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Overview

1.1 Active Power Loss and Electric Energy Loss

In an electric supply area, electric energy is supplied to customers through transmission, substation, and power grid distribution. During the transmission and distribution of electric energy, a certain quantity of active power loss and electric energy loss will be generated in all units of the power grids.

1.1.1 Main Types of Active Power Loss

According to the analysis based on electromagnetic field theory, the energy of an electromagnetic field is transmitted from the power source to the loads through the dielectric space of the electromagnetic field, and wires lead the energy of the electromagnetic field. The electric energy loss that goes into the wires and is then converted into heat energy is also supplied by the electromagnetic field.

According to the results of the analysis of a single core coaxial cable by using the Poynting vector of energy flow density in the case of AC transmission, while power is needed to transmit loads in the dielectric space, four types of active power loss are produced in the cable:

1. *Resistance heat loss* ΔP_1 *(W)*
   This is in direct proportion to the square of current, that is

$$\Delta P_1 = I^2 R \tag{1.1}$$

   Wherein: I – current passing the cable core (A);
   R – the sum of resistance of both the cable core and tegmen (Ω).
2. *Leakage loss* ΔP_2 *(W)*
   This is in direct proportion to the square of voltage, that is

$$\Delta P_2 = U^2 G \tag{1.2}$$

Line Loss Analysis and Calculation of Electric Power Systems, First Edition. Anguan Wu and Baoshan Ni.

$$G = \frac{2\pi l r}{\ln \frac{r_2}{r_1}} \tag{1.3}$$

Wherein: U – voltage between the cable core and tegmen (V);
G – leakage conductance of dielectric (1/Ω);
r – conductivity [1/(Ω·m)];
l – length of the cable (m);
r_1 – radius of the cable core (cm);
r_2 – inside radius of the cable tegmen (cm).

3. *Dielectric magnetizing loss* ΔP_3 *(W)*
   This is in direct proportion to the square of current and the frequency, that is

$$\Delta P_3 = I^2 \omega L \tan\delta \tag{1.4}$$

$$L = \frac{l\mu}{2\pi} \ln \frac{r_2}{r_1} \tag{1.5}$$

Wherein: ω – AC angular frequency (1/s);
L – inductance of the cable (Wb/A);
μ – magnetic conductivity of the cable dielectric (Ω·s/m);
$\tan\delta$ – repeated magnetizing loss tangent of the cable dielectric.

4. *Dielectric polarization loss* ΔP_4 *(W)*
   This is in direct proportion to the square of voltage and the frequency, that is

$$\Delta P_4 = U^2 \omega C \tan\delta \tag{1.6}$$

$$C = \varepsilon \frac{2\pi l}{\ln \frac{r_2}{r_1}} \tag{1.7}$$

Wherein: C – capacitance of the cable (F);
ε – dielectric constant of the cable dielectric (F/m);
$\tan\delta$ – repeated magnetizing loss tangent of the cable dielectric.

The above four types of active power loss represent the basic types of active power loss in the electric power system. In addition, corona loss may occur in high-voltage lines and high-voltage motors. This is a special type of active power loss caused by ionization of dielectric particles outside a conductor when the electric field intensity is too high in the surface of the conductor. It is related to the surface field intensity of the conductor and the air density. See Chapter 8, Section 8.2 for details.

1.1.2 Calculation of Electric Energy Loss

Electric energy loss ΔA (kW·h) is the integral of active power loss to time within a period, that is

$$\Delta A = \int_0^T \Delta P(t) \mathrm{d}t \times 10^{-3} \tag{1.8}$$

For resistance heat loss, Formula (1.8) can be rewritten to

$$\Delta A = \int_0^T I^2(t)R(t)\mathrm{d}t \times 10^{-3} \tag{1.9}$$

Within the period T, the load current and conductor resistance may vary, so it is more complicated to calculate the electric energy loss than the active power loss. When the period for calculation is long, it is difficult to use the method of point by point square accumulation to calculate the electric energy loss. If relevant parameters of the current load curve $I(t)$ or the active load curve $P(t)$ are used to calculate the electric energy loss, it is difficult to obtain satisfactorily accurate calculation results. This is an issue that should be focused on when studying the theory and calculation method of electric energy loss.

1.1.3 Electricity Line Loss and Line Loss Rate

The total quantity of electricity loss (including the allocated electricity loss in power grids, the electricity consumed by electric reactors and reactive compensation equipment, and unknown electricity loss) in transmission, substation, and distribution within a given period (day, month, quarter, or year) in an electric supply area or power grids is called electricity line loss or line loss. Although part of the electricity line loss can be determined by theoretical calculation or measured by tailor-made line loss meters, the total electricity line loss cannot be accurately determined. Therefore, the electricity line loss is usually calculated by subtracting the total "power sales quantity" from the total "electric supply" measured by the electric energy meter. In other words, the line loss is a margin, and its accuracy relies both on the accuracy of the electric energy metering system used to measure the electric supply and the power sales quantity, and on the scientific and reasonable system for recording the statistics of power sales quantity to customers.

The electric supply and the power sales quantity are a pair of interrelated concepts and are closely related to the scope of line loss. In the 1990s, before power plants and power grids were not separated, China's power supply enterprises and power generation enterprises were managed by a competent authority and administered by an electricity bureau at the provincial level. The line loss statistics were conducted at national, provincial, and prefectural levels. For prefectural power generation enterprises, their electric energy meters at the generation side and the supply side were managed by power supply agencies entrusted to the provincial electricity bureau. As a result, the electric supply in a given area refers to the electricity supplied by power plants, power supply areas, or power grids to customers, including the electricity line loss in transmission and distribution of electric energy. The formula[1] for calculating the electric supply is as follows:

$$A_{\mathrm{e.s}} = A_{\mathrm{e.p}} - A_{\mathrm{e.c}} - A_{\mathrm{out}} + A_{\mathrm{in}} \tag{1.10}$$

Wherein: $A_{\mathrm{e.s}}$ – electric supply of an electric supply area or power grids;
$A_{\mathrm{e.p}}$ – electric energy production of power plants in a local area or local power grids;
$A_{\mathrm{e.c}}$ – electricity consumption of power plants;
A_{out} – electricity output to other power grids;
A_{in} – electricity input from other power grids (including purchased electricity).

Power sales quantity refers to the electricity sold by electric power enterprises to customers and the electricity supplied by electric power enterprises for internal non-generation use (such as capital construction departments). Non-electricity generation departments of electric power enterprises should be treated as customers. Therefore, the power sales quantity in an electric supply area or power grids is the total electricity measured by the electric energy meters of customers.

