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Introduction

It is widely recognized that in almost all engineering applications, nonlinearities are inevitable and could not be eliminated thoroughly. Hence, the nonlinear systems have gained more and more research attention, and many results have been published. On the other hand, due to the wide appearance of stochastic phenomena in almost every aspect of our daily lives, stochastic systems that have found successful applications in many branches of science and engineering practice have stirred quite a lot of research interest during the past few decades. Therefore, control and filtering problems for nonlinear stochastic systems have been studied extensively in order to meet an ever-increasing demand toward systems with both nonlinearities and stochasticity.

In many engineering control/filtering problems, the performance requirements are naturally expressed by the upper bounds on the steady-state covariance, which is usually applied to scale the control/estimation precision, one of the most important performance indices of stochastic design problems. As a result, a large number of control and filtering methodologies have been developed to seek a convenient way to solve the variance-constrained design problems, among which the linear quadratic Gaussian(LQG) control and Kalman filtering are two representative minimum variance design algorithms.

On the other hand, in addition to the variance constraints, real-world engineering practice also desires the simultaneous satisfaction of many other frequently seen performance requirements, including stability, robustness, reliability, energy constraints, to name but a few key ones. This gives rise to the so-called multi-objective design problems, in which multiple cost functions or performance requirements are simultaneously considered with constraints being imposed on the system. An example of multi-objective control design would be to minimize the system steady-state variance indicating the performance of control precision, subject to a pre-specified external disturbance attenuation level evaluating system robustness. Obviously, multi-objective design methods have the ability to provide more flexibility in dealing with the trade-offs and constraints in a much more explicit manner on the pre-specified performance

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requirements than those conventional optimization methodologies like the LQG control scheme or H_∞ design technique, which do not seem to have the ability of handling multiple performance specifications.

When coping with the multi-objective design problem with variance constraints for stochastic systems, the well-known covariance control theory provides us with a useful tool for system analysis and synthesis. For linear stochastic systems, it has been shown that multi-objective control/filtering problems can be formulated using linear matrix inequalities (LMIs), due to their ability to include desirable performance objectives such as variance constraints, H_2 performance, H_∞ performance, and pole placement as convex constraints. However, as nonlinear stochastic systems are concerned, the relevant progress so far has been very slow due primarily to the difficulties in dealing with the variance-related problems resulting from the complexity of the nonlinear dynamics. A key issue for the nonlinear covariance control study is the existence of the covariance of nonlinear stochastic systems and its mathematical expression, which is extremely difficult to investigate because of the complex coupling of nonlinearities and stochasticity. Therefore, it is not surprising that the multi-objective control and filtering problems for nonlinear stochastic systems with variance constraints have not been adequately investigated despite the clear engineering insights and good application prospect.

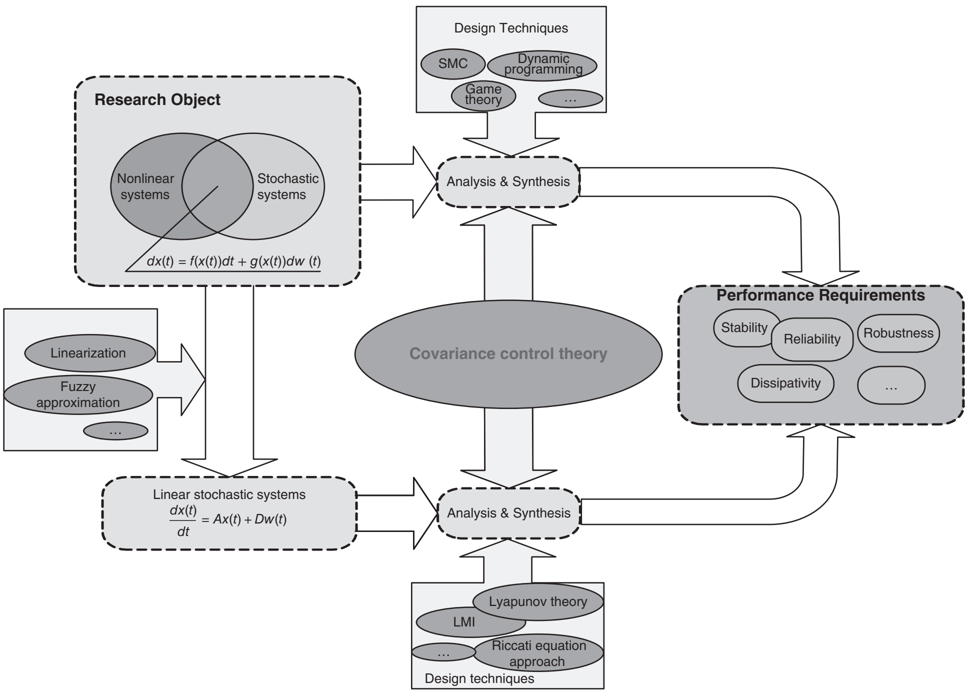

Figure 1.1 Architecture of surveyed contents.

In this chapter, we focus mainly on the multi-objective control and filtering problems for nonlinear systems with variance constraints and aim to give a survey on some recent advances in this area. We shall give a comprehensive discussion from three aspects, i.e., design objects (nonlinear stochastic system), design requirements (multiple performance specifications including variance constraints), and several design techniques. Then, as a special case of the addressed problem, mixed H_2/H_∞ design problems have been discussed in great detail with some recent advances. Subsequently, the outline of this book is given. The contents that are reviewed in this chapter and the architecture are shown in Figure. 1.1.

