1

Some basic concepts

1.1 Introduction

Particle physics is the study of the fundamental constituents of matter and their interactions. However, which particles are regarded as fundamental has changed with time as physicists' knowledge has improved. Modern theory – called the *standard model* – attempts to explain all the phenomena of particle physics in terms of the properties and interactions of a small number of particles of four distinct types: two spin-1/2 families of fermions called *leptons* and *quarks*; one family of spin-1 bosons – called *gauge bosons* – which act as 'force carriers' in the theory; and a spin-0 particle, called the *Higgs boson*, which explains the origin of mass within the theory, since without it, leptons, quarks and gauge bosons would all be massless. All the particles of the standard model are assumed to be *elementary*: that is they are treated as point particles, without internal structure or excited states.

The most familiar example of a lepton is the *electron* e^- (the superscript denotes the electric charge), which is bound in atoms by the *electromagnetic interaction,* one of the four fundamental forces of nature. A second well-known lepton is the *electron neutrino* ν_e, which is a light, neutral particle observed in the decay products of some unstable nuclei (the so-called β decays). The force responsible for the β decay of nuclei is called the *weak interaction.*

Another class of particles called *hadrons* is also observed in nature. Familiar examples are the neutron n and proton p (collectively called *nucleons*) and the three *pions* (π^+, π^-, π^0), where the superscripts again denote the electric charges. These are not elementary particles, but are made of quarks bound together by a third force of nature, the *strong interaction.* The theory is unusual in that

Particle Physics, Fourth Edition. B.R. Martin and G. Shaw.

Companion website: www.wiley.com/go/martin/particlephysics4

the quarks themselves are not directly observable, only their bound states. Nevertheless, we shall see in later chapters that there is overwhelming evidence for the existence of quarks and we shall discuss the reason why they are unobservable as free particles. The strong interaction between quarks gives rise to the observed strong interaction between hadrons, such as the nuclear force that binds nucleons in nuclei. There is an analogy here with the fundamental electromagnetic interaction between electrons and nuclei that also gives rise to the more complicated forces between their bound states, that is between atoms.

In addition to the strong, weak, and electromagnetic interactions, there is a fourth force of nature – gravity. However, the gravitational interaction between elementary particles is so small compared to those from the other three interactions that it can be neglected at presently accessible energies. Because of this, we will often refer in practice to the *three* forces of nature.

The standard model also specifies the origin of these three forces. Consider firstly the electromagnetic interaction. In classical physics this is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of spin-1 photons, which are the 'force carriers', or *gauge bosons*, of the electromagnetic interaction and, as we shall see presently, the long-range nature of the force is related to the fact that photons have zero mass. The use of the word 'gauge' refers to the fact that the electromagnetic interaction possesses a fundamental symmetry called *gauge invariance.* This property is common to all the three interactions of nature and has profound consequences, as we shall see.

The weak and strong interactions are also associated with the exchange of spin-1 particles. For the weak interaction, they are the charged $W^{\pm}$ and the neutral Z^0 *bosons,* with masses about 80–90 times the mass of the proton. The resulting force is very short range, and in many applications may be approximated by an interaction at a point. The equivalent particles for the strong interaction are called *gluons g.* There are eight gluons, all of which have zero mass and are electrically neutral, like the photon. Thus, by analogy with electromagnetism, the basic strong interaction between quarks is long range. The 'residual' strong interaction between the quark bound states (hadrons) is not the same as the fundamental strong interaction between quarks (but is a consequence of it) and is short range, again as we shall see later.

In the standard model, which will play a central role in this book, the main actors are the leptons and quarks, which are the basic constituents of matter; the 'force carriers' (the photon, the W and Z

bosons, and the gluons) that mediate the interactions between them; and the Higgs boson, which gives mass to the elementary particles of the standard model. In addition, because not all these particles are directly observable, quark bound states (i.e. hadrons) also play a very important role.

1.2 Antiparticles

In particle physics, high energies are needed both to create new particles and to explore the structure of hadrons. The latter requires projectiles whose wavelengths λ are at least as small as hadron radii, which are of order 10^{-15} m. It follows that their momenta $p = h/\lambda$ must be several hundred MeV/c ($1\,\text{MeV} = 10^6\,\text{eV}$), and hence their energies E must be several hundred MeV. Because of this, any theory of elementary particles must combine the requirements of both special relativity and quantum theory. This has the startling consequence that for every charged particle of nature, whether it is one of the elementary particles of the standard model, or a hadron, there exists an associated particle of the same mass, but opposite charge, called its *antiparticle*. This important theoretical prediction was first made for spin-1/2 particles by Dirac in 1928, and follows from the solutions of the equation he first wrote down to describe relativistic electrons. We therefore start by considering how to construct a relativistic wave equation.

1.2.1 Relativistic wave equations

We start from the assumption that a particle moving with momentum $\mathbf{p}$ in free space is described by a de Broglie wave function[1]

$$\Psi(\mathbf{r}, t) = N\, e^{i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar}, \tag{1.1}$$

with frequency $\nu = E/h$ and wavelength $\lambda = h/p$. Here $p \equiv |\mathbf{p}|$ and N is a normalisation constant that is irrelevant in what follows. The corresponding wave equation depends on the assumed relation between the energy E and momentum $\mathbf{p}$. Non-relativistically,

$$E = p^2/2m \tag{1.2}$$

and the wave function (1.1) obeys the non-relativistic Schrödinger equation

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t). \tag{1.3}$$

[1] We use the notation $\mathbf{r} = (x_1, x_2, x_3) = (x, y, z)$.

Relativistically, however,

$$E^2 = p^2c^2 + m^2c^4, \tag{1.4}$$

where m is the rest mass,[2] and the corresponding wave equation is

$$-\hbar^2 \frac{\partial^2 \Psi(\mathbf{r},t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi(\mathbf{r},t) + m^2c^4\Psi(\mathbf{r},t), \tag{1.5}$$

as is easily checked by substituting (1.1) into (1.5) and using (1.4). This equation was first proposed by de Broglie in 1924, but is now more usually called the *Klein–Gordon equation*.[3] Its most striking feature is the existence of solutions with negative energy. For every plane wave solution of the form

$$\Psi(\mathbf{r},t) = N \exp\left[i(\mathbf{p}\cdot\mathbf{r} - E_p t)/\hbar\right], \tag{1.6a}$$

with momentum $\mathbf{p}$ and positive energy

$$E = E_p \equiv +(p^2c^2 + m^2c^4)^{1/2} \geq mc^2$$

there is also a solution

$$\widetilde{\Psi}(\mathbf{r},t) \equiv \Psi^*(\mathbf{r},t) = N^* \exp\left[i(-\mathbf{p}\cdot\mathbf{r} + E_p t)/\hbar\right], \tag{1.6b}$$

corresponding to momentum $-\mathbf{p}$ and negative energy

$$E = -E_p = -(p^2c^2 + m^2c^4)^{1/2} \leq -mc^2.$$

Other problems also occur, indicating that the Klein–Gordon equation is not, in itself, a sufficient foundation for relativistic quantum mechanics. In particular, it does not guarantee the existence of a positive-definite probability density for position.[4]

The existence of negative energy solutions is a direct consequence of the quadratic nature of the mass–energy relation (1.4) and cannot be avoided in a relativistic theory. However, *for spin-1/2 particles* the other problems were resolved by Dirac in 1928, who looked for an equation of the familiar form

$$i\hbar \frac{\partial \Psi(\mathbf{r},t)}{\partial t} = H(\mathbf{r},\hat{\mathbf{p}})\Psi(\mathbf{r},t), \tag{1.7}$$

where H is the Hamiltonian and $\hat{\mathbf{p}} = -i\hbar\nabla$ is the momentum operator. Since (1.7) is first order in $\partial/\partial t$, Lorentz invariance requires that

[2] From now on, the word *mass* will be used to mean the rest mass.