The percentage of the electricity line loss in the electric supply is called the line loss rate, and its calculation formula is as follows:

$$\text{Line loss rate}\,(\%) = \frac{\text{electric supply} - \text{power sales quantity}}{\text{electric supply}} \times 100\% \tag{1.11}$$

During the operation management of power grids, the electricity line loss obtained by subtracting the total power sales quantity from the total electric supply is called the statistical electricity line loss, and the corresponding line loss rate is called the statistical line loss rate.

Part of the statistical electricity line loss cannot be avoided during the transmission and distribution of electric energy and is determined by the load conditions of power grids and the parameters of power supply equipment. Such electricity loss is called technical electricity loss and can be obtained by theoretical calculation. Therefore, it is also called the theoretical electricity line loss, and the corresponding line loss rate is called the theoretical line loss rate.

The electricity used by substations is also included in the statistical electricity line loss. This part of electricity is similar to the electricity used by power plants and is necessary for production. It does not really belong to the electricity line loss, but due to historical reasons, it is not managed by power grid enterprises as a production cost. Instead, it is included in the line loss for management and control. Given the lean management requirements, the electricity used by substations can be excluded from the electricity line loss and included in the production cost for management. This may play a positive role in standardizing cost management and promoting the assessment and theoretical calculation of line loss.

Part of the statistical electricity line loss is an unknown loss, also known as management loss, which can and should be avoided or reduced by means of necessary measures.

In the 1990s, after the reform of the separation between power plants and power grids, the power plants and power grid enterprises became independent operators. Power grid enterprises used the on-grid electricity of power plants as the main electric supply. China's two major power grid enterprises, namely the State Grid Corporation of China and China Southern Power Grid, manage their internal line loss by several levels of large regional power grid enterprises covering multiple provinces, provincial power grid enterprises, and prefectural power grid enterprises. The electric supply and power sales quantity in different ranges differ from Formula (1.10).

1. National, large regional, and provincial power grid enterprises do not sell electricity directly, and their power sales quantity is the sum of electricity sold by prefectural power grid enterprises. The exchange electricity between provincial power grid enterprises within a large region and between prefectural power grid enterprises within a province can, in a broad sense, be considered as the electric supply or power sales quantity. The electricity loss in power grid loss calculation units directly administered by large regional and provincial power grid enterprises plus the electricity line loss within the subordinate administration range is the total electricity loss at this level. The line loss rate at this level is calculated by comparing the electric supply at this level with the total electricity loss at this level.
2. For a prefectural power grid enterprise, in addition to calculating the on-grid electricity of power plants in the local area, the electricity input from other areas in the local province or other provinces should be considered as the electric supply, and the electricity output to other areas in the local province or other provinces should be considered as the power sales quantity. Accordingly, the electricity line loss in the local area is calculated.

   Given the current electric power management system and the statistical criteria, the calculation formula of statistical line loss rate is as follows:

$$\begin{aligned}\text{Statistical line loss rate}\,(\%) &= (\text{statistical electricity line loss}) / (\text{electric supply}) \times 100\% \\ &= (\text{on-grid electricity of power plants} + \text{electricity input} \\ &\quad - \text{electricity output} - \text{power sales quantity}) / \\ &\quad (\text{on-grid electricity of power plants} + \text{electricity input}) \times 100\%\end{aligned} \tag{1.12}$$

The calculation formula of theoretical line loss rate is as follows:

$$\begin{aligned}\text{Theoretical line loss rate }(\%) = {} & \text{theoretically calculated electricity line loss}\,/ \\ & \text{theoretically calculated electric supply} \times 100\% \end{aligned} \tag{1.13}$$

$$\text{Theoretically calculated electric supply} = \text{on-grid electricity of power plants} + \text{electricity input}$$

3. When a large regional power grid enterprise assesses the planned loss rate of one of its provincial power grid enterprises, the influence of the difference between the actual electricity of mutual supply and the planned electricity supply on the loss rate should be analyzed. This is a complicated loss allocation problem, which is addressed in Chapter 11, Section 11.3.

In line loss management, each level of a power grid enterprise should summarize the line loss information of their subordinate level of power grid enterprises and then include such information in the information of power grid loss calculation units at this level, thereby calculating the statistical line loss rate or the theoretical line loss rate of power grids at this level. The structure of the electricity line loss management information system can be designed to satisfy the requirements of such line loss management.

1.1.4 Calculation and Analysis of Line Loss

The planning of power grids, the comparison of power grid connection programs, and the design of substations require the theoretical calculation of line loss. The accuracy required for the line loss calculation during such planning and design is not high, but the calculation methods need to be simple and practical. Therefore, tabular methods and calculated curve methods are preferred. Local theoretical calculation of line loss can be used to predict the benefits of some technical measures of loss reduction, and the comparison in technology and economy is conducted to select an economical and reasonable loss reduction program. Relatively comprehensive and detailed theoretical calculation of line loss can determine the quantity and composition of the electricity line loss, and can also reveal the relationship between the technical electricity line loss and factors such as operating voltage level, load rate, and average power factor so as to establish technical measures of loss reduction more scientifically. The results of comprehensive theoretical calculation of line loss can also be compared with the statistical electricity line loss, so as to estimate the quantity of management electricity loss and provide a basis for reducing the management electricity loss.

The above three types of theoretical calculations of line loss are necessary for power supply agencies and industrial enterprises with independent power supply systems. Therefore, a comprehensive discussion of the theoretical calculation of line loss is very necessary.

The analysis of line loss with the results of a theoretical calculation of line loss is important for line loss management. According to Reference [88], three types of line loss analysis are required, namely statistical analysis, indicator analysis, and economic analysis.

1.1.4.1 Statistical Analysis

The electric energy loss during the transmission and substation in the main system is called power grid loss, and the electric energy loss during the transmission, substation and distribution in regional power grids is called regional line loss.

a. *Analysis of the composition of regional line loss.* The line loss in transmission and substation should be analyzed by voltage and line; the line loss in distribution should be analyzed by region, substation, line or distribution area (divided by the supply range of a distribution transformer). In addition, the no-load loss and load loss in regional power grids should be analyzed separately to calculate the no-load line loss rate and the load line loss rate.

b. *Analysis of the structure of power grids.* The line loss rate should be analyzed by voltage, and the electric supply and line loss rate for different electric supply structures should be analyzed, especially for the various step-down and coupling modes of two- and three-winding transformers, so as to find ways to improve the electric supply structure and reduce line loss.
c. *Analysis of the composition of power sales quantity.* The electric supply output to adjacent regions will increase the line loss in the local region, and the influence of such transit electric supply should be further analyzed. The power sales quantity of dedicated lines wherein the line loss is borne by customers and the bulk power sales quantity not considering loss are collectively called power sales quantity without loss. Obviously, the percentage of the power sales quantity without loss in the total power sales quantity directly affects the value of statistical line loss rate and should also be analyzed.