1.1 Analysis and Synthesis of Nonlinear Stochastic Systems

For several decades, nonlinear stochastic systems have been attracting increasing attention in the system and control community due to their extensive applications in a variety of areas ranging from communication and transportation to manufacturing, building automation, computing, automotive, and chemical industries, to mention just a few key areas. In this section, the analysis and synthesis problems for nonlinear systems and stochastic systems are recalled respectively, and some recent advances in these areas are also given.

1.1.1 Nonlinear Systems

It is well recognized that in almost all engineering applications, nonlinearities are inevitable and could not be eliminated thoroughly. Hence, nonlinear systems have gained more and more research attention, and many results have been reported; see, for example, Refs [1–3]. When analyzing and designing nonlinear dynamical systems, there are a wide range of nonlinear analysis tools, among which the most common and widely used is linearization because of the powerful tools we know for linear systems. It should be pointed out that, however, there are two basic limitations of linearization [4]. (1) As is well known, linearization is an approximation in the neighborhood of certain operating points. Thus, the resulting linearized system can only show the local behavior of the nonlinear system in the vicinity of those points. Neither non-local behavior of the original nonlinear system far away from those operating points nor global behavior throughout the entire state space can be correctly revealed after linearization. (2) The dynamics of a nonlinear system are much richer than that of a linear system. There are essentially nonlinear phenomena, like finite escape time, multiple isolated equilibria, subharmonic, harmonic or almost periodic oscillations, to name just a few key ones that can take place only in the presence of nonlinearity; hence, they cannot be described by linear models [5–8]. Therefore, as a compromise, during the past few decades, there has been tremendous interest in studying nonlinear systems, with nonlinearities being taken as the exogenous disturbance input to a linear system, since it could better illustrate the dynamics of the original nonlinear system than the linearized one with less sacrifice of the convenience on the application of existing mathematical tools. The nonlinearities emerging in such systems may arise from the linearization process of an originally highly nonlinear plant or may be an external nonlinear input, which would drastically degrade the system performance or even cause instability; see, for example, Refs [9–11].

On the other hand, in real-world applications, one of the most inevitable and physically important features of some sensors and actuators is that they are always corrupted by different kinds of nonlinearities, either from within the device themselves or from the external disturbances. Such nonlinearities generally result from equipment limitations as well as the harsh environments such as uncontrollable elements (e.g., variations in flow rates, temperature, etc.) and aggressive conditions (e.g., corrosion, erosion, and fouling, etc.) [12]. Since the sensor/actuator nonlinearity cannot be simply ignored and often leads to poor performance of the controlled system, a great deal of effort in investigating the analysis and synthesis problems has been devoted by many researchers to the study of various systems with sensor/actuator nonlinearities; see Refs [13–18].

Recently, the systems with randomly occurring nonlinearities (RONs) have started to stir quite a lot of research interest as it reveals an appealing fact that, instead of occurring in a deterministic way, a large amount of nonlinearities in real-world systems would probably take place in a random way. Some of the representative publications can be discussed as follows. The problem of randomly occurring nonlinearities was raised in Ref. [19], where an iterative filtering algorithm has been proposed for the stochastic nonlinear system in the presence of both RONs and output quantization effects. The filter parameters can be obtained by resorting to solving certain recursive linear matrix inequalities. The obtained results have quickly been extended to the case of multiple randomly occurring nonlinearities [20]. Such a breakthrough on how to deal with nonlinear systems with RONs has been well recognized and quickly followed by other researchers in the area. Using similar techniques, the filtering as well as control problems have been solved for a wide range of systems containing nonlinearities that are occurring randomly, like Markovian jump systems [21, 22], sliding mode control systems [23], discrete-time complex networks [24], sensor networks [25], time-delay systems [26], and other types of nonlinear systems [27–29], which therefore has proven that the method developed in Ref. [19] is quite general and is applicable to the analysis and synthesis of many different kinds of nonlinear systems.

It should be emphasized that, for nonlinearities, there are many different constraint conditions for certain aims, such as Lipschitz conditions, among which the kind of stochastic nonlinearities described by statistical means has drawn particular research focus since it covers several well-studied nonlinearities in stochastic systems; see Refs [27, 30–33] and the references therein. Several techniques for analysis and synthesis of this type of nonlinear system have been exploited, including the linear matrix inequality approach [30], the Riccati equation method [31], the recursive matrix inequality approach [32], gradient method [33], sliding mode control scheme [34], and the game theory approach [27].

