Introduction

1

Prerequisite Knowledge Needed for Chapter 1

No prior knowledge is required for this chapter. It is an advantage to have completed undergraduate courses in statics and dynamics, but is certainly not essential.

COPYRIGHTED MATERIAL

1.1 HISTORICAL PERSPECTIVE

A mechanism is a machine composed of rigid members that are joined together. Joints permit the members to interact with one another. Portions of the surfaces of the members that contact one another form the joints. The geometries of the contacting surface segments determine the properties of each joint.

Mechanisms are used for diverse purposes. Some are incorporated into items we use every day. Figure 1.1 shows a mechanism whose function is to magnify the force generated by a user squeezing the handles to a very large force exerted by the jaws. It is also designed to lock in place while generating that force, so the handles can be released while the jaws remain clamped on a work piece. This is an example of a planar mechanism because the members of the mechanism all move parallel to a single plane of motion. Many familiar mechanisms have this characteristic. Planar mechanisms are the primary focus of Chapters 3 through 7 of this book. Mechanisms whose primary function is transmitting force, like this one, are discussed in Chapter 14.

Other mechanisms are characterized by points in their members following paths that are curves in space. In Figure 1.2, the leg mechanisms allow the feet to be placed anywhere within a volume of space. Each mechanism is primarily used to generate a straight-line foot trajectory relative to the body, so that the machine can walk at a constant height with uniform speed. Notice that the critical function here is the ability to have a designated point generate a path of specified geometry (a straight line). This is known as a path generation problem. The leg mechanisms used here are called pantographs. A pantograph is a special kind of planar mechanism discussed in Section 8.1.3.

Figure 1.3 is the drive mechanism of an ornithopter, a vehicle that flies by flapping its wings like a bird. Here a spatial trajectory of the entire wing must be generated relative to the body, not just the path of a point, as was the case with Figure 1.2. The wings must flap relative to the body, but they must also rotate about the long axis of the wing at the top of the flap to allow the wing to generate lift. The wing must rotate back (feathering) at the bottom of the flap to minimize air resistance to the upstroke. This is an example of using a mechanism to generate a specified path in space of a whole body (the wing). We call this a motion-generation problem. There are many other ways in which mechanisms are used. Here the wings are flapped by two planar four-bar mechanisms that are geared

Kinematics, Dynamics, and Design of Machinery, Third Edition. Kenneth J. Waldron, Gary L. Kinzel, and Sunil K. Agrawal.
© 2016 John Wiley & Sons, Ltd. Published 2016 by John Wiley & Sons, Ltd.
Companion website: www.wiley.com/go/waldron/kinematics.

FIGURE 1.1
A pair of vice-grip pliers. A planar mechanism that multiplies the force applied by a user to the handles to apply a much greater force at the jaws. The mechanism is also designed to be locked in the closed position.

FIGURE 1.2
The Adaptive Suspension Vehicle [7]. Each leg is a planar pantograph mechanism hinged to the body about an axis parallel to the longitudinal axis of the vehicle. The feet can be placed anywhere within a volume of space, so this is, overall, a spatial mechanism. The pantograph mechanism allows the ankle joint to be moved in a straight line relative to the vehicle body by a single hydraulic cylinder.

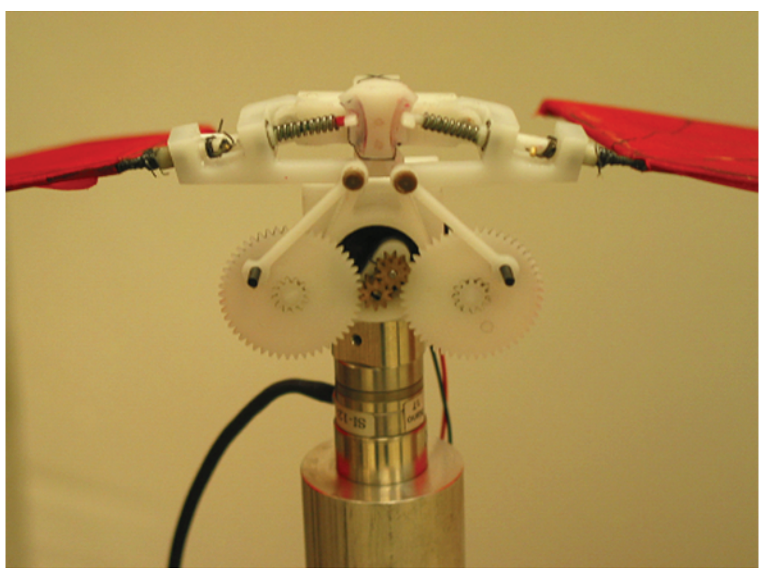


FIGURE 1.3
The drive mechanism of an ornithopter [1]. The mechanism must both flap the wings and rotate them about their long axes at the top and bottom of the flapping motion. A pair of four-bar mechanisms that are geared together accomplish the flapping motion. Cam and follower mechanisms are used to accomplish rotation of the wing at the top and bottom of the flapping motion.

together. The rotation about the wing axis is accomplished by a cam-and-follower mechanism. Cam mechanisms are discussed in Chapter 10.

Other common examples of ways in which mechanisms are used include the suspension of an automobile. A mechanism is used to maintain the wheels in a proper relationship with the body of the vehicle while allowing them to move to accommodate variations in the profile of the road. The suspension functions as a mechanical filter isolating the body of the vehicle and its occupants from bumps in the road.

FIGURE 1.4
Several hydraulic cylinders control a mechanical excavator.

Yet another example is the mechanism of an excavator, like the one shown in Figure 1.4, in which multiple hydraulic actuators are used to provide versatile paths of the bucket under direct control of the human operator to accomplish varied digging tasks.

The design of mechanisms is a technical area that is unique to mechanical engineering. Its history stretches back to prehistoric times. Artisans such as blacksmiths and carpenters also functioned as designers of mechanisms. One of the original functions of engineers was the design of mechanisms for both warfare and peaceful uses. During the Renaissance, Leonardo da Vinci [9] depicted a sophisticated variety of mechanisms, mostly for military purposes. Sometime thereafter, civil engineering and military engineering became distinct entities.

The modern era in mechanism design, along with the history of mechanical engineering as a distinct discipline, can be viewed as starting with James Watt [4]. However, the subject has not remained static. In fact, there have been dramatic changes in the practice of mechanism design in recent years.

Traditionally, machines were designed to be powered by a single “prime mover,” with all functions mechanically coordinated—a tradition that predates Watt. Developments in computer technology starting in the early 1970s, coupled with improvements in electric motors and other actuators, have made it possible to use a different approach. In this approach, machines are powered by multiple actuators coordinated electronically. The resulting machines are simpler, less expensive, more easily maintained, and more reliable. Another major change is in the techniques used in mechanism design. The use of interactive computer graphics has had a dramatic impact on design practice. One of our motivations in producing this book is to provide a treatment that reflects these changes in practice.

The functions for which mechanisms are used have changed with time, as have the methods used in designing them. Mechanisms were earlier used to generate irregular motion patterns. The Norden bombsight of World War II was a mechanical analog computer with dozens of mechanisms generating special functions mechanically [10]. Not only can those calculations now be performed more accurately and flexibly by a digital computer, but any desired motion can be generated easily and inexpensively with a computer-controlled electric motor, so there is little interest in using mechanisms in this way anymore.

1.2 KINEMATICS

Kinematics is the study of position and its time derivatives. Specifically, we are concerned with the positions, velocities, and accelerations of points and with the angular positions, angular velocities, and angular accelerations of solid bodies. The position of a body can be defined by the position of a nominated point of that body combined with the angular position of the body. In some circumstances, we are also interested in the higher time derivatives of position and angular position.

The subject of kinematics is a study of the geometry of motion—geometry with the element of time added. The bulk of the subject matter of this book can be referred to as the kinematics of mechanisms. Kinetics brings in the relationship between force and acceleration embodied in Newton's laws of motion is another important topic. Together, kinematics and kinetics constitute the subject known as dynamics. The subject matter is approached from a mechanical designer's perspective to present techniques that can be used to design mechanisms to meet specific motion requirements.

1.3 DESIGN: ANALYSIS AND SYNTHESIS

The material in this book falls into two sections. The first comprises methods for mathematically determining the geometry of a mechanism to produce a desired set of positions and/or velocities or accelerations. These are rational synthesis techniques. The second section discusses techniques to determine the positions, velocities, and accelerations of points in the members of mechanisms and the angular positions, velocities, and accelerations of those members. These are kinematic analysis techniques.

The creative activity that distinguishes engineering from science is design or, more formally, synthesis. Science is the study of what is; engineering is the creation of what is to be. The classical rational synthesis techniques developed by kinematicians offer a rather direct route to mechanism design that lends itself well to automation using computer-graphics workstations. However, these techniques represent only one way to design mechanisms, and they are relatively restrictive. Rational synthesis techniques exist only for specific types of mechanism design problems, and many practical mechanism design problems do not fit within the available class of solutions.

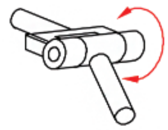
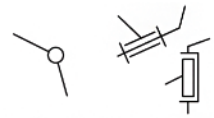
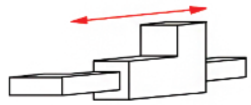
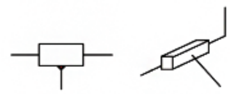
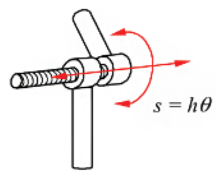
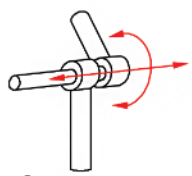
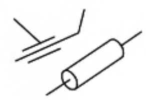
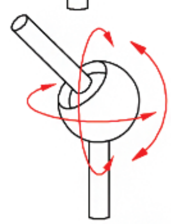
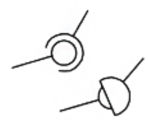
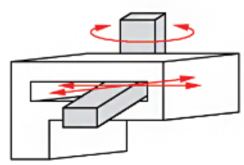
A new set of techniques that depend on the computational capabilities of modern graphical modeling software, in combination with a knowledge of motion geometry, are presented in Chapter 2 of this book. Yet another alternative is to use informal synthesis. This is a methodology used by engineers to solve design problems in many technical areas, not just in mechanism design. The basic procedure is to "guess" a set of dimensions and use analysis to check the resulting performance. The dimensions are then adjusted to attempt to match more closely the performance specifications and the mechanism is analyzed again. The process is repeated until an acceptably close match to the specifications is achieved. Thus, a primary use of the analysis material is also in mechanism design.

From an engineering point of view, it is not possible to treat mechanism design solely in terms of kinematics. The motivation for performing an acceleration analysis is often to enable inertia forces on the links to be calculated, allowing, in turn, computation of the forces transferred between links and the internal forces, or stresses, within the links. Mechanisms must usually drive loads, as well as generate motions. As soon as we introduce the concept of force, we leave the domain of pure kinematics and enter that of kinetics. Insofar as the largest forces in many mechanisms are inertia forces created by motion, it is convenient to study them within the general framework of kinematic techniques. There is also an important symmetry between the geometry of the force distribution and that of the velocity distribution that is particularly useful when working with spatial mechanisms. Thus, it is entirely appropriate to treat mechanism statics or kinetics within the general geometry of motion framework constructed to study mechanism kinematics. This treatment is presented in the later chapters of this book.

1.4 MECHANISMS

Mechanisms are assemblages of solid members connected by joints. Mechanisms transfer motion and mechanical work from one or more actuators to one or more "output" members. For the purposes of kinematic design, we idealize a mechanism to a kinematic linkage in which all the members are assumed to be perfectly rigid and are connected by kinematic joints. A kinematic joint is formed by direct contact between the surfaces of two members. One of the earliest codifications of mechanism

TABLE 1.1 Lower-Pair Joints

Connectivity (Number of Degrees of Freedom)	Names	Letter Symbol	Typical Form	Sketch Symbol
1	Revolute Hinge Turning pair	R		 (Planar) (Spatial)
1	Prismatic joint Slider Sliding pair	P		 (Planar) (Spatial)
1	Screw joint Helical joint Helical pair	H		 (Spatial)
2	Cylindric joint Cylindric pair	C		 (Spatial)
3	Spherical joint Ball joint Spherical pair	S		 (Spatial)
3	Planar joint Planar pair	P_L		 (Spatial)

kinematics was that of Reuleaux [8], and some of the basic terminology we use originated with him. He called a kinematic joint a "pair." He further divided joints into "lower pairs" and "higher pairs." A lower-pair joint is one in which contact between two rigid members occurs over geometrically congruent surfaces. The surface contact between lower pairs results in relatively low-contact stresses. A higher-pair joint is one in which contact occurs only at isolated points or along line segments. All other things being equal, a higher-pair joint will produce much higher stresses than will a lower-pair joint.

Joints are the most important aspect of a mechanism to examine during an analysis. They permit motion in some directions while constraining motion in others. The types of motion permitted are related to the number of degrees of freedom (dof) of the joint. The number of degrees of freedom of the joint is equal to the number of independent coordinates needed to specify uniquely the position of one link relative to the other, as constrained by the joint.

Lower-pair joints are necessarily restricted to a relatively small number of geometric types, because the requirement that surface contact be maintained constrains the geometry of the contacting surfaces. There are only six fundamentally different types of lower-pair joints, classified by the types of relative motion that they permit. There are, in contrast, an infinite number of possible higher-pair geometries. The lower-pair joint types are shown in Table 1.1. Some important examples of higher-pair joints are shown in Table 1.2.

Lower-pair joints are frequently used in mechanism design practice. They give good service because wear is spread out over the contact surface and because the narrow clearance between the surfaces provides good conditions for lubrication and a tight constraint on the motion. The change in the geometric properties of the joint with wear is slow for a lower pair. At least as important are the simple geometries of the relative motions that these joints permit. The simple geometries of the contacting surfaces also make them easy to manufacture.

Higher-pair joints that involve pure rolling contact, or that approximate that condition, are also used frequently. In pure rolling contact, the points in one of the two joint surfaces that are actually in contact with the other surface at any instant are at rest relative to that surface. Hence there is no relative sliding of the surfaces and joint friction and wear are minimized. Physically, the limitation of this kind of joint is the stress intensity that the material of the contacting bodies can support. Stresses are necessarily high because of the very small contact areas. If the bodies were perfectly rigid, contact would occur only at discrete points or along a line, the contact area would be zero, and the stresses would be locally infinite!

Lower-pair joints such as revolute joints and cylindrical joints are also often simulated by systems such as ball or roller bearings in which there are many elements acting in parallel. The actual contact joints in a ball bearing are rolling contacts, which are higher pairs. In this way, the low-friction properties of rolling contacts are exploited to obtain a joint with lower friction and higher load and relative speed capabilities than would be possible with a plain revolute joint. At the same time, the simple overall relative motion geometry of the revolute joint is retained. This is one example of a compound joint in which the joint is actually a complex mechanism but is regarded as kinematically equivalent to a simple revolute. Several examples of compound joints are shown in Table 1.3.

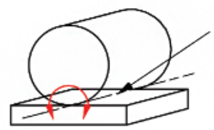
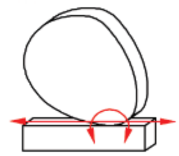
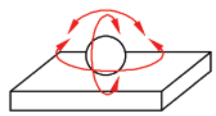
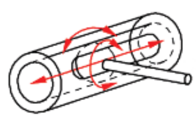
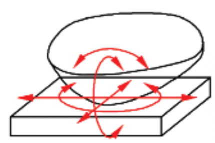
Single higher-pair joints are sometimes replaced by a series of lower-pair joints that permit equivalent motion (Figure 1.5). For example, the function of a pin-in-a-slot joint can be replaced by a combination of a revolute joint and a prismatic joint. Note that this involves adding extra members to the mechanism. When a rolling contact bearing or compound joint replaces a lower-pair joint, the two mechanisms are said to be *kinematically equivalent*. This means that the relative motions that are permitted between the bodies in the two cases are the same, even though the joint is physically quite different.

The number of degrees of freedom (connectivity) of a joint is the minimum number of independent parameters required to define the positions of all points in one of the bodies it connects relative to a reference frame fixed to the other. The term *connectivity* is used to denote this freedom of the body, even though the joint may be something very elaborate such as the antifriction bearing shown in Table 1.3 and Figure 1.6. If motion is restricted to a plane, the maximum number of degrees of freedom is three. In general spatial motion, the maximum number is six. The number of degrees of freedom permitted by each joint is listed in Tables 1.1, 1.2, and 1.3 in the first column.

1.5 PLANAR LINKAGES

A planar linkage is one in which the velocities of all points in all members are directed parallel to a plane, called the plane of motion. The only lower-pair joints that are properly compatible with planar motion are revolute and prismatic joints. The axes of rotation of all revolute joints must be normal to the plane of motion because points would not otherwise move in parallel planes. The directions of sliding of all prismatic joints must be parallel to the plane of motion, since all points in a member connected to another by a prismatic joint move on lines parallel to the sliding direction relative to the second member. Occasionally other lower-pair joints will appear in what is otherwise a planar linkage. However, they then function only as revolute or prismatic joints. For example, a spherical joint may be substituted for a revolute joint, but if the linkage is functionally planar, that spherical joint will operate as a revolute with rotation occurring only about the axis normal to the plane of motion. This type of situation will be discussed in more detail in the context of degrees of freedom and mobility.

TABLE 1.2 Examples of Higher-Pair Joints

Connectivity (Number of Degrees of Freedom)	Names	Typical Form	Comments
1	Cylindrical roller		Roller rotates about this line at this instant in its motion. Roller does not slip on the surface on which it rolls.
2	Cam pair		Cam rolls and slides on follower
3	Rolling ball		Ball rolls without slipping
4	Ball in cylinder		Ball can rotate about any axis through its center and slide along the cylinder axis
5	Spatial point contact		Body can rotate about any axis through the contact point and slide in any direction in the tangent plane

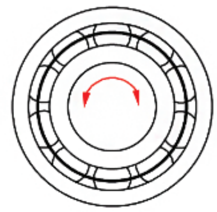
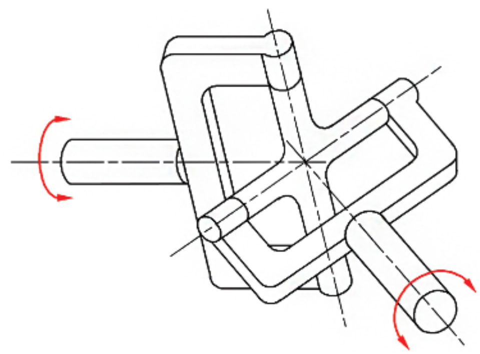
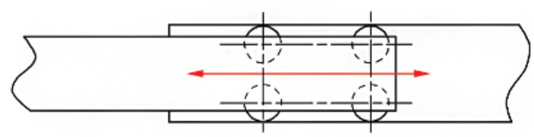
A common schematic method of representing planar linkages is to represent revolute joints by small circles as shown in Table 1.1. Binary links, those that have two joints mounted on them, are represented as lines joining those joints. Ternary links, those that have three joints mounted on them, are represented as triangles with the joints at the vertices, and so on. Examples of the resulting representation are shown in Figures 1.7 through 1.9. The link geometries may then be easily reproduced, giving an accurate view of the linkage in a specified position. Alternatively, the schematic may be used conceptually without accurate geometric data, to indicate the topology of the linkage. *Topology* is the branch of geometry that deals with issues of connectedness without regard to shape. Links with three or more joints should be shaded or crosshatched. Otherwise, the schematic for a quaternary link, one with four joints, cannot be distinguished from the schematic for a four-bar linkage loop.

A kinematic chain is any assemblage of rigid links connected by kinematic joints. A closed chain is one in which the links and joints form one or more closed circuits. Each closed circuit is a loop in which each link is attached to other links by at least two joints.

Prismatic joints are represented by means of a line in the direction of sliding, representing a slide, with a rectangular block placed on it. This produces linkage representations such as those shown in Figure 1.9. A *frame* or base member is a link that is fixed. That is, it has zero degrees of freedom relative to the fixed coordinate system. A *linkage* is a closed kinematic chain with one link selected as the frame.

In cases in which it is necessary to distinguish the base member of a linkage, it is customary not to show the base as a link in the normal manner but to indicate joints to base by “mounts,” as shown in Figures 1.10 and 1.11.

TABLE 1.3 Examples of Compound Joints

Connectivity	Names	Form
1	Ball bearing Anti-friction bearing Rolling contact bearing	
2	Universal joint Hooke joint Cardan joint	
1	Roller slide Roller guide	

The term *mechanism* is somewhat interchangeable with *linkage*. In normal usage, it is a somewhat more generic term encompassing systems with higher pairs, or combinations of lower- and higher-pair joints, whereas the term linkage tends to be restricted to systems that have only lower-pair joints. The links are numbered with the frame link usually taken as link 1.

Simple, single-loop linkages are given a symbolic designation by a sequence of letters denoting joint types written in clockwise order beginning and ending with the joints mounted to the frame link as shown in Figure 1.12. The letter designations for the different joints are given in Table 1.1.

The profiles of the contacting surfaces of higher pairs, such as cams and followers, are drawn in planar linkages producing representations such as that shown in Figure 1.13. Those surfaces must be general (not necessarily circular) cylinders whose straight-line generators are normal to the plane of motion. The profile drawn is, therefore, the generating curve of the cylinder shown in Figure 1.14. The cylinder is generated by translating that curve along a straight line in the direction normal to its plane. The familiar cylinder with a circular generating curve is called a right-circular cylinder.

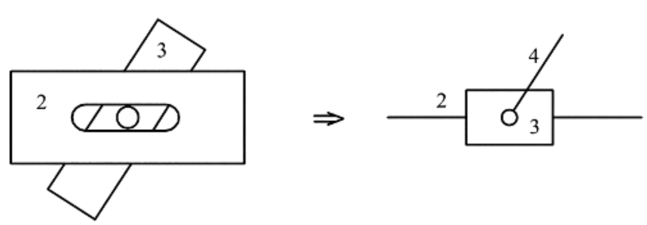


FIGURE 1.5
Replacement of a higher-pair joint by a kinematically equivalent combination of lower-pair joints.