The results of the above three types of statistical analysis should be compared with the results of theoretical calculation of line loss, in order to identify where the line loss is the biggest in transmission, substation and distribution systems and determine main measures of loss reduction.

1.1.4.2 Indicator Analysis

The indicator analysis basically includes the comparison of line loss rate indicators in the current period with those in last period, and the comparison of the difference between the statistical value of line loss rate in the current period and the planned value in last period. The indicator analysis can follow the five considerations below:

a. The increase/decrease in the power sales quantity, and changes in the electricity utilization category and the voltage composition.
b. Changes in the operating mode of the electric power system, the load flow distribution, and the structure of the power grids.
c. The influence of loss reduction measures and project production.
d. The influence of new large customers.
e. The influence of replacement of main system units.

1.1.4.3 Economic Analysis

The economic analysis mainly includes two types, namely the analysis of loss reduction benefits achieved in reactive compensation equipment intensively installed in substations and reactive compensation equipment dispersedly installed in distribution lines, and the analysis of benefits achieved in the assessment of peak power factors of large customers or the assessment of peak/valley power factors.

The Provisions issued in 2004 by the State Grid Corporation of China [75] set out the following requirements on the annual report summary and analysis of line loss:

1. The performance of line loss indicators.
2. The analysis of the composition of line loss by comprehensive line loss rate, loss rate and regional line loss rate; the analysis of line loss rate by voltage class; the analysis of line loss deducting the power sales quantity without loss and the bulk power sales quantity.
3. Existing problems and measures; the quantitative analysis of reasons for the increase/decrease in the line loss rate and the degree of influence of the increase/decrease in the line loss rate.
4. Solutions to the problems and key measures of subsequent work.

1.2 Calculation of AC Resistance

Overhead power lines generally use bare conductors which have larger AC resistance than DC resistance as a result of the skin effect of alternating current. Steel-cored aluminum wires have even larger AC resistance due to iron loss caused by magnetization of their steel cores. The increased resistance due to the skin effect can be theoretically calculated, while the increased resistance caused by the magnetization of steel cores must be determined through actual measurement. The calculation formula of AC resistance is as follows:

$$R = K_1 K_2 R_{dc}^{(2)} \tag{1.14}$$

$$K_1 = 0.996\,09 + 0.018\,578X_1 - 0.030\,263X_1^2 + 0.020\,735X_1^3 \tag{1.15}$$

$$K_2 = 0.999\,47 + 0.028\,895X_2 - 0.005\,934X_2^2 + 0.000\,422\,59X_2^3 \tag{1.16}$$

$$X_1 = \frac{D+2d}{D+d} \times 0.01\sqrt{\frac{8\pi f(D-d)}{R_{dc}(D+d)}} \tag{1.17}$$

$$X_2 = \frac{I}{S} \tag{1.18}$$

Wherein: R_{dc} – DC resistance of conductors at the calculation temperature (Ω/km);
K_1 – skin effect coefficient of conductors;
X_1 – parameter used to calculate the skin effect coefficient of conductors;
D, d – outer diameter and inner diameter of conductors (cm);
f – frequency of alternating current (Hz);
K_2 – iron loss coefficient of conductors;
X_2 – parameter used to calculate the iron loss coefficient of conductors;
I – current passing conductors (A);
S – sectional area of conductors (mm^2).

According to the calculation, the AC resistance is only ~0.02 to ~5.0% higher than the DC resistance of all aluminum conductors whose sectional areas range from 50 to 240 mm^2; the AC resistance is 1.3–4.6% higher than the DC resistance of steel-cored aluminum wires whose sectional areas range 25–240 mm^2. The lower limits above are calculated when the current-carrying capacity is 20% of the allowable value, while the upper limits above are calculated when the current-carrying capacity is the allowable value. This shows that, when the current of overhead line conductors is close to or exceeds the allowable value, factors causing higher AC resistance must be taken into account. In other circumstances, the DC resistance can be directly used to calculate the line loss, without leading to any significant error.

1.3 Influence of Temperature and Voltage Changes on Line Loss in the Measuring Period

1.3.1 Influence of Temperature Change on Line Loss in the Measuring Period

According to Formula (1.9), not only loads change with time, but also the resistance of conductors changes with temperature within a measuring period. Apparently, it is extremely complicated to take into account the two change factors at the same time for integral operation. To easily calculate the line loss, the influence of temperature change on the variable resistance can be considered first.

It is generally known that the resistance of conductors with temperature change can be calculated as per the following formula:

$$R_T = R_0(1+\alpha T) \tag{1.19}$$

Wherein: R_0 – resistance of conductors at 20 °C (Ω);
α – temperature coefficient of conductor resistance: generally $\alpha = 0.004$ for copper, aluminum and steel-cored aluminum wires;
T – air temperature (°C).

The record data of load current and temperature within one day (24 h) are substituted into Formulas (1.19) and (1.9), resulting in:

$$\begin{aligned}
\Delta A &= \int_0^{24} I^2(t)R(t)\mathrm{d}t \times 10^{-3} \\
&= \left[I_1^2 R_0(1+\alpha T_1) + I_2^2 R_0(1+\alpha T_2) + \cdots + I_{24}^2 R_0(1+\alpha T_{24})\right] \times 10^{-3} \\
&= \left[\left(I_1^2 + I_2^2 + \cdots + I_{24}^2\right) + \alpha\left(I_1^2 T_1 + I_2^2 T_2 + \cdots + I_{24}^2 T_{24}\right)\right] R_0 \times 10^{-3}
\end{aligned} \tag{1.20}$$

The weighted average temperature is defined as follows:

$$T_{we} = \frac{I_1^2 T_1 + I_2^2 T_2 + \cdots + I_{24}^2 T_{24}}{I_1^2 + I_2^2 + \cdots + I_{24}^2} \tag{1.21}$$

Then, Formula (1.20) can be rewritten to:

$$\begin{aligned}
\Delta A &= \left(I_1^2 + I_2^2 + \cdots + I_{24}^2\right)(1+\alpha T_{we}) R_0 \times 10^{-3} \\
&= \left(I_1^2 + I_2^2 + \cdots + I_{24}^2\right) R_{we} \times 10^{-3}
\end{aligned} \tag{1.22}$$

$$R_{we} = R_0(1+\alpha T_{we}) \tag{1.23}$$

Wherein: R_{we} – conductor resistance at the weighted average temperature.