1.1.2 Stochastic Systems

As is well known, in the past few decades there have been extensive studies and applications of stochastic systems because the stochastic phenomenon is inevitable and cannot be avoided in real-world systems. When modeling such kinds of systems, the way of neglecting the stochastic disturbances, which is a conventional technique in traditional control theory for deterministic systems, is no longer suitable. Having realized the necessity of introducing more realistic models, nowadays a great number of real-world systems such as physical systems, financial systems, ecological systems, as well as social systems are more suitable to be modeled by stochastic systems. Therefore, the stochastic control problem, which deals with dynamical systems described by difference or differential equations, and subject to disturbances characterized as stochastic processes, has drawn much research attention; see Ref. [35] and the references therein. It is worth mentioning that a kind of stochastic system represented as a deterministic system adding a stochastic disturbance characterized as white noise has gained special research interest and found extensive applications in engineering based on the fact that it is possible to generate stochastic processes with covariance functions belonging to a large class simply by sending white noise through a linear system; hence a large class of problems can be reduced to the analysis of linear systems with white noise inputs; see Refs [36–40] for examples.

Parallel to the control problems, the filtering and prediction theory for stochastic systems, which aims to extract a signal from observations of signal and disturbances, has been well studied and found to be widely applied in many engineering fields. It also plays a very important role in the solution of the stochastic optimal control problem. Research on the filtering problem originated in Ref. [41], where the well-known Wiener–Kolmogorov filter has been proposed. However, the Wiener–Kolmogorov filtering theory has not been widely applied mainly because it requires the solution of an integral equation (the Wiener–Hopf equation), which is not easy to solve either analytically or numerically. In Refs [42, 43], Kalman and Bucy gave a significant contribution to the filtering problem by giving the celebrated Kalman–Bucy filter, which could solve the filtering problem recursively. The Kalman–Bucy filter (also known as the H_2 filter) has been extensively adopted and widely used in many branches of stochastic control theory, due to the fast development of digital computers recently; see Refs [44–47] and the references therein.

1.2 Multi-Objective Control and Filtering with Variance Constraints

In this section, we first review the covariance control theory, which provides us with a powerful tool in variance-constrained design problems with multiple requirements specified by engineering practice. Then, we discuss several important performance specifications including robustness, reliability, and dissipativity. Two common techniques for solving the addressed problems for nonlinear stochastic systems are introduced. The mixed H_2/H_∞ design problem is reviewed in great detail as a special case of the multi-objective control/filtering problem with variance constraints.

1.2.1 Covariance Control Theory

As we have stated in the previous section, engineering control problems always require upper bounds on the steady-state covariances [39, 48, 49]. However, many control design techniques used in both theoretical analysis and engineering practice, such as LQG and H_∞ design, do not seem to give a direct solution to this kind of design problem since they lack a convenient avenue for imposing design objectives stated in terms of upper bounds on the variance values. For example, the LQG controllers minimize a linear quadratic performance index without guaranteeing the variance constraints with respect to individual system states. The covariance control theory [50] developed in late 1980s has provided a more direct methodology for achieving the individual variance constraints than the LQG control theory. The covariance control theory aims to solve the variance-constrained control problems while satisfying other performance indices [38, 45, 50, 51]. It has been shown that the covariance control approach is capable of solving multi-objective design problems, which has found applications in dealing with transient responses, round-off errors in digital control, residence time/probability in aiming control problems, and stability and robustness in the presence of parameter perturbations [51]. Such an advantage is based on the fact that several control design objectives, such as stability, time-domain and frequency-domain performance specifications, robustness and pole location, can be directly related to steady-state covariances of the closed-loop systems. Therefore, covariance control theory serves as a practical method for multi-objective control design as well as a foundation for linear system theory.

On the other hand, it is always the case in real-world applications such as the tracking of a maneuvering target that the filtering precision is characterized by the error variance of estimation [51, 52]. Considering its clear engineering insights, in the past few years the filtering problem with error variance constraints has received much interest and a great many research findings have been reported in the literature [42, 43, 53, 54]. The celebrated Kalman filtering approach is a typical method that aims to obtain state information based on the minimization of the variance of the estimation error [42, 43]. Nevertheless, the strict request of a highly accurate model seriously impedes the application of Kalman filtering as in many cases only an approximate model of the system is available. It therefore has brought about remarkable research interest to the robust filtering method, which aims to minimize the error variance of estimation against the system uncertainties or external unknown disturbances [55, 56]. Despite certain merits and successful applications, as in the case of the LQG control problem, the traditional minimum variance filtering techniques cannot directly impose the designing objectives stated in terms of upper bounds on the error variance values, by which we mean that those techniques try to minimize the filtering error variance in a mean-square sense rather than to constrain it within a pre-specified bound, which is obviously better able to meet the requirements of practical engineering. Motivated by the covariance control theory, in Ref. [57] the authors have proposed a more direct design procedure for achieving the individual variance constraint in filtering problems. Due to its design flexibility, the covariance control theory is capable of solving the error variance-constrained filtering problem while guaranteeing other multiple designing objectives [58]. Therefore, it always serves as one of the most powerful tools in dealing with multi-objective filtering as well as control problems [59].

It should be pointed out that most available literature regarding covariance control theory has been concerned with *linear time invariant* stochastic systems using the linear matrix inequality (LMI) approach. Moreover, when it comes to the variance-constrained controller/filter design problems for much more complicated systems such as *time-varying systems, nonlinear systems, Markovian Jump systems*, etc., unfortunately, the relevant results have been very few due primarily to the difficulties in dealing with the existence problem of the steady-state covariances and their mathematical expressions for those above-mentioned systems with complex dynamics. With the hope of resolving such difficulties, in recent years, special effort has been devoted to studying variance-constrained multi-objective design problems for systems of complex dynamics, and several methodologies for analysis and synthesis have been developed. For example, in Ref. [45], a Riccati equation method has been proposed to solve the filtering problem for linear time-varying stochastic systems with pre-specified error variance bounds. In Refs [60–62], by means of the technique of sliding mode control (SMC), the robust controller design problem has been solved for linear parameter perturbed systems, since SMC has strong robustness to matched disturbances or parameter perturbations. We shall return to this SMC problem later and more details will be discussed in the following section.

