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INTRODUCTION TO SPREADSHEET MODELS FOR OPTIMIZATION

This is a book about optimization with an emphasis on building models and using spreadsheets. Each facet of this theme—models, spreadsheets, and optimization—has a role in defining the emphasis of our coverage.

A *model* is a simplified representation of a situation or problem. Models attempt to capture the essential features of a complicated situation so that it can be studied and understood more completely. In the worlds of business, engineering, and science, models aim to improve our understanding of practical situations. Models can be built with tangible materials, or words, or mathematical symbols and expressions. A *mathematical model* is a model that is constructed—and also analyzed—using mathematics. In this book, we focus on mathematical models. Moreover, we work with *decision models*, or models that contain representations of decisions. The term also refers to models that support decision-making activities.

A *spreadsheet* is a row-and-column layout of text, numerical data, and logical information. The spreadsheet version of a model contains the model's elements, linked together by specific logical information. Electronic spreadsheets, like those built using Microsoft® Office Excel®, have become familiar tools in the business, engineering, and scientific worlds. Spreadsheets are relatively easy to understand, and people often rely on spreadsheets to communicate their analyses. In this book, we focus on the use of spreadsheets to represent and analyze mathematical models.

This text is written for an audience that already has some familiarity with Excel. Our coverage assumes a level of facility with Excel comparable to a beginner's level. Someone who has used other people's spreadsheets and built simple spreadsheets for some purpose—either personal or organizational—has probably developed this skill level. Box 1.1 describes the Excel skill level assumed. Readers without this level of background are encouraged to first work through some introductory materials, such as the books by McFedries (1) and Walkenbach (2).

Optimization Modeling with Spreadsheets, Third Edition. Kenneth R. Baker.
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BOX 1.1 *Excel Skills Assumed as Background for This Book*

Navigating in workbooks, worksheets, and windows
Using the cursor to select cells, rows, columns, and noncontiguous cell ranges
Entering text and data; copying and pasting; filling down or across
Formatting cells (number display, alignment, font, border, and protection)
Editing cells (using the formula bar and cell edit capability [F2])
Entering formulas and using the function wizard
Using relative and absolute addresses
Using range names
Creating charts and graphs

Optimization is the process of finding the best values of the variables for a particular criterion or, in our context, the best decisions for a particular measure of performance. The elements of an optimization problem are a set of decisions, a criterion, and perhaps a set of required conditions, or *constraints*, that the decisions must satisfy. These elements lend themselves to description in a mathematical model. The term optimization sometimes refers specifically to a procedure that is implemented by software. However, in this book, we expand that perspective to include the model-building process as well as the process of finding the best decisions.

Not all mathematical models are optimization models. Some models merely describe the logical relationship between inputs and outputs. Optimization models are a special kind of model in which the purpose is to find the best value of a particular output measure and the choices that produce it. Optimization problems abound in the real world, and if we're at all ambitious or curious, we often find ourselves seeking solutions to those problems. Business firms are very interested in optimization because making good decisions helps a firm run efficiently, perform profitably, and compete effectively. In this book, we focus on optimization problems expressed in the form of spreadsheet models and solved using a spreadsheet-based approach.

1.1 ELEMENTS OF A MODEL

To restate our premise, we are interested in mathematical models. Specifically, we are interested in two forms—algebraic and spreadsheet models. In the former, we use algebraic notation to represent elements and relationships, and in the latter, we use spreadsheet entries and structure. For example, in an algebraic statement, we might use the variable x to represent a quantitative decision, and we might use some function $f(x)$ to represent the measure of performance that results from choosing decision x. Then, we might adopt the letter z to represent a criterion for decision making and construct the equation $z = f(x)$ to guide the choice of a decision. Algebra is the basic language of analysis largely because it is precise and compact.

As an introductory modeling example, let's consider the price decision in the scenario of Example 1.1.

EXAMPLE 1.1 *Price, Demand, and Profit*

Our firm's production department has carried out a cost accounting study and found that the unit cost for one of its main products is $40. Meanwhile, the marketing department has estimated the relationship between price and sales volume (the so-called demand curve for the product) as follows:

$$y = 800 - 5x \tag{1.1}$$

where y represents quarterly demand and x represents the selling price per unit. We wish to determine a selling price for this product, given the information available. ■

In Example 1.1, the decision is the unit price, and the consequence of that decision is the level of demand. The demand curve in Equation 1.1 expresses the relationship of demand and price in algebraic terms. Another equation expresses the calculation of profit contribution, by multiplying the demand y by the unit profit contribution $(x-40)$ on each item

$$z = (x-40)y \tag{1.2}$$

where z represents our product's quarterly profit contribution.

We can substitute Equation 1.1 into 1.2 if we want to write z algebraically as a function of x alone. As a result, we can express the profit contribution as

$$z = 1{,}000x - 5x^2 - 32{,}000 \tag{1.3}$$

This step embodies the algebraic principle that simplification is always desirable. Here, simplification reduces the number of variables in the expression for profit contribution. Simplification, however, is not necessarily a virtue when we use a spreadsheet model.