As mentioned above, if the electric energy loss is calculated based on the weighted average temperature and Formula (1.22), then the influence of temperature change is completely considered.

As per Formula (1.21), if the load is constant, then $T_{\mathrm{we}} = T_{\mathrm{av}}$ (average temperature). As the daily temperature changes in a unimodal manner and the daily load normally changes with two different peaks, T_{av} is very close to T_{we} within a period of one day and night or more than one day and night, and the replacement of T_{we} with T_{av} will not produce any larger negative error.

According to the analysis of relative error of resistance as per Formula (1.23), as the temperature coefficient α of conductor resistance is very small, even if the replacement of T_{we} with T_{av} produces a certain relative error, relative errors of resistance and electric energy loss are still very small. If the measuring period is one month or one year, Formula (1.9) used to calculate the electric energy loss of a three-phase symmetrical unit can be rewritten to the following by using T_{av} to calculate the resistance:

$$\Delta A = 3R\int_0^T I^2(t)\mathrm{d}t \times 10^{-3} \tag{1.24}$$

1.3.2 *Influence of Voltage Change on Line Loss in the Measuring Period*

When the measured load data represents active power and reactive power instead of the current, the voltage change should be considered for the calculation of line loss. If the measuring period is one day and night, then Formula (1.24) can be rewritten to:

$$\Delta A = R\int_0^{24} \frac{[P_2(t) + Q^2(t)]}{U^2(t)}\mathrm{d}t \times 10^{-3} \tag{1.25}$$

Wherein: R – resistance calculated per Formula (1.23) by considering the temperature change (Ω);
$P(t)$, $Q(t)$ – active power (kW) and reactive power (kvar) at the same measuring point;
$U(t)$ – voltage at active and reactive power measuring point (kV).

The active power weighted average voltage and the reactive power weighted average voltage can be defined with the squared values of active power and reactive power of one day and night as the weight, that is:

$$\left.\begin{aligned} \frac{1}{U_{we.P}^2} &= \left(\frac{P_1^2}{U_1^2} + \frac{P_2^2}{U_2^2} + \cdots + \frac{P_{24}^2}{U_{24}^2}\right) \Big/ (P_1^2 + P_2^2 + \cdots + P_{24}^2 \\ \frac{1}{U_{we.Q}^2} &= \left(\frac{Q_1^2}{U_1^2} + \frac{Q_2^2}{U_2^2} + \cdots + \frac{Q_{24}^2}{U_{24}^2}\right) \Big/ Q_1^2 + Q_2^2 + \cdots + Q_{24}^2 \end{aligned}\right\} \tag{1.26}$$

Then, Formula (1.25) can be rewritten to:

$$\Delta A = R\left[\int_0^{24} P^2(t)\mathrm{d}t/U_{we.P}^2 + \int_0^{24} Q^2(t)\mathrm{d}t/U_{we.Q}^2\right] \times 10^{-3} \tag{1.27}$$

According to the calculations with the measured data of 220, 110, and 35 kV systems with different voltage and load variations, if the average voltage U_{av} is used to replace weighted average voltages $U_{we.P}$ and $U_{we.Q}$, the error generally does not exceed minus 1%, so Formula (1.27) can be further rewritten to:

$$\Delta A = \frac{R}{U_{av}^2}\left[\int_0^{24} P^2(t)\mathrm{d}t + \int_0^{24} Q^2(t)\mathrm{d}t\right] \times 10^{-3} \tag{1.28}$$

Under the condition of normal operation, as the long-term voltage variation is not large, the replacement of $U_{we.P}$ and $U_{we.Q}$ with U_{av} is still feasible, that is

$$\Delta A = \frac{R}{U_{av}^2}\left[\int_0^T P^2(t)\mathrm{d}t + \int_0^T Q^2(t)\mathrm{d}t\right] \times 10^{-3} \tag{1.29}$$

A former Soviet Union scholar once studied the relationship between voltage deviation and electric energy loss in the distribution network [3]. The results show that, regardless of the voltage deviation, if the average voltage is used to calculate the electric energy loss the error is related to the correlation coefficient between the voltage and current changes, the signs, and the voltage deviation, that is

$$\left.\begin{aligned} \delta(\Delta A)\% &= \left|r_{U/I}\right|\sigma_U\% \\ \delta(\Delta A)\% &= \left(\frac{\Delta A_2 - \Delta A_1}{\Delta A_1}\right) \times 100\% \end{aligned}\right\} \tag{1.30}$$

Wherein: ΔA_1 – electric energy loss considering the voltage deviation (kW·h);
ΔA_2 – electric energy loss calculated with the average voltage (kW·h);
$r_{U/I}$ – correlation coefficient between voltage and current changes, $-1 \le r_{U/I} \le 1$;
$\sigma_U\%$ – mean square error of voltage change, which is calculated by percentage.

The distribution of voltage deviations in the distribution network is close to a “normal distribution”, so

$$\sigma U\% = \frac{1}{6}\left(\frac{U_{max} - U_{min}}{U_{av}}\right) \times 100\% \tag{1.31}$$

Wherein U_{max} and U_{min} – maximum and minimum voltages.

According to Formula (1.31), when the voltage change in the distribution network reaches up to 20%, the error resulted from the calculation of line loss with the average voltage will not exceed 3.3% which is allowable in engineering calculation.

1.4 Influence of Load Curve Shape on Line Loss

1.4.1 Load Curve and Load Duration Curve

Load changes are recorded with time sequence, which can be used to produce a normal load curve. Within a period T, a derivative curve arranged by load value and its duration rather than time sequence is called load duration curve.