[3] These authors incorporated electromagnetic interactions into the equation, in a form now known to be appropriate for charged spin-0 bosons.

[4] For a discussion of this point, see for example pp. 467–468 of Schiff (1968).

it also be first order in spatial derivatives. Dirac therefore proposed a Hamiltonian of the general form

$$H = -i\hbar c \sum_{i=1}^{3} \alpha_i \frac{\partial}{\partial x_i} + \beta mc^2 = c\,\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta mc^2, \tag{1.8}$$

in which the coefficients β and $\alpha_i (i = 1, 2, 3)$ are determined by requiring that solutions of the Dirac equation (1.8) are also solutions of the Klein–Gordon equation (1.5). Acting on (1.7) with $i\hbar\partial/\partial t$ and comparing with (1.5), leads to the conclusion that this is true if, and only if,

$$\alpha_i^2 = 1,\ \beta^2 = 1, \tag{1.9a}$$

$$\alpha_i \beta + \beta \alpha_i = 0 \tag{1.9b}$$

and

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 0 \qquad (i \neq j). \tag{1.9c}$$

These relations cannot be satisfied by ordinary numbers, and the simplest assumption is that β and $\alpha_i (i = 1,\ 2,\ 3)$ are matrices, which must be hermitian so that the Hamiltonian is hermitian. The smallest matrices satisfying these requirements have dimensions 4×4 and are given in many books,[5] but are not required below. We thus arrive at an interpretation of the Dirac equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi = -i\hbar c \sum_i \alpha_i \frac{\partial \Psi}{\partial x_i} + \beta mc^2 \Psi \tag{1.10}$$

as a four-dimensional matrix equation in which the Ψ are four-component wave functions

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \psi_1(\mathbf{r}, t) \\ \psi_2(\mathbf{r}, t) \\ \psi_3(\mathbf{r}, t) \\ \psi_4(\mathbf{r}, t) \end{pmatrix}, \tag{1.11}$$

called *spinors.* Plane wave solutions take the form

$$\Psi(\mathbf{r}, t) = u(\mathbf{p})\ \exp\left[i(\mathbf{p} \cdot \mathbf{r} - Et)/\hbar\right], \tag{1.12}$$

where $u(\mathbf{p})$ is also a four-component spinor satisfying the eigenvalue equation

$$H_{\mathbf{p}} u(\mathbf{p}) \equiv (c\,\alpha \cdot \mathbf{p} + \beta mc^2) u(\mathbf{p}) = Eu(\mathbf{p}), \tag{1.13}$$

obtained by substituting (1.11) into (1.10). This equation has four solutions:[6] two with positive energy $E = +E_p$ corresponding to the two possible spin states of a spin-1/2 particle (called ‘spin up’ and

[5] See, for example, pp. 473–475 of Schiff (1968).
[6] A proof of these results is given in, for example, pp. 475–477 of Schiff (1968).

'spin down', respectively) and two corresponding negative energy solutions with $E = -E_p$.

The problem of the negative-energy solutions will be resolved in the next section. Here we note that the positive-energy solutions of the Dirac equation lead to many predictions that have been verified experimentally to a very high precision. Notable among these are relativistic corrections in atomic spectroscopy, including spin-orbit effects and the prediction that point-like spin-1/2 particles of mass m and charge q have a Dirac magnetic moment

$$\boldsymbol{\mu}_D = q\,\mathbf{S}/m, \tag{1.14}$$

where $\mathbf{S}$ is the spin vector. This is a key result. It not only yields the correct value for the electron but provides a simple test for the point-like nature of any other spin-1/2 fermion. For the proton and neutron, the experimental values are

$$\boldsymbol{\mu}_p = 2.79\,e\mathbf{S}/m_p \quad \text{and} \quad \boldsymbol{\mu}_n = -1.91\,e\mathbf{S}/m_n, \tag{1.15}$$

in disagreement with Equation (1.14). Historically, the measurement of the proton magnetic moment by Frisch and Stern in 1933 was the first indication that the proton was not a point-like elementary particle.

1.2.2 Hole theory and the positron

The problem of the negative energy states remains. They cannot be ignored, since their existence leads to unacceptable consequences. For example, if such states are unoccupied, then transitions from positive to negative energy states could occur, leading to the prediction that atoms such as hydrogen would be unstable. This problem was resolved by Dirac, who postulated that the negative energy states are almost always filled. For definiteness, consider the case of electrons. Since they are fermions, they obey the Pauli exclusion principle, and the Dirac picture of the vacuum is a so-called 'sea' of negative energy states, each with two electrons (one with spin 'up', the other with spin 'down'), while the positive energy states are all unoccupied (see Figure 1.1). This state is indistinguishable from the usual vacuum with $E_V = 0$, $\mathbf{p}_V = \mathbf{0}$, etc. This is because for each state of momentum $\mathbf{p}$ there is a corresponding state with momentum $-\mathbf{p}$, so that the momentum of the vacuum $\mathbf{p}_V = \Sigma\,\mathbf{p} = \mathbf{0}$. The same argument applies to spin, while, since energies are measured relative to the vacuum, $E_V \equiv 0$ by definition. Similarly, we may define the charge $Q_V \equiv 0$, because the constant electrostatic potential produced by the negative energy sea is unobservable. Thus this state has all the *measurable* characteristics of the naive vacuum, and the 'sea' is unobservable.

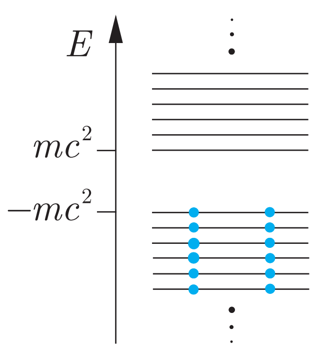

Figure 1.1 Dirac picture of the vacuum. The sea of negative energy states is totally occupied with two electrons in each level, one with spin 'up' and one with spin 'down'. The positive energy states are all unoccupied.

Dirac's postulate solves the problem of unacceptable transitions from positive energy states, but has other consequences. Consider what happens when an electron is added to, or removed from, the vacuum. In the former case, the electron is confined to the positive energy region since all the negative energy states are occupied. In the latter case, *removing* a negative energy electron with $E = -E_p < 0$, momentum $-\mathbf{p}$, spin $-\mathbf{S}$ and charge $-e$ from the vacuum (which has $E_V = 0, \mathbf{p}_V = 0, \mathbf{S}_V = 0, Q_V = 0$) leaves a state (the sea with a 'hole' in it) with positive energy $E = E_p > 0$, momentum $\mathbf{p}$, spin $\mathbf{S}$ and charge $+e$. This state cannot be distinguished by any measurement from a state formed by *adding* to the vacuum a particle with momentum $\mathbf{p}$, energy $E = E_p > 0$, spin $\mathbf{S}$ and charge $+e$. The two cases are equivalent descriptions of the same phenomena. Using the latter, Dirac predicted the existence of a spin-1/2 particle e^+ with the same mass as the electron, but opposite charge. This particle is called the *positron* and is referred to as the *antiparticle* of the electron.[7]

The positron was subsequently discovered by Anderson, and by Blackett and Ochialini, in 1933. The discovery was made using a device of great historical importance, called a *cloud chamber*. When a charged particle passes through matter, it interacts with it, losing energy. This energy can take the form of radiation or of excitation and ionisation of the atoms along the path.[8] It is the aim of *track chambers* – of which the cloud chamber is the earliest example – to produce a visible record of this trail and hence of the particle that produced it.