When it comes to nonlinear stochastic systems, limited work has been done in the covariance-constrained analysis and design problems, just as we have anticipated. A multi-objective filter has been designed in Ref. [63] for systems with Lipschitz nonlinearity, but the variance bounds cannot be pre-specified. Strictly speaking, such an algorithm cannot be referred to as variance-constrained filtering in view of the lack of capability for directly imposing specified constraints on variance. An LMI approach has been proposed in Ref. [30] to cope with robust filtering problems for a class of stochastic systems with nonlinearities characterized by statistical means, attaining an assignable H_2 performance index. In Ref. [59], for a special class of nonlinear stochastic systems, namely, systems with multiplicative noises (also called bilinear systems or systems with state/control dependent noises), a state feedback controller has been put forward in a unified LMI framework in order to ensure that the multiple objectives, including stability, H_∞ specification, and variance constraints, are simultaneously satisfied. This paper is always regarded as the origination of covariance control theory for nonlinear systems, for within the established theoretical framework quite a lot of performance requirements can be taken into consideration simultaneously. Furthermore, with the developed techniques, the obtained elegant results could be easily extended to a wide range of nonlinear stochastic systems; see, for example, Refs [27, 33, 64–66].

We shall return to such types of nonlinear stochastic systems later to present more details of recent progress in Section 1.2.3.

1.2.2 Multiple Performance Requirements

In the following, several performance indices originating from engineering practice and frequently applied in multi-objective design problems are introduced.

1.2.2.1 Robustness

In real-world engineering practice, various reasons, such as variations of the operating point, aging of devices, identification errors, etc., would lead to the parameter uncertainties that result in the perturbations of the elements of a system matrix when modeling the system in a state-space form. Such a perturbation in system parameters cannot be avoided and would cause degradation (sometimes even instability) to the system. Therefore, in the past decade, considerable attention has been devoted to different issues for linear or nonlinear uncertain systems, and a great number of papers have been published; see Refs [2, 46, 67–70] for some recent results.

On another research frontier of robust control, the H_∞ design method, which is used to design controller/filter with guaranteed performances with respect to the external disturbances as well as internal perturbations, has received an appealing research interest during the past decades; see Refs [71–74] for instance. Since Zames' original work [71], significant advances have been made in the research area of H_∞ control and filtering. The standard H_∞ control problem has been completely solved by Doyle *et al.* for linear systems by deriving simple state-space formulas for all controllers [72]. For nonlinear systems, the H_∞ performance evaluation can be conducted through analyzing the L_2 gain of the relationship from the external disturbance to the system output, which is a necessary step when deciding whether further controller design is needed. In the past years, the nonlinear H_∞ control problem has also received considerable research attention, and many results have been available in the literature [73–77]. On the other hand, the H_∞ filtering problem has also gained considerable research interest along with the development of H_∞ control theory; see Refs [75, 78–82]. It is well known that the existence of a solution to the H_∞ filtering problem is in fact associated with the solvability of an appropriate algebraic Riccati equality (for linear cases) or a so-called Hamilton–Jacobi equation (for nonlinear ones). So far, there have been several approaches for providing solutions to nonlinear H_∞ filtering problems, few of which, however, is capable of handling multiple performance requirements in an H_∞ optimization framework.

It is worth mentioning that, in contrast to the H_∞ design framework within which multiple requirements can hardly be under simultaneous consideration, the covariance control theory has provided a convenient avenue for the robustness specifications to be perfectly integrated into the multi-objective design procedure; see Refs [59, 76] for example. For nonlinear stochastic systems, control and filtering problems have been solved with the occurrence of parameter uncertainties and stochastic nonlinearities while guaranteeing the H_∞ and variance specifications; see Refs [33, 64, 65, 76] for some recent publications.

1.2.2.2 Reliability

In practical control systems, especially networked control systems (NCSs), due to a variety of reasons, including erosion caused by severe circumstances, abrupt changes of working conditions, intense external disturbances, and the internal physical equipment constraints and aging, the process of signal sampling and transmission has always been confronted with different kinds of failures, such as measurements missing, signal quantization, sensor and actuator saturations, and so on. Such a phenomenon is always referred to as incomplete information, which would drastically degrade the system performance. In recent years, as requirements increase toward the reliability of engineering systems, the reliable control problem, which aims to stabilize the systems accurately and precisely in spite of incomplete information caused by possible failures, has therefore attracted considerable attention. In Refs [83, 84], binary switching sequences and Markovian jump parameters have been introduced to model the missing measurements phenomena. A more general model called the multiple missing measurements model has been proposed in Ref. [85] by employing a diagonal matrix to characterize the different missing probabilities for individual sensors. The incomplete information caused by sensor and actuator saturations has also received considerable research attention and some results have been reported in the literature[18, 86, 87], where the saturation has been modeled as so-called sector-bound nonlinearities. As far as signal quantization is mentioned, in Ref. [88] a sector-bound scheme has been proposed to handle the logarithmic quantization effects in feedback control systems, and such an elegant scheme has then been extensively employed later on; see, for example, Refs [89, 90] and the references therein.

