CHAPTER 1

STATICS REVIEW

1.1 STATICS REVIEW

In construction management and civil engineering programs, students are required to take statics and strength of material classes in preparation for their successor. The successor might be a generic "structures" course, a temporary structure course, or maybe no successor course at all. Whichever direction the curriculum goes, the basics of statics and strength of materials is the common denominator.

This book has been written under the assumption that the student has a background in statics and strength of materials and these skills only need to be refined. Temporary structures utilizes many of the less complicated aspects of statics and strength of materials, so even if the student did not master the two prerequisites, he should still be successful in the subject matter of this book. In addition, temporary structure design is a very practical subject, and the student should be energized to see that the challenges that this book covers are real construction situations that the student will experience for his or her entire career.

1.2 UNITS OF MEASURE

At the time of this writing, local and state projects in the United States continue to use the English "Imperial" unit system (feet, pounds, etc.). While most of Europe and the rest of the world use the metric system, the United States has resisted this movement. Even the California Department of Transportation, which had converted current and future projects to the Imperial system of measures late in the 20th century, has gone back to using the Imperial system in the early 21st century. Since England

TABLE 1.1 Units of Measure

Unit Name	Unit of Measure
Length	
Foot	ft (′)
Inches	in (″)
Area	
Square feet	SF, ft^2
Square inches	in^2
Volume	
Cubic feet	CF, ft^3
Cubic inches	in^3
Force and Pressure	
Pound	lb, #
Kip	k (1000 lb)
Pounds per ft	lb/ft
Kips per ft	k/ft
Pounds per SF	lb/SF, psf
Pounds per linear foot	lb/ft
Kips per linear foot	kpf
Kips per SF	k/SF, ksf
Pounds per CF	lb/CF, pcf
Moment	
Foot-pounds	ft-lb
Inch-pounds	in-lb
Foot-kips	ft-k
Inch-kips	in-k
Stress	
Pounds per ft^2	psf, lb/ft^2
Pounds per in^2	psi, lb/in^2
Kips per ft^2	ksf, k/ft^2
Kips per in^2	ksi, k/in^2
Temperature	
Degree Fahrenheit	°F

has also gone to the metric system, their "English" Imperial system is now referred to as the U.S. units. Because this text has been written for students in the United States, examples will be given in U.S. units only. Table 1.1 shows most of the common units of measure used in this book.

1.2.1 Common Units of Measure

With any engineering subject, the use of variables to represent different engineering values is standard. These symbols derive either from the Greek alphabet or English letters. Regardless, a great number of symbols are necessary to represent the various engineering concepts. Table 1.2 shows the notations and symbols most used in this book.

TABLE 1.2 Notation and Symbols

Subject	Symbol (Variable)	Description
Properties	S	Section modulus
	I	Moment of inertia
	E	Modulus of elasticity
	A	Cross-sectional area
	r	Radius of gyration
	R	Radius of a circle
	e	Eccentricity
	a	Moment arm distance
	b	Beam width
	c	Distance from centroid to top or bottom edge
	d	Depth of beam
	D	Diameter of a circle
	g	Acceleration of gravity
	h	Height or depth
	K	Distance from top of beam to tangent of web
	e	Effective length of a column or strut, distance from top of beam to web tangent
Stress	f_b	Bending stress
	f_v	Shear stress
	f'_v	Twice shear stress used for short-term shear loading
	$f_{c\parallel}$	Normal compression stress parallel to the grain of the wood
	f_c	Normal compression stress perpendicular to the grain of the wood
Soil Mechanics	c	Cohesion (psf)
	ϕ	Angle of internal friction (degrees)
	β	Passive slip plane angle (degrees)
	α	Active slip plane angle (degrees)
	μ	Coefficient of friction
	γ	Unit weight (pcf)
	π	$\pi = 3.1416$
	k_a	Active soil coefficient
	k_p	Passive soil coefficient
	T	Temperature

1.3 STATICS

Statics is the study of an object that is not moving, hence static or equilibrium. A force is a motion or change of motion in a body. A common force that is produced on Earth naturally is gravity. Gravity is the tendency of the weight of a body to be attracted to the center of the Earth. The mass of some unit weight is placed in motion by gravity or other means. The force of the mass originates at the center of gravity of the body in question. Thus, there is direction and a known weight. Another way to describe a force is something that has magnitude and direction.