The cloud chamber was devised by C.T.R. Wilson, who noticed that the condensation of water vapour into droplets goes much faster in the presence of ions. It consisted of a vessel filled with air almost saturated with water vapour and fitted with an expansion piston. When the vessel was suddenly expanded, the air cooled and became supersaturated. Droplets were then formed preferentially along the trails of ions left by charged particles passing through the chamber. The chamber was illuminated by a flash of light immediately after expansion, and the tracks of droplets so revealed were photographed before they had time to disperse.

Figure 1.2 shows one of the first identified positron tracks observed by Anderson in 1933. The band across the centre of the picture is a 6 mm lead plate inserted to slow particles down. The track is curved due to the presence of a 1.5 T applied magnetic field

[7] This prediction, which is now regarded as one of the greatest successes of theoretical physics, was not always so enthusiastically received at the time. For example, in 1933 Pauli wrote: 'Dirac has tried to identify holes with antielectrons. We do not believe that this explanation can be seriously considered' (Pauli, 1933).
[8] This will be discussed in detail in Chapter 4.

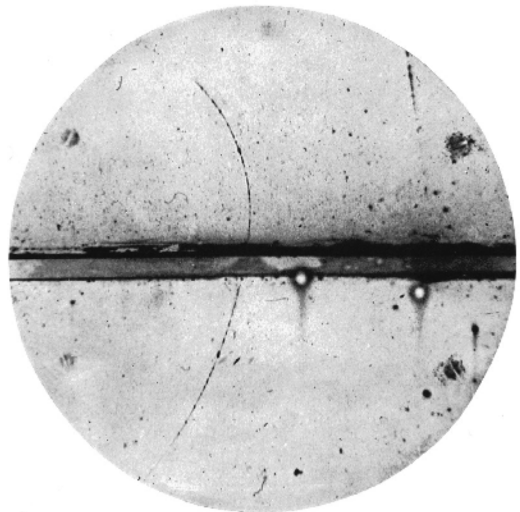

Figure 1.2 One of the first positron tracks observed by Anderson in a Wilson cloud chamber (see text for details). (Anderson 1933. Reproduced with permission from the American Physical Society.)

$\mathbf{B}$, and since the curvature of such tracks increases with decreasing momentum, we can conclude that the particle enters at the bottom of the picture and travels upwards. The sign of the particle's charge q then follows from the direction of the Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, where $\mathbf{v}$ is the particle's velocity, and hence of the curvature; it is positive.

That the particle is a positron and not a proton follows essentially from the range of the upper track. The rate of energy loss of a charge particle in matter depends on its charge and velocity. (This will be discussed in Chapter 4.) From the curvature of the tracks, one can deduce that the momentum of the upper track is 23 MeV/c, corresponding to either a slow-moving proton with speed $v \ll c$ or a relativistic ($v \approx c$) positron. The former would lose energy rapidly, coming to rest in a distance of about 5 mm, comparable with the thickness of the lead plate. The observed track length is more than 5 cm, enabling a limit $m_+ \leq 20m_e \ll m_p$ to be set on the mass m_+ of the particle, which Anderson suggested was a positron. Many other examples were found, especially by Blackett and Ochialini, and by 1934 Blackett, Ochialini and Chadwick had established that $m_+ = m_e$ within experimental errors of order 10%. The interpretation of the light positive particles as positrons was thus established beyond all reasonable doubt.

The Dirac equation applies to any spin-1/2 particle, and hole theory predicts that *all* charged spin-1/2 particles, whether they are elementary or hadrons, have distinct antiparticles with opposite charge, but the same mass. The argument does not extend to bosons, because they do not obey the exclusion principle on which hole theory depends, and to show that charged bosons also have antiparticles of opposite charge requires the formal apparatus of relativistic quantum field theory.[9] We shall not pursue this here, but note that the basic constituents of matter – the leptons and quarks – are not bosons, but are spin-1/2 fermions. The corresponding results on the antiparticles of hadrons, irrespective of their spin, can then be found by considering their quark constituents, as we shall see in Chapter 3. For neutral particles, there is no general rule governing the existence of antiparticles, and while some neutral particles have distinct antiparticles associated with them, others do not. The photon, for example, does not have an antiparticle (or, rather, the photon and its antiparticle are identical) whereas the neutron does. Although the neutron has zero charge, it has a non-zero magnetic moment, and distinct antineutrons exist in which the sign of this magnetic moment is reversed relative to the spin direction. The neutron is also characterised by other quantum numbers (that we will meet later) that change sign between particle and antiparticle.

In what follows, if we denote a particle by P, then the antiparticle is in general written with a bar over it, that is $\overline{P}$. For example, the antiparticle of the proton p is the antiproton $\bar{p}$, with negative electric charge, and associated with every quark, q, is an antiquark, $\bar{q}$. However, for some common particles the bar is usually omitted. Thus, for example, in the case of the positron e^+, the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle, the electron.

1.3 Interactions and Feynman diagrams

By analogy with chemical reactions, interactions involving elementary particles and/or hadrons are conveniently summarised by 'equations', in which the different particles are represented by symbols. Thus, in the reaction $\nu_e + n \to e^- + p$, an electron neutrino ν_e interacts with a neutron n to produce an electron e^- and a proton p; while $e^- + p \to e^- + p$ represents an electron and proton interacting to give the same particles in the final state, but in general travelling in different directions. The latter, where the initial and final particles are the same, is an example of *elastic scattering*, whereas $\nu_e + n \to e^- + p$, where the initial and final particles differ, is an example of an *inelastic reaction.* The forces producing the above interactions are due to the exchange of particles and a convenient way of illustrating this is to use the pictorial technique of *Feynman diagrams.* These were introduced by Feynman in the 1940s and are now one of the cornerstones of elementary particle physics.

[9] For an introduction, see for example Mandl and Shaw (2010).

Associated with them are mathematical rules and techniques that enable the calculation of the quantum mechanical probabilities for given reactions to occur. Here we will avoid the mathematical detail but use the diagrams to understand the main features of particle reactions. We will introduce Feynman diagrams by firstly discussing electromagnetic interactions.

1.3.1 Basic electromagnetic processes

The electromagnetic interactions of electrons and positrons can all be understood in terms of eight basic processes. In hole theory, they arise from transitions in which an electron jumps from one state to another, with the emission or absorption of a single photon. The interpretation then depends on whether the states are both of positive energy, both of negative energy or one of each.

The basic processes whereby an electron either emits or absorbs a photon are

$$\text{(a)}\, e^- \to e^- + \gamma \quad \text{and} \quad \text{(b)}\, \gamma + e^- \to e^-.$$

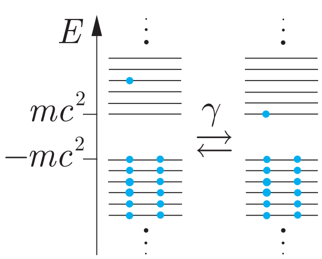

Figure 1.3 Hole theory representation of the processes $e^- \rightleftarrows e^- + \gamma$.