It should be pointed out that for nonlinear stochastic systems, the relevant results of reliable control/filtering with variance constraints are relatively fewer, and some representative results can be summarized as follows. By means of the linear matrix inequality approach, a reliable controller has been designed for a nonlinear stochastic system in Ref. [64] against actuator faults with variance constraints. In the case of sensor failures, the gradient method and the LMI method have been applied respectively in Refs [33] and [65] to design multi-objective filters respectively satisfying multiple requirements including variance specifications simultaneously. However, despite its clear physical insight and importance in engineering applications, the control problem for nonlinear stochastic systems with incomplete information has not yet been studied sufficiently.

1.2.2.3 Dissipativity

In recent years, the theory of dissipative systems, which plays an important role in system and control areas, has been attracting a great deal of research interest and many results have been reported so far; see Refs [91–96]. Originating in Ref. [95], the dissipative theory serves as a powerful tool in characterizing important system behaviors such as stability and passivity, and has close connections with bounded real lemma, passivity lemma and circle criterion. It is worth mentioning that, due to its simplicity in analysis and convenience in simulation, the LMI method has gained particular attention in dissipative control problems. For example, in Refs [94, 96], an LMI method was used to design the state feedback controller, ensuring both asymptotic stability and strictly quadratic dissipativity. For singular systems, Ref. [91] has established a unified LMI framework to satisfy admissibility and dissipativity of the system simultaneously. In Ref. [93], the dissipative control problem has been solved for time-delay systems.

Although the dissipativity theory provides us with a useful tool for the analysis of systems with multiple performance criteria, unfortunately, when it comes to nonlinear stochastic systems, not much literature has been concerned with the multi-objective design problem for nonlinear stochastic systems, except for Ref. [97], where a multi-objective control law has been proposed to simultaneously meet the stability, variance constraints, and dissipativity of a closed-loop system. So far, the variance-constrained design problem with dissipativity taken into consideration has not yet been studied adequately and still remains challenging.

1.2.3 Design Techniques for Nonlinear Stochastic Systems with Variance Constraints

The complexity of nonlinear system dynamics challenges us to come up with systematic design procedures to meet control objectives and design specifications. It is clear that we cannot expect one particular procedure to apply to all nonlinear systems; therefore, quite a lot of tools have been developed to deal with control and filtering problems for nonlinear stochastic systems, including the Takagi–Sugeno (T-S) fuzzy model approximation approach, linearization, gain scheduling, sliding mode control, backstepping, to name but a few of the key ones. In the sequel, we will investigate two nonlinear design tools that can be easily combined with the covariance control theory for the purpose of providing a theoretical framework within which the variance-constrained control and filtering problems can be solved systematically for nonlinear stochastic systems.

1.2.3.1 Takagi–Sugeno Fuzzy Model

The T-S fuzzy model approach occupies an important place in the study of nonlinear systems for its excellent capability in nonlinear system descriptions. Such a model allows one to perfectly approximate a nonlinear system by a set of local linear sub systems with certain fuzzy rules, thereby carrying out the analysis and synthesis work within the linear system framework. Therefore, the T-S fuzzy model is extensively applied in both theoretical research and engineering practice of nonlinear systems; see Refs [98–101] for some of the latest publications. However, despite its engineering significance, not much literature has taken the system state variance into consideration, mainly due to the technical difficulties in dealing with the variance-related problems. Some tentative work can be summarized as follows. In Ref. [102], a minimum variance control algorithm as well as direct adaptive control scheme has been applied in a stochastic T-S fuzzy ARMAX model to track the desired reference signal. However, as we mentioned above, such a minimum variance control algorithm lacks the ability of directly imposing design requirements on the variances of an individual state component. Therefore, in order to cope with this problem, in Ref. [103] a fuzzy controller has been designed to stabilize a nonlinear continuous-time system, while simultaneously minimizing the control input energy and satisfying variance constraints placed on the system state. The result has then been extended in Ref. [104] to the output variance constraints case. Recently, such a T-S fuzzy model based variance-constrained algorithm has found successful application in nonlinear synchronous generator systems; see Ref. [105] for more details.

1.2.3.2 Sliding Mode Control

In the past few decades, the sliding mode control (also known as the variable structure control) problem, originated in Ref. [106], has been extensively studied and widely applied, because of its advantage of strong robustness against model uncertainties, parameter variations, and external disturbances. In the sliding mode control, trajectories are forced to reach a sliding manifold in finite time and to stay on the manifold for all future time. It is worth mentioning that in the existing literature about the sliding mode control problem for nonlinear systems, the nonlinearities and uncertainties taken into consideration are mainly under matching conditions, that is, when nonlinear and uncertain terms enter the state equation at the same point as the control input, the motion on the sliding manifold is independent of those matched terms; see Refs [107, 108] for examples. Under such an assumption, the covariance-constrained control problems have been solved in Refs [60–62] for a type of continuous stochastic system with matching condition nonlinearities.