FIGURE 1.1 Force not centered on beam.

Active forces are those forces created by the magnitudes and directions (P) such as the resultant force of a load of concrete. Reactions (R) are also forces, but they are the by-product of the sum of the resultant forces, such as two beams supporting another beam on each of its ends. For instance, Figure 1.1 shows an active P (10 k) force acting downward on the beam, 4 ft from the left side. The reactions are the forces at the supports pushing back (upward). The reactions between the left and right side are not the same because the active force is not centered on the beam.

Forces can be in line with the longitudinal axis of a member or act eccentrically. Axial, compression, and tension forces typically act in line with the linear axis of the member. Eccentric loading introduces forces coming from different directions from the normal forces. The forces are not always vertical or horizontal but sometimes act in directions that are not normal to either axis. Standard structural members are designed to best transfer forces along their linear axes; however, we are not always able to comply with this optimum design.

For the purpose of temporary structures, forces are either uniform (linear) or concentrated. Concentrated loads are also known as point loads and are represented by force arrows and force value in pounds or kips as shown above. Linear loads come in different forms. They can be triangular, trapezoidal, or rectangular depending on the material creating the forces. These forces are represented by a sequence of arrows and a value in pounds per linear foot (lb/ft) or kips per linear foot (kips/ft) as shown in Figure 1.2. Different types of forces created in temporary structures will be discussed in Chapter 3.

1.3.1 Centroids/Center of Gravity

When moment of inertia is introduced to the statics student, it is typically discussed after center of gravity is understood. The student should spend time understanding where and why each object has a center of gravity. In this book, the center of gravity (COG) and "centroid" or "centroidal axis" is used synonymously.

The center of gravity of a square, circle, or rectangle is in the center of each dimension of a material of uniform density. The vertical is referred to as the y axis and the horizontal is referred to the x axis. The center of gravity of circles (pipes) and rectangles is illustrated in Figure 1.3.

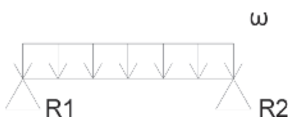

FIGURE 1.2 Uniform load on beam.

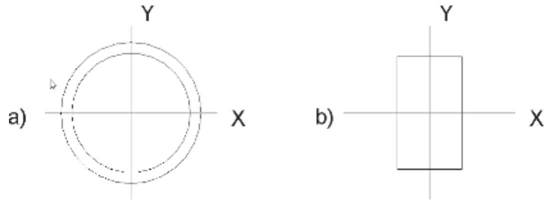

FIGURE 1.3 Center of gravity of pipes and rectangles.

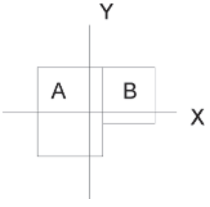

FIGURE 1.4 Center of gravity of a composite shape.

Shapes that are not symmetrical have centers of gravity that are also not symmetrical. For instance, if the shape in Figure 1.4 is considered, where would the center of gravity be in the x and in the y axis?

Example 1.1 If A has a base of $4''$ and a height of $6''$, and B has a base of $4''$ and a height of $3''$ and the top of A and B are flush, determine the composite's COG in the x and y directions.

Step 1: Calculate the area of A and B and determine each shape's individual center of gravity:

$$A = 4'' \times 6'' = 24 \text{ in}^2$$

A's COG is half the base and half the height, $2''$ and $3''$:

$$B = 4'' \times 3'' = 12 \text{ in}^2$$

B's COG is half the base and half the height, $2''$ and $1.5''$. The sum of the two areas is $24 + 12 = 36 \text{ in}^2$.

Step 2: Determine the distance of the x axis to the bottom of the composite by adding the moments of each about the bottom plane. Figure 1.5 illustrates this dimension.

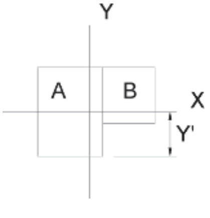

FIGURE 1.5 Center of gravity determining y'.

