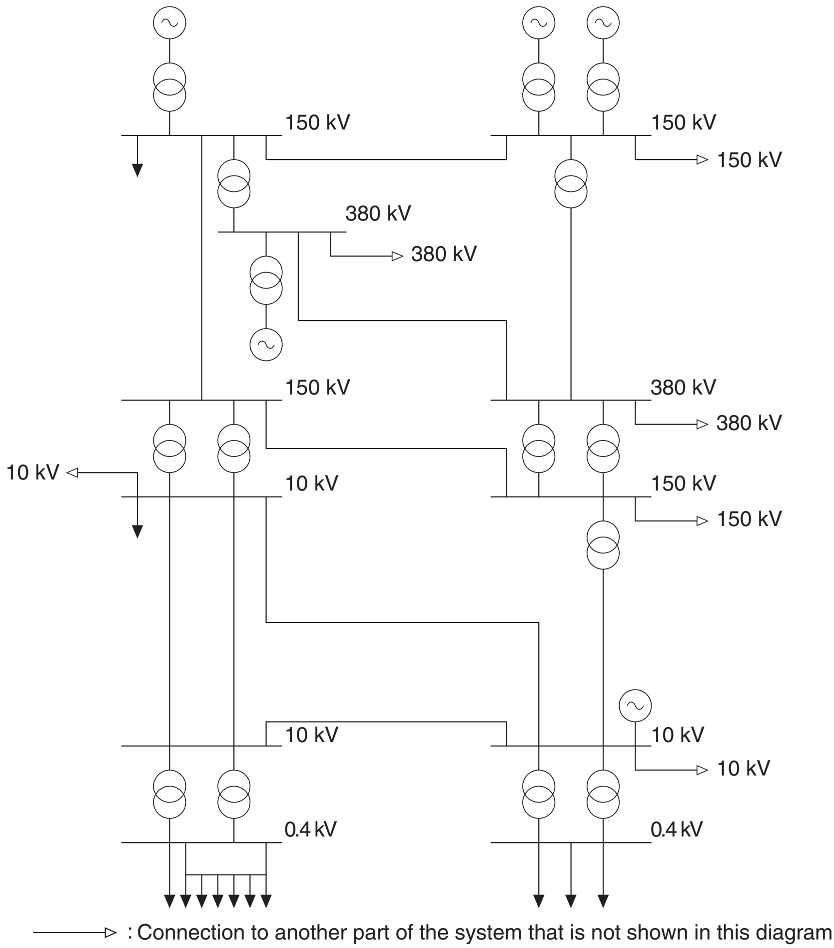


Figure 1.37 One-line diagram of a section of a power system [10].

transmission and distribution are highlighted. The utilization of electricity is described in Chapter 4. Without controls, the power system cannot function, and the controls implemented are presented in Chapter 5. In the control center, the generation, transmission, and distribution of electrical energy are monitored, coordinated, and controlled by means of the energy management system (EMS). The EMS is the interface between the operator and the power system, and its fundamentals are outlined in Chapter 6. The electricity market, where all the previously treated items meet each other in a commercial sense, is introduced in Chapter 7. The book concludes by questioning the vertical structure of the power system in Chapter 8, when some thoughts on future power systems are offered. The relation between Maxwell's laws and lumped-element modeling is the topic of the first appendix. The lumped-element models for the power transformer, the synchronous machine, the induction machine, and the overhead lines and underground cables are derived in the other appendices.

Problems

1.1 For a specific node in a 50 Hz single-phase circuit, the voltage equals $v(t) = 325.27 \sin(\omega t + \varphi)$ V, and the injected current is $i(t) = 141.42 \sin(\omega t - \varphi)$ A, where $\varphi = \pi/6$ rad.

a. Calculate for $v(t)$ and $i(t)$ the peak value and the RMS value.
b. Express the voltage and current as phasors.
c. Is the circuit capacitive or inductive? Explain your answer.

1.2 In a three-phase circuit, the voltages of the phases a and b, with respect to the neutral n, are $V_{an} = 140\angle 45°$ V and $V_{bn} = 90\angle -15°$ V. Calculate V_{ba}.

1.3 In a balanced three-phase system, with a phase sequence abc, the Y-connected impedances are $Z = 10\angle 30°\ \Omega$. If $V_{bc} = 400\angle 90°$ V, calculate

a. V_{cn}
b. I_{cn}
c. S consumed by the impedances

1.4 Two ideal voltage sources, $E_1 = 100\angle 0°$ V and $E_2 = 100\angle 30°$, are connected through an impedance $Z = j5\ \Omega$. For both voltage sources the generator convention is used, which means that the power delivered by the voltage sources is positive.

a. Calculate the currents I_1 and I_2 delivered by both sources.
b. Calculate the active power and the reactive power consumed by both sources.
c. Which of the two sources is the generator?
d. Calculate the losses.

1.5 In a balanced three-phase system, the power injected at node m equals $S = 100 + j60$ MVA, while the line-to-line voltage equals $380\angle 0°$ kV.

a. Determine the power factor of the supplied power.
b. Give an expression for the current phasor magnitude.

1.6 In a balanced three-phase system, the voltage between phases *a* and *b* is $V_{ab} = 173.2\angle 0°$ V. The Y-connected load is $Z = 10\angle 0°\ \Omega$. The phase sequence is *abc*.
a. Calculate all phase-to-neutral voltages.
b. Calculate all phase currents.

1.7 A three-phase Y-connected load consumes 250 kW, with a power factor of 0.8 lagging from a 400 V line. In parallel with this load is a three-phase capacitor bank connected, which delivers 60 kvar.
a. Calculate the total phase current (combined load and capacitor bank).
b. What is the resulting power factor?

1.8 The equivalent circuit to represent a load on a 110 kV transmission line consists of three Y-connected impedances of $Z = 80 + j30\ \Omega$. Assuming an RMS line-to-line voltage of 100 kV,
a. What are the phase currents drawn by the load?
b. What is the active and reactive power drawn by the load?

1.9 A three-phase transmission line can be represented by a phase impedance of $Z = 5 + j60\ \Omega$/phase. The complex power, measured at the sending end of the line, is $S_1 = 210 + j30$ MVA. The line has a fixed line-to-line voltage of 220 kV at the sending end.
a. Calculate the voltage and the complex power at the receiving end of the line.
b. What are the transmission losses?

1.10 Redo Problem 1.9 and use per-unit values. The base quantities are
- $|S_b| = 300$ MVA (three phase)
- $|V_b| = 130$ kV (line-to-neutral)

1.11 In a base system $|S_{b,1}|$ and $|V_{b,1}|$, the base impedance equals $|Z_{b,1}|$. $|Z_{b,2}|$ is the base impedance in a base system using $|S_{b,2}|$ and $|V_{b,2}|$.
Show that the following equation holds:

$$|Z_{b,2}| = |Z_{b,1}| \frac{|S_{b,1}|}{|S_{b,2}|} \frac{|V_{b,2}|^2}{|V_{b,1}|^2}$$

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