FIGURE 1.6
Various rolling-element and plain bearings. As discussed in the text, rolling-element bearings are compound joints that replace kinematically equivalent plain joints.

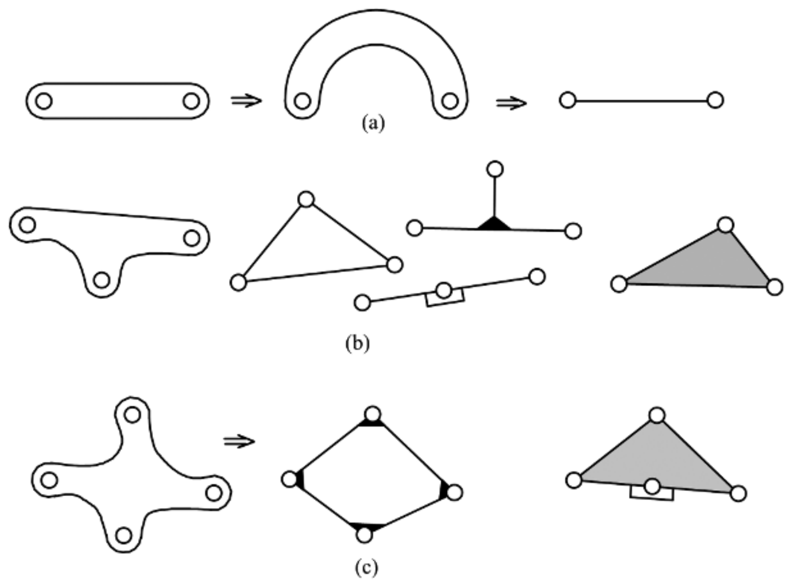


FIGURE 1.7
Representations of links in linkages: (a) Binary links: those to which two joints are mounted; (b) ternary links; and (c) quaternary links. These have three and four joints, respectively.

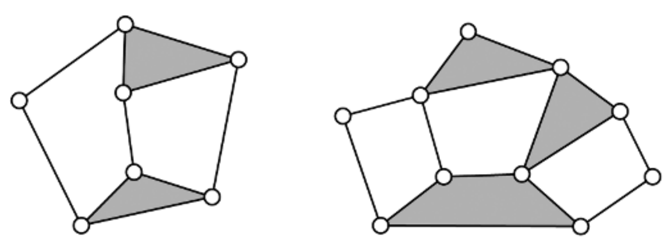


FIGURE 1.8
Conventional representations of planar linkages. Circles indicate revolute joints. Binary links, those with two joints mounted on them, are represented by line segments. Ternary links, with three joints, are represented by triangles, and so on.

1.6 VISUALIZATION

Because linkage motion is inextricably intertwined with geometry, it is always important to the designer to visualize the motion. In this respect, planar linkages are relatively easy to work with because their geometry and loci representing their motion can be drawn on a two-dimensional surface. Nevertheless, it can be very difficult to visualize successive positions of the links of a planar linkage from a drawing of that linkage in a representative position. Yet this succession of positions and the relative locations of all the links in each of the positions are very important when trying to predict effects such as interference with each other and with other machine parts. Mechanism

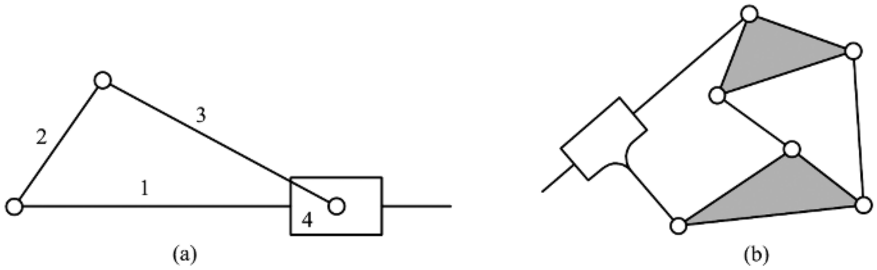


FIGURE 1.9
Representations of planar linkages with prismatic joints. (a) A four-bar slider-crank mechanism. Note that the sliding "block" is a binary member of the mechanism with a revolute joint and a prismatic joint providing the connections to adjacent members in the loop. The fillets connecting the block to a binary member represented by a line in (b) represent a rigid connection. Thus, the combination is, in this case, a binary member of the linkage.

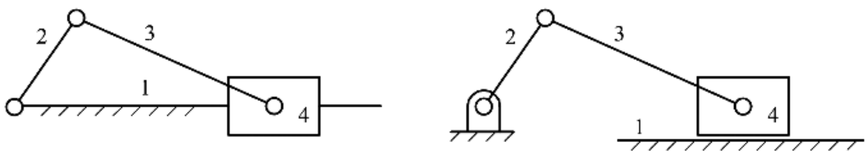


FIGURE 1.10
Selection of a frame member converts the chain of Figure 1.9(a) into a linkage. This linkage is known as a slider-crank linkage.

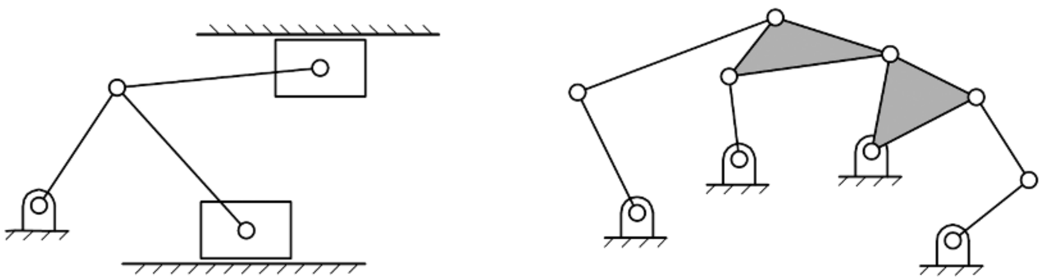
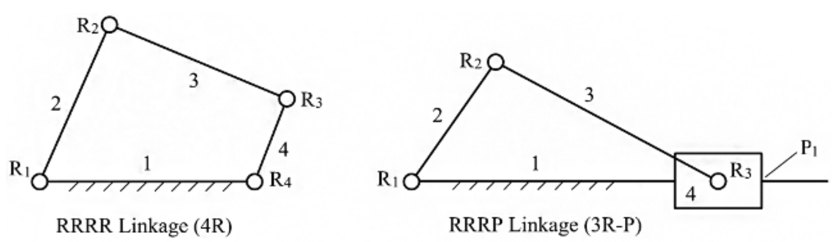


FIGURE 1.11
Representations of planar linkages with the base link not shown in the same form as the other links. The page can be thought of as representing the base link. Hatched "mounts" indicate the joints to the base link.

FIGURE 1.12
Designation of single loop linkages by means of their joints. The joints are taken in clockwise order around the loop, starting and finishing with a joint to frame.



designers have traditionally solved this problem by constructing simple physical models with the links cut from cardboard (e.g., a manila folder) and revolute joints formed by pins (e.g., thumbtacks) or grommets. Prototyping kits or even children's construction toys (Figure 1.15) provide an alternative that requires more construction time but gives a more functional model.

When mechanisms are designed using computer graphics systems, animation on a computer can be used to visualize the motion of the mechanism, rather than construction of

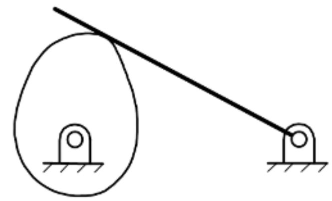


FIGURE 1.13
Representation of a plate cam with a rocker follower. The face of the follower is a plane, so it is represented by a line. The cam is represented by its profile curve.

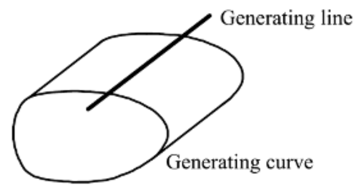


FIGURE 1.14
General cylinder. The generating curve is a plane curve. Its plane is normal to the generating line. The surface may be generated by moving the generating curve so that a point on it moves along the generating line. Alternatively, the surface may be generated by moving the generating line so that a point on it traverses the generating curve.

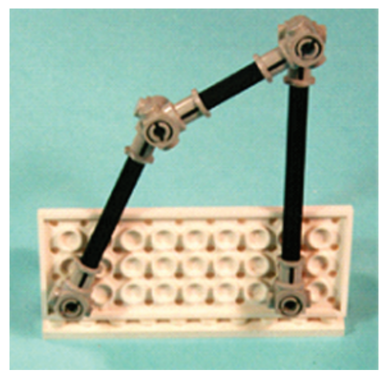


FIGURE 1.15
Planar four-bar linkage model made with LEGO Technics [6].

a physical model. Animation should be used with caution, however. As will be seen in a later section of this chapter, there are important interference effects that do not lend themselves to planar representation but which become immediately apparent in a physical model. Furthermore, adding realistic boundary profiles to the representations of links on computer graphic systems is often time consuming and simply not worth the effort when trying a variety of different alternative linkage configurations. Instead, quick physical visualization models may be a more efficient alternative. The reader is urged to get into the habit of constructing simple models to visualize the motions of linkages that are being designed or analyzed, and to make use of computer animation when it is available. If the appropriate resources are available, physical models can be produced from solid models on the computer by means of 3D printing techniques. Figure 1.16 shows such a physical model.

FIGURE 1.16
A spatial mechanism model manufactured using a 3D printing system.

1.7 CONSTRAINT ANALYSIS

The number of degrees of freedom (dof) of a body is the number of independent coordinates needed to specify uniquely the position of that body relative to a given reference frame. Similarly, we call the minimum number of coordinates needed to specify uniquely the positions of all of the members of a system of rigid bodies the number of degrees of freedom of that system. We will use the concept of the number of degrees of freedom in three distinct but closely related ways. The first is the number of degrees of freedom of a body relative to a specified reference frame, which is defined as above. The second is the number of degrees of freedom of a kinematic joint. The third is the number of degrees of freedom of the entire linkage or mechanism.

Both because "number of degrees of freedom" is such a mouthful and because we are using a distinct concept, we will refer to the number of degrees of freedom of a joint as its *connectivity*. In addition, the term connectivity will apply to the number of relative freedoms between two bodies. Likewise, we will refer to the number of degrees of freedom of a linkage as the *mobility* of that linkage. These terms may be formally defined as follows:

If a kinematic joint is formed between two rigid bodies that are not otherwise connected, the *connectivity* of that joint is the number of degrees of freedom of motion of (either) one of the two bodies joined relative to the other.

The *mobility* of a mechanism is the minimum number of coordinates needed to specify the positions of all members of the mechanism relative to a particular member chosen as the base or frame.

The mobility, or number of degrees of freedom of a linkage, is used to determine how many pair variables must be specified before the positions of all of the points on all of the members of the linkage can be located as a function of time. A linkage has a mobility of one or more. Traditionally, almost all linkages had one degree of freedom. However, in modern design practice, linkages with two or more degrees of freedom are becoming very common. Coordination to achieve a desired motion is then done using actuators that are digitally controlled. How this is done is discussed in Chapter 17. If the mobility is zero or is negative, as determined by the constraint equations developed below, the assemblage is a structure. If the mobility is zero, the structure is statically determinate. If the mobility is negative, the structure is statically indeterminate.

To compute the mobility, let us consider the planar case first and then extend the results to the spatial case. As indicated in Figure 1.17, in the plane, a body moving freely has three degrees of freedom. Suppose that in a given linkage, there are n links. If they are all free to move independently, the system has mobility $3n$. If one link is chosen as the frame link, it is fixed to the base reference frame and loses all of its degrees of freedom. Therefore the total mobility of the system is $3(n-1)$ with no joints formed between the members.

If a joint with connectivity f_i (f_i degrees of freedom) is formed between two bodies, the mobility of the system is diminished since those two bodies originally had three degrees of freedom of motion relative to one another. After formation of the joint, they have only f_i degrees of freedom of relative motion. Hence the reduction in the system mobility is $3-f_i$. If joints continue to be formed until there are j joints, the loss of system mobility is

$$(3-f_1)+(3-f_2)+\ldots+(3-f_j)=\sum_{i=1}^{j}(3-f_i)=3j-\sum_{i=1}^{j}f_i$$

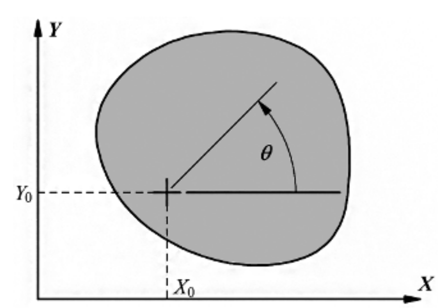


FIGURE 1.17
One set of three coordinates that can be used to describe planar motion. The number of degrees of freedom of a body is the number of independent coordinates needed to specify its position. Therefore, a body moving freely in a plane has three degrees of freedom.

Then the total mobility of the linkage will be

$$M = 3(n-1) - \left(3j - \sum_{i=1}^{j} f_i\right) = 3(n-j-1) + \sum_{i=1}^{j} f_i \tag{1.1}$$

Equation 1.1 is called a constraint criterion. There are many different-appearing versions of this relationship to be found in the literature. They all, in fact, are equivalent to one another, except that some are restricted to a subset of the cases covered by Equation 1.1.

A problem arises in some cases in which the same joint apparently connects more than two members. Typically, three or more members are pinned together by the same shaft and are free to rotate relative to one another about the same revolute axis. This difficulty is readily resolved if we recall that a kinematic joint is formed by contact between the surfaces of *two* rigid bodies. This is the reason for Reuleaux's name "pair" for what we here call a "joint." Considering the present case, we see that there is not one joint but several between the bodies. In fact, if p members are connected at a "common" joint, the connection is equivalent to $p-1$ joints all of the same type. Inclusion of this number in j, and $(p-1)f$ in the connectivity sum of Equation 1.1 will ensure correct results. This is illustrated in Example 1.3.

EXAMPLE 1.1
Degrees of Freedom in a Simple Four-Bar Linkage

Determine the mobility of the planar four-bar linkage shown in Figure 1.18.

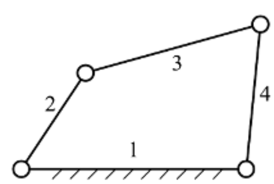


FIGURE 1.18
Mechanism for Example 1.1.

Solution:

$$n = j = 4$$

$$\sum_{i=1}^{j} f_i = j \times 1 = 4$$

$$M = 3(n-j-1) + \sum_{i=1}^{j} f_i = 3(4-4-1) + 4 = 1$$

A special case that deserves attention occurs when the mobility in Equation 1.1 is set to one and all joints have connectivity one ($f_i = 1$). Then, Equation 1.1 gives

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(n - j - 1) + j = 1$$

EXAMPLE 1.2
Degrees of Freedom in a Complex Mechanism

Determine the mobility of the linkage shown in Figure 1.19. The linkage is planar and all joints have connectivity one.

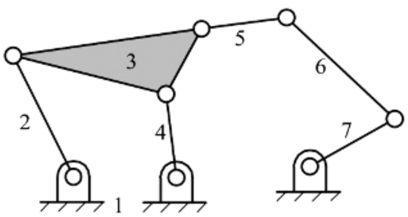


FIGURE 1.19
Two-loop planar linkage model for Example 1.2.

Solution:

Notice that the base member must always be counted even when it is not shown in the same way as the other members but just by a set of "bearing mounts." The solution is

$$n = 7;\ j = 8$$

$$\sum_{i=1}^{j} f_i = j \times 1 = 8$$

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(7 - 8 - 1) + 8 = 2$$

EXAMPLE 1.3
Degrees of Freedom When Joints Are Coincident

Determine the mobility of the linkage shown in Figure 1.20. The linkage is planar and all joints have connectivity one. Links 3, 4, and 5 are connected at the same revolute joint axis.

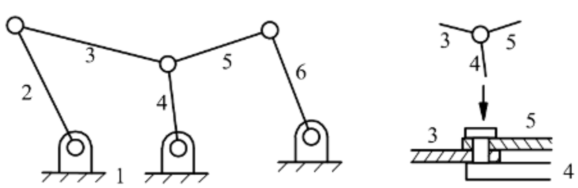


FIGURE 1.20
Mobility analysis of a linkage when more than two members come together at a single point location.

Solution:

When p members are connected at the same joint axis, then $p - 1$ joints are associated with the same axis. Hence the location where links 3, 4, and 5 come together counts as two revolute joints. As indicated in Figure 1.20, members 3 and 5 can be thought of as being connected to link 4 by two separate revolute joints that have the same axis of rotation. The solution to the problem is

$$n = 6;\ j = 7$$

$$\sum_{i=1}^{j} f_i = j \times 1 = 7$$

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(6 - 7 - 1) + 7 = 1$$

EXAMPLE 1.4
Degrees of Freedom for a Mechanism Containing a Higher Pair

Determine the mobility of the linkage shown in Figure 1.21. The linkage is planar and not all of the joints have connectivity one.

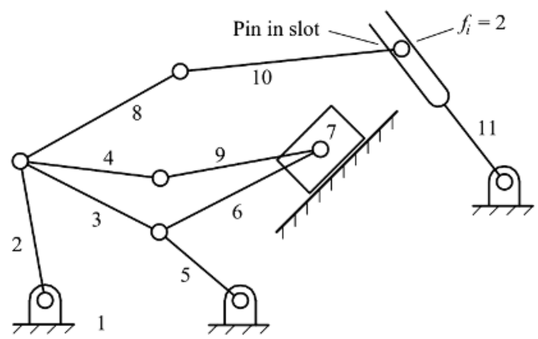


FIGURE 1.21
Mobility analysis of a linkage with various types of joints.

Solution:

In this mechanism, there are three places where more than two links come together at the same revolute joint location. In addition, there is a pin-in-a-slot joint that permits two degrees of freedom (connectivity equals 2). Therefore, the joints must be counted carefully. When this is done, we find n and j to be

$$n = 11, \; j = 14$$

and

$$\sum_{i=1}^{j} f_i = 13 \times 1 + 1 \times 2 = 15$$

Then

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(11 - 14 - 1) + 15 = 3$$

or

$$3n - 2j = 4 \tag{1.2}$$

Because n and j are integers, n must be even because 4 and $2j$ are both even numbers. This is an example of a *Diophantine* equation. That is, one that admits only integral solutions. Written as an expression for j in terms of n, the equation becomes

$$j = 3n/2 - 2$$

Some of the possible solutions are listed in Table 1.4. In each case, the joints may be either revolute or prismatic joints, since they are the only lower-pair joints that can properly be included in planar linkages.

Solution 1 gives the rather trivial case of two bodies connected by a single revolute or slider joint. This is shown in Figure 1.22(a). Actually, this mechanism is very common. For example, a door, its hinges, and the doorframe form an open kinematic chain and a mechanism of this type.

TABLE 1.4 Different Integer Solutions to Equation 1.2 for Mobility of One

Solution Number	*n*	*j*	Number of Configurations
1	2	1	1
2	4	4	1
3	6	7	2
4	8	10	16
5	10	13	230
6	12	16	6856

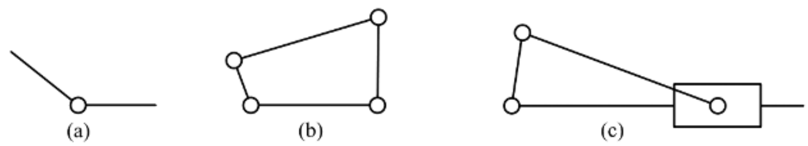


FIGURE 1.22 Solutions of the planar mobility equation for $M=1$ when $n=2$ and $n=4$.

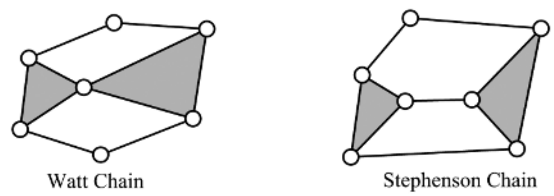


FIGURE 1.23 The two solutions of the planar mobility equation for seven revolute joints. $M=1$ and each kinematic chain has six members.

Solution 2 gives a single, closed loop of four members with four joints. Two forms are shown in Figure 1.22(b) and (c). The one in Figure 1.22(b) is the planar four-bar linkage that forms a major element in planar linkage theory. The one in Figure 1.22(c) is the slider-crank linkage, which has also been extensively studied.

Solution 3 presents two new features: First members with more than two joints mounted on them appear. Second, even when only revolute joints are included, there are two possible, topologically distinct, configurations of six members with seven joints. These are respectively named the Watt and Stephenson six-bar chains and are shown in Figure 1.23.

Solution 4 gives 16 possible different topological configurations (shown in Figure 1.24), and solution 5 gives 230. The number increases very rapidly with larger numbers of members. For example, solution 6 gives 6856 configurations [5].

From the above, it should be apparent why we spend so much effort on the design of four-link mechanisms. The four-link arrangement is the simplest possible nontrivial linkage. It turns out that most design requirements can be met by four- or six-link mechanisms.

Note that, in the previous discussion, the type of the joints was not specified. All that was specified was that the joints have connectivity one and that the linkage is planar and has mobility one. Although the joints pictured in Figures. 1.22–1.24 are all revolute, rolling contact joints could be substituted for any of the joints, and prismatic joints could be substituted for some of them. Thus, even if the joints are confined to lower pairs, the four-link, four-joint solution represents the four different chains shown in Figure 1.25. The scotch yoke, based on the 2R-2P chain, is shown in Figure 1.26.

Furthermore, as discussed later in this chapter, the important concept of inversion generates several different linkages from any mechanism based on the 3R-P and 2R-2P chains. An inversion is a different mechanism derived from a given mechanism or linkage. “Different” means that the motion relative to the frame that can be produced by the inversion is different from that provided in the original mechanism, that is, the inversion produces a different general form for the paths of points on the different links or a different input-output function.