They correspond in hole theory to transitions between positive energy states of the electron and are represented pictorially in Figure 1.3. They may also be represented diagrammatically by Figures 1.4(a) and (b), where by convention time runs from left to right. These are examples of Feynman diagrams.

Similar diagrams may be drawn for the corresponding positron processes

$$\text{(c)}\, e^+ \to e^+ + \gamma \quad \text{and} \quad \text{(d)}\, \gamma + e^+ \to e^+,$$

and are shown in Figures 1.4(c) and (d). Time again flows to the right, and we have used the convention that an arrow directed towards the right indicates a particle (in this case an electron) while one directed to the left indicates an antiparticle (in this case a positron). The corresponding hole theory diagram, analogous to Figure 1.3, is left as an exercise for the reader. Finally, there are processes in which an electron is excited from a negative energy state to a positive energy state, leaving a 'hole' behind, or in which a positive energy electron falls into a vacant level (hole) in the negative energy sea. These are illustrated in Figure 1.5, and correspond to the production or annihilation of e$^+$e- pairs. In both cases, a photon may either be absorbed from the initial state or emitted to the final state, giving the four processes

$$\begin{array}{ll} \text{(e)}\, e^+ + e^- \to \gamma, & \text{(f)}\, \gamma \to e^+ + e^-, \\ \text{(g) vacuum} \to \gamma + e^+ + e^-, & \text{(h)}\, \gamma + e^+ + e^- \to \text{vacuum}, \end{array}$$

represented by the Feynman diagrams of Figures 1.4(e) to (h).

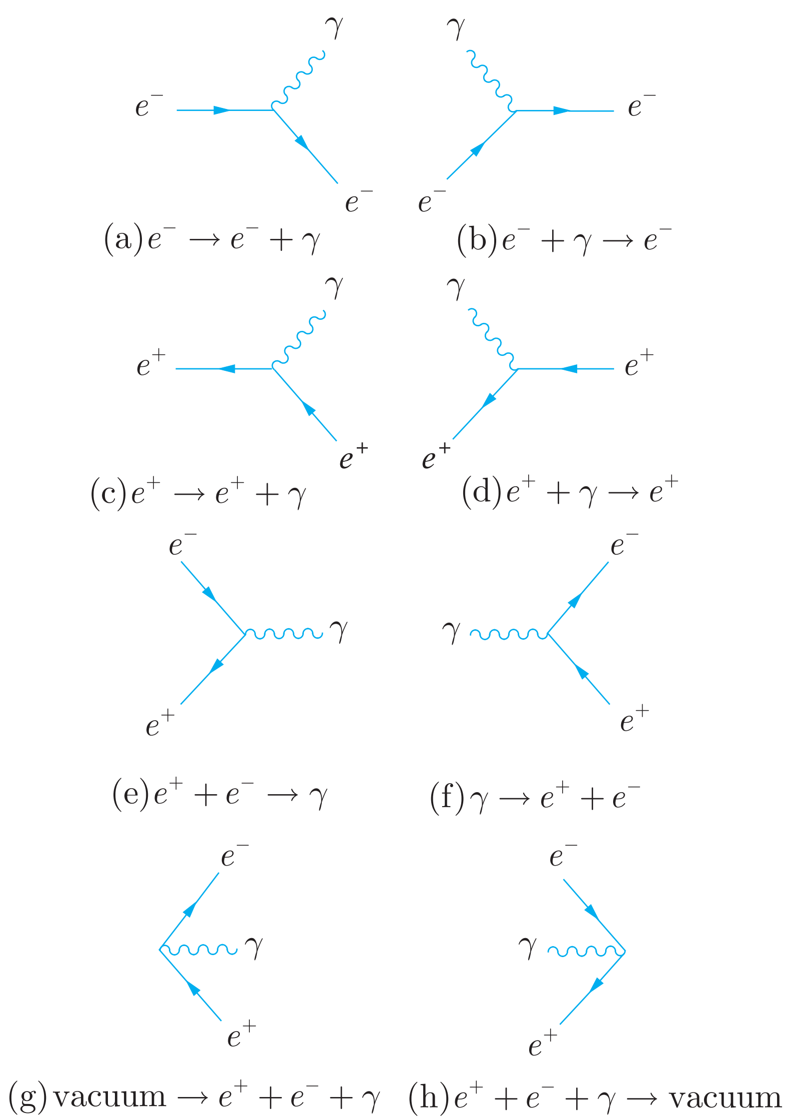

Figure 1.4 Feynman diagrams for the eight basic processes whereby electrons and positrons interact with photons. In all such diagrams, time runs from left to right, while a solid line with its arrow pointing to the right (left) indicates an electron (positron).

This exhausts the possibilities in hole theory, so that there are just eight basic processes represented by the Feynman diagrams, Figures 1.4(a) to (h). Each of these processes has an associated probability of the order of the electromagnetic fine structure constant

$$\alpha \equiv \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}. \tag{1.16}$$

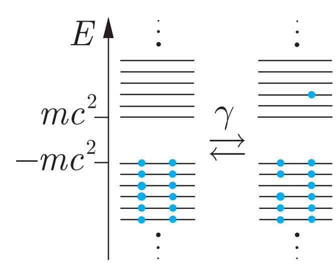

Figure 1.5 Hole theory representation of the production or annihilation of e^+e^- pairs.

1.3.2 Real processes

In each of the diagrams in Figures 1.4(a) to (h), each vertex has a line corresponding to a single photon being emitted or absorbed,

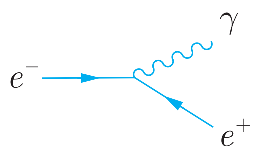

Figure 1.6 The forbidden vertex $e^- \to e^+ + \gamma$.

while one fermion line has the arrow pointing toward the vertex and the other away from the vertex, implying charge conservation at the vertex.[10] For example, a vertex like Figure 1.6 would correspond to a process in which an electron emitted a photon and turned into a positron. This would violate charge conservation and is therefore forbidden.

Momentum and angular momentum are also assumed to be conserved at the vertices. However, in free space, energy conservation is violated. For example, if we use the notation $(E, \mathbf{k})$ to denote the total energy and three-momentum of a particle, then in the rest frame of the electron, reaction (a), is

$$e^-(E_0, \mathbf{0}) \to e^-(E_k, -\mathbf{k}) + \gamma(ck, \mathbf{k}), \tag{1.17}$$

where $k \equiv |\mathbf{k}|$ and momentum conservation has been imposed. In free space, $E_0 = mc^2$, $E_k = (k^2c^2 + m^2c^4)^{1/2}$ and $\Delta E \equiv E_k + kc - E_0$ satisfies

$$kc < \Delta E < 2kc \tag{1.18}$$

for all finite k.

Similiar arguments show that energy conservation is violated for all the basic processes. They are called *virtual* processes to emphasise that they cannot occur in isolation in free space. To make a real process, two or more virtual processes must be combined in such a way that energy conservation is only violated for a short period of time τ compatible with the energy–time uncertainty principle

$$\tau\, \Delta E \approx \hbar. \tag{1.19}$$

In particular, the initial and final states – which in principle can be studied in the distant past ($t \to -\infty$) and future ($t \to +\infty$), respectively – must have the same energy. This is illustrated by Figure 1.7(a), which represents a process whereby an electron emits a photon that is subsequently absorbed by a second electron. Although energy conservation is violated at the first vertex, this can be compensated by a similar violation at the second vertex to give exact energy conservation overall. Figure 1.7(a) represents a contribution to the physical elastic scattering process

$$e^- + e^- \to e^- + e^-$$

from a single-photon exchange. There is also a second contribution, represented by Figure 1.7(b) in which the other electron emits the exchanged photon. Both processes contribute to the observed scattering.