Figure 1.1a shows the load curve and Figure 1.1b shows the load duration curve, as follows:

$$\begin{cases} \overline{cd} = \overline{c'd'}, \overline{ef} + \overline{gh} = \overline{e'h'} \\ I_0 = I'_0\ I_1 = \mathrm{I}'_1,\ I_2 = I'_2 \end{cases}$$

Obviously, the load square curve has its corresponding load duration square curve, so

$$I_0^2 = \left(I'_0\right)^2, I_1^2 = \left(I'_1\right)^2, I_2^2 = \left(I'_2\right)^2$$

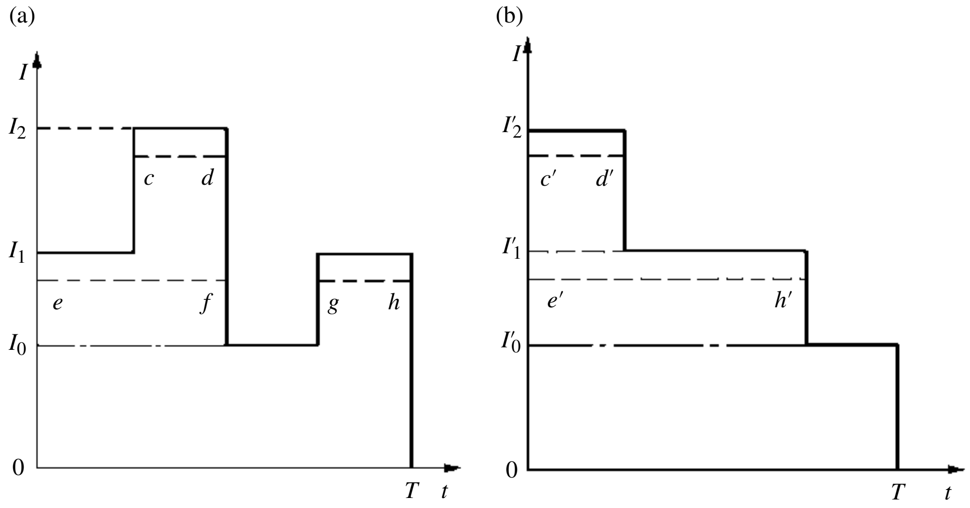

Figure 1.1 Load curve and load duration curve. (a) Load curve. (b) Load duration curve.

Assume that the power factor of a three-phase symmetrical unit is 1.0, and that the voltage is constant within the measuring period, with the load curve as shown in Figure 1.1a. Then, the electric energy passing through this unit is $A = \sqrt{3}U\int_0^T I(t)\mathrm{d}t$ within the measuring period. As per Figure 1.1,

$$
\begin{aligned}
A &= \sqrt{3}U\int_0^T I(t)\mathrm{d}t \\
&= \sqrt{3}U\left[I_0T + (I_1 - I_0)\left(\overline{ef} + \overline{gh}\right) + (I_2 - I_1)\overline{cd}\right] \\
&= \sqrt{3}U\left[I'_0T + \left(I'_1 - I'_0\right)\overline{e'h'} + \left(I'_2 - I'_1\right)\overline{c'd'}\right. \\
&= \sqrt{3}U\int_0^T I'(t)\mathrm{d}t = A'
\end{aligned}
$$

Therefore, the load duration curve is an equivalent transformation graph to the load curve, both of which have the same area.

Assume that the resistance of the above unit is R per phase. Then, the electric energy loss of this unit is $\Delta A = 3R\int_0^T I^2(t)\mathrm{d}t \times 10^{-3}$ within the measuring period. According to the definition of the load duration square curve, the following relationship can be derived as per Figure 1.1:

$$
\begin{aligned}
\Delta A &= 3R\int_0^T I^2(t)\mathrm{d}t \times 10^{-3} \\
&= 3R\left[I_0^2T + (I_1 - I_0)^2\left(\overline{ef} + \overline{gh}\right) + (I_2 - I_1)^2\overline{cd}\right] \\
&= 3R\left[\left(I'_0\right)^2T + \left(I'_1 - I'_0\right)^2\overline{e'h'} + \left(I'_2 - I'_1\right)^2\overline{c'd'}\right. \\
&= 3R\int_0^T [I'(t)]^2\mathrm{d}t = \Delta A'
\end{aligned}
$$

Therefore, the load duration square curve is an equivalent transformation graph to the load square curve, both of which have the same area.

Because the load duration curve and the load curve have double equivalence in terms of electric energy and electric energy loss, the load duration curve is taken as the main analysis target during the theoretical calculation and analysis of line loss, and the derived conclusion is applicable to the corresponding load curve.

1.4.2 Parameters of Characterization Load Curve

1.4.2.1 *Load Factor* f

Load factor f (see Figure 1.2) is the ratio of average load to maximum load within the measuring period, that is

$$f_P = \frac{P_{av}}{P_{max}} \text{ or } f_I = \frac{I_{av}}{I_{max}} \tag{1.32}$$

Wherein: P_{av} and P_{max} – average active power and maximum active power of load (kW);
I_{av} and I_{max} – average current and maximum current of load (A).

The load rate reflects the average utilization of power system equipment and serves as an important indicator to assess the operation of the power system.

1.4.2.2 *Minimum Load Rate* β

Minimum load rate β (see Figure 1.2) is the ratio of minimum load to maximum load within the measuring period, that is

$$\beta = \frac{I_{min}}{I_{max}} \tag{1.33}$$

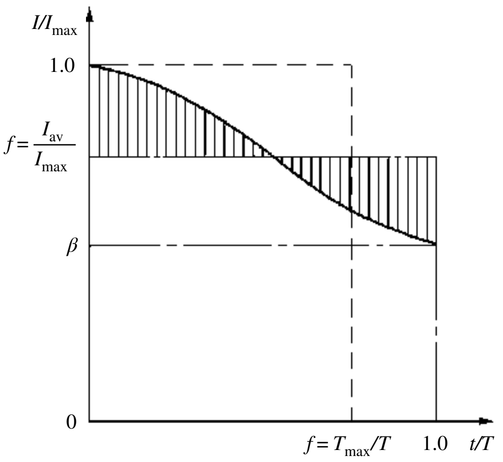

Figure 1.2 Meaning of load factor f and minimum load rate β.

1.4.2.3 *Maximum Load Utilization Time* $\mathbf{T}_{max}$ *and Maximum Loss Time* τ_{max}

Assume that the voltage and power factor remain the same within the measuring period, and that the electric energy passing through a unit under varying current within the period T equals the electric energy passing through the unit under maximum current within $T_{\max}$.