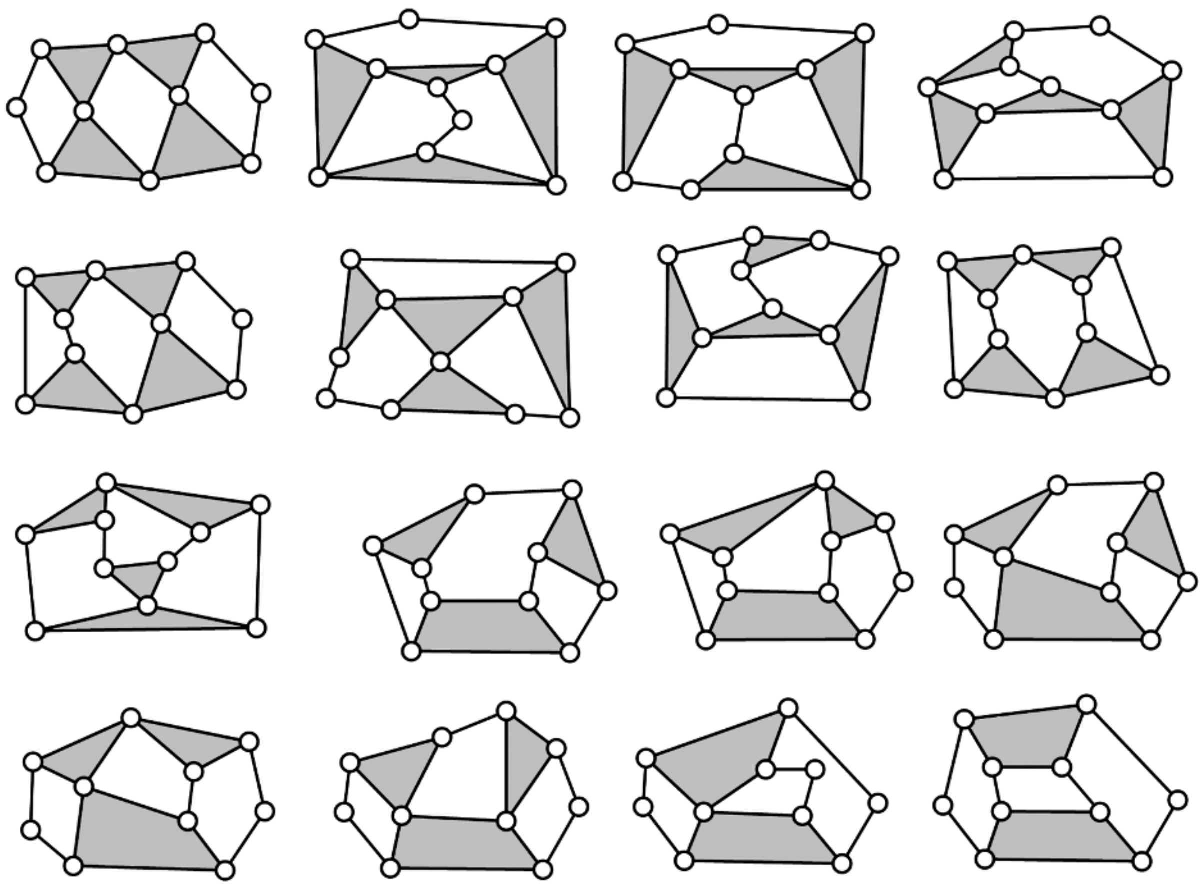


FIGURE 1.24
The 16 solutions of the planar mobility equation for 10 revolute joints. $M = 1$ and each kinematic chain has eight members.

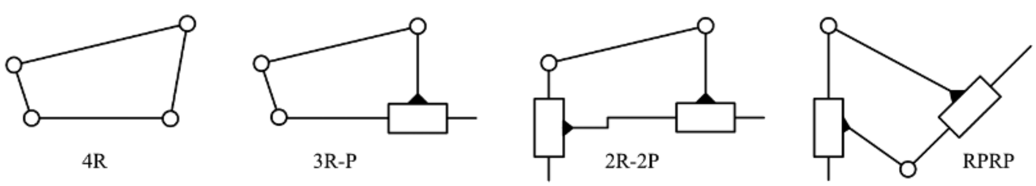


FIGURE 1.25
Four different forms of four-bar chains with combinations of revolute and prismatic joints.

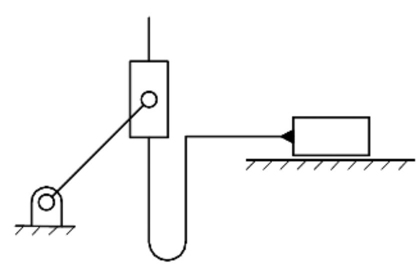


FIGURE 1.26
The 2R-2P chain as a scotch-yoke mechanism.

1.8 CONSTRAINT ANALYSIS OF SPATIAL LINKAGES

In spatial motion, each body that moves freely has six degrees of freedom rather than three. Using exactly the same reasoning as was used in the planar case, the constraint criterion equation becomes

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i \tag{1.3}$$

This is called the Kutzbach criterion. If only lower pair joints are involved, each with connectivity one, the equation becomes

$$M = 6(n - j - 1) + j = 6n - 5j - 6$$

If the linkage is required to have mobility one, this gives

$$6n = 7 + 5j \tag{1.4}$$

Equation 1.4 corresponds to Equation 1.2 derived in the case of planar motion. Like that equation, it is a Diophantine equation that admits only integral values of the variables. Evidently, j must be odd because $5j$ must be odd to combine with the odd number 7 to produce the even number $6n$. The sum $7 + 5j$ must also be divisible by three. Solutions to Equation 1.4 are a little harder to generate than those of Equation 1.2. The simplest solution is given by $j = 1$ and $n = 2$. This is exactly the same as the simplest solution in the planar case depicted in Figure 1.22(a). The next allowable solution is $j = 7$ and $n = 7$. This is a single, closed loop with seven members and seven joints. It bears the same relationship to general spatial linkage topologies that the planar four-bar linkage does to planar ones. The next order solution is $j = 13$, $n = 12$. There are three distinct topological forms in this case. For spatial mechanisms, the complexity increases with the number of members and joints even more rapidly than it does for planar mechanisms.

EXAMPLE 1.5
Degrees of Freedom in a Spatial Mechanism

Determine the mobility of the linkage shown in Figure 1.27. The linkage is spatial. The joints are lower pairs of the types labeled.

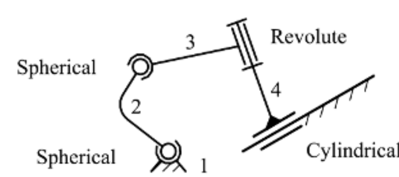


FIGURE 1.27
A four-member, single loop, spatial linkage.

Note how the three-dimensional joints are drawn. There is no formalism that is more or less universally recognized for representing spatial mechanisms as there is for planar linkages; however, we will follow the symbols shown in Table 1.1.

Solution:

$$n = j = 4$$

$$\sum_{i=1}^{j} f_i = 2 \times 3 + 1 \times 1 + 1 \times 2 = 9$$

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(4 - 4 - 1) + 9 = 3$$

Another way of looking at the constraint criterion is in terms of closures. Imagine building up the linkage by starting with the base link and successively adding members and joints. If a joint connects an additional member to the system, the number of degrees of freedom is increased by f_i, if f_i is the connectivity of that joint, and the numbers of members and joints are both increased by one. If a joint is made between two members that are already part of the linkage, the total number of degrees of freedom is decreased by the number of constraints imposed by that joint. The number of constraints imposed by a joint is the number of degrees of freedom lost by the system when that joint is formed. For a spatial mechanism, it is $6 - f_i$ since two bodies have six degrees of freedom of motion relative to one another when they are free of each other and only f_i degrees of freedom of relative motion after the joint is formed. Also, in this case, the formation of the joint results in the formation of a closed loop of members and joints within the linkage. This is called a closure. Proceeding in this manner, the mobility of the linkage can be expressed as

$$M = \sum_{i=1}^{j} f_i - 6c$$

where c is the number of closures. Now, when a closure is formed, the number of members does not increase, whereas the number of joints increases by one. If there are no closures (open kinematic chains), the number of link members is given by

$$n = j + 1$$

the additional member being the base member. Therefore, if there are c closures in the linkage

$$c = j + 1 - n$$

Thus, substitution for c in the expression for the mobility leads to Figure 1.28. The relationship among c, j, and n is illustrated in the figure.

The reason for looking at the constraint criterion from this viewpoint is that it relates to the position analysis of a spatial linkage. When a closure is formed, a set of six algebraic equations called *closure* equations can be written. The formulation of these equations will be briefly treated in Chapter 9, although their study is largely beyond the scope of this book. The quantity $6c = 6(j + 1 - n)$ is therefore the number of equations available for position analysis of the mechanism. The variables in those equations are the joint parameters, the variables needed to fix the relative positions of the bodies connected by each joint. There are f_i of these joint parameters for joint i. Therefore the total number of variables in the linkage is

$$\sum_{i=1}^{j} f_i$$

In this way, it may be seen that Equation 1.3 expresses the mobility of the linkage as the number of variables less the number of equations for the system.

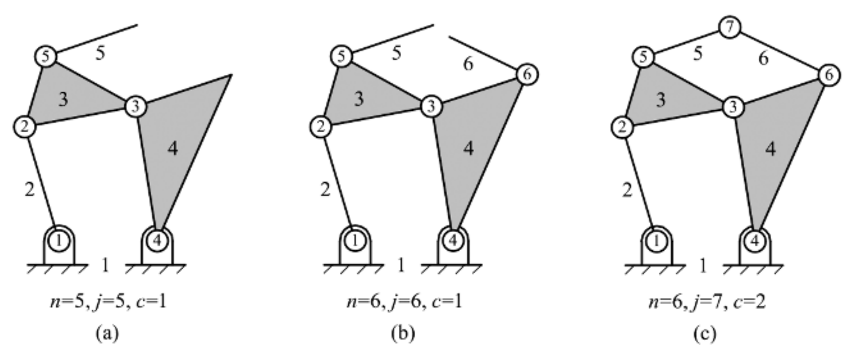


FIGURE 1.28
The effect of adding a member to a linkage together with a joint (b) and of adding a joint without an additional member (c). Adding a joint without a member always closes a loop within the linkage.

Yet another viewpoint on the constraint criterion that is productive to pursue is that of static force analysis. Free body diagrams can be drawn for all members except the base. Six static equilibrium equations can be written for each free body. Hence there are $6(n-1)$ equations describing the system. At each joint there is a number of reaction force and torque components that is equal to the number of constraints of that joint. These force components are the variables in a static force analysis. Since the number of constraints at joint i is $6-f_i$, the number of variables is

$$\sum_{i=1}^{j}(6-f_i) = 6j - \sum_{i=1}^{j} f_i$$

Therefore, the difference between the number of variables and the number of equations is

$$6j - \sum_{i=1}^{j} f_i - 6(n-l) = -M$$

Thus, the mobility is meaningful from the point of view of static force analysis also. If $M=0$, the linkage is not movable and is a structure. The position problem can be solved to obtain the joint positions that cannot vary. The static equilibrium problem can be solved for all of the reaction force and torque components. The structure is statically determinate since there is a unique solution to the static equilibrium problem.

If the mobility is -1, the number of equations for the position problem exceeds the number of variables. Therefore, in general there is no solution to the position problem. For a solution to exist, it is necessary for the equations to be dependent. This means that the geometry of the mechanism must satisfy the conditions needed for the equations to be dependent. Physically, this means that, in general, it is not possible to assemble the linkage. One or more of the closures cannot be made. However, if the link geometry is changed to bring the surfaces for the closing joint into alignment, the linkage may be assembled.

From the viewpoint of force analysis, the mobility is the number of static equilibrium equations less the number of force variables: the converse of the situation for position analysis. Thus, if $M=-1$ there is one more force variable than the number of force equations. Therefore, in this case, solutions of the system exist, but there is no unique solution. The force problem cannot be solved without additional information relating the forces in the system. The linkage is a statically indeterminate structure. If the links are modeled as elastic, rather than rigid solids, compatibility of their deflections under load provides the necessary additional relationships.

Conversely, if the mobility is one or more, the number of position variables is greater than the number of position equations. Solutions to the system exist, but there is no unique solution. The number of force equations is greater than the number of force variables, so, in general, no solution to the static force problem exists. In practice, application of an arbitrary set of loads to the linkage would lead to rapid, uncontrolled acceleration, and the system behavior could not be described without writing dynamic equations. However, this invalidates the assumption of a static model.

Specification of the value of a joint parameter is equivalent to fixing that joint. Physically, putting an actuator on that joint which would hold it in position might do it. The joint can now support a force, or torque. The effect is to increase the number of unknown force variables by one. If a linkage has mobility one, fixing the position of a joint with connectivity one converts it into a structure. It also converts the static force problem from one in which there is one more equation than there are variables to one in which the number of variables is the same as the number of equations. That is, it is statically determinate.

Fixing the torque applied about a revolute joint, or the force applied by an actuator at a prismatic joint, has a quite different effect. It does not change the number of variables or the number of equations in either the position or the force problem. This is because having a passive joint is already equivalent to fixing the force or torque variable about that joint. The torque applied at a passive revolute joint is fixed to zero. Changing it to any other value does not affect the number of unknown variables. It does, however, affect the values of the unknown force variables.

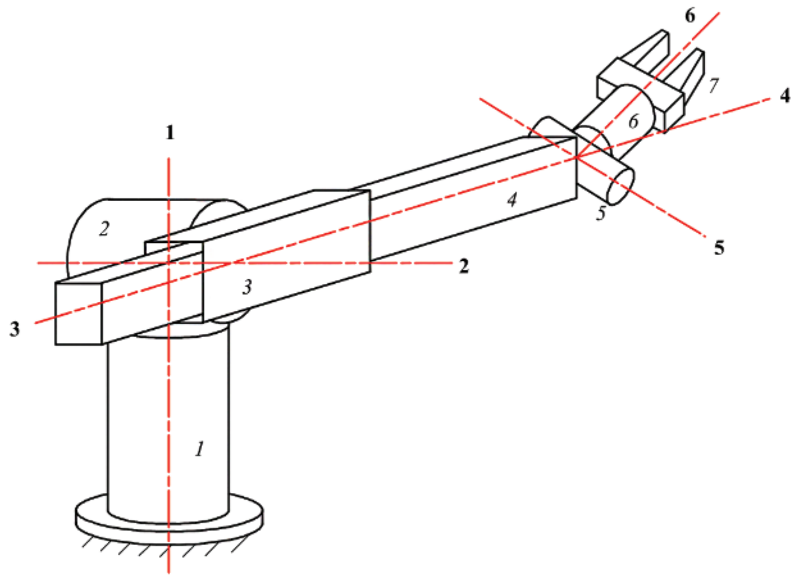


FIGURE 1.29
A robotic manipulator that is used to produce general spatial motions of its gripper.

This is important in practical applications of multiply actuated mechanisms. Consider the manipulator arm shown in Figure 1.29. It has seven members (italic numbers) and six joints. The red dashed lines with bold numbers indicate the joint axes. Joints 1, 2, 4, 5, and 6 are revolute joints. Joint 3 is a prismatic joint. The axes of joints 3 and 4 are the same. Member one is the base member.

Applying the constraint criterion to this mechanism, we have: $n=7$, $j=6$, $\sum_{i=1}^{j} f_i = 6$ so

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(7 - 6 - 1) + 6 = 6$$

If we actuate all of the joints so that we can specify their positions, the position of the mechanism is uniquely specified.

Consider now what happens if the manipulator grips an object that is fixed relative to the base member, as is shown in Figure 1.30. It is assumed that the gripper grasps the object tightly so that no

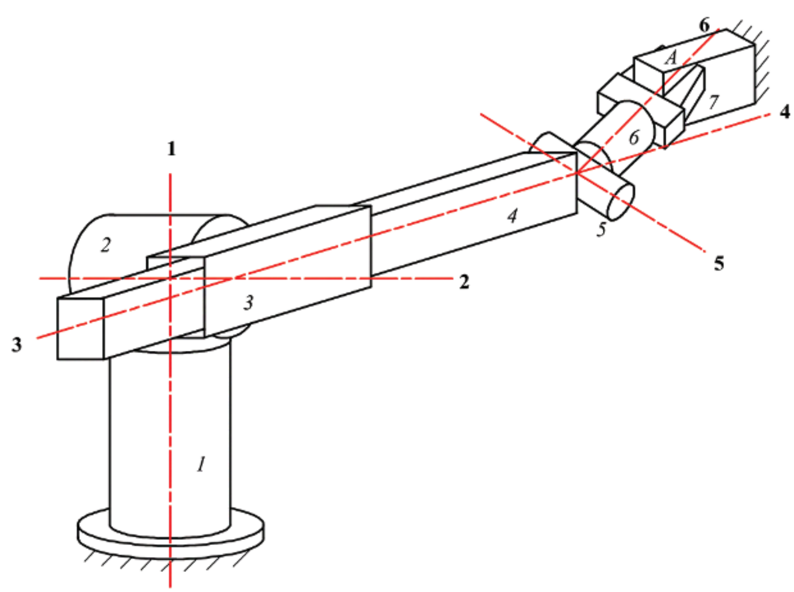


FIGURE 1.30
The robotic manipulator of Figure 1.29 gripping a fixed object. If the gripper grasps the object so that no relative motion is possible, the gripper becomes fixed to member one. This reduces the number of members in the system to six and closes a loop.

relative motion is possible. The effect is to make link 7 a part of link 1. Therefore, application of the constraint equation gives $n=6$, j = 6, $\sum_{i=1}^{j} f_i = 6$

so

$$M = 6(n-j-1) + \sum_{i=1}^{j} f_i = 6(6-6-1) + 6 = 0$$

The mechanism is now a structure, and we do not have the liberty of setting the joint variables to any value we wish. Attempting to control the mechanism by commanding joint positions, as is done when the manipulator is moving freely, is not effective in this case. Since most manipulator structures are very stiff, a small position error results in very large forces on the actuators. The usual result is that the actuator controllers become unstable, producing violent vibratory behavior. However, if the actuators are commanded to produce specified forces or torques, there is no problem. The actuator torques and forces can be set to any desired set of values. In this way it is possible to apply a specified force system to the fixed object A by means of the manipulator. Notice that commanding forces and torques all the time is not a solution. If actuator forces are commanded when the manipulator is moving freely, the number of static equilibrium equations exceeds the number of variables by six and the manipulator will perform rapid uncontrolled movements, violating the assumption of static stability.

1.9 IDLE DEGREES OF FREEDOM

Equation 1.4 sometimes gives misleading results. There are several reasons for this. One is the phenomenon of idle degrees of freedom. Consider the linkage shown in Figure 1.31. This linkage has four members and four joints. Two of the joints are revolutes. The other two are spherical joints. This mechanism is quite often used in situations such as the steering mechanisms of automobiles. Applying the constraint criterion, we have $n=4$, $j=4$ so

$$\sum_{i=1}^{j} f_i = 2 \times 1 + 2 \times 3 = 8$$

Therefore

$$M = 6(n-j-1) + \sum_{i=1}^{j} f_i = 6(4-4-1) + 8 = 2$$

Nevertheless, practical experience with this mechanism shows that there is a unique value of the output joint angle, ϕ, for any given value of the input angle, θ. How can this be explained?

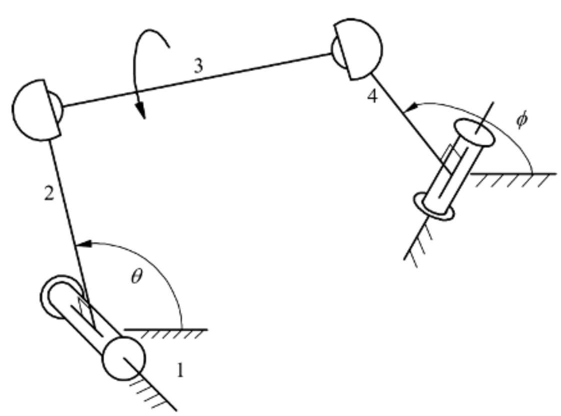


FIGURE 1.31
A spatial four-member, four-joint linkage. Two of the joints are revolutes. The other two are spherical joints. θ is the input joint angle and ϕ is the output joint angle. The linkage has an idle degree of freedom since member 3 can spin about the line joining the centers of the spherical joints without affecting the relationship between θ and ϕ.

Examination of the mechanism reveals that the coupler member is free to spin about the line through the centers of the two spherical joints. This motion can take place in any position of the linkage without affecting the values of the input and output joint angles. It is what is termed an idle degree of freedom. That is, it is a degree of freedom that does not affect the input-output relationship of the linkage.

The real problem here is that usually we are not really interested in the mobility of the entire linkage, that is, of all of its links. Rather, we are interested in the connectivity that the linkage provides as a joint between two of its members. This is a new use of the term *connectivity*. Previously we applied it only to simple joints at which the members contact each other directly. However, a mechanism constrains the number of degrees of freedom of relative motion of any two of its members. Therefore it can be regarded as forming a kinematic joint between any two of its members. We can define its connectivity as a joint between those members and as the number of degrees of freedom of relative motion that it permits between the members.

In the example of Figure 1.31, the connectivity of the linkage as a joint between the input and output members is one, even though the mobility of the linkage is two, and the connectivity between links 3 and 1 is two. The mobility places an upper bound on the connectivity of the mechanism as a joint between any two of its members. There is no simple method of directly determining connectivity, so the mobility equation is used. If the mobility is one and the linkage is predictable by means of Equation 1.3, there is no problem. The connectivity of the linkage as a joint between any two of its members is also one. If the mobility is greater than one, strictly speaking, all that can be said is that the connectivity between any given pair of members may be equal to the mobility or may be less than that number. Fortunately, idle degrees of freedom can usually be identified by inspection.