A's COG is $\frac{1}{2}$ (6″) = 3″ from the bottom.
B's COG is 3″ + $\frac{1}{2}$ (3″) = 4.5″ from the bottom.

Now considering their areas, total the areas about the bottom and make them equal to the combined area and its unknown distance to the new centroid (y'). This will be how far upward the horizontal centroid is:

$$[(24\ \text{in}^2)(3'')] + \left[\left(12\ \text{in}^2\right)(4.5'')\right] = [(36\ \text{in}^2)(y')]$$

Solve for y':

$$y' = \frac{(24\ \text{in}^2)(3'') + (12\ \text{in}^2)(4.5'')}{36\ \text{in}^2} \qquad y' = 3.5''$$

Step 3: Determine the distance of y' from the left side of the composite by summing the moments of each about the left side. Figure 1.6 illustrates this dimension.

A's COG is $\frac{1}{2}$ (4″) = 2″ from the left.
B's COG is 4″ + $\frac{1}{2}$ (4″) = 6″ from the left.

Once again considering their areas, sum the areas about the left side and make them equal to the combined area and its unknown distance to the new centroid (x'). This will be how far to the right the vertical centroid is:

$$[(24\ \text{in}^2)(2'')] + [(12\ \text{in}^2)(6'')] = [(36\ \text{in}^2)(x')]$$

Solve for x':

$$x' = \frac{(24\ \text{in}^2)(2'') + (12\ \text{in}^2)(6'')}{36\ \text{in}^2} \qquad x' = 3.33''$$

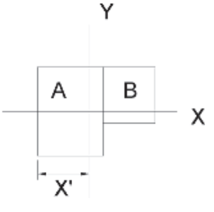

FIGURE 1.6 Center of gravity determining X.

Figure 1.7 shows the solution graphically.

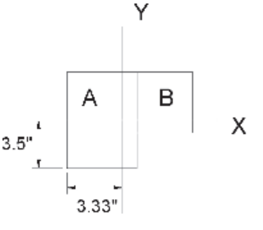

FIGURE 1.7 Center of gravity determining solution.

Figure 1.8 shows some sample calculations for some common shapes.

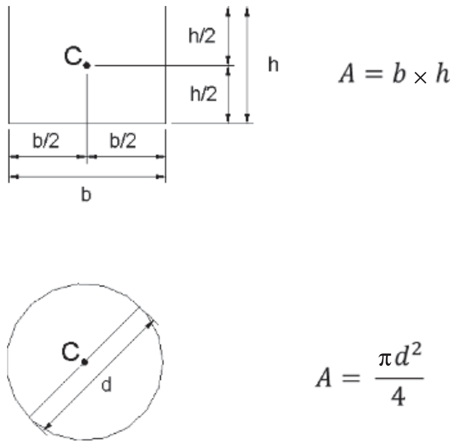

FIGURE 1.8 Centroids of area of common shapes.

1.3.2 Properties of Sections

Cross-Sectional Area The cross-sectional area of a beam consists of the depth times the width or the combination of these if there is an irregular shape. For a rectangular or square section, it is defined as

$$A = b \times d$$

where b = beam width
d = beam depth

Centroidal Axis The centroid of a beam section is important as one begins to study moment of inertia. The centroid is the center of gravity of the beam and can be measured about the x or the y axis. The distance from the centroid to the outermost surface of a beam is important because this is the location where the highest stress occurs. When a beam is placed in a bending condition, its stress is measured by the beams section modulus value. The section modulus is a derivative of the beams moment of inertia and the distance from the centroid to the outermost fiber (surface).

Moment of Inertia Moment of inertia is introduced in statics following the understanding of centroids and center of gravity. These basic concepts are key to understanding how beam sections are influenced by how far their sections extend beyond the centroidal axis. Moment of inertia of beams increases as the beams cross-sectional properties are farther from the centroidal axis. Moment of inertia is used to determine deflection in beams and is paramount to the value of section modulus, a very important component of a beam's bending resistance (to be covered in the next section).