Then, $T_{\max}$ is called the maximum load utilization time (see Figure 1.3), that is

$$\sqrt{3}U\cos\varphi\int_0^T I(t)\mathrm{d}t = \sqrt{3}UI_{\max}\cos\varphi T_{\max}$$

$$T_{\max} = \frac{\int_0^T I(t)\mathrm{d}t}{I_{\max}} \tag{1.34}$$

$\tau_{\max}$, the maximum loss time, is defined as: when the voltage and power factor remain unchanged within the measuring period, if the electric energy loss of current passing through a unit under varying current equals the electric energy loss of current passing through the unit under maximum current within the time of $\tau_{\max}$ (see Figure 1.3), that is

$$\tau_{\max}I_{\max}^2 3R = 3R\int_0^T I^2(t)\mathrm{d}t$$

$$\tau_{\max} = \frac{\int_0^T I^2(t)\mathrm{d}t}{I_{\max}^2} \tag{1.35}$$

As $I_{\mathrm{av}} = \int_0^T I(t)\mathrm{d}t/T$, $I_{\max} = \int_0^T I(t)\mathrm{d}t/T_{\max}$ is derived from Formula (1.34) and substituted into Formula (1.32), resulting in:

$$f = \frac{I_{\mathrm{av}}}{I_{\max}} = \frac{T_{\max}}{T} \tag{1.36}$$

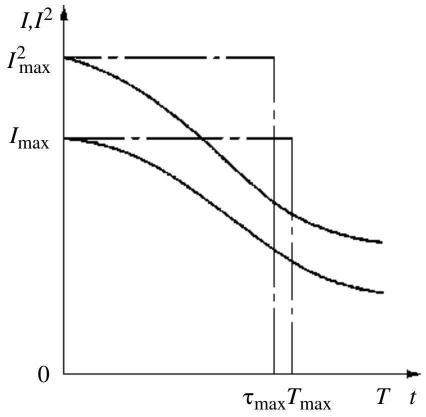

Figure 1.3 Definition of $T_{\max}$ and $\tau_{\max}$.

According to Formula (1.36), the load rate is equal to the per unit value at the maximum load utilization time.

1.4.2.4 *Loss Factor* F

Loss factor F is the ratio of maximum loss time to measuring period T, that is

$$F = \frac{\tau \max}{T} = \frac{\int_0^T I^2(t)\mathrm{d}t / I_{\max}^2}{T} = \frac{I_{\mathrm{rms}}^2}{I_{\max}^2} \tag{1.37}$$

$$I_{\mathrm{rms}} = \sqrt{\int_0^T I^2(t)\mathrm{d}t / T} \tag{1.38}$$

Wherein I_{rms} – rms current.

According to Formula (1.37), the loss factor is also equal to the ratio of rms current square to maximum current square. The meaning of loss factor F is shown in Figure 1.4.

The maximum load of power supply equipment is the focus of operation monitoring. The parameter of loss factor together with the maximum load can be used to calculate the line loss, so Formula (1.24) can be rewritten to:

$$\Delta A = 3I_{\max}^2 FRT \times 10^{-3} \tag{1.39}$$

1.4.2.5 *Form Coefficient* K

$\sqrt{F} = I_{\mathrm{rms}}/I_{\max}$ can be derived from Formula (1.37) and it is called loss equivalent load factor in Japan. As I_{rms} is the "equivalent loss" current, the ratio of it to $I_{\max}$ can be regarded as another type of load factor.

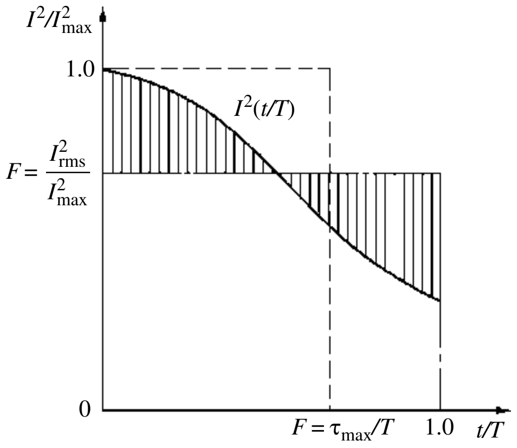

Figure 1.4 Meaning of loss factor F.

Form coefficient is defined as the ratio of rms current to average current, that is

$$K = I_{\mathrm{rmx}}/I_{\mathrm{av}} \tag{1.40}$$

$I_{\mathrm{rms}} = \sqrt{F} I_{\max}$, so

$$K = \sqrt{F}/(I_{\mathrm{av}}/I_{\max}) = \sqrt{F}/f \tag{1.41}$$

In summary, the loss equivalent load rate is the ratio between rms current of equivalent loss and the maximum current, and the load rate is the ratio of average value to the maximum value in the current load curve. The form coefficient is the ratio of such two load rates and also the ratio of the loss equivalent current to the electric energy equivalent current, comprehensively reflecting the features of load loss (square) curve and load curve.

The above five parameters are defined based on the current variable. Similar coefficients can be derived for three load curves expressed by active power, reactive power and apparent power. For the sake of distinction, different subscripts may be used.

1.4.3 Relationship Between Loss Factor and Load Factor

Figure 1.5a shows two special load curves.

For load curve 1, the maximum load duration is $t_{\max} \approx 0$, and the load is constant as the minimum load during most of the time. For load curve 2, the maximum load duration is $t_{\max}$, and the load is close to zero during the rest of the time. The y-coordinate of the load curve (see Figure 1.5a) refers to the per unit value of load current based on the maximum current, and the x-coordinate refers to the per unit value of time based on the measuring period T.

According to load curve 1 in Figure 1.5,

$$f = \frac{\beta \times 1.0}{1.0} = \beta, F = \frac{(\beta \times 1.0)^2}{1.0^2} = \beta^2$$

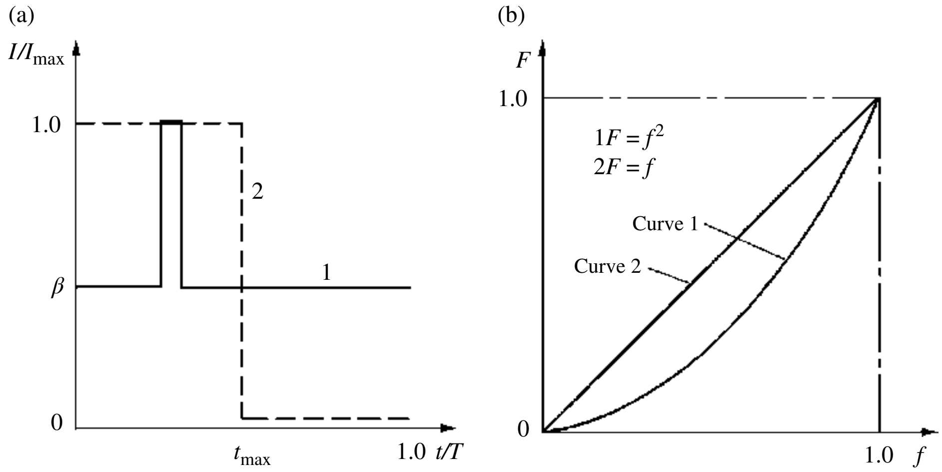

Figure 1.5 Relationship between loss factor and load rate. (a) Two special load curves. (b) Value range of F and f.