Another example is shown in Figure 1.32. This is one form of the Stewart-Gough platform mechanism, discussed in detail in Chapter 9. This mechanism is commonly used to produce general spatial motions in aircraft simulators for training pilots. The output member is connected to the base by six "limbs," each of which has an actuated prismatic joint in the middle and two spherical joints at either end. There are 14 members: 2 in each of the limbs plus the base and output members. There are 18 joints: 6 prismatic joints and 12 spherical joints. Hence

$$\sum_{i=1}^{j} f_i = 6 \times 1 + 12 \times 3 = 42$$

Therefore

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(14 - 18 - 1) + 42 = 12$$

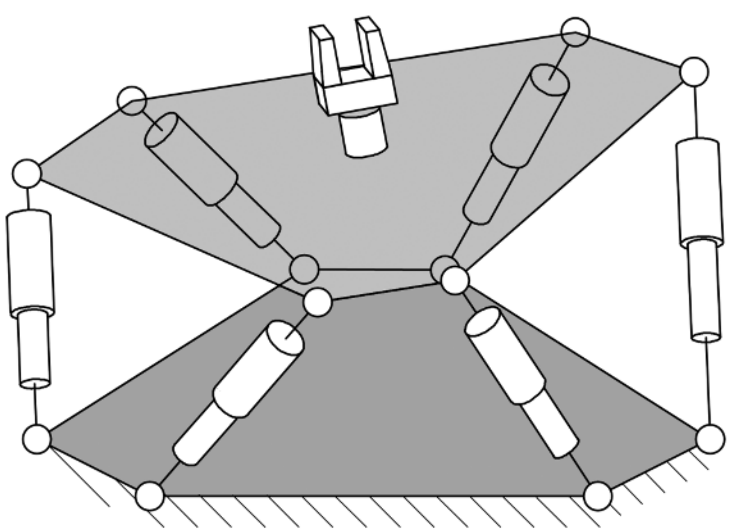


FIGURE 1.32
A Stewart-Gough platform. The upper and lower members are plane hexagons connected by six limbs, each having an actuated prismatic joint. The limbs are connected to the upper and lower hexagons by spherical joints.

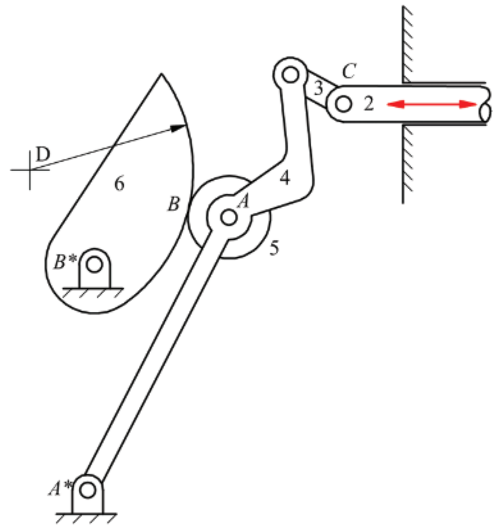


FIGURE 1.33
A planar mechanism with an idle degree of freedom. The roller follower can rotate without affecting the relative positions of the other members.

However, it is easily seen that each limb is free to spin about the line joining the centers of its spherical joints without affecting the position of the output member relative to the base. Therefore, the mechanism has six idle degrees of freedom, and its connectivity as a joint between base and output member is

$$C = M - 6 = 6$$

By positioning the actuated prismatic joints, the output member can be placed in any position within its working volume.

While idle degrees of freedom are most common in spatial linkages, they can also occur in planar linkages. Typically, this occurs when cam roller followers are involved. For example, if the mobility of the linkage in Figure 1.33 is computed, it will be found to be 1 if there is rolling contact between the roller (link 5) and the cam (link 6) at point B. However, if there is cam contact at B, the mobility will be 2. The extra degree of freedom is associated with the free rotation of link 5 relative to the frame. Usually, this rotation will be of no interest because the motion of all of the other links in the mechanism will be unaffected by this rotation.

To locate the idle degrees of freedom, it is first necessary to identify the input link and output link. Then check to determine if a single link or a combination of connected links can move without altering the relative position of the input and output links. Idle degrees of freedom are dependent both on geometry and on the choice of the input and output links. In some cases, idle degrees of freedom will exist for one choice of input/output but not for a different choice.

1.10 OVERCONSTRAINED LINKAGES

A second reason why the constraint criteria—Equations 1.1 and 1.3—sometimes give misleading results is the phenomenon of *overconstraint*. A mechanism can be overconstrained either locally or generally. If the mechanism is overconstrained locally, a portion of the system may be a structure, but the entire mechanism can move. When this happens, we must replace that portion of the linkage with a single rigid body and recompute the mobility of the mechanism. An example is shown in the planar system of Figure 1.34(a).

Here $n = 9$ and $j = 2\times 1 + 2\times 2 + 2\times 3 = 12$. Note that there are two joints at which three members are connected and two at which four members are connected

$$\sum_{i=1}^{j} f_i = j = 12$$

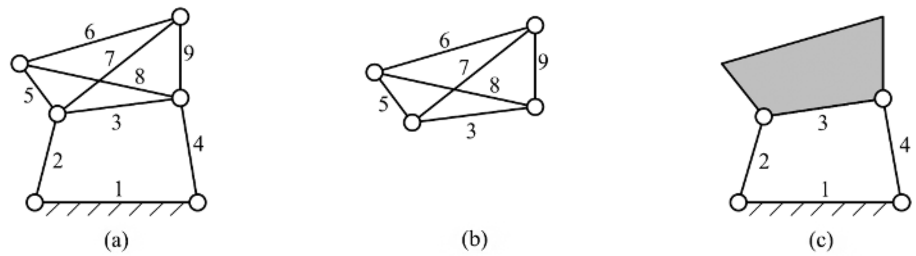


FIGURE 1.34
(a) A planar mechanism in which part of the mechanism is a structure, leading to a misleading value of mobility. All joints are revolutes. (b) The part of the mechanism that is a statically indeterminate structure. (c) A modified model of the linkage that gives the correct mobility value.

Hence

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(9 - 12 - 1) + 12 = 0$$

However, it is obvious that the portion of the linkage consisting of members 3, 5, 6, 7, 8, and 9 is a statically indeterminate structure. This portion is shown in Figure 1.34(b). Here $n = 6$ and, because three members are connected at each joint location, $j = 4 \times 2 = 8$. Also

$$\sum_{i=1}^{j} f_i = j = 8$$

Therefore

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i = 3(9 - 12 - 1) + 12 = 0$$

This reveals the statically indeterminate nature of the structure and the source of the error in the mobility value. A portion of the linkage that is a statically *determinate* structure does not cause an error in calculating mobility.

To compute a correct value of mobility, the linkage is remodeled as shown in Figure 1.34(c) with the portion that is a structure replaced by a single, rigid member. The linkage is now revealed to be a planar four-bar linkage for which the mobility is one.

Mechanisms, and especially spatial mechanisms, can also be generally overconstrained. Figure 1.35 shows a spatial linkage with four members and four revolute joints. It has a special geometry. The opposite members are identical, and the normals to the pairs of axes in the links intersect at the joint axes. The lengths of those normals (a and b) are related to the angles between successive axes (α and β) by the relationship

$$a \sin \beta = b \sin \alpha$$

As was demonstrated over a hundred years ago by Bennett [4], this linkage has mobility one. However, if we apply the constraint criterion with $n = j = 4$ and $\sum_{i=1}^{j} f_i = 4$, the result is

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(4 - 4 - 1) + 4 = -2$$

In this case, because of the special geometry, the position equations of the linkage turn out to be dependent in all positions. For this reason, the effective number of equations is only three, rather than

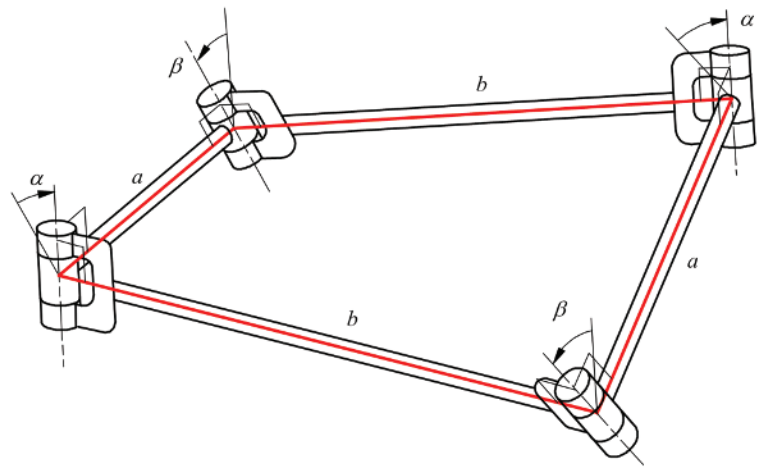


FIGURE 1.35
The Bennett mechanism is a spatial four-bar linkage that is movable despite violating the constraint criterion. The side lengths and twist angles obey the relationship $a \sin\beta = b \sin\alpha$.

the six that would be expected for a single closed spatial loop. Because the constraint criterion calculates the difference between the number of position variables and the number of available equations, it miscounts the mobility by three degrees of freedom. It turns out that a rather large number of linkages have anomalous mobility like the Bennett mechanism. Another example is the linkage shown on Figure 1.16. Here $n = 6$ and $j = 6$, so M should be 0, yet the mechanism is movable with one degree of freedom. Such linkages are called overconstrained linkages. Many of these are largely curiosities. However, there are several very important families of overconstrained linkages that are exceedingly common in engineering practice.

The most common example of overconstraint is the family of planar linkages. There is no *a priori* reason why planar linkages should not obey the general spatial mobility criterion. Nevertheless, as we saw in Section 1.7, they do not. Equation 1.3 gives a value for M that is always $3c$ less than the correct value, where c is the number of independent closure equations for the linkage. The fact that planar linkages obey Equation 1.1, that has the same form as Equation 1.3 but with the coefficient 6 replaced by 3, indicates that only three of the six equations produced by any closure are independent for a planar linkage.

Another common family of overconstrained linkages is the family of spherical linkages. These are linkages whose joints are all revolutes. The axes of those joints all pass through a single point. Figure 1.36 shows a spherical four-bar linkage.

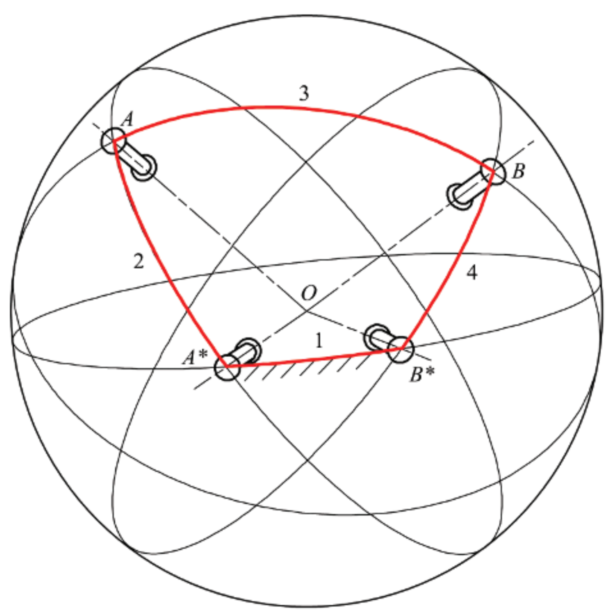


FIGURE 1.36
A spherical four-bar linkage. Here the axes of the four revolute joints intersect at a single point, O.

FIGURE 1.37
A front-end loader. If analyzed using the planar mobility equations, the mechanism will be found to have less than one degree of freedom. Parallel actuators are used on both sides of the machine to balance the load and increase stiffness. The loader part of the machine has two degrees of freedom.

Spherical linkages obey the same form of constraint criterion as planar linkages and the Bennett linkage. Thus, three of the equations resulting from each closure in a spherical linkage are always dependent.

Compared with properly constrained linkages (those that obey Equation 1.3) overconstrained linkages have properties that are different in important and practical ways. They tend to be much stiffer and stronger in supporting loads, particularly those orthogonal to the direction of motion at the point of application. However, they are sensitive to dimensional accuracy in their members. This requires manufacture to relatively tight tolerances, which can increase cost. Conversely, properly constrained linkages are completely insensitive to link geometry, as far as mobility is concerned. This means that, in lightly loaded situations, they can absorb abuse that deforms links and still function, at least after a fashion. This is an important property in situations such as the control linkages of agricultural machinery. In heavily loaded situations, the design engineer will often deliberately increase the degree of overconstraint to improve stiffness and strength. An example is the bucket support linkage of a front-end loader. A photograph of the loader is shown in Figure 1.37, and one of the bucket support linkages is identified in Figure 1.38.

In principle, only one of the two planar-inverted, slider-crank linkages is needed to lift/support the bucket. In this case, we would have

$$n = j = \sum_{i=1}^{j} f_i = 4$$

and

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(4 - 4 - 1) + 4 = -2$$

Since the true mobility is 1, the degree of overconstraint is $1 - (-2) = 3$. However, the mechanism is doubled up with identical linkages supporting each end of the bucket. This gives $n = 7$ and $j = \sum_{i=1}^{j} f_i = 8$. Thus

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i = 6(7 - 8 - 1) + 8 = -4$$

Therefore, for the doubled linkage the degree of overconstraint is $1 - (-4) = 5$. The result is a much stronger mechanism since the individual planar loops do not have to support the large out-of-plane

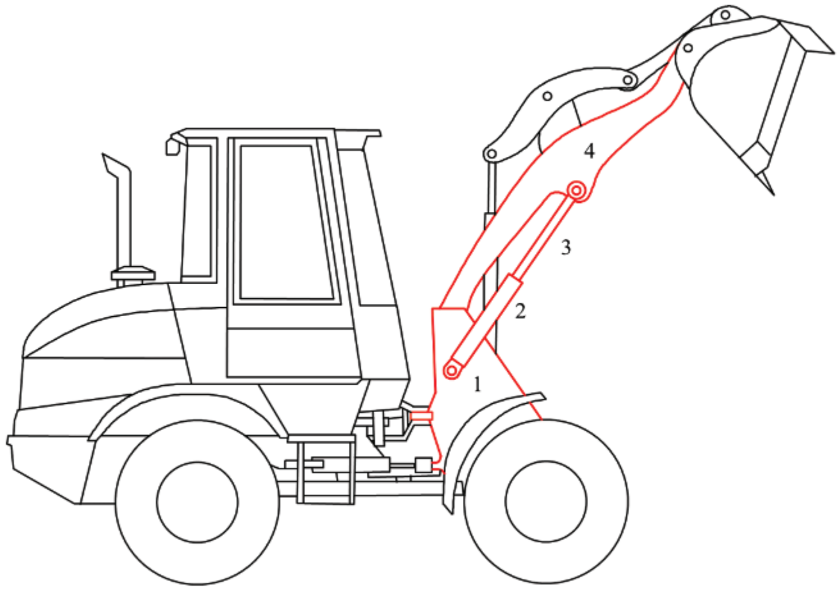


FIGURE 1.38
Schematic drawing of the bucket-support linkage for a front-end loader similar to that shown in Figure 1.37.

moments that a single linkage would have to support. The cost is that the axes of the corresponding joints on both sides must be collinear to a high degree of accuracy, requiring careful manufacturing.

1.11 USES OF THE MOBILITY CRITERION

The mobility criterion is most useful to the engineer when examining an unfamiliar mechanical system. It allows a quick check to determine whether the links, joints, and actuators identified are consistent with system function. Inconsistency may indicate that some elements have been misidentified or that passive degrees of freedom are present. As already discussed, overconstraint may also need to be considered. In particular, if the linkage is planar or spherical, the appropriate form of the constraint equation should be used in place of the general form.

It is possible to formulate expressions for the mobility that accommodate overconstrained closures of arbitrary type. These expressions are equivalent to the form

$$M = \sum_{k=1}^{c} b_k + \sum_{i=1}^{j} f_i \tag{1.5}$$

where $c = n - j - 1$ is the number of closures of the linkage.

Unfortunately, unless the values of b_k associated with the different closures can be identified by inspection, such expressions have no value. The reason is that the mobility equation gives a quick check of the number of position variables and independent equations without the need to develop those equations. However, the only way to verify an overconstrained closure of a type not identifiable by inspection is to develop the closure equations and analyze them for dependency. Therefore the quick-check advantage of the mobility equation disappears, and there is no way to derive information about the linkage without performing a complete position analysis.

1.12 INVERSION

A commonly used tactic in studying mechanism kinematics is *inversion*. This is a change of the fixed reference frame from one link to another that causes different characteristics of the motion relative to the frame to become evident. For example, Figure 1.39 shows the different inversions of a slider-crank linkage, and Figure 1.40 shows the inversions of a pin-in-a-slot mechanism. The pin-in-a-slot inversions are often used as inexpensive substitutes for the slider-crank inversions. The motion

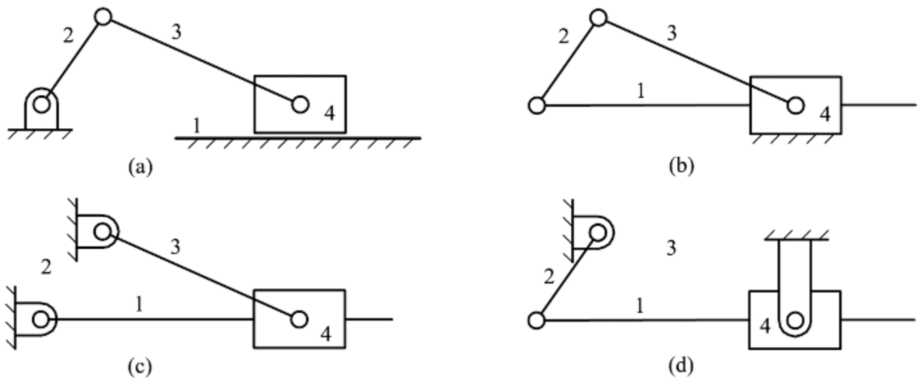


FIGURE 1.39
Inversions of the slider-crank linkage. The linkage in (a) is the original linkage and those in (b), (c), and (d) are the inversions. Note that the motion properties of the inversions are quite different from those of the original linkage, and from each other, but yet are related in useful ways.

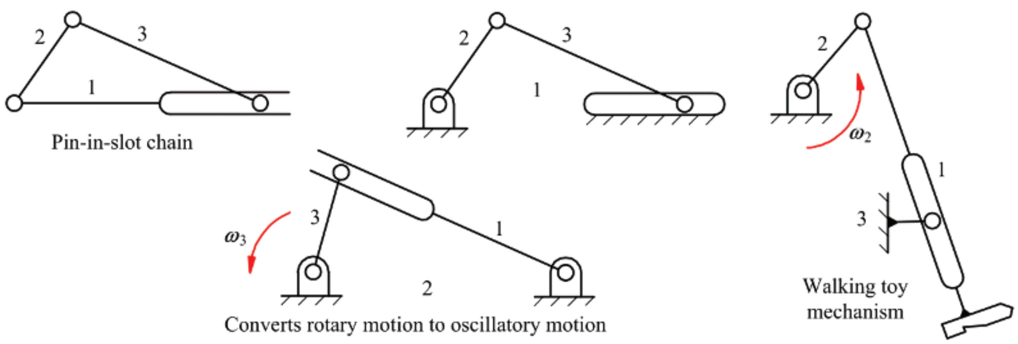


FIGURE 1.40
Uses of inversions of a pin-in-a-slot linkage.

characteristics of the coupler links for each of the mechanisms are all very different. Nevertheless, the linkage topology and the relative angular relationships among the links are the same in all cases. Therefore, useful information obtained from the study of the linkage in one inversion can be transferred to the study of other inversions. Note that in Figure 1.39, the relative positions of all of the joints are the same for the position chosen. It is only when the mechanisms move that the different motion characteristics are revealed. To determine the inversions of a mechanism, it is convenient to start with the chain from which the mechanism is formed. A different linkage results whenever a different link is selected as the frame.

1.13 REFERENCE FRAMES

It is necessary to be careful about reference frames when working with systems of many bodies. A reference frame can be attached to each body, and we can express positions, velocities, and accelerations relative to any or all of them.

As far as kinematics is concerned, there is no restriction on the use of reference frames. All frames are equally viable. We can invert from one frame to another without restriction.

It is only when we introduce forces and enter the realm of kinetics that a restriction appears. It is then that Newton's first and second laws, that relate motion properties to force, are true only if all motion properties are referred to a common reference frame. This common reference frame must be of a special type, called an *inertial reference frame*. For the purposes of mechanism design, the inertial reference frame is almost always fixed to the earth. There are, however, engineering problems, such as the design of mechanisms to be carried on spacecraft, for which the primary inertial reference frame must be used. The primary inertial reference frame of Newtonian mechanics is fixed relative to the fixed stars. A more complete discussion of inertial reference frames can be found in most texts on rigid-body dynamics. Einstein showed that in a space-time framework all reference frames are equally valid, thereby removing the Newtonian distinction between inertial reference frames and others. However, in the domain in which mechanical engineers usually operate, Newtonian mechanics provides a very accurate simplification of relativistic mechanics that is of great practical utility.

It is important to remember that position and motion properties can be expressed only relative to a reference frame. A habit has grown up in this subject of referring to a velocity or acceleration of a point relative to another point. This convention will be found to be convenient in some types of problems, particularly in graphical analysis, and there is no harm in using it provided that it is clearly understood that it is a shorthand expression for the velocity or acceleration of the first point relative to a *reference frame* in which the second point is fixed. The identity of that reference frame should always be kept in mind.

In some discussions in the following, it will be convenient to have a notation that explicitly states the reference frame in which a particular vector is expressed. A method that is often used is to indicate the reference frame by means of a superscript placed in front of the symbol for the vector. For example, ${}^1\boldsymbol{v}_A$ indicates the velocity of point A relative to reference frame 1, and ${}^2\boldsymbol{\omega}_3$ indicates the angular velocity of member 3 relative to reference frame 2.

Usually, we will associate one reference frame with each member of a linkage and will number it to agree with the number of the member. Reference frame 1 will usually refer to the fixed link. Unfortunately, the use of superscripts to indicate reference frames complicates expressions and makes them more difficult to read. For this reason, the superscripts will be dropped whenever the reference frame to which motions are referred is evident.

1.14 MOTION LIMITS

A member of a linkage that is connected to the base by a revolute joint and that rotates completely as the linkage moves through its motion cycle is called a crank. Usually, there will also be members in the linkage that look exactly like cranks because they are connected to the base by a revolute joint, but which cannot rotate completely.