Example 1.1 has some important features. First, our model contains three numerical inputs: 40 (the unit cost), −5 (the marginal effect of price on demand), and 800 (the maximum demand). Numerical inputs such as these are called *parameters*. In some models, parameters correspond to raw data, but in many cases, parameters are summaries drawn from a more primitive data set. They may also be estimates made by a knowledgeable party, forecasts derived from statistical analyses, or predictions chosen to reflect a future scenario.

Our model also contains a decision—an unknown quantity yet to be determined. In traditional algebraic formulations, unknowns are represented as variables. Quantitative representations of decisions are therefore called *decision variables*. The decision variable in our model is the unit price x.

Our model contains the equation that relates demand to price. We can think of this relationship as part of the model's logic, prescribing a necessary relationship between two variables—price and demand. Thus, in our model, the only admissible values of x and y are those that satisfy Equation 1.1.

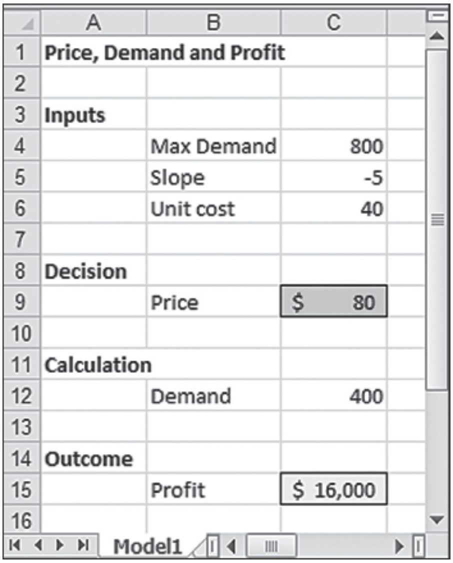
Finally, our model contains a calculation of quarterly profit contribution, which is the performance measure of interest and a quantity that we wish to maximize. This output variable measures the consequence of selecting any particular price decision in the model. In optimization models, we are concerned with maximizing or minimizing some measure of performance, expressed as a mathematical function, and we refer to it as the *objective function*, or simply the *objective*.

1.2 SPREADSHEET MODELS

Algebra is an established language that works well for describing problems, but not always for obtaining solutions. Algebraic solutions tend to occur in formulas, not numbers, but numbers most often represent decisions in the practical world. By contrast, spreadsheets represent a practical language—one that works very effectively with numbers. Like algebraic models, spreadsheets can be precise and compact, but there are also complications that are unique to spreadsheets. For example, there is a difference between form and content in a spreadsheet. Two spreadsheets may look the same in terms of the numbers displayed on a computer screen, but the underlying formulas in corresponding cells could differ. Because the information behind the display can be different even when two spreadsheets have the same on-screen appearance, we can't always determine the logical content from the form of the display. Another complication is the lack of a single, well-accepted way to build a spreadsheet representation of a given model. In an optimization model, we want to represent decision variables, an objective function, and constraints. However, that still leaves a lot of flexibility in choosing how to incorporate the logic of a particular model into a spreadsheet. Such flexibility would ordinarily be advantageous if the only use of a spreadsheet were to help individuals solve problems. But spreadsheets are perhaps even more important as vehicles for communication. When we use spreadsheets in that role, flexibility can sometimes lead to confusion and disrupt the intended communication.

We will try to mitigate these complications with some design guidelines. For example, it is helpful to create separate modules in the spreadsheet for decision variables, objective function, and constraints. To the extent that we follow such guidelines, we may lose some flexibility in building a spreadsheet model. Moving the design process toward standardization will, however, make the content of a spreadsheet more understandable from its form, so differences between form and content become less problematic.

With optimization, a spreadsheet model contains the analysis that ultimately provides decision support. For this reason, the spreadsheet model should be intelligible to its users, not just to its developer. On some occasions, a spreadsheet might come into routine use in an organization, even when the developer moves on. New analysts may inherit the responsibilities associated with the model, so it is vital that they, too, understand how the spreadsheet works. For that matter, the decision maker may also move on. For the organization to retain the learning that has taken place, successive decision makers must also understand the spreadsheet. In yet another scenario, the analyst develops a model for one-time use but then discovers a need to reuse it several months later in a different context. In such a situation, it's important that the analyst understands the original model, lest the passage of time obscure its purpose and logic. In all of these cases, the spreadsheet model fills a significant



	A	B	C
1	**Price, Demand and Profit**		
2			
3	**Inputs**		
4		Max Demand	800
5		Slope	-5
6		Unit cost	40
7			
8	**Decision**		
9		Price	$ 80
10			
11	**Calculation**		
12		Demand	400
13			
14	**Outcome**		
15		Profit	$ 16,000
16			

FIGURE 1.1 Spreadsheet model for determining price.

communications need. Thus, it is important to keep the role of communication in mind while developing a spreadsheet.