So if any value is taken for β, then

$$F = f^2 \tag{1.42}$$

According to load curve 2 in Figure 1.5,

$$f = \frac{1.0 \times t_{\max}}{1.0} = t_{\max}, F = \frac{1.0^2 \times t_{\max}}{1.0^2} = t_{\max}$$

So if any value is taken for $t_{\max}$, then

$$F = f \tag{1.43}$$

The $F(f)$ curve expressed by Formula (1.42) and Formula (1.43) is shown in Figure 1.5b. When the load curve is mutant (curve 1), $F = f^2$; when the load curve is stable (curve 2), $F = f$. Any actual load curve falls between such two extreme cases, so the relationship between F and f for any load curve must fall in the range between $F = f$ and $F = f^2$.

Figure 1.5b also indicates that, in addition to such two end values as 0 and 1, the same load factor f can correspond to different F values, *which shows that a single load factor parameter cannot accurately reflect the loss conditions of a unit*. In other words, even with the same load factor, the loss factor varies with the shape of the load curve. When $f = 0.5$, the loss factor uncertainly leads to the largest absolute error and relative error, each of which is as follows:

$$\Delta F_{\max} = F_2 - F_1 = 0.5 - 0.5^2 = 0.25$$

$$(\delta F)_{\max} = \frac{\Delta F_{\max}}{F_1} \times 100\% = \frac{0.25}{0.25} \times 100\% = 100\%$$

This indicates that, as the load curve differs in thousands of ways, a significant error must be produced if a single load factor parameter is used to calculate the loss factor. To avoid this, a loss factor formula with multiple parameters is required, which will be further analyzed in Chapter 2.

1.5 Influence of Load Power Factor and Load Distribution on Line Loss

1.5.1 Influence of Load Power Factor

If the average voltage during the measuring period T is U_{av} and the conductor resistance is R, then the electric energy loss of a three-phase line can be calculated by:

$$\begin{aligned} \Delta A &= \frac{R}{U_{av}^2}\left\{\int_0^T \left[P^2(t) + Q^2(t)\right]\mathrm{d}t\right\} \times 10^{-3} \\ &= \frac{R}{U_{av}^2}\int_0^T P^2(t)\left[1 + \tan^2\varphi(t)\right]\mathrm{d}t \times 10^{-3} \end{aligned} \tag{1.44}$$

1. If the power factor remains the same during the measuring period, then Formula (1.44) can be rewritten to:

$$\begin{aligned} \Delta A &= \frac{R}{U_{av}^2}\left(1 + \tan^2\varphi\right)\int_0^T P^2(t)\mathrm{d}t \times 10^{-3} \\ &= \frac{R}{U_{av}^2 \cos^2\varphi}\int_0^T P^2(t)\mathrm{d}t \times 10^{-3} \end{aligned} \tag{1.45}$$

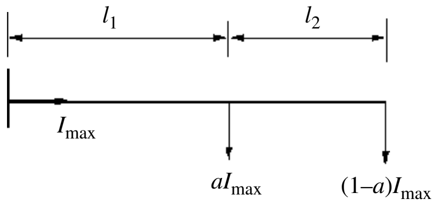

Figure 1.6 Influence of load distribution.

2. If the power factor is varying during the measuring period, then Formula (1.44) can be rewritten to:

$$\Delta A = \frac{R}{U_{av}^2}\left[\int_0^T P^2(t)\mathrm{d}t + \int_0^T P^2(t)\tan^2\varphi(t)\mathrm{d}t\right] \times 10^{-3} \tag{1.46}$$

According to the above formulas, the load power factor has a complicated influence on the calculation of line loss, which will be analyzed in detail in Chapter 4.

1.5.2 Influence of Load Distribution of Multi-Branch Line

Assume that there are two customers in a line, both of which have the same shape of load curve. The whole line uses conductors with the same sectional area. The distance from the branch point to the start end and the tail end of the line is l_1 and l_2, respectively. The maximum current at the start end and at the two customers is I_{max}, aI_{max}, and $(1–a)I_{max}$, respectively, as shown in Figure 1.6.

Because the load curves of the two customers are exactly the same, $F_1 = F_2 = F$. The electric energy loss of the whole line during the measuring period T can be calculated as per the following formula:

$$\begin{aligned}\Delta A &= \Delta A_1 + \Delta A_2 \\ &= 3\left[I_{max}^2 F l_1 r_0 + (1-a)^2 I_{max}^2 F l_2 r_0\right] \times T \times 10^{-3} \\ &= 3I_{max}^2 F r_0\left[l_1 + (1-a)^2 l_2\right] T \times 10^{-3}\end{aligned} \tag{1.47}$$

Wherein r_0 – resistance per unit length of conductor (Ω/km).

According to Formula (1.47), the smaller a is, the bigger the electric energy loss ΔA is. When a is positivecertain, the bigger l_2 is, and the bigger ΔA is. Therefore, two factors influence the electric energy loss of a multi-branch line: one is the shunt condition, that is the distribution of load values; the other is the distance from each load point to the start end of the line, that is the special distribution of load. Given the differences in the shape of load curve and power factor of multi-branch loads and in the voltage at multi-branch points, it is very complex to accurately calculate the electric energy loss of a multi-branch line. To do the line loss calculation, a series of approximate assumptions must be made for simplified calculation and analyzed in terms of their influence on the calculation results. These contents will be explained in Chapter 5.