Consider the four-bar linkage shown in Figure 1.41(a) in which the link A^*A is a crank rotating fully about the revolute joint A^*. It will be assumed to rotate continuously in the counterclockwise direction. Complete revolution of this link requires that it pass through the positions shown in Figure 1.41(b) and (c). Now consider the motion of the revolute joint B^*. Prior to reaching the position of Figure 1.41b, link B^*B was rotating counterclockwise about joint B^*. In the position of Figure 1.41(b), further rotation of B^*B about B^* in the counterclockwise direction is not possible. B^*B comes to rest and reverses its direction of motion. Similarly, before entering the position of Figure 1.41(c), the link B^*B is rotating clockwise about the joint B^*. In this position, further

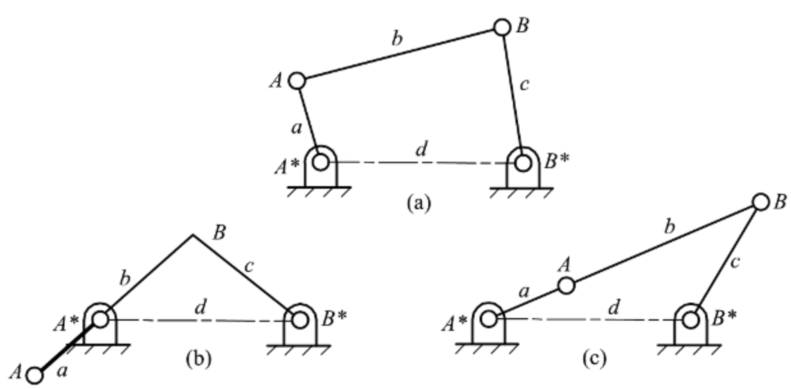


FIGURE 1.41
The limiting positions of joint B of a four-bar linkage.

rotation in this direction is not possible and the link comes to rest and then reverses direction. The positions shown in Figure 1.41(b) and (c) are called motion limit positions for the joint B^*. The link B^*B does not perform a full rotation but simply oscillates between these positions. That is, it is not a crank, it is a rocker.

In Figure 1.41, we have introduced a joint nomenclature that we will generally use throughout this book. Both rockers and cranks will contain joints that rotate about joints fixed to the frame. When possible, we will represent the moving joint and fixed joint as a set where the moving joint or pivot is designated by a letter (e.g., A), and the fixed joint or pivot is designated by the same letter with an asterisk (e.g., A^*). This nomenclature is especially useful when discussing mechanism synthesis.

1.15 CONTINUOUSLY ROTATABLE JOINTS

At this point it is necessary to introduce some terminology to describe the different members of a four-bar linkage. The fixed link, that is, the member to which the frame of reference is attached, is called the base or frame. The two members that are connected to the base by revolute joints are called turning links. The link that is jointed to both turning links and has no direct connection to the base is called the coupler. The turning links may be further distinguished by the terms crank, for a link capable of complete revolution relative to the base, and rocker, for a link that is only capable of oscillating between motion limits.

A linkage is actuated, or driven, by applying a force to one of its moving links or a torque about one of the axes. This may be done in a variety of ways, as is evident from the number of different types of commercial actuators (Figure 1.42). Actuators are more fully discussed in Chapter 17. It is frequently convenient for the powered link to be connected to the base by a revolute joint. The linkage may then be actuated by applying a torque to that link, or, more precisely, applying a torque between the base and that link. In this case it is usually also preferable that the link be continuously rotatable since it may then be actuated by means of a continuously rotating motor. For this reason, it is important to be able to identify four-bar linkages that have continuously rotatable joints and to locate those joints. This may be done by means of a simple set of rules called Grashof's rules [2].

Grashof distinguished two fundamentally different types of four-bar linkage by means of the inequality

$$s + l < p + q \tag{1.6}$$

FIGURE 1.42
Photographs of a variety of actuators.

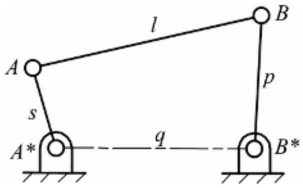


FIGURE 1.43
The nomenclature used when discussing Grashof's inequality.

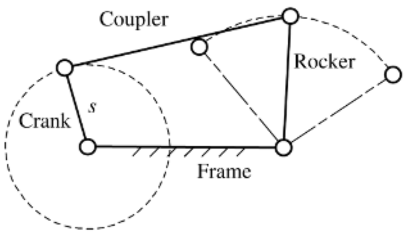


FIGURE 1.44
Crank-rocker subtype of Grashof type 1 linkage. This linkage type occurs when the shortest link is jointed to the base of the linkage.

where, as shown in Figure 1.43, s is the length of the shortest side, l is the length of the longest side, and p and q are the lengths of the other two sides. Linkages that obey this inequality (Grashof type 1 linkages) have two joints that perform complete rotations and two that oscillate between motion limits. The two fully rotatable joints are those on the ends of the shortest link. Linkages that do not obey the inequality (Grashof type 2 linkages) have no fully rotatable joints. All four joints then oscillate between motion limits.

The behavior of a linkage that obeys the Grashof inequality is strongly dependent on the locations of the fully rotatable joints relative to the base link. That is, it is dependent on the inversion of the linkage. The following additional rules distinguish three subtypes that have different behavior:

1. If the shortest link is jointed to the base, the linkage is a crank-rocker (Figure 1.44). The joint between the shortest link and the base is fully rotatable. Hence, that link is a crank. The other fully rotatable joint connects that crank to the coupler. Hence, the other joint connected to the base is not fully rotatable, and the link it connects to the base oscillates. It is the rocker. A crank-rocker can be conveniently driven about the joint connecting the crank to the base.
2. If the shortest link is the base, both joints at the base are fully rotatable, and so both links connected to the base are cranks (Figure 1.45). The linkage is a double-crank, also known as a drag-link. It may be conveniently actuated at either of the base joints.
3. If the shortest link is the coupler, neither base joint is fully rotatable (Figure 1.46). The linkage is a type 1 double-rocker. Its behavior is different from that of type 2 double-rockers, those that do not satisfy the inequality, because in the type 1 linkage the two floating joints can rotate completely. The result is that the coupler performs a complete rotation relative to the base while the two frame mounted links simply rock. The angular motion of the coupler of a type 2 double-rocker is an oscillation relative to the base.

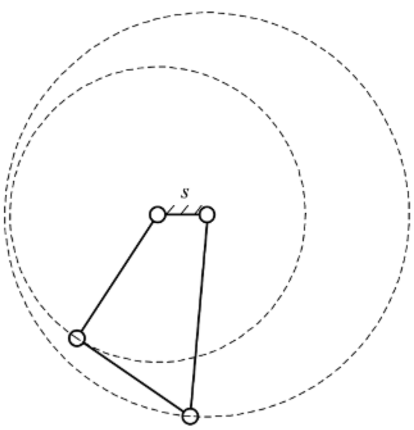


FIGURE 1.45
The double-crank subtype of Grashof type 1 linkage. This linkage type is also called a drag-link. It occurs when the shortest link is the base.

The Grashof inequality may be proved as follows: Consider the linkage shown in Figure 1.47(a). In order to perform a complete rotation it must pass through the positions shown in Figure 1.47(b) and (c). Let a be the length A^*A, b the length AB, c the length B^*B, and d the length A^*B^*. It is assumed that

$$a < d$$

The triangle inequality states that the sum of the lengths of any two sides of a triangle is greater than that of the third. It may be applied three times to Figure 1.47(b) to give

$$a + d < b + c \qquad \text{(a)}$$

$$b < c + a + d \qquad \text{(b)}$$

$$c < b + a + d \qquad \text{(c)}$$

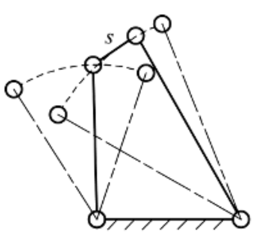


FIGURE 1.46
The type 1 double-rocker. This subtype occurs when the shortest link is the coupler.

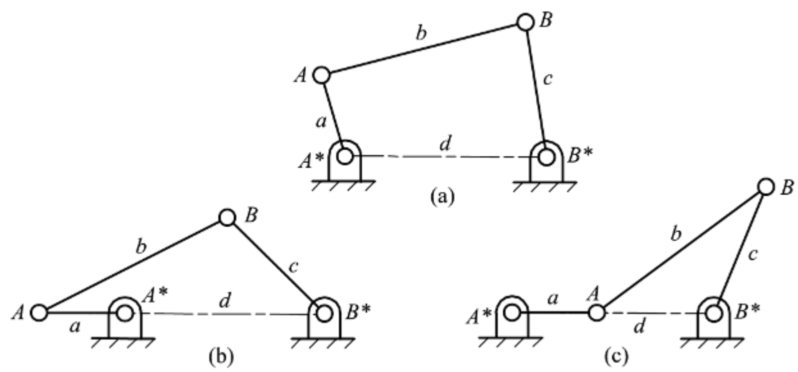


FIGURE 1.47
(b) and (c) are the extreme positions during the motion of the four-bar linkage (a). The triangle inequality is applied to these positions during the proof of the Grashof inequality.

The triangle inequality may also be applied three times to Figure 1.47(c) to give

$$d - a < b + c \tag{d}$$

$$b < c + d - a \tag{e}$$

$$c < b + d - a \tag{f}$$

Examination of these inequalities reveals that if (e) is true then (b) is certainly true, because the right-hand side of (b) is that of (e) plus $2a$. We say that inequality (e) is stronger than inequality (b). Hence inequality (b) can be eliminated. By adding a to both sides, inequality (e) can be written in the form

$$a + b < c + d \tag{e$'$}$$

Similarly, inequality (c) is certainly true if inequality (f) is true. Once again, the right-hand side of inequality (c) is larger by $2a$. If a is added to both sides, inequality (f) assumes the form

$$a + c < b + d \tag{f$'$}$$

Inequality (d) is certainly true if inequality (a) is true, since its left-hand side is less than that of inequality (a) by $2a$. Hence, the six inequalities are reduced to three: (a), (e′) and (f′). Addition of both sides of inequalities (a) and (e′) gives

$$2a + b + d < 2c + b + d$$

so that

$$a < c$$

Likewise, addition of both sides of inequalities (a) and (f′) gives

$$2a + c + d < 2b + c + d$$

so that

$$a < b$$

Since a has also been assumed to be less than d, it follows that a is the shortest link length. Now, whichever of the inequalities (a), (e′), and (f′) has the longest link length on the left added to a will be the strongest. That is, the left-hand side is largest and the right-hand side is smallest. Whichever one this is assumes the form

$$s + l < p + q$$

where $s = a$ is the shortest link length, l is the longest link length, and p and q are the two remaining link lengths.

It must be remembered that we assumed that a was less than d. It is also necessary to deal with the case in which a is larger than d. This can be handled by inverting the linkage so that AB becomes the base link and A^*B^* becomes the link jointed to it by the continuously rotatable joint. Pursuing the application of the triangle inequality then results in d being the shortest link length, and the Grashof inequality again results.

What we have shown so far is that the Grashof inequality is a necessary condition for the presence of a fully rotatable joint, and that joint is always at one end of the shortest link. Now, there can never be just one fully rotatable joint in a four-bar linkage. There must always be at least two. If there were just one fully rotatable joint, a topological contradiction would result when the rotation of A^*A relative to the other links after one cycle were to be considered. If that link were to perform a complete rotation about joint A^*, and joints A, B, and B^* were to oscillate back to their initial positions, A^*A would have performed a complete rotation relative to each of the other links. That is, it would have performed a complete revolution relative to AB. However, joint A has not performed a complete revolution but, rather, has performed zero net rotation. Hence there cannot be just one completely rotatable joint. Since we have shown that any completely rotatable joint must be at one end of the shortest link, it follows that there are two completely rotatable joints, and they are at either end of the shortest link. This completes the proof of Grashof’s rules.

The shortest link of a type 1 linkage performs a complete revolution in each motion cycle relative to the other members. The net rotations of the fully rotatable joints on both ends of that link cancel one another so that the net rotations of the remaining links relative to one another are zero for a complete motion cycle.

Sometimes it is not necessary for the mechanism to perform a complete motion cycle. A restricted range of driving joint motion may be adequate. In that case, linear actuators, such as hydraulic or pneumatic cylinders acting across the driving joint, may be used. However, it is still necessary that the driving joint not pass through a motion limit within the necessary range of motion. Grashof’s rules are often useful in ensuring that this does not happen.

Occasionally it is necessary to drive a crank-rocker linkage by oscillating the rocker through a part of its motion range. In this case the linkage is usually referred to as a rocker-crank.

The reasons associated with the use of type 2 double-rocker linkages, or with the use of type 1 linkages driven by rockers rather than cranks, will be better understood after a discussion of linkage synthesis. Often, a linkage that is synthesized to produce a specific motion cannot be driven through that motion without the driving joint passing through a motion limit position. In that case, a solution might be to drive the other base joint.

A special case arises when

$$s + l = p + q$$

This is called a transition linkage or Grashof neutral linkage. In this case, the linkage can assume a “flattened” configuration as shown in Figure 1.48. When passing through this position, it can change from one to the other of the two configurations in which the linkage can be assembled for a given driving crank angle. In practice, this is often undesirable because it leads to unpredictable behavior and possibly large loads on the members and joints.

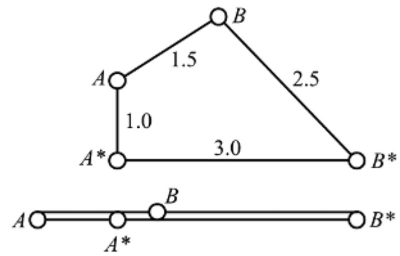


FIGURE 1.48
A Grashof neutral linkage. A characteristic of such linkages is the ability to assume the flattened position shown.

1.16 COUPLER-DRIVEN LINKAGES

In some applications, linkages are actuated not by applying a force or torque to one of the links jointed to the base but rather by applying a force or torque to the coupler, the member that has no direct connection to the base. Everyday examples are not uncommon (e.g., polycentric hinges for heavy doors or for automotive hood and trunk lids and oscillating fans).

It is still important for a coupler-driven mechanism not to pass through a motion limit within the desired motion range. The motion limit positions for a coupler drive are quite different from those for a crank drive. They are the positions in which the two rotating links become parallel, as shown in Figure 1.49. In these positions the angular motion of the coupler ceases and must reverse if motion is to continue. Elimination of these motion limits produces a linkage whose coupler performs a complete revolution relative to the base link. Because in a type 1 linkage the shortest link rotates completely relative to the remaining links, that link must be either the coupler or the base. It follows that the Grashof subtypes for which complete rotation of the coupler relative to the base is possible are the type 1 double-rocker and drag-link subtypes.

1.17 MOTION LIMITS FOR SLIDER-CRANK MECHANISMS

The limits for a slider-crank mechanism can be determined by considering the combinations of link lengths that will cause the linkage to lock up. A typical slider-crank is shown in Figure 1.50.

The limit positions of the rotating link a of Figure 1.50 are determined when the coupler link is perpendicular to the direction of slider travel. The limiting assembly position occurs for one of the four geometries shown in Figure 1.51.

From the four limit positions shown in Figure 1.51, it is apparent that the following relationships must be maintained if it is to be possible to drive the slider-crank for a full 360° rotation of the crank

$$b > a$$

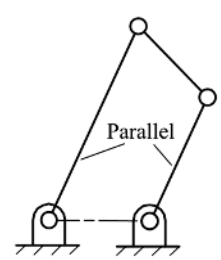


FIGURE 1.49
The motion limit for the coupler of a coupler-driven four-bar linkage.

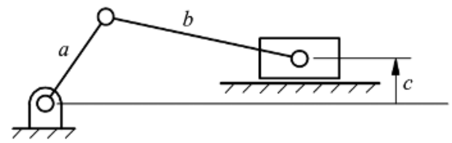


FIGURE 1.50
General slider-crank mechanism with offset dimension c.

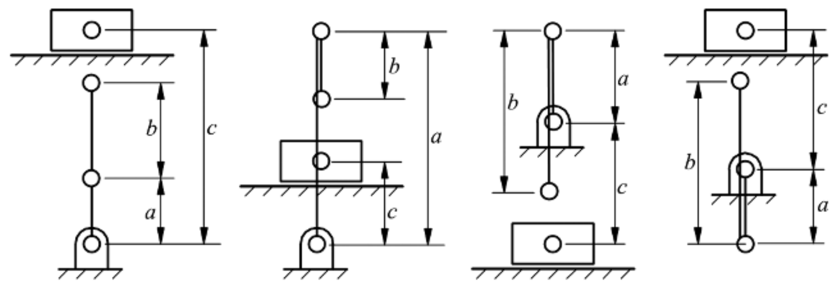


FIGURE 1.51
Positions for which the slider-crank mechanism represented in Figure 1.50 cannot be assembled.

and

$$b - a > c$$

where

a = length of the crank
b = length of the coupler
c = offset distance from crank-ground pivot to slider pin (measured positive upward)

EXAMPLE 1.6
Using Grashof's Equation

In the Watt six-bar linkage shown in Figure 1.52, the joint between links 5 and 6 must be placed on the arc indicated. Using Grashof's rule, determine the region for joint D that will allow the full rotation of link 6. The critical dimensions are

$A^*A = 1.14$ in	$AB = 2.26$ in	$A^*C = 1.74$ in	$EF = 1.2$ in	$FG = 0.57$ in
$A^*E = 2.00$ in	$CD = 2.68$ in	$b = 0.8$ in	$c = 1.09$ in	

In the position shown, link 6 is symmetric about the vertical axis through E

Solution

Consider first the slider-crank mechanism (A^*AB) even though the crank A^*A does not rotate 360°. Clearly, if the crank A^*A can rotate through 360°, it will not lock up in any intermediate position. Based on the dimensions given

$$AB > A^*A$$

and

$$AB - A*A = 1.12.$$

Therefore

$$AB - A*A > c$$

and the crank of the slider-crank mechanism can rotate a full 360°.

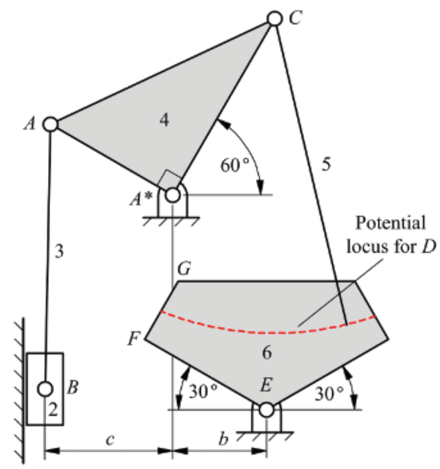


FIGURE 1.52
Mechanism for which range for point D is to be determined.

Next consider the crank-rocker mechanism (A^*CDE). For a crank rocker, link 6 must be the crank, which means that DE must be the shortest link. The longest link is CD. Therefore, based on Equation 1.6, for A^*CDE to be a crank-rocker mechanism

$$DE + CD < A{*}E + AC$$

or

$$DE + 2.68 < 2.00 + 1.74$$

or

$$DE < 1.06 \text{ in}$$

The allowable range for D is shown on the Figure 1.53.

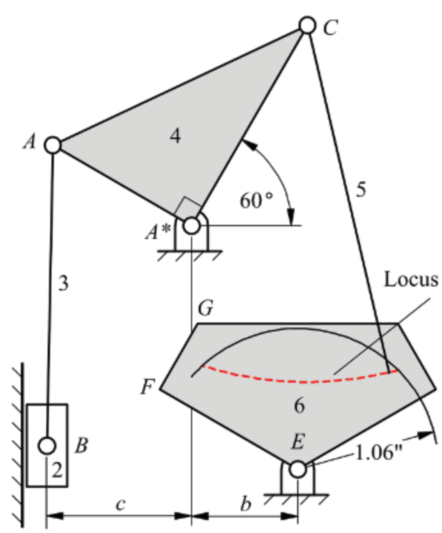


FIGURE 1.53
Allowable range for point D.

1.18 INTERFERENCE

This is a topic that is often ignored in courses and texts on mechanism design. That is unfortunate since a full-cycle motion capability can be prevented by topological interference even when Grashof's rules indicate that it is possible. An understanding of topological interference is particularly important at the present time, when linkages are often designed using CAD systems and their functioning checked by animation rather than by construction of physical models. It is very difficult to represent topological interference adequately on a planar display. For this reason, the reader is urged to construct models using cardboard and thumbtacks, or whatever other appropriate materials are available, when reading this section. That is the best way to gain an understanding of the nature of topological interference. There is also a tendency to regard interference as a result of the physical shape of the links and as something that can be avoided if enough care is given to the design of the physical link geometry. That is not what we are talking about here. Topological interference is a fundamental property of a linkage configuration in the same way that Grashof type is. It cannot be avoided by simply reshaping the links.

Topological interference really affects only the capability of executing a complete motion cycle using a rotary input. If oscillatory motion over a partial cycle is all that is required, topological interference can usually be circumvented.

The topological and physical limitation that the links cannot pass through each other creates difficulties in arranging for input and output motion transfer to and from type 1 linkages. When a simple, type 1 four-bar linkage is viewed as a three-dimensional structure with revolute joint axes of finite length, there is only one way in which it can be assembled to avoid any of the links or joint axes having to pass through each other. This is shown in Figure 1.54. The problem is the fully rotatable joints.

The oscillatory joints of a type 1 linkage never pass through positions in which their joint angles ϕ and ψ, shown in Figure 1.54, become either zero or π. If either one did so, the joint diagonally opposite it would be at a motion limit, preventing it from performing a complete rotation. Consequently, the axes of these joints never cross the lines of the links AB and A^*B^*, so there is no interference. However, when joint A^* is fully rotated, the axis of joint A must cross the line A^*B^* and, since A^*A is the shortest link, it must cross between B^* and A^*. Likewise, when joint A is fully rotated, the axis of joint A^* will cross the line AB between A and B. Viewing the linkage from a direction normal to the joint axes, it can be seen that, if joint A is on one side of the link A^*A, the link A^*B^* must be on the other side, otherwise the link will cut the joint axis. Similarly, joint A^* must be on the opposite side of A^*A to link AB. It follows that, in the direction along the link axes, AB and A^*B^* are on opposite sides of A^*A and B^*B. This may seem to be dependent on the physical realization of the links, but it is, in fact, a fundamental topological property of the loop.

The simplest situation for motion transfer is when both input and output motions are rotary. Motion can then be transferred into and out of the linkage by means of shafts attached to the turning links. Interference constrains the arrangement of the input and output shafts, as shown in Figure 1.55.