The moment of inertia for a rectangle or square section can be solved by the following simple formula that is also referred to as the first moment:

$$I = \frac{bd^3}{12}$$

where b = beam width (in)
d = beam depth (in)

For composite sections, the moment of inertia is determined by the sum of the first and second moments. The first moment is the moment of inertia of the individual components of the composite (web and flanges). A composite section consists of different shapes (usually rectangles) that are put together to create one common unit made of the same material in most cases. The top and bottom horizontal components are referred to as flanges and the vertical components are referred to as webs or webbing. The second moment takes into account the distance of the composite components to the centroid. This is referred to by engineers as the parallel axis theorem. Large moments of inertia are developed when the individual components become farther from the centroid. Therefore, a tall beam would have a higher moment of inertia as the top and bottom flange get farther from the centroid. The moment of inertia of a composite beam can be solved with the following formula referred to by engineers as the parallel axis theorem:

$$I = \Sigma I + \Sigma Ad^2$$

where I = first moment (in^4)
A = area of an individual component (flange or web)
d = distance from the centroid to the center of the individual component

Example 1.2 Determine the moment of inertia about the horizontal axis for the composite beam section shown in Figure 1.9.

Beam properties:

The thickness of the flange and the web are both 2″.
The flange is 6″ wide (A).
The web is 8″ tall, not including the flange thickness (B).

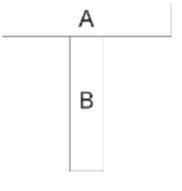

FIGURE 1.9 Section drawing of composite section.

Step 1: Determine the centroid (COG) about the horizontal axis.

To do this, total the moments of each section A and B about the bottom of the composite. Refer to Figure 1.10.

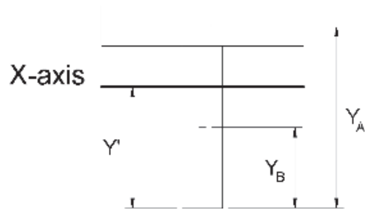

FIGURE 1.10 Section drawing of composite section.

Area of $A = 2 \times 6 = 12\ \text{in}^2$
Area of $B = 2 \times 8 = 16\ \text{in}^2$

Distance from bottom to center of A:

$$Y_A = 8 + 1 = 9''$$

Distance from bottom to center of B:

$$Y_B = \frac{8}{2} = 4''$$

$$(28)(y') = (12)(9) + (16)(4);\ y' = 6.14''$$

Step 2: Determine the first and second moments using Table 1.3. Calculate d, which is the distance from the section to the centroid. Table 1.3 summarizes the different values that make up the moment of inertia. It is recommended that one always use a similar table to organize the information to avoid mistakes.

$$\sum I + \sum Ad^2 = 260.75\ \text{in}^4$$

The first and second moments add up to 260.75 in^4.

TABLE 1.3 Moment of Inertia Summary

Section	Area	d (in)	Ad^2 (in^4)	I (in^4)	$I + Ad_2$ (in^4)
A	12	$9 - 6.14 = 2.86$	98.15	4	102.15
B	16	$6.14 - 4 = 2.14$	73.27	85.33	158.60

Section Modulus Section modulus is a derivative of moment of inertia. It increases or decreases based on the distance from its centroid to its outermost fiber, either top or bottom of the beam section. Section modulus is used when determining bending stress of a beam. Section modulus can be derived by the following formula or obtained from a beam section (see Appendix 1). This appendix comes from values calculated in the *Manual of Steel Construction* (MSC). The authors of the MSC have calculated all the beam properties for all available beams manufactured in the United States and some other countries. For wood sections, the National Design Specification for Wood Construction (NDS) provides values for the most common wood sections provided. Appendix 6 illustrates these values, which are rectangular

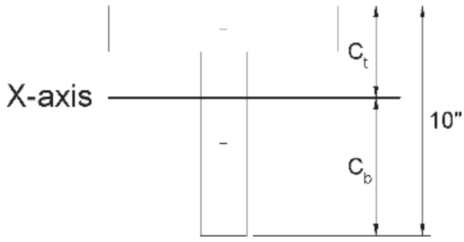

FIGURE 1.11 Distance to extreme fibers.

and square sections. If the moment of inertia and the centroid are known, then the section modulus can be derived from both. Figure 1.11 illustrates the distance *c* from either top or bottom of the composite.

$$S = \frac{I}{c}$$

where *I* is the moment of inertia and *c* is the distance from the centroid to the outermost fiber of the beam. If the composite beam in Example 1.1 was used as a simply supported beam, the top would be in compression and the bottom would be in tension. The beam height is 10 in, the center of gravity is 6.14″ from the bottom. Therefore, the center of gravity is (10 – 6.14) = 3.86″ from the top.