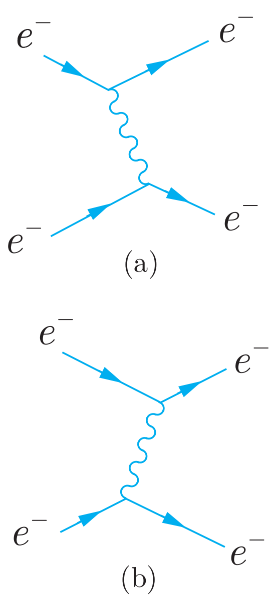

Figure 1.7 Single-photon exchange contributions to electron–electron scattering. Time as usual runs from left to right.

[10] Compare Kirchhoff's laws in electromagnetism.

Scattering can also occur via multi-photon exchange and, for example, one of the diagrams corresponding to two-photon exchange is shown in Figure 1.8. The contributions of such diagrams are, however, far smaller than the one-photon exchange contributions. To see this, we consider the number of vertices in each diagram, called its *order*. Since each vertex represents a basic process whose probability is of order $\alpha \approx 1/137 \ll 1$, any diagram of order n gives a contribution whose probability is of order α^n. On comparing Figures 1.7 and 1.8, we see that single-photon exchange is of order α^2, two-photon exchange is of order α^4 and, more generally, n-photon exchange is of order α^{2n}. To a good approximation multi-photon exchanges can be neglected, and we would expect the familiar electromagnetic interactions used in atomic spectroscopy, for example, to be accurately reproduced by considering only one-photon exchange. Detailed calculation confirms that this is indeed the case.

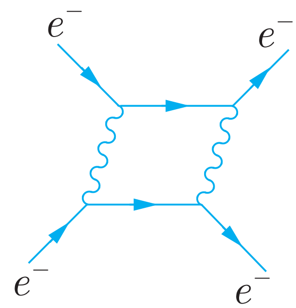

Figure 1.8 A contribution to electron–electron scattering from two-photon exchange.

1.3.3 Electron–positron pair production and annihilation

Feynman diagrams like Figures 1.7 and 1.8 play a central role in the analysis of elementary particle interactions. The contribution of each diagram to the probability amplitude for a given physical process can be calculated precisely by using the set of mathematical rules mentioned previously (the *Feynman rules*), which are derived from the quantum theory of the corresponding interaction. For electromagnetic interactions, this theory is called *quantum electrodynamics* (or *QED* for short) and the resulting theoretical predictions have been verified experimentally with quite extraordinary precision. The Feynman rules are beyond the scope of this book and our use of Feynman diagrams will be much more qualitative. For this, we need consider only the lowest-order diagrams contributing to a given physical process, neglecting higher-order diagrams like Figure 1.8 for electron–electron scattering.

To illustrate this we consider the reactions $e^+ + e^- \to p\gamma$, where to conserve energy at least two photons must be produced, $p \geq 2$. In lowest order, to produce p photons we must combine p vertices from the left-hand side of Figure 1.4. For $p = 2$,

$$e^+ + e^- \to \gamma + \gamma$$

and there are just two such diagrams: Figure 1.9(a), which is obtained by combining Figures 1.4(a) and (e), and Figure 1.9(b), which combines Figures 1.4(c) and (e). These two diagrams are closely related. If the lines of Figure 1.9(a) were made of rubber, we could imagine deforming them so that the top vertex occurred after, instead of before, the bottom vertex, and it became Figure 1.9(b). Figures 1.7(a) and (b) are related in the same way. Diagrams related in this way are called different 'time orderings' and, in practice, it is

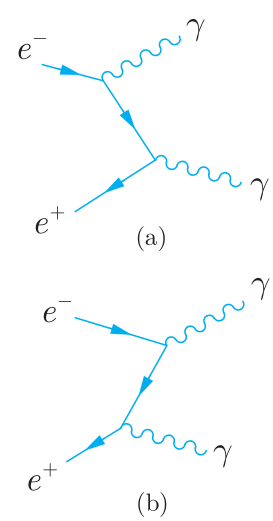

Figure 1.9 Lowest-order contributions to $e^+ + e^- \to \gamma + \gamma$. The two diagrams are related by 'time ordering', as explained in the text.

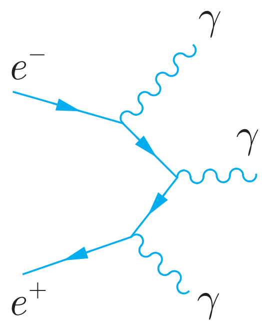

Figure 1.10 The process $e^+ + e^- \to \gamma + \gamma + \gamma$ in lowest order. Only one of the six possible time orderings is shown, leaving the other five implied.

usual to draw only one time ordering (e.g. Figure 1.9(a)) leaving the other(s) implied. Thus for p = 3,

$$e^+ + e^- \to \gamma + \gamma + \gamma,$$

and a possible diagram is that of Figure 1.10, obtained by combining Figures 1.4(a), (c) and (e). Since there are three vertices, there are $3! = 6$ different ways of ordering them in time. We leave it as an exercise for the reader to draw the other five time-ordered diagrams whose existence is implied by Figure 1.10.

In general, the process $e^+ + e^- \to p\gamma$ is of order p, with an associated probability of order α^p. From just the order of the diagrams (i.e. the number of vertices), we therefore expect that many-photon annihilation is very rare compared to few-photon annihilation, and that[11]

$$R \equiv \frac{\text{Rate}(e^+e^- \to 3\gamma)}{\text{Rate}(e^+e^- \to 2\gamma)} = \mathrm{O}\,(\alpha). \tag{1.20}$$

For very low energy e^+e^- pairs, this prediction can be tested by measuring the 2γ and 3γ decay rates of positronium, which is a bound state of e^+ and e^- analogous to the hydrogen atom.[12] In this case, the experimental value of R is 0.9×10^{-3}. This is somewhat smaller than $\alpha = 0.7 \times 10^{-2}$, and indeed some authors argue that $\alpha/2\pi = 1.2 \times 10^{-3}$ is a more appropriate measure of the strength of the electromagnetic interaction. We stress that (1.20) is only an order-of-magnitude prediction, and we will in future use $10^{-2} - 10^{-3}$ as a rough rule-of-thumb estimate of what is meant by a factor of order α.

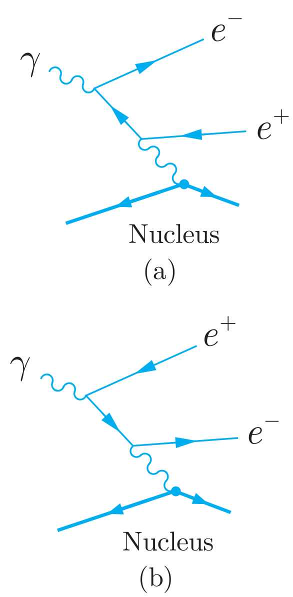

Figure 1.11 The pair production process $\gamma + (Z, A) \to e^+ + e^- + (Z, A)$ in lowest order. The two diagrams represent ‘distinct’ contributions and are not related by time ordering.