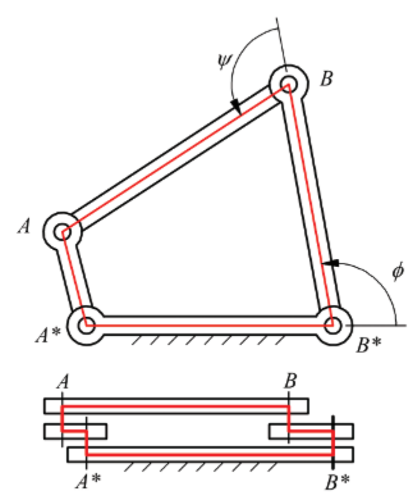


FIGURE 1.54
Assembly order of type 1 linkage needed to avoid topological interference. Notice that here the vital information is about the order in which the links are placed *along the joint axes*.

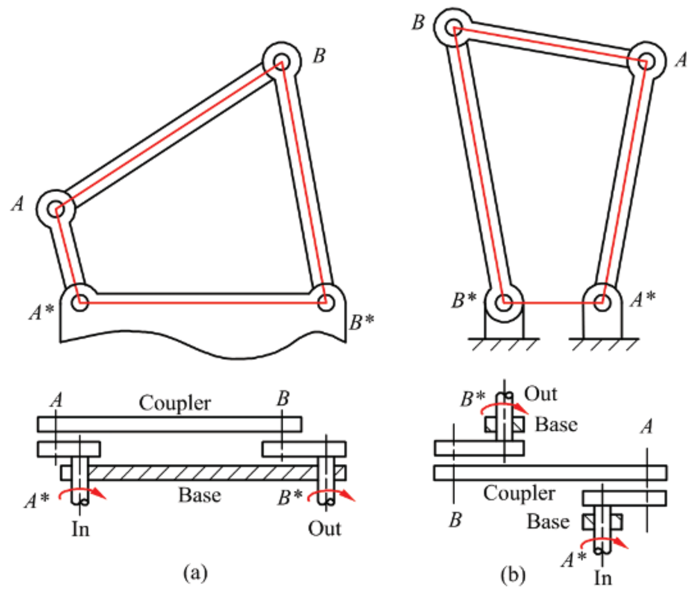


FIGURE 1.55
Drive shaft configurations needed for crank-rocker (a) and drag-link (b) linkages.

If the linkage is a *crank-rocker* both shafts must enter from the same side in order to avoid interference between the shafts and the coupler link [Figure 1.55(a)]. Notice that the shafts must pass through the base link to get to the turning links, which are on the inside of the linkage in this inversion. Physically, the shafts are supported in bearings mounted in the base link.

If the linkage is a *drag-link*, the shafts may be attached directly to the turning links, one on either side, since those links are on the outside of the linkage in this inversion. If this is done, the fixed bearings may be moved to the outside, essentially turning the base link inside out. The base link becomes a pair of fixed bearing mounts on either side of the linkage, as is shown in Figure 1.55(b). A *drag-link* linkage must always be mounted in this manner to achieve full cycle motion, regardless of the means of input or output, since otherwise the coupler must pass through the base.

The discussion of rotary input and output to *type 1 double-rocker* linkage will be left until later because it is not possible to achieve full-cycle motion with a crank drive in this type of linkage.

A more complex case is that in which the input is rotary and motion must be transferred from a point on the coupler link. This is easy enough to arrange in the *crank-rocker* case, as shown in Figure 1.56, because the base and coupler are on the outside of the linkage.

Much more difficult is the case in which motion must be transferred from a point on a crank or on the coupler of a *drag-link*. Because the coupler moves between the cranks there is no way to avoid

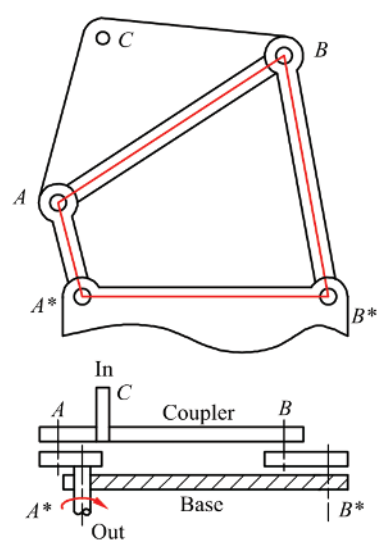


FIGURE 1.56
Configuration necessary for transfer of motion from a point on the coupler of a crank-rocker mechanism.

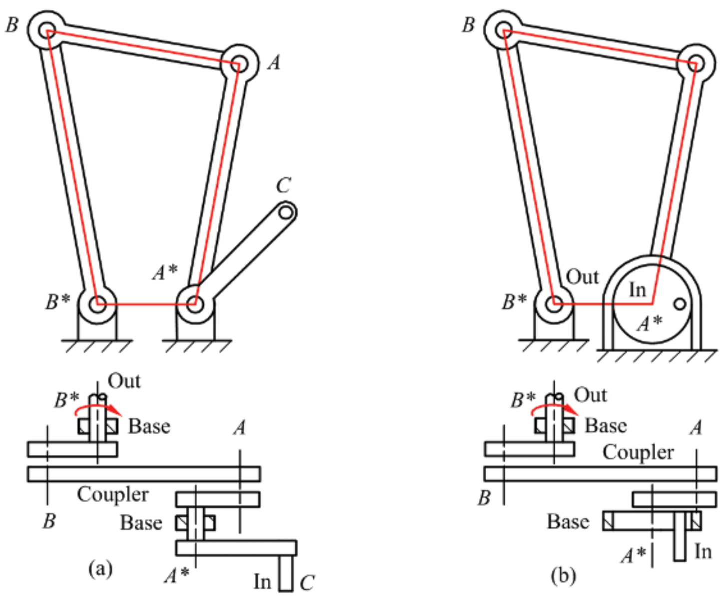


FIGURE 1.57
Alternative arrangements for motion transfer from a point on the crank of a drag-link.

interference of a shaft coming off the coupler with those cranks in full-cycle motion. Furthermore, because the two parts of the base are outside the cranks, there is also a problem of interference with the base. This latter problem also affects the transfer of motion to or from points on the cranks.

It is possible to circumvent the interference problem for motion transfer from points on a crank by “doubling” the crank. This is shown in Figure 1.57(a). If the transfer point is reasonably close to the base joint of the crank, the result in Figure 1.57(a) can be achieved by using a bearing of sufficiently large diameter to encompass the transfer point. This is shown in Figure 1.57(b). The crank is essentially duplicated on the outside of the base bearing. A shaft rigidly fixed to both the crank and the duplicate passes through the bearing forming the base joint and ensures that both move together. This effectively makes points on the crank available outside the base, where additional links can be attached at motion transfer joints. In this way, multiloop linkages such as that shown in Figure 1.58 can be built up and driven by the driving crank of the master *drag-link* loop.

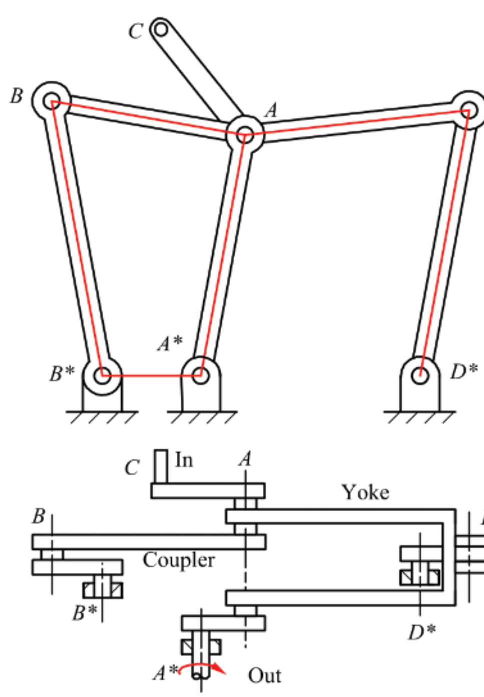


FIGURE 1.58
Six-bar modification necessary to achieve motion transfer from the coupler of a drag-link linkage.

There is no simple way to transfer motion from a point on the coupler of the *drag-link* without preventing full-cycle mobility. It can be done by splitting one of the joints between crank and coupler, allowing the linkage loop to pass through itself. This requires the addition of at least one auxiliary link, so the mechanism, strictly speaking, is no longer a four-bar. The yoke shown in Figure 1.58 carries the two bearings that replace the simple joint of the original linkage. It is undesirable to leave this link unconstrained, so it is usual to add a second link connecting it to base, as shown in the figure. This allows the coupler to be moved outside the crank. However, it is still not possible to transfer motion directly from a point on the coupler because of interference with the yoke. For this reason, the coupler is doubled in the same way that the crank was in Figure 1.57, producing the six-bar arrangement shown. As can be seen, this is quite an extensive modification.

The situation for coupler drive, in which the driving torque is applied to the coupler link and the output motion is taken off that same link, is quite similar. The two linkage types that can, in principle, perform complete motion cycles in the coupler drive mode are the *type 1 double-rocker* and *drag-link* types. Both present a problem because the base and coupler are inside the cranks in the basic loop. The *type 1 double-rocker* can be made to allow full cycle motion, without interference, by doubling the coupler.

Once again, the *drag-link* presents additional problems because of the necessity of splitting the base, resulting in the base joints being outside the cranks. Coupler-driven full-cycle motion of a simple *drag-link* is not possible because of interference. The six-bar arrangement of Figure 1.58 can be used for full-cycle motion that is identical to that of the *drag-link* with coupler drive.

All of the foregoing discussion relates only to full-cycle motion, that is, to motion in which the driving link performs a complete rotation. If oscillation through a partial motion cycle is adequate for the application, interference can always be avoided by modifying the physical shapes of the links. This is true even for the *drag-link* type. Type 2 linkages can also be used in this mode. One only has to ensure that it is not necessary for such linkages to pass through motion limit positions of the driving link when traversing the desired segment of the motion cycle.

1.19 PRACTICAL DESIGN CONSIDERATIONS

1.19.1 Revolute Joints

A rubbing contact between two members, here called a kinematic joint, is also known as a bearing. Design of bearings to perform satisfactorily for long periods under a load is a focus of the subject of tribology. Although an in-depth treatment of tribology is beyond the scope of this book, it is necessary for the mechanism designer to be aware of the limitations that may be placed on the design by the necessity of having bearings.

Revolute joints perform well under many conditions. As with all the lower pairs, the distribution of contact over a surface distributes and normally slows wear. The closed geometry of the joint provides good conditions for trapping lubricant between the joint surfaces.

A revolute joint that is in continuous, unidirectional rotation at relatively high speeds can enter a regime called hydrodynamic lubrication in which the relative movement of the bearing elements acts to entrain lubricant and maintain a separation between the solid journal elements. The entrainment action creates an area of elevated pressure in the lubricant that supports the load on the bearing. The establishment of hydrodynamic action is often assisted by pumping lubricant into the bearing. In principal, once hydrodynamic action is established, there is no contact between the solid bearing elements, and hence there is no wear. The effective friction is solely due to viscous resistance in the lubricant and is, therefore, low. Typically wear occurs only when the machine is started up and shut down. The crankshaft support bearings, and the bearings between the crankshaft and the connecting rods of an automotive engine are good examples of hydrodynamic bearings.

Another type of bearing that has some of the characteristics of a hydrodynamic bearing, but is free of some of its limitations, is a hydrostatic bearing. Here the objective remains the same, to carry the bearing load by a pressure differential in the lubricant, and maintain separation between solid bearing journals at all times. However, in this case, the pressure to support the bearing load is provided by pumping the lubricant into the bearing on the loaded side at an elevated pressure.

Hydrostatic bearings do not rely on continuous rotation to maintain bearing action. Therefore, they can be used when the rotation speeds are low, or when the direction of rotation reverses. They do tend to be expensive, due to a need for close tolerances to minimize lubricant leakage out of the bearing, and because of the need for a relatively high capacity lubricant pump. Hydrostatic bearings are usually used for the main rotor bearings on large turbo-generator sets.

When the speed of rotation is slow, or reverses, a greased bushing or a solid contact bearing may be used. These bearing types are geometrically similar, and differ only in the use of a liquid lubricant. Usually that lubricant will be a viscous grease. The high viscosity both promotes some hydrostatic action and diminishes leakage out the sides of the bearing. Frequent lubrication is, nevertheless, necessary for this type of bearing. The materials should also be chosen to provide adequate performance and wear resistance even in the absence of lubricant.

Solid contact bearings rely on the choice of contacting materials to provide both low friction and wear resistance. Teflon has a low coefficient of friction with most metals. It is also relatively hard and highly temperature resistant for a plastic material. Consequently it is frequently chosen for one element of a bearing pair. Other plastic materials such as nylon and delrin are also used. Note that the same material should never be used for both journal members of a solid bearing pair. The reason is that journals with similar materials can weld together at small asperities when driven under load, resulting in high friction and rapid wear. This is why bronze bushings are frequently partnered with steel journals for greased bearings. Generally harder materials wear better than softer ones. Hence steel is preferred to aluminum for bearing journals. One final caution is that some materials should never be lubricated with petroleum-based lubricants. Nylon tends to absorb oil and swell and fail. Solid lubricants such as graphite or molybdenum disulphide can be used when petroleum based lubricants are not an option because of material or other constraints.

Rolling element bearings provide another alternative for the support of rotary motion. Here the load is transferred between the journals by hardened steel balls or rollers trapped between the journals. The contact between one of these rolling elements and the journal is, of course, a higher-pair joint. However, the combined effect of all the balls or rollers rolling on the journals is kinematically equivalent to a revolute joint. Lubricants are used, but the way in which they work is somewhat different from that of the lubricants in other types of joints. This kind of action is called *boundary lubrication*. The lubricant is squeezed to very high pressures between the rolling element and journal and plays a role in distributing the load over both elements. The contact between a ball and journal is a point, if both are perfectly rigid, and that between a roller and journal is a line. In either case, the stress is ideally locally infinite. Of course, elastic deformation of the elements acts to distribute the load over a finite area. The boundary lubrication mechanism assists in this load distribution.

Because pure rolling contact does not involve sliding of one member over another, wear, of the type found in other bearings, is not an issue for rolling contact bearings. Also, the effective friction can be very low, and rolling element bearings work well with motion cycles that stop or reverse. However, the very high contact stresses in the elements require very hard and very accurately manufactured rolling elements and journals. Consequently, rolling element bearings can be relatively expensive. They are also relatively bulky and are not well suited to situations where space is limited. The principal failure mode of a rolling element bearing is basically fatigue due to the high contact, or Hertzian stresses, in the rolling elements and journals. This leads to subsurface cracking and eventual spalling, or breaking out of pieces from the surface of a rolling element. Once this process starts, the bearing tends to fail quite rapidly. The nature of the failure mode is such that all rolling elements have finite life. Unfortunately that life is statistically distributed over a significant range, making it relatively difficult to predict failure and apply preventive maintenance procedures to change bearings before failure occurs.

1.19.2 Prismatic Joints

As compared to revolute joints, prismatic joints are much more problematic in their application. As will be shown next, they are sensitive to the direction and manner of load application. Also, a prismatic joint cannot be infinite in length so all prismatic joints experience motion reversals which precludes the use of fully established hydrodynamic lubrication.

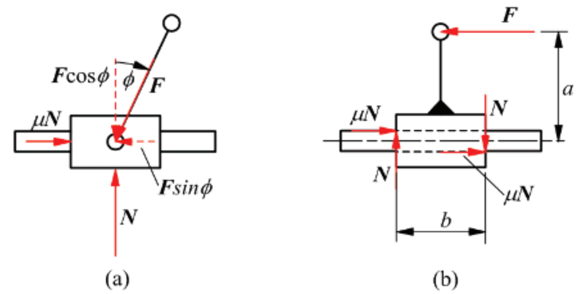


FIGURE 1.59
Jamming in sliding joints. In (a), the slide will jam if the angle between the applied force F and the direction of sliding becomes too great. The slider will jam when the angle ϕ is less than $\tan^{-1}\mu$. In (b), the slider will jam due to the offset force if $2\mu a > b$.

If a sliding joint is loaded by a connecting rod, as in a slider crank mechanism, the loading force is applied along the line through the bearing center, as shown in Figure 1.59(a). The friction force along the joint direction is proportional to the normal force. If the friction force exceeds the component of the applied force along the slide direction the joint will jam. That is, if the angle between the axis of the connecting rod and the normal to the joint direction is less than the friction angle

$$\phi = \tan^{-1}\mu$$

where μ is the coefficient of friction, the joint will jam.

Figure 1.59(b) shows another effect that may lead to jamming of the slider. Applying a load offset from the slider surfaces results in an applied moment that must be resisted by a couple composed of normal forces, as shown in the figure. In a real prismatic joint, there must be a small clearance between the members. The application of the offset force, F, causes the block to angulate slightly relative to the shaft so that contact actually occurs only at the ends of the joint. Thus, the block is subject to normal and friction forces at the locations shown in the figure. The joint will jam if

$$F \leq 2\mu N$$

However, for horizontal force and moment equilibrium

$$2\mu N = F \text{ and } aF = bN$$

Therefore, the joint will jam if

$$b < 2\mu a$$

Offset loads and loading directions at large angles to the joint direction also combine to produce jamming.

Jamming is best avoided by shunning designs that have sliding joints with poor loading geometries. If such a geometry cannot be avoided, jamming due to offset loading may be alleviated by increasing the length of the prismatic joint, if space allows. Increasing b until it is greater than any expected value of $2\mu a$ should avoid the problem. Reducing the effective coefficient of friction is effective in either case. That might be done by lubrication, or by choice of a low-friction material combination. Lubrication, as a solution, may be problematic if jamming is a catastrophic failure mode. Sooner or later, the joint is likely to have too little lubricant.

The best solution, in many cases, is to use a rolling contact joint to minimize the effective coefficient of friction. Roller on rail configurations that are kinematically equivalent to a prismatic joint are available. A ball bushing is a relatively inexpensive and compact device. It must roll on a smooth, hardened steel shaft. A ball bushing does not provide any restraint on twisting about the shaft axis. For this reason, when ball bushings are used, the bushings and shafts are usually configured in parallel pairs.

1.19.3 Higher Pairs

Pure rolling contact may not require any lubrication, or special attention, as is the case of the contact between a vehicle tire and the ground. Sliding contacts, however, can result in very rapid wear, jamming, and failure unless carefully designed and lubricated. Combined rolling and sliding, as in a gear mesh, also requires careful attention to lubrication at any but the lowest loads and speeds. Gears that carry significant loads and power flows are normally enclosed in gearboxes to allow lubricant to be actively splashed or pumped over them. The gearbox allows lubricant to run off the gears and collect in the bottom of the box, or sump, for recycling.

Cam and follower pairs are particularly demanding with respect to lubrication, particularly if flat-faced followers are used. The valve timing cams in an automotive engine are housed in a sealed chamber so they can be bathed in lubricant. Oil is often pumped through the rocker shaft to ports in the faces of the followers to ensure lubrication, and some hydrodynamic action, over the rubbing surfaces.

1.19.4 Cams versus Linkages

As will be seen, both cams and linkages are used to generate desired irregular motions. As solutions to design problems requiring irregular motions, they each have their strengths and weaknesses. Cams are usually easier to design geometrically, but much harder to make work satisfactorily. The lubrication issues involved in rubbing contact are referred to above. In low volume, cams are expensive to manufacture. However, if the volume of parts needed is high enough to justify manufacture of a die, and production of the cams by near net shape methods such as injection molding, die casting, or powder metallurgy becomes feasible, cam mechanisms can be very economical. Cams are particularly convenient for timing mechanisms, such as valve lifters. They are easily designed to dwell in a set position for a set portion of the motion cycle.

Linkages are robust and inexpensive, particularly if only revolute joints are needed. They are economical to manufacture in either large or small volumes. Lubrication is, relatively speaking, very easy. However, they do not allow as much freedom to the designer as cams. It is quite difficult to design a high quality dwell mechanism using only linkages. Also, linkages often consume more space than cam mechanisms. Given an irregular motion generation problem, most experienced machine designers will seek a linkage solution first, unless the problem is clearly better suited to a cam.

REFERENCES

1. Agrawal, Sunil K., Madangopal, Rajkiran, and Khan, Zaeem A. (2004). "Biologically Inspired Design of Small Flapping Wing Air Vehicles Using Four-Bar Mechanisms and Quasi-Steady Aerodynamics." *Journal of Mechanical Design*, pp. 1–8.
2. Grashof, Franz. (1883). *Theoretische Machinenlehre*, Vol. **2**, Leipzig: L. Voss.
3. Harrisberger, Lee. (1965). "A Number Synthesis Survey of Three-Dimensional Mechanisms." *Trans. ASME, J. of Eng. for Ind.*, May, pp. 213–220.
4. Hartenberg, R. S., and Denavit, J. (1964). *Kinematic Synthesis of Linkages*, New York, NY: McGraw-Hill.
5. Hunt, K. H. (1978). *Kinematic Geometry of Mechanisms*. Oxford, UK: Oxford University Press, p. 40.
6. LEGO Systems, Inc., 555 Taylor Road, Enfield, CT.
7. McGhee, R. B., and Waldron, K. J. (1985). "The Adaptive Suspension Vehicle Project: A Case Study in the Development of an Advanced Concept for Land Locomotion." *Unmanned Systems*, Vol. **4**, No. 1, Summer, pp. 34–43.
8. Reuleaux, Franz. (1963). *The Kinematics of Machinery* (Transl. and ed. by Alexander B. W. Kennedy). New York, NY: Dover Publications, Inc.
9. Richter, Irma A. (1952). *Leonardo Da Vinci Notebooks*. New York, NY: Oxford University Press, Inc.
10. U.S. War Department. (1944). Bombardiers' Information File, AAF Form 24B, Washington, DC.

PROBLEMS

Linkage Structure

1.1 Find a mechanism as an isolated device or in a machine and make a realistic sketch of the mechanism. Then make a freehand sketch of the kinematic schematics for the mechanism chosen.

1.2 Cabinet hinges use various types of linkages for the folding mechanism. Identify three types of cabinet hinges and make a freehand sketch of the kinematic mechanism used.