Based on the section modulus formula above:

$$S_{\text{bottom}} = \frac{260.75 \text{ in}^4}{6.14''} = 42.50 \text{ in}^3$$

$$S_{\text{top}} = \frac{260.75 \text{ in}^4}{10'' - 6.14''} = 67.55 \text{ in}^3$$

For a rectangular or square section, since the centroid is in the center of the section, the section modulus is as follows:

$$S = \frac{bd^2}{6}$$

where b = beam width
d = beam depth

Example 1.3 Determine the section modulus of the section shown where the base dimension is 3″ and the depth of the section is 9″. Refer to Figure 1.12 for details.

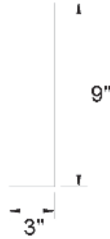

FIGURE 1.12 Vertical rectangular section.

Since $S = bd^2/6$, therefore, $S = (3)(9)^2/6 = 40.5$ in^3.

And since $I = bd^3/12$, therefore, $I = (3)(9)^3/12 = 182.3$ in^4.

If this section is turned 90° and its base becomes 9″ and depth becomes 3″, this would dramatically change the beam's section modulus (and moment of inertia). Let's recalculate the section modulus of the beam in the weak direction. Refer to Figure 1.13 for details.

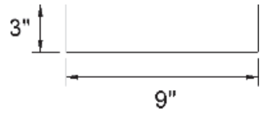

FIGURE 1.13 Horizontal rectangular section.

Since $S = bd^2/6$, therefore, $S = (9)(3)^2/6 = 13.5$ in^3.

And since $I = bd^3/12$, therefore, $I = (9)(3)^3/12 = 20.25$ in^4.

The bending resistance (S) between the strong axis and the weak axis is three times higher. The deflection and buckling resistance (I) between the strong and the weak axes is nine times higher.

Modulus of Elasticity Modulus of elasticity is a measurement of stiffness for a particular material. Every material has a different value for modulus of elasticity. The most common materials in temporary structures will be discussed in succeeding chapters. Modulus of elasticity is used in temporary structures to determine deflection in beams and allowable buckling stress in columns. This will be discussed in more detail in the following chapters.

Radius of Gyration Radius of gyration is a distance from the centroid toward the outer portions of the beam or column that resists buckling. Rectangles (lumber and tube steel) have a strong and a weak radius of gyration value when the x and y axes are different dimensions. Squares (lumber and tube steel) have the same radius of gyration in both the x and y axis Circles have a radial radius of gyration; it's the same for 360°. Finally, angle iron and similar sections have three axes of radius of gyration, and the weak axis for buckling is the z axis. This z-axis radius of gyration for angle sections should be used for buckling calculations if the designer wants to use the worst-case scenario. Figure 1.14 shows these three axes. The radius of gyration is a

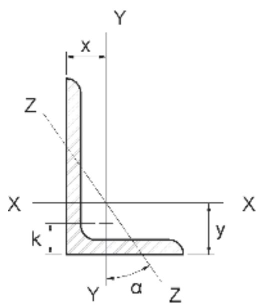

FIGURE 1.14 Angle section showing three axes.

derivative of the cross-sectional area (A) of a section and its moment of inertia (I). Radius of gyration is calculated as follows:

$$r = \sqrt{I/A}$$

where I = moment of inertia
A = cross-sectional area

Notice that the units work out to inches from the center of the cross section outward. There is also a different radius of gyration for each axis, the strong and weak. Usually when column buckling is a concern, the weak r value is used because that is the direction that buckling will occur.