As a second example, consider the pair production reaction $\gamma \to e^+ + e^-$. This basic process cannot conserve both energy and momentum simultaneously, but is allowed in the presence of a nucleus, that is

$$\gamma + (Z, A) \to e^+ + e^- + (Z, A),$$

where Z and A are the charge and mass number (the number of nucleons) of the nucleus, respectively. Diagrammatically, this is shown in Figure 1.11, where the two diagrams are not different time orderings. (There is no way that (a) can be continuously deformed into (b).) Since one of the vertices involves a charge Ze, the corresponding factor $\alpha \to Z^2\alpha$, and the expected rate is of order $Z^2\alpha^3$. This Z dependence is confirmed experimentally, and is exploited in devices

[11] We will often use the symbol O to mean ‘order’ in the sense of ‘order-of-magnitude’.
[12] This is discussed in Section 5.5.

for detecting high-energy photons by the pair production that occurs readily in the field of a heavy nucleus.[13]

1.3.4 Other processes

Although we have introduced Feynman diagrams in the context of electromagnetic interactions, their use is not restricted to this. They can also be used to describe the fundamental weak and strong interactions. This is illustrated by Figure 1.12(a), which shows a contribution to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$ due to the exchange of a Z^0, and by Figure 1.12(b), which shows the exchange of a gluon g (represented by a coiled line) between two quarks, which is a strong interaction.

Feynman diagrams that involve hadrons can also be drawn. As an illustration, Figure 1.13 shows the decay of a neutron via an intermediate charged W boson. In later chapters we will make extensive use of Feynman diagrams in discussing particle interactions.

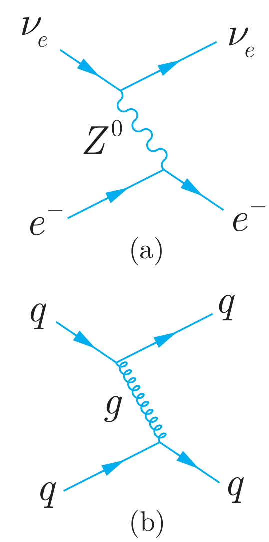

Figure 1.12 Contributions of (a) Z^0 exchange to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$ and (b) gluon exchange contribution to the strong interaction $q + q \to q + q$.

1.4 Particle exchange

In Section 1.1 we said that the forces of elementary particle physics were associated with the exchange of particles. In this section we will explore the fundamental relation between the range of a force and the mass of the exchanged particle and show how the weak interaction can frequently be approximated by a force of zero range.

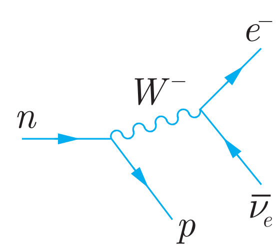

Figure 1.13 The decay $n \to p + e^- + \bar{\nu}_e$ via an intermediate W meson.

1.4.1 Range of forces

The Feynman diagram of Figure 1.14 represents the elastic scattering of two particles A and B, of masses M_A and M_B, that is $A + B \to A + B$, via the exchange of a third particle X of mass M_X, with equal coupling strengths g to particles A and B. In the rest frame of the incident particle A, the lower vertex represents the virtual process

$$A(M_A c^2,\ \mathbf{0}) \to A(E_A,\ \mathbf{p}) + X(E_X,\ -\mathbf{p}), \tag{1.21}$$

where

$$E_A = \left(p^2c^2 + M_A^2 c^4\right)^{1/2} \quad \text{and} \quad E_X = \left(p^2c^2 + M_X^2 c^4\right)^{1/2}, \tag{1.22}$$

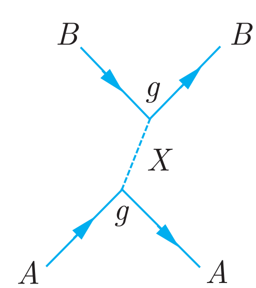

Figure 1.14 Contribution to the reaction $A + B \to A + B$ from the exchange of a particle X.

[13] These detectors will be discussed in Chapter 4.

and $p = |\mathbf{p}|$. The energy difference between the final and initial states is given by

$$\Delta E = E_X + E_A - M_A c^2 \to 2pc, \qquad p \to \infty, \qquad \to M_X c^2, \qquad p \to 0, \tag{1.23}$$

and thus $\Delta E \geq M_X c^2$ for all p. By the uncertainty principle, such an energy violation is allowed, but only for a time $\tau \approx \hbar/\Delta E$, so we immediately obtain

$$r \approx R \equiv \hbar/M_X c \tag{1.24}$$

as the maximum distance over which X can propagate before being absorbed by particle B. This constant R is called the *range* of the interaction and this was the sense of the word used in Section 1.1.

The electromagnetic interaction has an infinite range because the exchanged particle is a massless photon. The strong force between quarks also has an infinite range because the exchanged particles are massless gluons.[14] In contrast, the weak interaction is associated with the exchange of very heavy particles, the W and Z bosons, with masses

$$M_W = 80.4 \text{ GeV/c}^2 \text{ and } M_Z = 91.2 \text{ GeV/c}^2 \quad (1 \text{ GeV} = 10^9 \text{ eV}) \tag{1.25}$$

corresponding to ranges that from (1.24) are of order

$$R_{W,Z} \equiv \frac{\hbar}{M_W c} \approx 2 \times 10^{-3} \text{fm} \quad (1\text{fm} = 10^{-15}\text{m}). \tag{1.26}$$

In many applications, this range is very small compared with the de Broglie wavelengths of all the particles involved. The weak interaction can then be approximated by a zero-range or point interaction, corresponding to the limit $M_X \to \infty$, as shown in Figure 1.15.

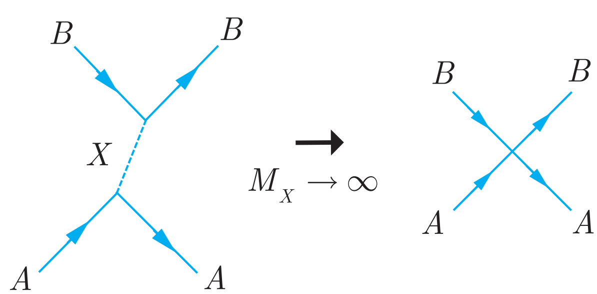

Figure 1.15 The zero-range or point interaction resulting from the exchange of a particle X in the limit $M_X \to \infty$.

[14] The force between hadrons is much more complicated, because it is not due to single-gluon exchange. It has a range of approximately (1–2) fm, where 1 fm = 10^{-15} m.

1.4.2 The Yukawa potential

In the limit that M_A becomes large, we can regard B as being scattered by a static potential of which A is the source. This potential will in general be spin dependent, but its main features can be obtained by neglecting spin and considering X to be a spin-0 boson, in which case it will obey the Klein–Gordon equation,

$$-\hbar^2 \frac{\partial^2 \phi(\mathbf{r},t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi(\mathbf{r},t) + M_X^2 c^4 \phi(\mathbf{r},t). \tag{1.27}$$

The static solution of this equation satisfies

$$\nabla^2 \phi(\mathbf{r}) = \frac{M_X^2 c^2}{\hbar^2} \phi(\mathbf{r}), \tag{1.28}$$

where $\phi(\mathbf{r})$ is interpreted as a static potential. For $M_X = 0$ this equation is the same as that obeyed by the electrostatic potential, and for a point charge $-e$ interacting with a point charge $+e$ at the origin, the appropriate solution is the Coulomb potential

$$V(r) = -e\,\phi(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}, \tag{1.29}$$

where $r = |\mathbf{r}|$ and ε_0 is the dielectric constant. The corresponding solution in the case where $M_X^2 \neq 0$ is easily verified by substitution to be

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-r/R}}{r}, \tag{1.30}$$

where R is the range defined in (1.24) and we assume equal coupling constants g for particle X to the particles A and B. It is conventional to introduce a dimensionless strength parameter

$$\alpha_X = \frac{g^2}{4\pi\hbar c} \tag{1.31}$$

that characterises the strength of the interaction at short distances, in analogy to the fine structure constant of QED.