1.3 The drawings shown in Figure P1.3 are pictorial representations of real mechanisms that are commonly encountered. Make a freehand sketch of the kinematic schematic representation of each mechanism.

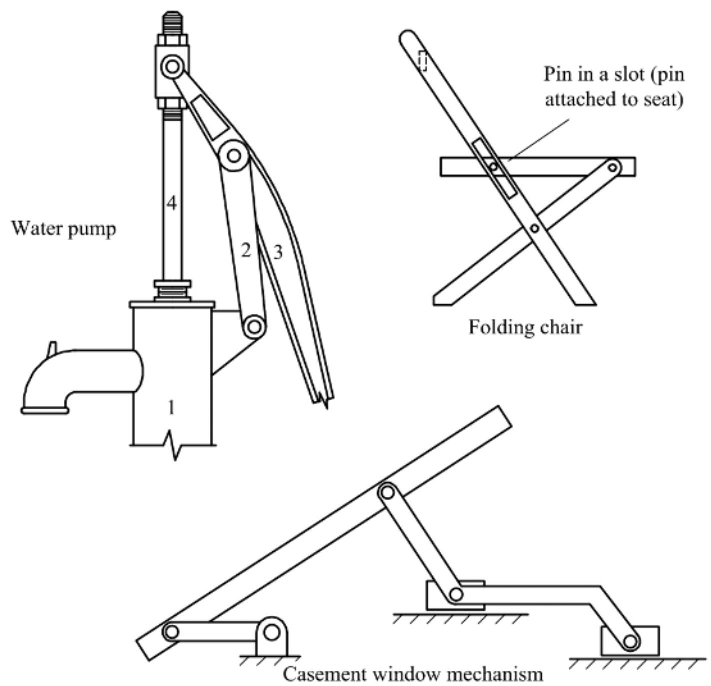


FIGURE P1.3
Linkages for Problem 1.3.

1.4 Linkages are often used to guide devices such as computer keyboards in and out of cabinets. Find three such devices, and make a freehand sketch of the kinematic mechanisms used for the devices.

1.5 Four-bar linkages are used in common devices around the home and businesses. Locate six such devices and make a freehand sketch of the device and describe its function.

1.6 Figure P1.6 is an elliptical trainer machine. The mechanism is a planar linkage. There are linkages on both sides of the machine. The linkage on the right is a mirror image of the one on the left and the linkages are connected together so that they are always 180° out of phase with each other. For the left side linkage identify the moving joints and links. There is a handle that rotates about a fixed pivot. There is also a foot pedal that floats in that it is not connected to the frame of the machine. Sketch the topology of the linkage. How many links and joints are there? How many binary links? How many ternary links? How many four-bar loops can you identify? Which linkage topology in Figure 1.23 or 1.24 does the topology match? Identify any joints that perform complete rotations as the mechanism is cycled.

FIGURE P1.6
Linkage for Problem 1.6.

1.7 Figure P1.7 shows another type of elliptical trainer machine. The mechanism is a planar linkage that includes a slider joint. There are linkages on both sides of the machine. The linkage on the right is a mirror image of the one on the left and the linkages are connected together so that they are always 180° out of phase with each other. For the left side linkage identify the moving joints and links. There is a handle that rotates about a fixed pivot. There is also a foot pedal that floats in that it is not connected to the frame of the machine. Sketch the topology of the linkage. How many links and joints are there? How many binary links? How many ternary links? Identify any joints that perform complete rotations as the mechanism is cycled.

FIGURE P1.7 Linkage for Problem 1.7.

1.8 Figure P1.8 shows a bicycle suspension linkage. If the shock absorber is considered, the linkage can be represented as a six bar mechanism. Draw the back suspension linkage and identify the chain in Figure 1.23 to which the topography corresponds.

FIGURE P1.8
Bicycle for Problem 1.8.

1.9 A small excavator is shown in Figure P1.9. The machine has a swing linkage but the main mechanism is planar. Draw the planar excavation linkage. Treat each hydraulic cylinder as a slider in a tube.

FIGURE P1.9
Excavator linkage for Problem 1.9.

1.10 Since the mid-1940s, modern tractors used for farming, construction, and landscape work have used a three-point hitch to attach implements to the rear of the tractor. This allows the operator to control the height and orientation of the implement using the tractor hydraulic system. A three-point hitch is shown in Figure P1.10 along with a schematic of the linkage. The kinematics of the system can be studied using a planar schematic as shown. In the schematic, link 2 is rotated using a hydraulic motor, and the implement being controlled is link 6. Characterize the linkage as a Watt's or Stephenson's six-bar linkage.

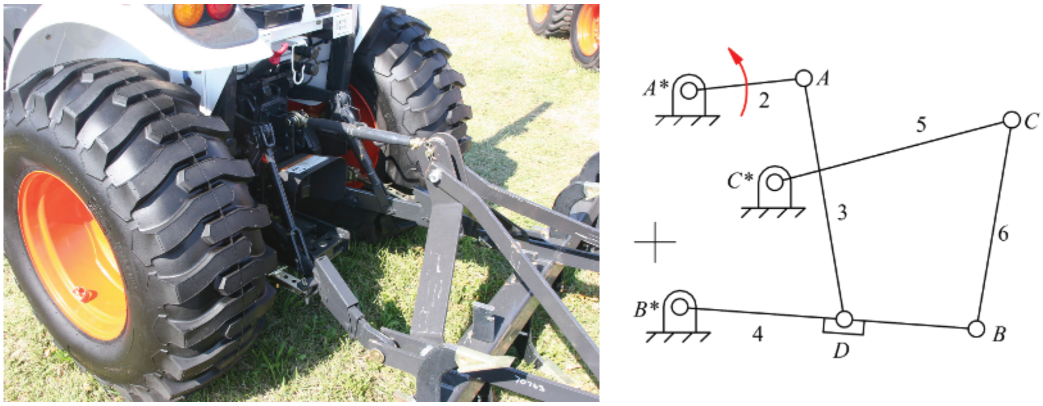


FIGURE P1.10
Tractor three-point hitch linkage for Problem 1.10.

Mechanism Mobility for Planar Mechanisms

1.11 Calculate the mobility, or number of degrees of freedom, of each of the mechanisms in Problem 1.3.

1.12 What is the number of members, number of joints, and mobility of each of the planar linkages shown in Figure 1.12.

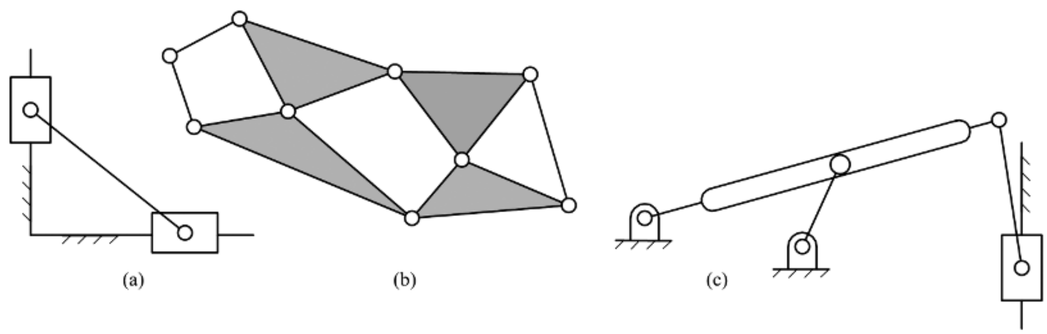


FIGURE P1.12
Linkages for Problem 1.12.

1.13 What are the number of members, number of joints, and mobility of each of the planar linkages shown in Figure P1.13?

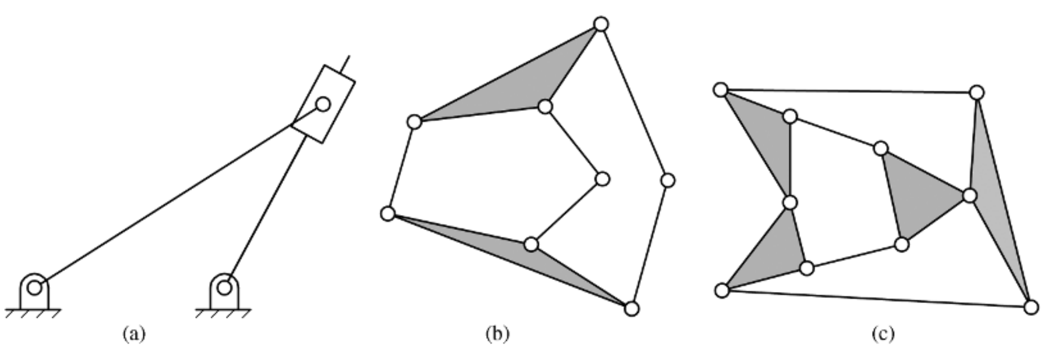


FIGURE P1.13
Linkages for Problem 1.13.

1.14 Determine the mobility and the number of idle degrees of freedom of each of the planar linkages shown in Figure P1.14. Show the equations used and identify the input and output links assumed when determining your answers.

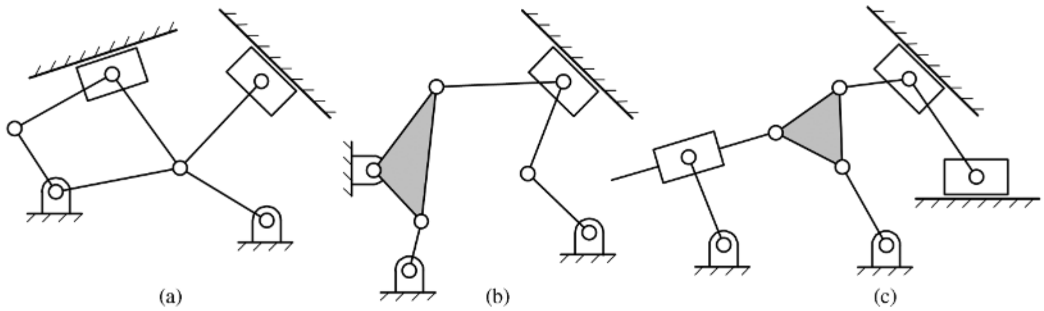


FIGURE P1.14
Linkages for Problem 1.14.

1.15 Determine the mobility and the number of idle degrees of freedom of the linkages in Figure P1.15. Show the equations used and identify any assumptions made when determining your answers.

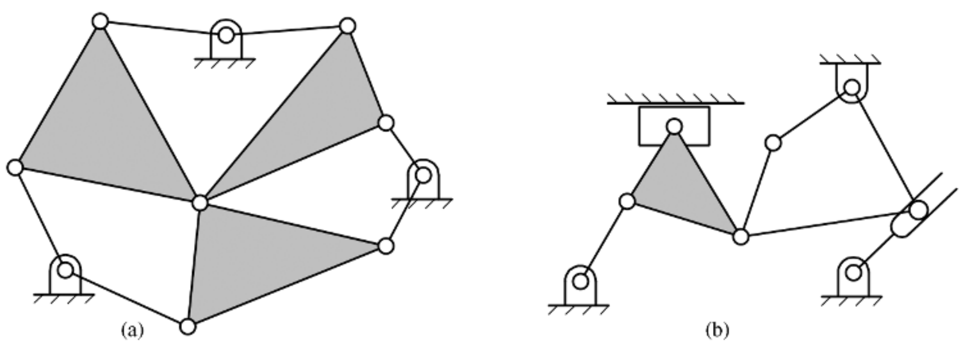


FIGURE P1.15
Linkages for Problem 1.15.

1.16 Determine the mobility and the number of idle degrees of freedom associated with the mechanism in Figure P1.16. Show the equations used and identify any assumptions made when determining your answers.

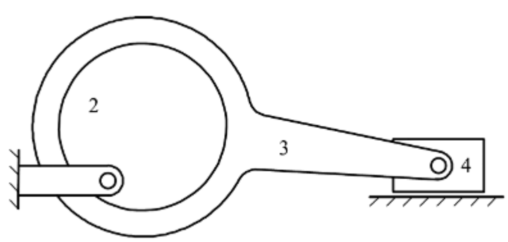


FIGURE P1.16 Linkage for Problem 1.16.

1.17 Determine the mobility of each of the planar linkages shown in Figure P1.17. Show the equations used to determine your answers.

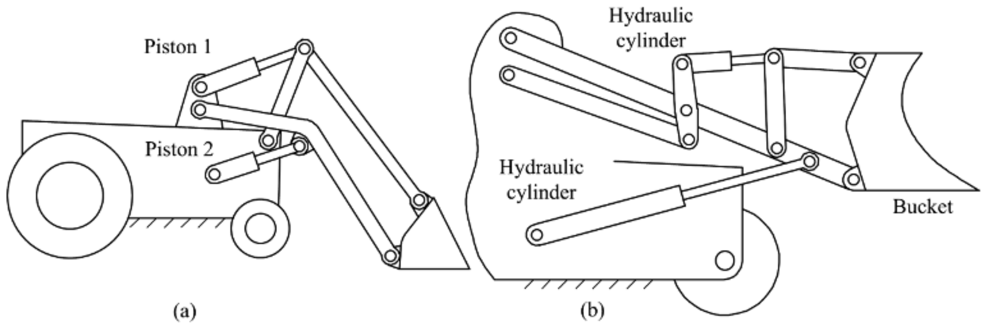


FIGURE P1.17
Linkages for Problem 1.17.

1.18 Determine the mobility and the number of idle degrees of freedom of each of the planar linkages shown in Figure P1.18. Show the equations used and identify any assumptions made when determining your answers.

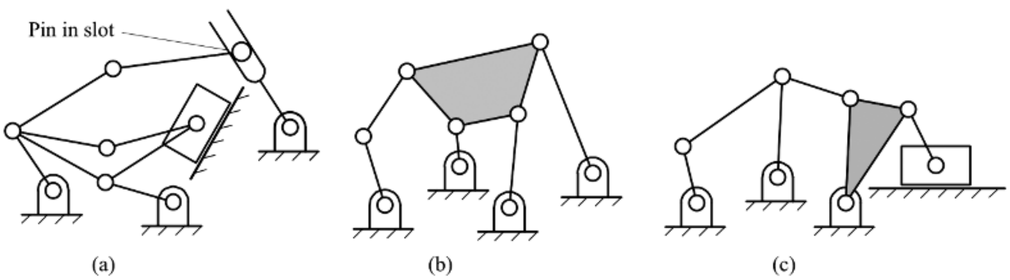


FIGURE P1.18
Linkages for Problem 1.18.

1.19 Determine the mobility and the number of idle degrees of freedom of each of the planar linkages shown in Figure P1.19. Show the equations used to determine your answers.

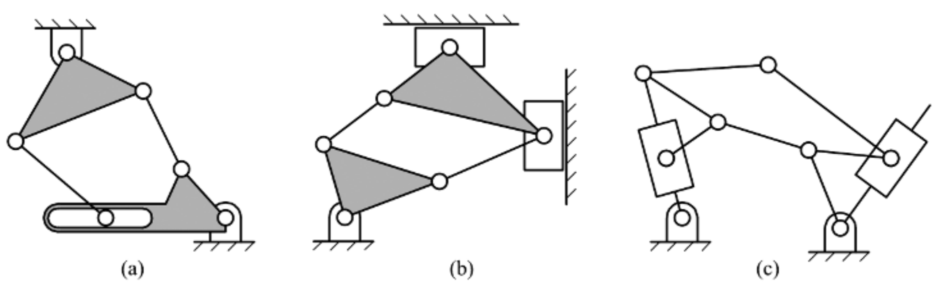


FIGURE P1.19
Linkages for Problem 1.19.

1.20 Determine the mobility and the number of idle degrees of freedom of each of the planar linkages shown in Figure P1.20. Show the equations used and identify any assumptions made when determining your answers.

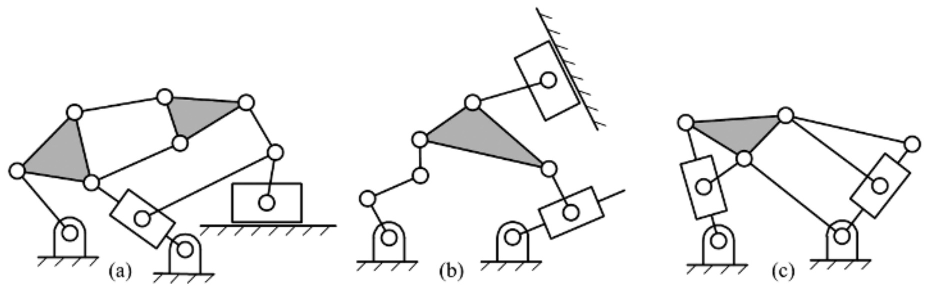


FIGURE P1.20
Linkages for Problem 1.20.

1.21 If position information is available for all points in the planar linkage shown in Figure P1.21, can all of the velocities be determined uniquely if the value of ω is given? Explain your answer.

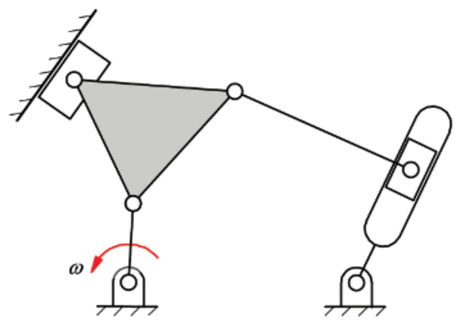


FIGURE P1.21 Linkage for Problem 1.21.

1.22 Determine the mobility and the number of idle degrees of freedom associated with each mechanism in Figure P1.22. Show the equations used and identify any assumptions made when determining your answers.

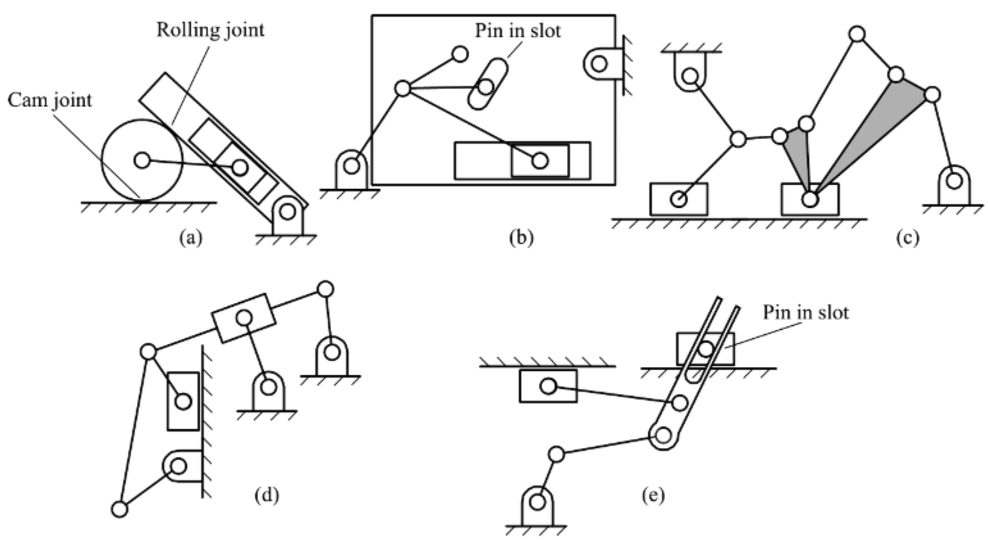


FIGURE P1.22
Linkages for Problem 1.22.

1.23 Determine the mobility and the number of idle degrees of freedom for each of the mechanisms shown in Figure P1.23. Show the equations used and identify any assumptions made when determining your answers.

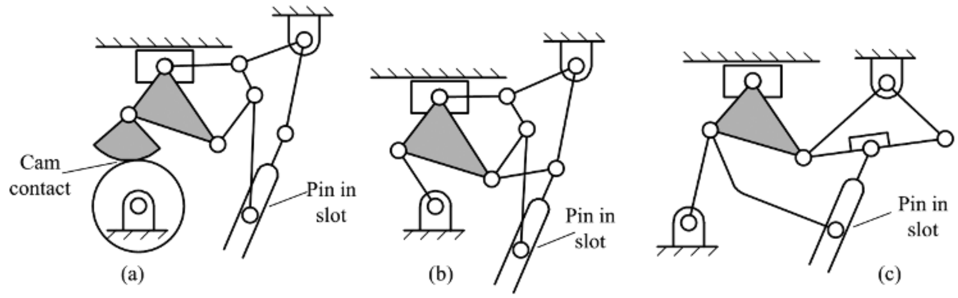


FIGURE P1.23
Linkages for Problem 1.23.

1.24[1] Determine the mobility and the number of idle degrees of freedom associated with the mechanism shown in Figure P1.24. The mechanism is a side-dumping car that consists of body 2 and truck 3 connected together by two six-bar linkages, *ABCDEF* and *AGHKLMN*. Link *NM* is designed as a latch on its free end (see left drawing). When jack 1 is operated, body 3 is lifted to the dumping position shown in the right-hand drawing. Simultaneously, the six-bar linkage *AGHKLMN* opens the latch on link *NM* and raises link *GH*. Linkage *ABCDEF* swings open side *BC* and the load can be dumped at some distance from the car (see right-hand drawing). Show the equations used to determine your answers.

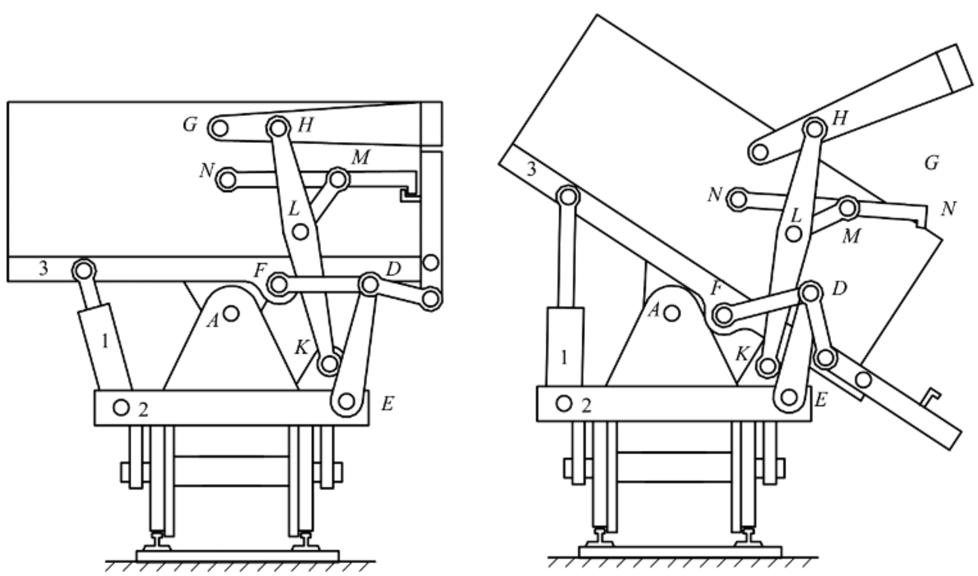


FIGURE P1.24
Linkage for Problem 1.24.