Example 1.4 What is the radius of gyration of a 4″ diameter standard pipe with the following properties?

$$A = 3.17\ \text{in}^2 \quad \text{and} \quad I = 7.23\ \text{in}^4$$

$$\sqrt{7.23\ \text{in}^4/3.17\ \text{in}^2} = 1.51\ \text{in}$$

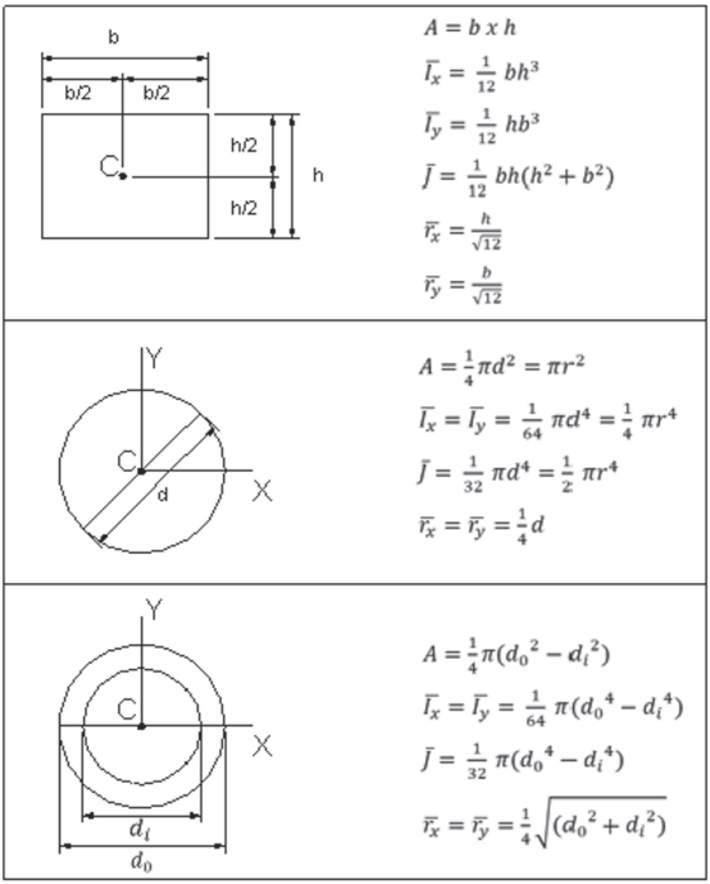

FIGURE 1.15 Common shape properties.

The radius of gyration for this 4″ pipe determines how much buckling resistance the pipe would have if used as a column or strut.

Figure 1.15 shows common shapes used in temporary structures and illustrates the important properties.

Resultants The resultant of a force system is the system converted to a single force. The location of the resultant is determined by the shape of the system. Figure 1.16 shows examples of some simple resultants from a few different force systems.

The resultant of the rectangular force is located in the center.

The resultant of the triangular force is located one third the distance from the left.

The resultant of the trapezoidal force is located a distance (x) somewhere between the triangular resultant and the rectangular resultant.

Statics students typically spend a great deal of time on force vectors, force triangles, and truss analysis. This text does not review these concepts as they are not used with basic temporary structure analyses. This will become evident in Chapter 3 when the different forces in temporary structures are introduced. In statics, it was also very important to understand the difference between different types of supports: roller, fixed, hinged, and the like. Temporary structure supports are hinged the majority of the time. The rest of the time, the supports or connections are fixed by welding or bolting. The reason this is not common is because temporary structures are "temporary," and any effort to fix connections cost money in labor and materials both while installing and while removing them.

Free-body diagrams (FBDs) are drawings that summarize the forces (concentrated), their directions, and the beam geometry and conditions. As learned in statics, the free-body diagram can be the whole structure or part of the structure. This text tends to draw FBDs of the whole structure. Those teaching temporary structures should always insist on a properly drawn FBD accompanying all temporary structures computations. Mistakes most often occur when students have not taken time to create a complete FBD of the structure. A typical FBD is illustrated in Figure 1.17.

Moments Created by Forces A moment is a tendency of a force to create rotation. The point at which the moment is rotating is the center of the moment.