1.6 Influence of Measuring Instrument Error on Line Loss

1.6.1 Composition of Electric Energy Metering System and Constitution of Metering Error

Low-voltage residential electricity and small power electricity are directly measured by a single-phase watt-hour meter and a three-phase three-wire or three-phase four-wire watt-hour meter respectively, while large power commercial electricity and industrial electricity must be measured by a current transformer. Industrial and agricultural irrigation electricity with voltage over 0.4 kV may be measured by a current transformer together with a voltage transformer. In this case, a voltage transformer with a rated secondary voltage of 100 V, a current transformer with a rated secondary current of 5 A, a supporting watt-hour meter, and wires connecting the foregoing three constitute a complete electric energy metering system.

The metering error of a complete electric energy metering system consists of the error of the watt-hour meter, the ratio error, the phasor angle error of transformers, and the error caused by the voltage drop of connecting wires. For star three-phase loads of star connection, if the electric energy metering system uses the three-phase, four-wire watt-hour meter as the main measuring element, then its composition error consists of the error of the transformers, the error caused by the secondary voltage drop of wires, and the error of the watt-hour meter, as shown below [34]:

$$\varepsilon = 0.003(f_1 + f_2 + f_3) + 0.009\,7(\delta_1 + \delta_2 + \delta_3)\tan\varphi + (\Delta f + \tan\delta') + \varepsilon_0 \tag{1.48}$$

$$f_i = f_{Ui} + f_{Ii}$$

$$\delta_i = \alpha_i - \beta_i$$

Wherein: f_i – sum of voltage transformer ratio error and current transformer ratio error of various phases;
δ_i – difference between current transformer angle error and voltage transformer angle error of various phases;
Δf, $\tan\delta'$ – additional ratio error and angle error caused by secondary voltage drop of wires;
ε_0 – error of the watt-hour meter.

1.6.2 Composition of Electronic Watt-Hour Meter Error

1.6.2.1 Structural Features of Electronic Watt-Hour Meter

The multiplier used in the hardware system of the increasingly widely used electronic watt-hour meter includes time division multiplier and AC/DC direct sampling multiplier. The time division multiplier uses field effect transistors (FETs) to control amplitude modulation circuits, which easily suffers interference from the spike effect of FETs. The production of a high-precision time division multiplier imposes quite high requirements on components and entails complicated auxiliary circuits and higher costs. Complicated auxiliary circuits are generally not set for the electronic watt-hour meter which is largely used for civil purpose with low precision, so its anti-interference capability is rather weak.

The traditional AC/DC direct sampling multiplier generally adopts a broken line approximate expression function generator composed of high-frequency diodes to obtain the product of voltage signal and current signal. Temperature compensation is required for the diodes. Its circuits are complicated and its precision is not high. In recent years, the A/D converter has been used to digitize voltage, current, and their differences; and microcomputer software has been used to realize square operation, forming a digital square multiplier; as complicated analog circuits are not used to realize square operation and the accuracy of the converter is highly improved, the high precision pulse (electronic) watt-hour meter based on the principle of digital square multiplier has been widely used for gateway electric energy metering in the electric power system.

1.6.2.2 Metering Error of Electronic Watt-Hour Meter

Both of the above multipliers have input circuits and generally use transformer input, so each electronic watt-hour meter has ratio error and angle error in voltage, current input voltage change coefficients K_u and K_i, and the square multiplier has amplitude error and phase error in the transfer coefficient. As a result, an electronic watt-hour meter itself has many factors that may cause errors. Even if instrument initialization and calibration of standard constants are well conducted at delivery, errors may also occur due to external interference in the operating environment, a change in the performance of internal transformers, and other random factors. Therefore, like a traditional electromechanical watt-hour meter, an electronic watt-hour meter may have random errors.

1.6.3 Influence of Metering System Error on the Calculation of Line Loss Rate

The existing electric energy measuring procedures set accuracy classes for various watt-hour meters, current transformers, and voltage transformers of electric energy metering systems by their quantity of electric energy within a measuring period. Since the reform of the separation between power plants and power grids in the electric power management system, power grid enterprises have had higher requirements for the accuracy of power purchase quantity metering systems than for power sales quantity metering systems. Therefore, in recent years, provincial, local, and municipal power grid enterprises have installed centralized meter reading systems featuring real-time monitoring. With respect to power purchase quantity and power sales quantity metering facilities with larger electric energy quantity, the satisfactory accuracy of electric energy metering can be achieved by selecting appropriate transformer accuracy, secondary wire design, and electronic watt-hour meter. Because a complete electric energy metering system has random errors caused by various factors, there are metering errors in the power purchase quantity and power sales quantity of a relatively independent municipal power grid enterprise, affecting the statistical accuracy of the line loss rate within a certain measuring period.

According to the results of long-term testing of an electromechanical watt-hour meter, positive and negative random errors of a heavily operated watt-hour meter are similar, and the long-term statistics show that the random errors of a watt-hour meter comply with a normal distribution in probability theory. In other words, the accuracy of aggregate metering values of n watt-hour meters with the same accuracy class is $1/\sqrt{n}$ of the accuracy of a single watt-hour meter. Given the lack of in-depth understanding of the error statistical law of an electronic watt-hour meter, assuming there are both electromechanical and electronic watt-hour meters and that metering errors in the power purchase quantity and power sales quantity still comply with the normal distribution statistical law, the influence of electric energy metering errors on the line loss rate may be estimated within a certain statistical period.

For example, if the total number of power purchase meters of a power grid enterprise is n_1 with a weighted average accuracy of Δ_1, the total number of power sales meters is n_2 with a weighted average accuracy of Δ_2, and the actual line loss rate is $\Delta A\%$, then the relative error can be calculated by the following formula as the meter accuracy affects the accuracy of the statistical line loss rate:

$$\delta(\Delta A\%) = \left(\frac{1-\Delta A\%}{\Delta A\%}\right)\left(\frac{\Delta_1}{\sqrt{n_1}} + \frac{\Delta_2}{\sqrt{n_2}}\right) \tag{1.49}$$

According to Formula (1.49), there is a metering error between the statistical line loss rate and the actual line loss rate, so a certain fluctuation range is allowed for the line loss rate. For example, considering a municipal power supply enterprise, the number of its power purchase gateway meters is $n_1 = 50$, $\Delta_1 = 0.50\%$; the total number of 10 kV distribution transformer electric energy meters and 10 kV large customers is $n_2 = 1300$, $\Delta_2 = 2.5\%$; and the planned line loss rate of up to 10 kV level is $\Delta A\% = 4.3\%$. Substituting these into Formula (1.49), $\delta(\Delta A\%) = 3.12\%$ is obtained. The allowable range of statistical line loss rate is 4.434 – 4.166%, and the allowable fluctuation range is ±3.12% of the planned value.