Along with the development of continuous-time sliding mode control theory, in recent years, as most control strategies are implemented in a discrete-time setting (e.g., networked control systems), the sliding mode control problem for discrete-time systems has gained considerable research interest and a large amount of literature has appeared on this topic. For example, in Refs [109, 110] the integral type SMC schemes have been proposed for sample-data systems and a class of nonlinear discrete-time systems respectively. Adaptive laws were applied in Refs [111, 112] to synthesize sliding mode controllers for discrete-time systems with stochastic as well as deterministic disturbances. In Ref. [113], a simple methodology for designing sliding mode controllers was proposed for a class of linear multi-input discrete-time systems with matching perturbations. Using the dead-beat control technique, Ref. [114] presented a discrete variable structure control method with a finite-time step to reach the switching surface. In cases when the system states were not available, the discrete-time SMC problems were solved in Refs [115, 116] via output feedback. It is worth mentioning that in Ref. [117], the discrete-time sliding mode reaching condition was first revised and then a new reaching law was developed, which has proven to be a convenient way to handle robust control problems; see Refs [118, 119] for some of the latest publications. Recently, for discrete-time systems that are not only confronted with nonlinearities but also corrupted by more complicated situations like propagation time-delays, randomly occurring parameter uncertainties, and multiple data packet dropouts, the SMC strategies have been designed in Refs [23, 77, 80] to solve the robust control problems and have shown good performances against all the mentioned negative factors. Currently, the sliding mode control problems for discrete-time systems still remain a hotspot in systems and control science; however, when it comes to variance-constrained problems, the related work is much fewer. As preliminary work, Ref. [34] has proposed an SMC algorithm guaranteeing the required H_2 specification for discrete-time stochastic systems in the presence of both matched and unmatched nonlinearities. In this paper, although only the H_2 performance is handled, it is worth mentioning that with the proposed method, other performance indices can be considered simultaneously within the established unified framework by similar design techniques.

1.2.4 A Special Case of Multi-Objective Design: Mixed H_2/H_∞ Control/Filtering

As a special case of multi-objective control problem, mixed H_2/H_∞ control/filtering has gained a great deal of research interest for several decades. So far, there have been several approaches to tackle the mixed H_2/H_∞ control/filtering problem. For linear deterministic systems, the mixed H_2/H_∞ control problems have been extensively studied. For example, an algebraic approach has been presented in Ref. [120] and a time-domain Nash game approach has been proposed in Refs. [37, 121] to solve the addressed mixed H_2/H_∞ control/filtering problems respectively. Moreover, some efficient numerical methods for mixed H_2/H_∞ control problems have been developed based on a convex optimization approach in Refs [40, 122–124], among which the linear matrix inequality approach has been employed widely to design both linear state feedback and output feedback controllers subject to H_2/H_∞ criterion, due to its effectiveness in numerical optimization. It is noted that the mixed H_2/H_∞ control theories have already been applied to various engineering fields [47, 125, 126].

Parallel to the mixed H_2/H_∞ control problem, the mixed H_2/H_∞ filtering problem has also been well studied and several approaches have been proposed to tackle the problem. For example, Bernstein and Haddad [120] transformed the mixed H_2/H_∞ filtering problem into an auxiliary minimization problem. Then, by using the Lagrange multiplier technique, they gave the solutions in terms of an upper bound on the H_2

filtering error. In Refs [127] and [128], a time-domain game approach was proposed to solve the mixed H_2/H_∞ filtering problem through a set of coupled Riccati equations. Recently, the LMI method has been widely employed to solve the multi-objective mixed H_2 and H_∞ filtering problems; see Refs [58, 129] for examples.

As far as nonlinear systems are concerned, the mixed H_2/H_∞ control problem as well as the filtering problem have gained some research interest; see, for example, Refs [130–132]. For nonlinear deterministic systems, the mixed H_2/H_∞ control problem has been solved with the solutions characterized in terms of the cross-coupled Hamilton–Jacobi-Issacs (HJI) partial differential equations. Since it is difficult to solve the cross-coupled HJI partial differential equations either analytically or numerically, in Ref. [131] the authors have used the Takagi and Sugeno (T-S) fuzzy linear model to approximate the nonlinear system, and solutions to the mixed H_2/H_∞ fuzzy output feedback control problem has been obtained via an LMI approach. For nonlinear stochastic systems, unfortunately, the mixed H_2/H_∞ control and filtering problem has not received a full investigation and few results have been reported. In Ref. [36], for a special type of nonlinear stochastic system, which is known as a bilinear system (also called systems with state-dependent noise or systems with multiplicative noise), a stochastic mixed H_2/H_∞ control problem has been solved and sufficient conditions have been provided in terms of the existence of the solutions of cross-coupled Ricatti equations. Very recently, an LMI approach has been proposed in Ref. [132] to solve the mixed H_2/H_∞ control problem for a class of nonlinear stochastic system that includes several well-studied types of nonlinear systems. For the stochastic systems with much more complicated nonlinearities, by means of the game theory approach, the mixed H_2/H_∞ control problem has been solved for systems with RONs in Ref. [27] and Markovian jump parameters in Ref. [66] respectively. Nevertheless, to the best of authors' knowledge, the mixed H_2/H_∞ control and filtering problems for general nonlinear systems have not yet received enough investigation and still remain challenging topics.