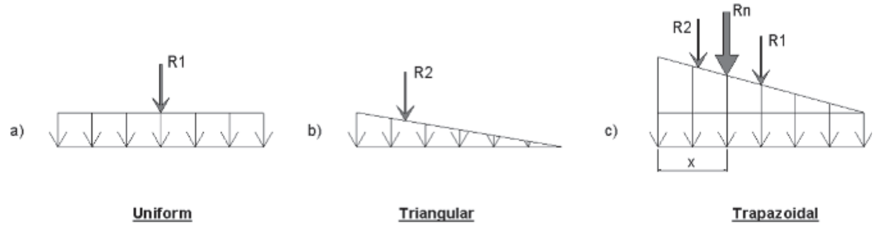

FIGURE 1.16 Resultant force location.

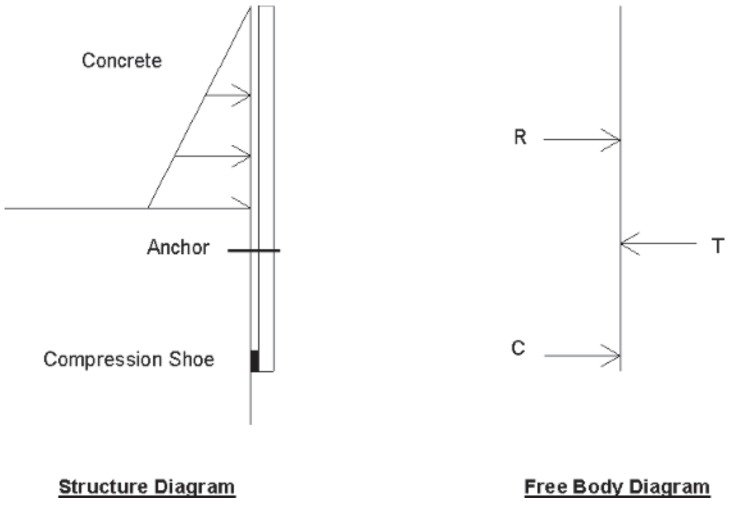

FIGURE 1.17 Free-body diagram example.

FIGURE 1.18 Static equilibrium.

The perpendicular distance between the force line of action and the center of the moment is the moment arm. Therefore, a moment is the force times the moment arm distance. If force is represented in pounds and the moment arm is represented by feet, then the moment units are in foot-pounds (ft-lb).

Laws of Equilibrium This text focuses on static equilibrium. Static equilibrium is when a body is at rest and all forces cancel all other forces out. To achieve static equilibrium, three laws form the cornerstone to temporary and permanent design. These are shown in Figure 1.18.

All forces in the horizontal direction must equal zero, $\Sigma Fx = 0$.
All forces in the vertical direction must equal zero, $\Sigma Fy = 0$.
All moments must equal zero, $\Sigma M = 0$.

Example 1.5 Determine the *X* and *Y* forces and the moment created in the simple structure at support *A* with the forces shown in Figure 1.19.

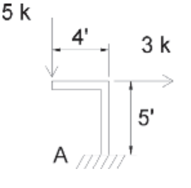

FIGURE 1.19 Example 3 sketch.

$$\Sigma Fy = 0 \qquad -5\text{ k} + F_{\text{VA}} = 0 \qquad F_{\text{VA}} = 5\text{ k}$$

$$\Sigma Fx = 0 \qquad +3\text{ k} - F_{\text{HA}} = 0 \qquad F_{\text{HA}} = -3\text{ k}$$

$$\Sigma M = 0 \qquad +(5\text{ k} \times 4\text{ ft}) - (3\text{ k} \times 5\text{ ft}) = 0 \qquad \text{MA} = +5\text{ ft-k}$$

F_{VA} = vertical force at point A

F_{HA} = horizontal force at point A

M_A = moment at point A

ΣF_X = sum of forces horizontally

ΣF_Y = sum of forces vertically

Determining Reactions If we go back to Figure 1.1 and ask what are the reaction forces at the left support and the right support, this would be possible by adding the moments about each end of the beam.