The form of $V(r)$ in (1.30) is called a *Yukawa potential*, after the physicist who first introduced the idea of forces due to massive particle exchange in 1935. As $M_X \to 0$, $R \to \infty$ and the Coulomb potential is recovered from the Yukawa potential, while for very large masses the interaction is approximately point-like (zero range). The effective coupling strength in this latter approximation and its range of validity are best understood by considering the corresponding scattering amplitude.

1.4.3 The zero-range approximation

In lowest-order perturbation theory, the probability amplitude for a particle with initial momentum $\mathbf{q}_i$ to be scattered to a final state with momentum $\mathbf{q}_f$ by a potential $V(\mathbf{r})$ is proportional to the *scattering amplitude*[15]

$$\mathcal{M}(\mathbf{q}) = \int V(\mathbf{r}) \exp(i\mathbf{q}\cdot\mathbf{r}/\hbar)\, \mathrm{d}^3\mathbf{r}\,, \tag{1.32}$$

where $\mathbf{q} \equiv \mathbf{q}_i - \mathbf{q}_f$ is the momentum transfer. The integration may be done using polar coordinates (r, θ, ϕ). Taking $\mathbf{q}$ in the z direction, gives

$$\mathbf{q}\cdot\mathbf{r} = |\mathbf{q}|\; r\cos\theta \tag{1.33}$$

and

$$\mathrm{d}^3\mathbf{r} = r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}r\, \mathrm{d}\phi, \tag{1.34}$$

where $r = |\mathbf{r}|$. For the Yukawa potential, the integral (1.32) gives

$$\mathcal{M}(\mathbf{q}) = \frac{-g^2\hbar^2}{|\mathbf{q}|^2 + M_X^2 c^2}. \tag{1.35}$$

In using (1.35) for the scattering amplitude we have assumed potential theory, treating the particle A as a static source. The particle B then scatters through some angle without loss of energy, so that $|\mathbf{q}_i| = |\mathbf{q}_f|$ and the initial and final energies of particle B are equal, $E_i = E_f$. While this is a good approximation at low energies, at higher energies the recoil energy of the target particle cannot be neglected, so that the initial and final energies of B are no longer equal. A fully relativistic calculation taking account of this is more difficult, but the result is surprisingly simple. Specifically, in lowest-order perturbation theory, one obtains

$$\mathcal{M}(q^2) = \frac{g^2\hbar^2}{q^2 - M_X^2 c^2}, \tag{1.36}$$

where

$$q^2 \equiv (E_f - E_i)^2 - (\mathbf{q}_f - \mathbf{q}_i)^2 c^2 \tag{1.37}$$

is called the squared energy–momentum transfer, or squared four-momentum transfer. In the low-energy limit, $E_i = E_f$ and (1.36) reduces to (1.35). However, in contrast to (1.35), which was derived in the rest frame of particle A, the form (1.36) is explicitly Lorentz invariant and holds in all inertial frames of reference.

[15] This is called the Born approximation. For a discussion, see for example pp. 397–399 of Gasiorowicz (1974).

The amplitude (1.36) corresponds to the exchange of a single particle, as shown for example in Figure 1.14. (Multiparticle exchange corresponds to higher orders in perturbation theory and higher powers of g^2.) In the zero-range approximation, (1.36) reduces to a constant. To see this, we note that this approximation is valid when the range $R = \hbar/M_X c$ is very small compared with the de Broglie wavelengths of all the particles involved. In particular, this implies $q^2 \ll M_X^2 c^2$, and neglecting q^2 in (1.36) gives

$$\mathcal{M}(q^2) = -G, \tag{1.38}$$

where the constant G is given by

$$\frac{G}{(\hbar c)^3} = \frac{1}{\hbar c}\left(\frac{g}{M_X c^2}\right)^2 = \frac{4\pi\alpha_X}{(M_X c^2)^2} \tag{1.39}$$

and has the dimensions of inverse energy squared. Thus we see that in the zero-range approximation, the resulting point interaction between A and B (see Figure 1.15) is characterised by a single dimensioned coupling constant G and not g and M_X separately. As we shall see in the next chapter, this approximation is extremely useful in weak interactions, where the corresponding *Fermi coupling constant* G_F, measured for example in nuclear β-decay, is given by

$$G_F\Big/(\hbar c)^3 = 1.166 \times 10^{-5}\,\text{GeV}^{-2}. \tag{1.40}$$

1.5 Units and dimensions

In previous sections, we have quoted numerical values for some constants, for example G_F above, and given formulas for others. Before continuing our discussion, we consider more carefully the question of units.

Units are a perennial problem in physics, since most branches of the subject tend to adopt a system that is convenient for their own purpose. Elementary particle physics is no exception, and adopts so-called *natural units* (*nu*), chosen so that the fundamental constants

$$c = 1 \quad \text{and} \quad \hbar = 1. \tag{1.41}$$

In other words, c and $\hbar$ are used as fundamental units of velocity and action (or angular momentum), respectively. To complete the definition, a third unit must be fixed, which is chosen to be the unit of energy. This is taken to be the electron volt (eV), defined as the energy required to raise the electric potential of an electron or proton by one volt. The abbreviations keV (10^3 eV), MeV (10^6 eV), GeV (10^9 eV) and TeV (10^{12} eV) are also in general use.

Quantum mechanics and special relativity play crucial roles in elementary particle physics, so that $\hbar$ and c occur frequently in formulas. By choosing natural units, all factors of $\hbar$ and c may be omitted from equations using (1.41), which leads to considerable simplifications. For example, the relativistic energy relation

$$E^2 = p^2c^2 + m^2c^4$$

becomes

$$E^2 = p^2 + m^2,$$

while the Fermi coupling constant (1.40) becomes

$$G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}. \tag{1.42}$$

In natural units (nu) *all* quantities have the dimensions of a power of energy, since they can all be expressed in terms of $\hbar$, c and an energy. In particular, masses, lengths and times can be expressed in the forms

$$M = E/c^2, \quad L = \hbar c/E, \quad T = \hbar/E,$$

so that a quantity with mks dimensions $M^pL^qT^r$ has the nu dimensions E^{p-q-r}. Since $\hbar$ and c are suppressed in natural units, this is the only dimension that is relevant, and dimensional checks and estimates are very simple. The nu and mks dimensions are listed for some important quantities in Table 1.1. We note that in natural units many different quantities (e.g. mass, energy and momentum) all have the same dimension.