[1] Problem courtesy of Joseph Davidson, Arizona State University.

1.25 Determine the mobility and the number of idle degrees of freedom associated with the mechanism in Figure P1.25. The round part rolls without slipping on the pieces in contact with it.

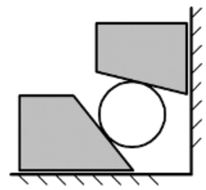


FIGURE P1.25 Linkage for Problem 1.25.

1.26 Determine the mobility and the number of idle degrees of freedom for each of the mechanisms shown in Figure P1.26. Show the equations used and identify any assumptions made when determining your answers.

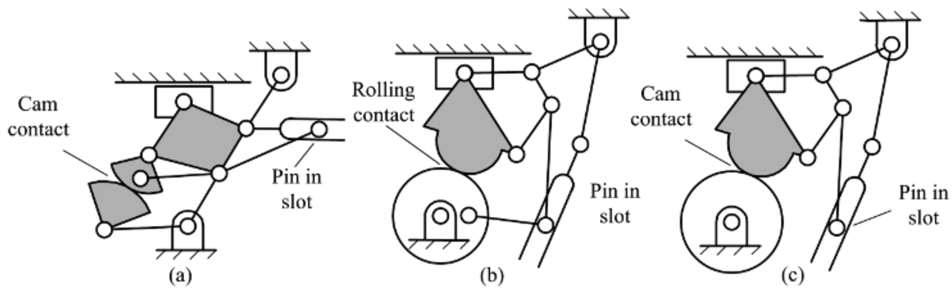


FIGURE P1.26
Linkages for Problem 1.26.

1.27[2] Determine the mobility and the number of idle degrees of freedom associated with the mechanism in Figure P1.27. The figure is a schematic of the entire linkage for a large power shovel used in strip mining. It can cut into a bank 20 m high and can dump to a height of 14.5 m. Link 7 is connected to link 8 with a revolute joint. Show the equations used and identify any assumptions made when determining your answers.

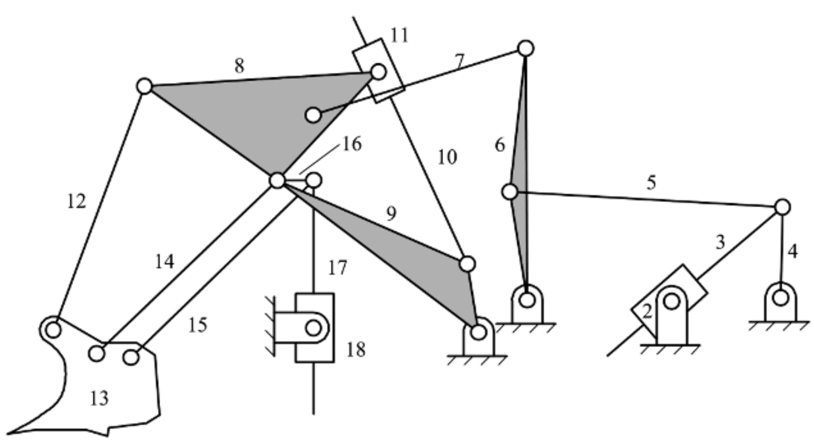


FIGURE P1.27
Linkage for Problem 1.27.

[2] Problem courtesy of Joseph Davidson, Arizona State University

1.28 In Figure P1.28 is a portion of the support mechanism for the dipper on a large earth-moving machine used in removing overburden in strip mining operations. The fixed centers for the portion of the mechanism shown really move, but useful information can be obtained by observing the dipper motion relative to the "frame" as shown in the sketch. Both links 4 and 5 are mounted at B^*. Links 4 and 6 are parallel and of equal length. The dipper is moved by a hydraulic cylinder-driving crank 5 about its fixed axis. Determine the mobility of the mechanism.

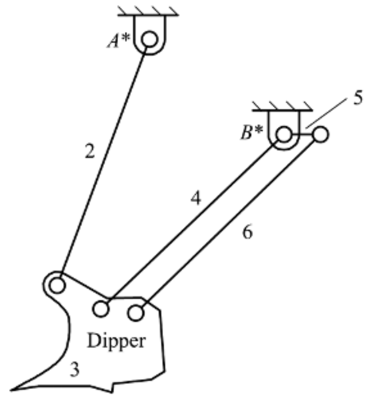


FIGURE P1.28 Linkage for Problem 1.28.

Mechanism Mobility for Spatial Mechanisms

1.29 What are the number of members, number of joints, mobility, and the number of idle degrees of freedom of each of the spatial linkages shown in Figure P1.29? For the idle degrees of freedom, identify the input and output links assumed.

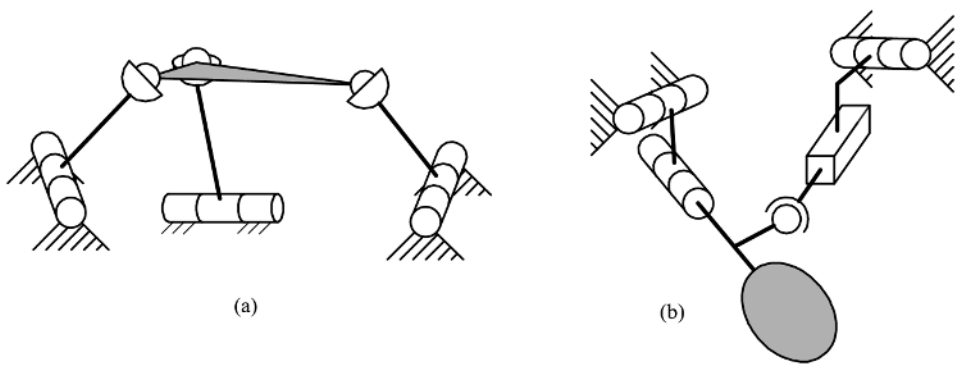


FIGURE P1.29
Linkages for Problem 1.29.

1.30 Determine the mobility and the number of idle degrees of freedom of the spatial linkages in Figure P1.30. Show the equations used to determine your answers. For the idle degrees of freedom, identify the input and output links assumed.

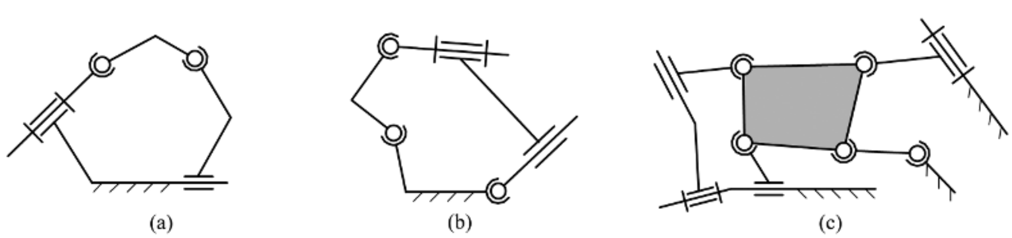


FIGURE P1.30
Linkages for Problem 1.30.

1.31 Determine the mobility and the number of idle degrees of freedom of the spatial linkages shown in Figure P1.31. Show the equations used to determine your answers. For the idle degrees of freedom, identify the input and output links assumed.

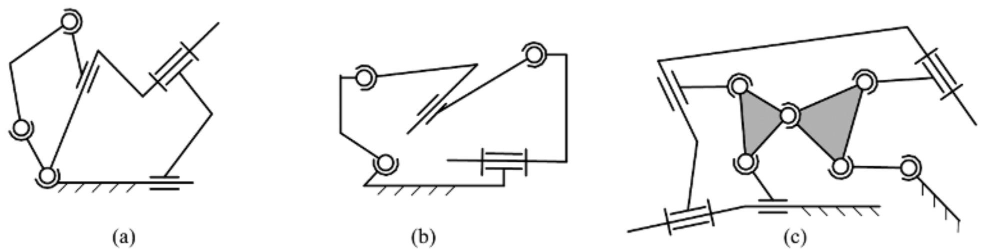


FIGURE P1.31
Linkages for Problem 1.31.

1.32 Determine the mobility and the number of idle degrees of freedom for each of the mechanisms shown in Figure P1.32. Show the equations used to determine your answers. For the idle degrees of freedom, identify the input and output links assumed.

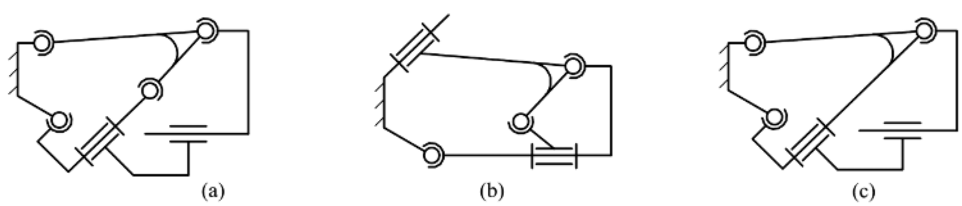


FIGURE P1.32
Linkages for Problem 1.32.

1.33 Determine the mobility and the number of idle degrees of freedom for each of the mechanisms shown in Figure P1.33. Show the equations used to determine your answers. For the idle degrees of freedom, identify the input and output links assumed.

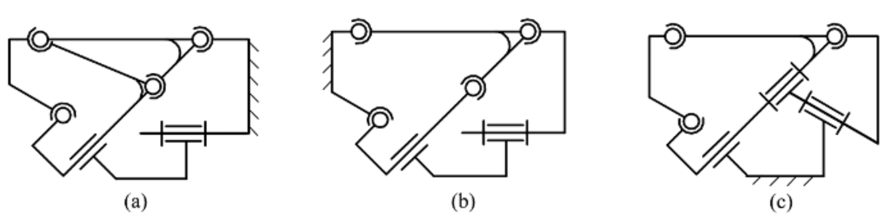


FIGURE P1.33
Linkages for Problem 1.33.

1.34[3] Determine the mobility and the number of idle degrees of freedom associated with each mechanism in Figure P1.34. Show the equations used to determine your answers.

[3] Problem based on [3].

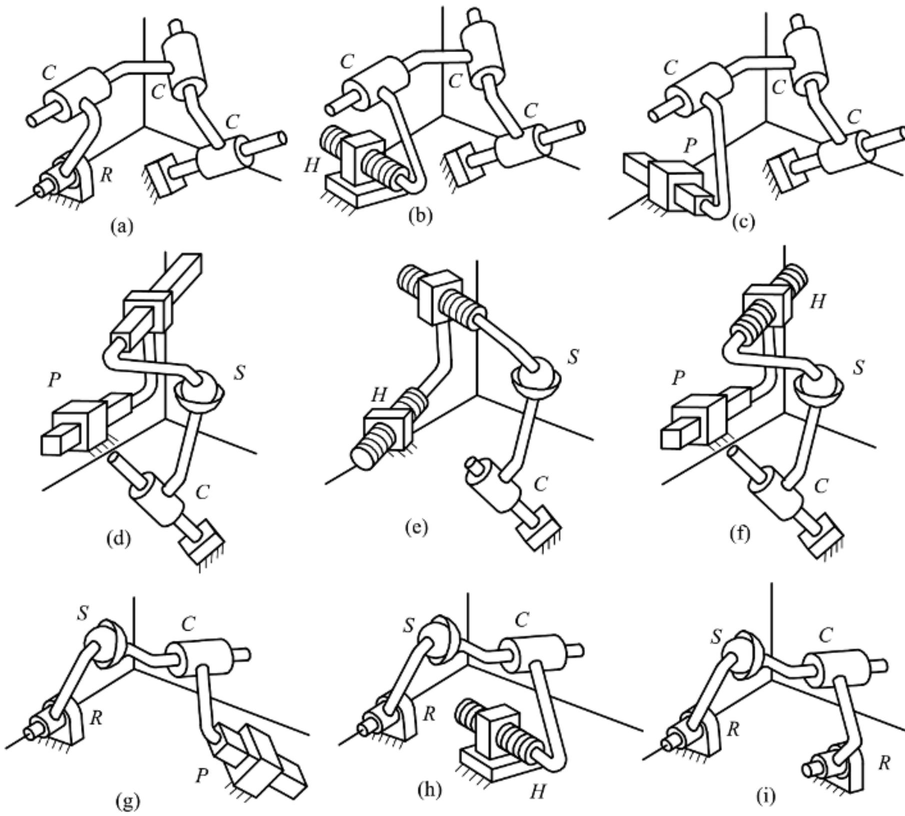


FIGURE P1.34
Linkages for Problem 1.34.

1.35 In the spatial linkages shown in Figure P1.35, (a) through (d) are all known to have mobility 1. The joints are revolutes in all cases except for one spherical joint in each of (c) and (d). In each case, determine if the linkage is properly constrained or overconstrained. Justify your answers. If the linkage is overconstrained, what geometrical specializations can you see that might result in mobility.

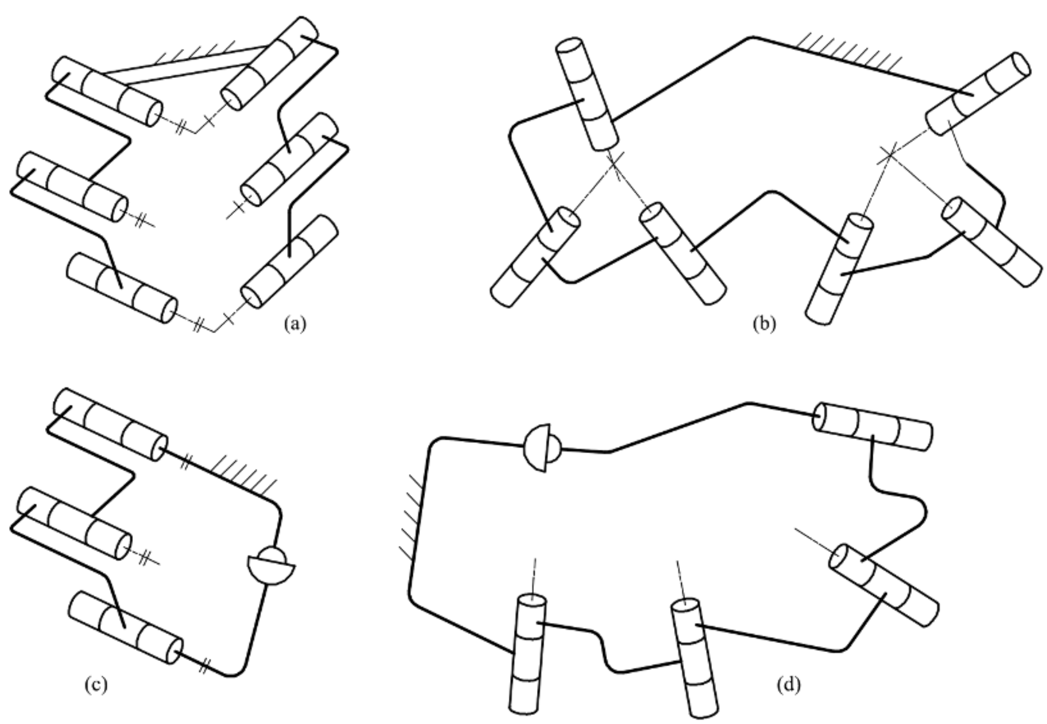


FIGURE P1.35
Linkages for Problem 1.35.

Four-Bar Linkage Type (Grashof's Equation)

1.36 Determine which (if either) of the following linkages can be driven by a constant-velocity motor. For the linkage(s) that can be driven by the motor, indicate the driver link.

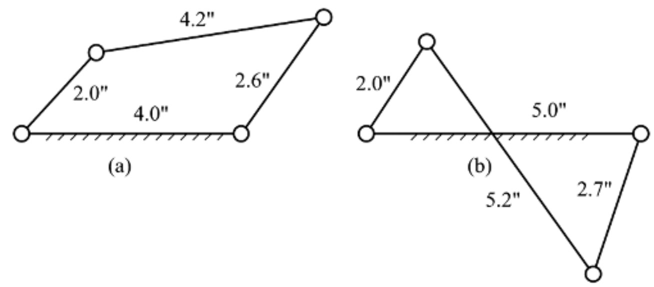


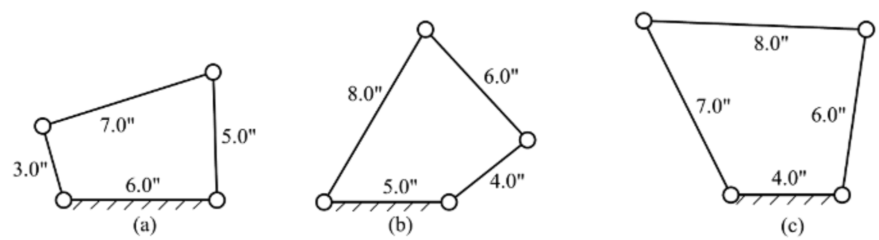
FIGURE P1.36
Linkages for Problem 1.36.

1.37 Assume that you have a set of links of the following lengths: 2 in, 4 in, 5 in, 6 in, 9 in. Design a four-bar linkage that can be driven with a continuously rotating electric motor. Justify your answer with appropriate equations, and make a scaled drawing of the linkage. Label the crank, frame, coupler, and rocker (follower).

1.38 Assume that you have a set of links of the following lengths: 20 mm, 30 mm, 45 mm, 56 mm, 73 mm. Design a four-bar linkage that can be driven with a continuous-rotation electric motor. Justify your answer with appropriate equations, and make a freehand sketch (labeled) of the resulting linkage. Label the crank, frame, coupler, and rocker (follower).

1.39 For the four-bar linkages in Figure P1.39, indicate whether they are Grashof type 1 or 2 and whether they are crank-rocker, double-crank, or double-rocker mechanisms.

FIGURE P1.39
Linkages for Problem 1.39.



1.40 You are given a set of three links with lengths 2.4 in, 7.2 in, and 3.4 in. Select the length of a fourth link and assemble a linkage that can be driven by a continuously rotating motor. Is your linkage a Grashof type 1 or Grashof type 2 linkage? (Show your work.) Is it a crank-rocker, double-rocker, or double-crank linkage? Why?

1.41 You have available a set of eight links from which you are to design a four-bar linkage. Choose the links such that the linkage can be driven by a continuous-rotation motor. Sketch the linkage and identify the type of four-bar mechanism resulting.

$$L_1 = 2'',\ L_2 = 3'',\ L_3 = 4'',\ L_4 = 6'',\ L_5 = 7'',\ L_6 = 9.5'',\ L_7 = 13'',\ \text{and}\ L_8 = 9''$$

1.42 Determine the number of fully rotating cranks in the planar mechanisms shown in Figure P1.42. Show your calculations.

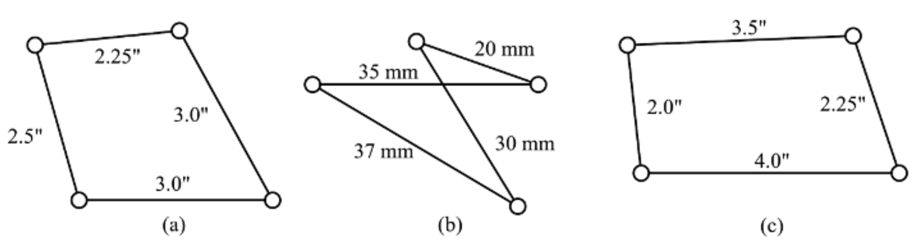


FIGURE P1.42
Linkages for Problem 1.42.

1.43 If the link lengths of a four-bar linkage are $L_1 = 1$ mm, $L_2 = 3$ mm, $L_3 = 4$ mm, and $L_4 = 5$ mm and link 1 is fixed, what type of four-bar linkage is it? Also, is the linkage a Grashof type 1 or 2 linkage? Answer the same questions if $L_1 = 2$ mm.

1.44 You are given two sets of links. Select four links from each set such that the coupler can rotate fully with respect to the others. Sketch the linkage and identify the type of four-bar mechanism.

a) $L_1 = 5''$, $L_2 = 8''$, $L_3 = 15''$, $L_4 = 19''$, and $L_5 = 28''$

b) $L_1 = 5''$, $L_2 = 2''$, $L_3 = 4''$, $L_4 = 3.5''$, and $L_5 = 2.5''$

1.45 The mechanisms shown in Figure P1.45 are drawn to scale.

a) Sketch kinematic schematics showing the relationships between the members and joints.

b) Determine the Grashof type of each four-bar linkage in each mechanism.

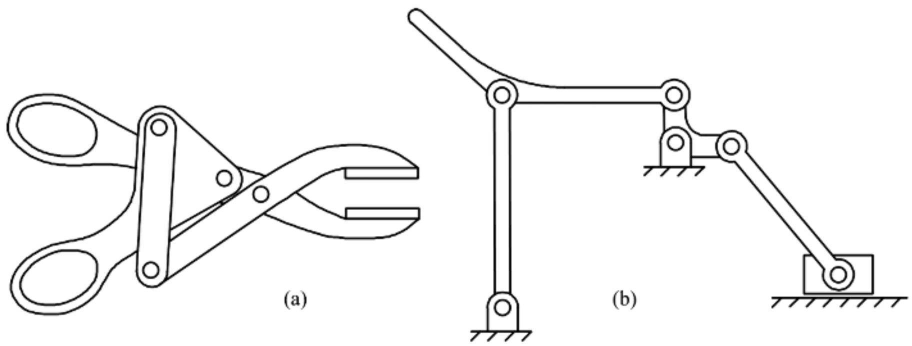


FIGURE P1.45
Linkages for Problem 1.45.