1.3 Outline

The outline of this book is given as follows. It it worth mentioning that Chapter 2 to Chapter 7 mainly focus on the variance-constrained control and filtering problems for nonlinear time-invariant systems, while in Chapter 9 to Chapter 12, the research objects are nonlinear time-varying systems. The framework of this book is shown in Figure 1.2.

- In Chapter 1, the research background is firstly introduced, which mainly involves the nonlinear stochastic systems, covariance control, multiple performance requirements, several design techniques including the T-S fuzzy model and sliding mode control, and a special case of variance-constrained multi-objective design problem–mixed H_2/H_∞ control/filtering problem. Then the outline of the book is listed.

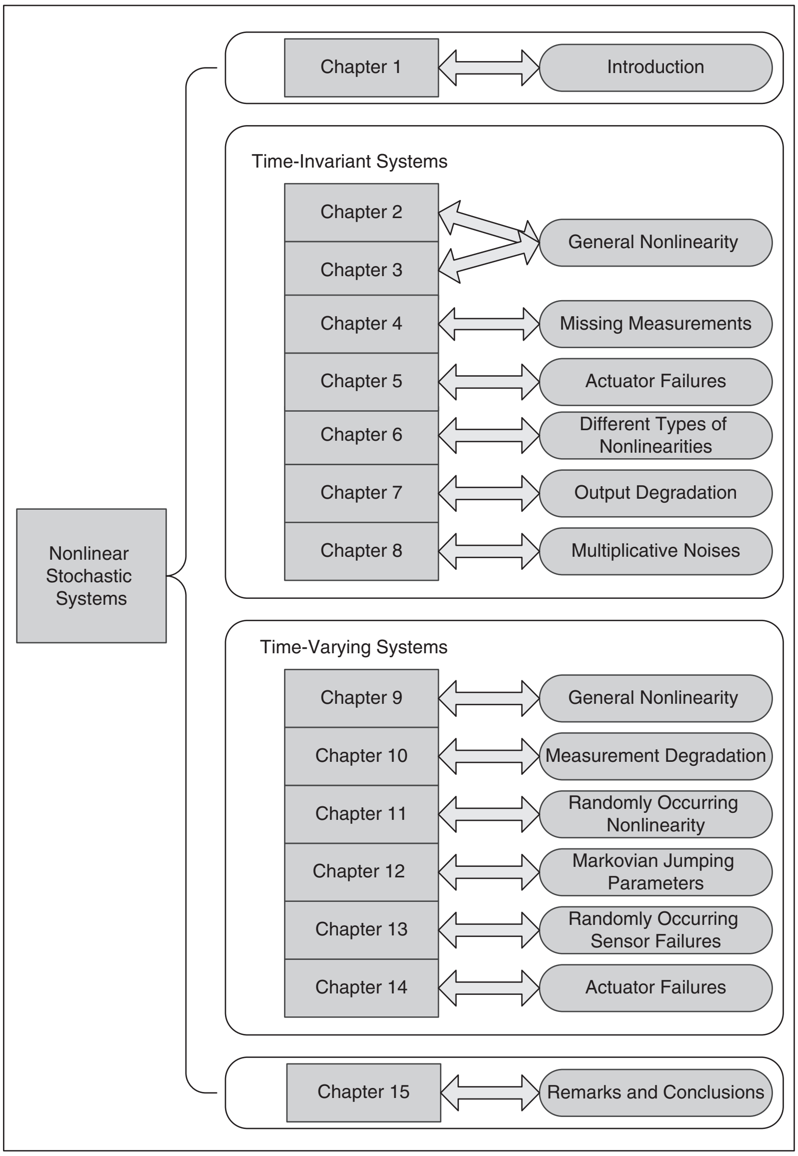

Figure 1.2 The framework of this book

- In Chapter 2, a robust H_∞ variance-constrained controller has been designed for a class of uncertain discrete-time nonlinear stochastic systems. A unified framework has been established to solve the controller design problem, which requires the simultaneous satisfaction of exponential stability, H_∞ performance index, and individual variance constraints. Two types of optimization problems have been proposed either optimizing H_∞ performance or the system state variances.