Natural units are very convenient for theoretical arguments, as we will see. However, we must still know how to convert from natural units to the 'practical' units in which experimental results are invariably stated. This is done in two steps. We first restore the $\hbar$

Table 1.1 The mks dimension $M^pL^qT^r$ and the nu dimensions $E^n = E^{p-q-r}$ of some quantities.

	mks			nu
Quantity	p	q	r	n
Action ($\hbar$)	1	2	–1	0
Velocity (c)	0	1	–1	0
Mass	1	0	0	1
Length	0	1	0	–1
Time	0	0	1	–1
Momentum	1	1	–1	1
Energy	1	2	–2	1
Fine structure constant (α)	0	0	0	0
Fermi coupling constant (G_F)	1	5	–2	–2

and c factors by dimensional arguments and then use the conversion factors

$$\hbar = 6.582 \times 10^{-22}\,\mathrm{MeV\,s} \tag{1.43a}$$

and

$$\hbar c = 1.973 \times 10^{-13}\,\mathrm{MeV\,m} \tag{1.43b}$$

to evaluate the result.

We illustrate this by using the nu expression for the cross-section σ for Thomson scattering,[16] that is Compton scattering from free electrons when the photon energy is much less than the electron rest energy. This is given by[17]

$$\sigma = \frac{8\pi\alpha^2}{3m_e^2}.$$

To convert it to practical units, we write

$$\sigma = \frac{8\pi\alpha^2}{3m_e^2}\hbar^a c^b$$

and demand that σ has the dimensions of length squared. This gives $a = 2, b = -2$, so that

$$\sigma = \frac{8\pi\alpha^2(\hbar c)^2}{3(m_e c^2)^2} = 6.65 \times 10^{-29}\mathrm{m}^2, \tag{1.44}$$

using (1.43b) and $m_e = 0.51\,\mathrm{MeV}/c^2$.

Cross-sections are usually quoted in *barns* b ($10^{-28}\mathrm{m}^2$). For example, the Thomson cross-section (1.44) is 0.665b while the total cross-section for *np* scattering is typically of order 30–40 mb, depending on the energy, where $1\,\mathrm{mb} = 10^{-3}\,\mathrm{b} = 10^{-31}\,\mathrm{m}^2$. Other abbreviations commonly used are *microbarns* mb ($10^{-34}\mathrm{m}^2$) and *picobarns* ($10^{-40}\,\mathrm{m}^2$). Similiarly, lengths are often quoted in *fermis* fm (10^{-15}m) rather than metres. In these units, the radius of the proton is about 0.8 fm, while the range of the weak force is of order 10^{-3} fm. Energies are measured in MeV, GeV, etc., while momenta are measured in MeV/c, etc. and masses in MeV/c^2, etc., as in Equation (1.25) for the weak boson mass. This should be compared to natural units, where energy, momentum and mass all have the dimension of energy (see Table 1.1) and are all measured in, for example, MeV.

A list of some useful physical constants and conversion factors is given inside the back cover of this book. *From now on natural units will be used in formulas throughout the book unless explicitly stated otherwise.*

---

[16] Cross-sections and related quantities are formally defined in Appendix B.
[17] See, for example, p. 20 of Mandl and Shaw (2010).

Problems 1

1.1 Write down equations in symbol form that describe the following interactions:
(a) the elastic scattering of an electron antineutrino and a positron;
(b) the elastic scattering of proton and a neutron;
(c) the annihilation of a proton and an antiproton to an electron, a positron and a photon.

1.2 The matrices α_i ($i = 1, 2, 3$) and β in the Dirac equation (1.8) can be chosen to have the explicit forms

$$\alpha_i = \begin{pmatrix} \mathbf{0} & \sigma_i \\ \sigma_i & \mathbf{0} \end{pmatrix}, \quad \beta = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}, \tag{1.45a}$$

where $\mathbf{1}$ and $\mathbf{0}$ are the 2×2 unit matrix and null matrix, respectively, and the *Pauli spin matrices* σ_i are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.45b}$$

Check that these forms satisfy the anticommutation relations

$$[\alpha_i, \beta] \equiv \alpha_i\beta + \beta\alpha_i = 0 \quad \text{and} \quad [\alpha_i, \alpha_j] = 0, \quad \text{where } i \neq j = 1, 2, 3.$$

1.3 For a free particle of momentum $\mathbf{p} = (p_x, p_y, p_z)$, the Dirac equation has four independent solutions (1.12), where the independent spinors can be chosen to be of the forms

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ a_1 \\ b_1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ a_2 \\ b_2 \end{pmatrix}, \begin{pmatrix} a_3 \\ b_3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} a_4 \\ b_4 \\ 0 \\ 1 \end{pmatrix},$$

where a_i, b_i are finite coefficients, if we assume the explicit form of the Dirac matrices given in Problem 1.2. Verify that solutions of these forms exist and find the values of the coefficients a_i, b_i and the corresponding energies.

1.4 Draw the topologically distinct Feynman diagrams that contribute to the following process in lowest order:
(a) $\gamma + e^- \to \gamma + e^-$;
(b) $e^+ + e^- \to e^+ + e^-$;
(c) $\nu_e \bar{\nu}_e$ elastic scattering.
(*Hint:* There are two such diagrams for each reaction.)

1.5 Draw one fourth-order diagram for each of the reactions (a) $\gamma + e^- \to \gamma + e^-$ and (b) $e^+ + e^- \to e^+ + e^-$.

1.6 Show that the Yukawa potential of Equation (1.30) is the only spherically symmetric solution of the static Klein–Gordon equation (1.28) that vanishes as r goes to infinity.

1.7 Verify by explicit integration that substituting the Yukawa potential (1.30) into (1.32) leads to (1.35) for the corresponding scattering amplitude.

1.8 In lowest order, the process $e^+ + e^- \to \gamma + \gamma$ is given by the Feynman diagrams of Figure 1.9. Show that for electrons and positrons almost at rest, the distance between the two vertices is typically of order m^{-1}

in natural units, where m is the electron mass. Convert this result to practical units and evaluate it in fermis.

1.9 In lowest order, the process $e^+ + e^- \to \mu^+ + \mu^-$ is given by the Feynman diagram of Figure 1.16. Estimate the typical distance between the vertices at energies much larger than the masses of any of the particles in (a) the rest frame of the electron and (b) the centre-of-mass frame. Check the consistency of these estimates by considering the Lorentz contraction in going from the rest frame, in which the initial electron is stationary, to the centre-of-mass frame.

1.10 Parapositronium is an unstable bound state of an electron and a positron. Its lifetime is given in natural units by $\tau = 2/m\alpha^5$, where m is the mass of the electron and α is the fine structure constant. Restore the factors of $\hbar$ and c by dimensional arguments and evaluate in seconds.

1.11 For total centre-of-mass energies of a few GeV, the cross-section for the reaction $\nu_\mu + e^- \to \mu^- + \nu_\mu$ is given by $\sigma = G_F^2 E^2/\pi$ in natural units, where G_F is the Fermi coupling constant (1.42). What is the cross-section in picobarns at an energy $E = 3\,\mathrm{GeV}$?

1.12 (a) The *classical radius* of an electron is defined as $r_c = e^2/4\pi\varepsilon_0 m_e$ in natural units. What is its value in fm?

(b) A simple classical model of the electron is to regard it as a sphere of uniform charge density, whose radius R is such that the electrostatic potential accounts for the whole electron mass. Derive the value of R, expressing it in terms of the classical radius defined above.

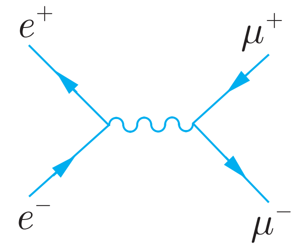

Figure 1.16 Lowest-order diagram for $e^+ + e^- \to \mu^+ + \mu^-$.