Example 1.6 Determine the left and right reactions of the following beam where $P = 5$ k, the beam is15 ft long, and the concentrated force is located 4 ft from the left side as shown in Figure 1.20.

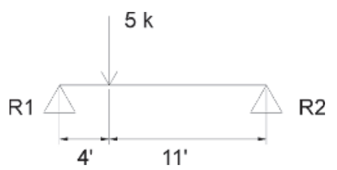

FIGURE 1.20 Simple beam reaction example.

To determine the reaction forces at R_1 and R_2, total the moments about one of the two reactions. Let's first add up the moments about R_1.

$$\Sigma MR_1 = 0 \qquad -[(5\,\text{k})(4')] + [(R_2)(15')] = 0$$

therefore, $R_2 = 1.33$ k.

Now for the other reaction, we can use one of two methods. We can either total the moments about R_2, in order to get R_1 (similar to what we just did), or we can add up the forces in the vertical direction:

$$\Sigma MR_1 = 0 \qquad +[(5\,\text{k})(11')] + [(R_{1V})(15')] = 0$$

therefore, $R_{1V} = 3.67$ k.

$$\Sigma F_V = 0 \qquad -5\text{ k} + 1.33\text{ k} + R_{2V} = 0$$

therefore, $R_{2V} = 3.67$ k.

FIGURE 1.21 Simple beam resultant force solution.

It is recommended that the moment method by itself be used or that both methods be used as a double check. The vertical force method can produce errors if the R_1 calculation was incorrect. The engineer should also do a visual check. This means, do the results make sense? For example, if we reversed the reactions by mistake and did a visual check, we would realize that the larger reaction force is farther from the 5-k force and that would not be logical. The larger reaction force has to be on the side that the 5-k force is closest to. Figure 1.21 illustrates the resultant forces of Example 1.6.

Example 1.7 Now let's look at a more complicated example. Figure 1.22 contains multiple loads and load types.

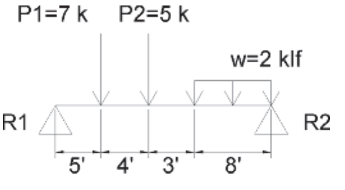

FIGURE 1.22 Simple beam with various loads.

This beam is 20 ft long; the 7-k (P_1) load is 5 ft from the left side; the 5-k (P_2) load is 9 ft from the left side; and the uniform load is 8 ft long, starts 12 ft from the left, and extends to the far right of the beam.

Step 1: Since there is a uniform load, first it must be converted to a concentrated load.

$$P_3 = wL$$

where P_3 = force in klf
w = uniform load
L = length of the beam

$$P_3 = 2 \text{ klf} \times 8 \text{ ft} = 16 \text{ k}$$

Step 2: Place the P_3 load of 16 k at the center of the uniform load and draw a new FBD. Double check that the dimensions equal 20 ft. Draw an FBD similar to Figure 1.23.

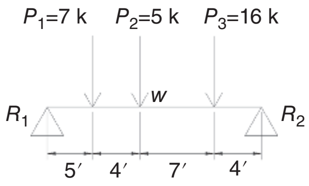

FIGURE 1.23 Simple beam various loads resultant forces.

Step 3: Add the moments about the left reaction (R_1) in order to calculate R_2.

$$\Sigma MR_1 = 0 \qquad -[(7\,\text{k})(5')] - [(5\,\text{k})(9')] - [(16\,\text{k})(16')] + [(R_2)(20')] = 0$$

therefore, $R_2 = 16.8$ k.

Step 4: Determine R_1 using both methods shown above in Example 1.1.

$$\Sigma MR_2 = 0 \qquad +[(7\,\text{k})(15')] + [(5\,\text{k})(11')] + [(16\,\text{k})(4')] - [(R_1)(20')] = 0$$

therefore, $R_1 = 11.2$ k.

$$\Sigma F_V = 0 \qquad -7\text{ k } - 5\text{ k } - 16\text{ k} + 16.8\text{ k} + R_{1V} = 0$$

therefore, $R_{1V} = 11.2$ k.

The visual test is not as easy this time, but with the 16-k load close to the right support, it would be expected that the right side would have a larger reaction.