- Chapter 3 investigates the mixed H_2/H_∞ filtering problem for systems with deterministic uncertainties and stochastic nonlinearities, where the nonlinearities are characterized by statistical means. It is the objective to design a filter such that, for all admissible stochastic nonlinearities and deterministic uncertainties, the overall filtering process is exponentially mean-square quadratically stable, the H_2 filtering performance is achieved, and the prescribed disturbance attenuation level is guaranteed in an H_∞ sense.
- Chapter 4 is concerned with the robust filtering problem for a class of nonlinear stochastic systems with missing measurements and parameter uncertainties. The missing measurements are described by a binary switching sequence satisfying a conditional probability distribution and the nonlinearities are expressed by statistical means. We aim to seek a sufficient condition for the exponential mean-square stability of the filtering error system, which is first derived and then an upper bound of the state estimation error variance is obtained, in order to obtain the explicit expression of the desired filters parameterized.
- In Chapter 5, we deal with a robust fault-tolerant controller design problem with variance constraints for a class of nonlinear stochastic systems with both norm-bounded parameter uncertainties and possible actuator failures. The nonlinearity taken into consideration can be expressed by statistic means, which can cover several types of important nonlinearities, and the actuator failure model adopted here is more practical than the conventional outage case.
- Chapter 6 is concerned with a robust SMC design for uncertain nonlinear discrete-time stochastic systems and the H_2 performance constraint has been studied. The nonlinearities considered in this chapter contain both matched and unmatched forms. We also have proposed an improved form of discrete switching function with which several typical classes of stochastic nonlinearities can be dealt with via the SMC method.
- In Chapter 7, a dissipative control problem has been solved for a class of nonlinear stochastic systems while guaranteeing simultaneously exponential mean-square stability, variance constraints, system dissipativity, and reliability. An algorithm has been proposed to convert the original nonconvex feasibility problem into an optimal minimization problem, which is much easier to solve by standard numerical software.
- In Chapter 8, the robust variance-constrained H_∞ control problem is considered for uncertain stochastic systems with multiplicative noises. The norm-bounded parametric uncertainties enter into both the system and output matrices. The purpose of the problem is to design a state feedback controller such that, for all admissible parameter uncertainties, (1) the closed-loop system is exponentially mean-square quadratically stable; (2) the individual steady-state variance satisfies given upper bound constraints; and (3) the prescribed noise attenuation level is guaranteed in an H_∞ sense with respect to the additive noise disturbances. A general framework is established to solve the addressed multi-objective problem by using a linear matrix inequality (LMI) approach, where the required stability, the H_∞ characterization,

and variance constraints are all easily enforced. Within such a framework, two additional optimization problems are formulated: one is to optimize the H_∞ performance and the other is to minimize the weighted sum of the system state variances. A numerical example is provided to illustrate the effectiveness of the proposed design algorithm.

- In Chapter 9, we are concerned with the robust H_∞ control problem for a class of uncertain nonlinear discrete time-varying stochastic systems with the covariance constraint. All the system parameters are time-varying and the uncertainty enters into the state matrix. An output feedback controller has been designed by means of recursive linear matrix inequalities (RLMIs), satisfying the H_∞ disturbance rejection attenuation level in the finite horizon and an individual upper bound of state covariance at each time point.
- When it comes to the finite-horizon multi-objective filtering for time-varying nonlinear stochastic systems, Chapter 10 has proposed a technique that could handle H_∞ performance and variance constraint at the same time. It is worth mentioning that the design algorithm developed in this chapter is forward in time, which is different from those in most of the existing literature, where the H_∞ problem can be solved only backward in time and thus can be combined with the variance design and is suitable for online design.
- In Chapter 11, the mixed H_2/H_∞ controller design problem has been dealt with for a class of nonlinear stochastic systems with randomly occurring nonlinearities that are characterized by two Bernoulli distributed white sequences with known probabilities. For the multi-objective controller design problem, the sufficient condition of the solvability of the mixed H_2/H_∞ control problem has been established by means of the solvability of four coupled matrix-valued equations. A recursive algorithm has been developed to obtain the value of the feedback controller step by step at every sampling instant.
- Chapter 12 is concerned with the mixed H_2/H_∞ control problem over a finite horizon for a class of nonlinear Markovian jump systems with both stochastic nonlinearities and probabilistic sensor failures. The failure probability for each sensor is governed by an individual random variable satisfying a certain probability distribution over a given interval for each mode. The purpose of the addressed problem is to design a controller such that the closed-loop system achieves the expected H_2 performance requirement with a guaranteed H_∞ disturbance attenuation level. The solvability of the addressed control problem is expressed as the feasibility of certain coupled matrix equations. The controller gain at each time instant k can be obtained by solving the corresponding set of matrix equations. A numerical example is given to illustrate the effectiveness and applicability of the proposed algorithm.
- In Chapter 13, the robust variance-constrained H_∞ control problem is concerned for a class of nonlinear systems with possible sensor failures occurring in a random way, called randomly occurring sensor failures (ROSFs). The occurrence of the nonlinearities under consideration is governed by a Bernoulli distributed variable. The purpose of the addressed problem is to design an output-feedback controller

such that, for certain systems with ROSFs, (1) the closed-loop systems meets the desired H_∞ performance in the finite horizon and (2) the state covariance is under an individual upper bound at each time point. A sufficient condition for the existence of the addressed controller is given and a numerical computing algorithm is developed to meet the aforementioned requirements by means of recursive linear matrix inequalities (RLMIs). An illustrative simulation example is provided to show the applicability of the proposed algorithm.
- Chapter 14 deals with the fault-tolerant control problem for a class of nonlinear stochastic time-varying systems with actuator failures. Both H_2 and H_∞ performance requirements are taken into consideration. The proposed actuator failure model is quite general and covers several frequently seen actuator failure phenomena as special cases. The stochastic nonlinearities are quite general and could stand for several nonlinear systems. It is the purpose of this chapter to find equilibrium strategies of a two-player Nash game, and meanwhile both H_2 and H_∞ performances are achieved via a proposed state feedback control scheme, which is characterized by the solution to a set of coupled matrix equations. The feedback gains can be solved recursively backward in k. A numerical computing algorithm is presented and then a simulation example is given to illustrate the effectiveness and applicability of the proposed algorithm.
- In Chapter 15, the conclusions and some potential topics for future work are given.