A spreadsheet version of our pricing model might look like the one in Figure 1.1. This spreadsheet contains a cell (C9) that holds the unit price, a cell (C12) that holds the level of demand, and a cell (C15) that holds the total profit contribution. Actually, cell C12 holds Equation 1.1 in the form of the Excel formula `=C4+C5*C9`. Similarly, cell C15 holds Equation 1.2 with the formula `=(C9-C6)*C12`. In cell C9, the unit price is initially set to $80. For this choice, demand is 400, and the quarterly profit contribution is $16,000.

In a spreadsheet model, there is usually no premium on being concise, as there is when we use algebra. In fact, when conciseness begins to interfere with a model's transparency, it becomes undesirable. Thus, in Figure 1.1, the model retains the demand equation and displays the demand quantity explicitly; we have not tried to incorporate Equation 1.3. This form allows a user to see how price influences profit contribution through demand because all of these quantities are explicit. Furthermore, it is straightforward to trace the connection between the three input parameters and the calculation of profit contribution.

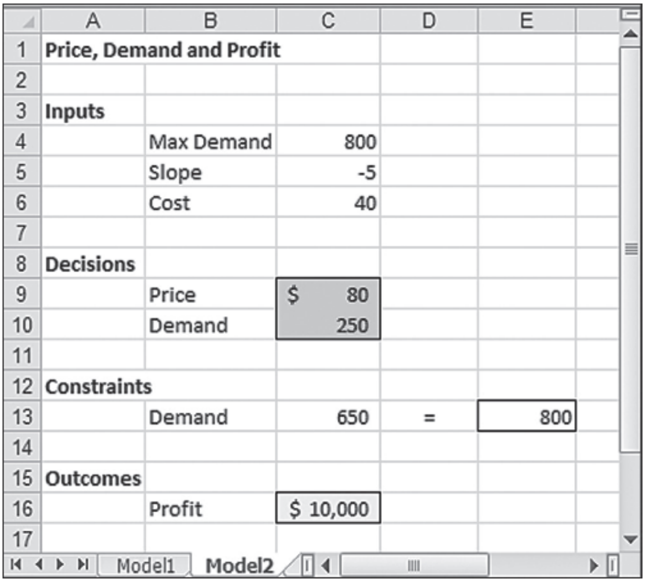
To summarize, our model consists of three parameters and a decision variable, together with some intermediate calculations, all leading to an objective function that we want to maximize. In algebraic terms, the model consists of Equations 1.1 and 1.2, with the prescription that we want to maximize Equation 1.2. In spreadsheet terms, the model consists of the spreadsheet in Figure 1.1, with the prescription that we want to maximize the value in cell C15.

The spreadsheet is organized into four modules: inputs, decision, calculation, and outcome, separating different kinds of information. In spreadsheet models, it is a good idea to separate input data from decisions and decisions from outcome measures. Intermediate calculations that do not lead directly to the outcome measure should also be kept separate.

In the spreadsheet model, cell borders and shading draw attention to the decision (cell C9) and the objective (cell C15) as the two most important elements of the optimization model. No matter how complicated a spreadsheet model may become, we want the decisions and the objective to be located easily by someone who looks at the display.

In the spreadsheet of Figure 1.1, the input parameters appear explicitly. It would not be difficult to skip the Inputs section entirely and express the demand function in cell C12 with the formula =800 − 5 * C9 or to express the profit contribution in cell C15 with the formula =(C9 − 40) * C12. This approach, however, places the numerical parameters in formulas, so a user would not see them at all when looking at the spreadsheet. Good practice calls for displaying parameters explicitly in the spreadsheet, as we have done in Figure 1.1, rather than burying them in formulas.

The basic version of our model, shown in Figure 1.1, is ready for optimization. But let's look at an alternative, shown in Figure 1.2. This version contains the four modules, and the numerical inputs are explicit but placed differently than in Figure 1.1. The main difference is that demand is treated as a decision variable and the demand curve is expressed as an explicit constraint. Specifically, this form of the model treats both price and demand as variables in cells C9:C10, as if the two choices could be made arbitrarily. However, the constraints module describes a



	A	B	C	D	E
1	Price, Demand and Profit				
2					
3	Inputs				
4		Max Demand	800		
5		Slope	-5		
6		Cost	40		
7					
8	Decisions				
9		Price	$ 80		
10		Demand	250		
11					
12	Constraints				
13		Demand	650	=	800
14					
15	Outcomes				
16		Profit	$ 10,000		
17					

FIGURE 1.2 Alternative spreadsheet model for determining price.

relationship between the two variables in the form of Equation 1.1, which can equivalently be expressed as

$$y + 5x = 800 \tag{1.4}$$

We can meet this constraint by forcing cell C13 to equal cell E13, a condition that does not yet hold in Figure 1.2. Cell C13 contains the formula on the left-hand side of Equation 1.4, and cell E13 contains a reference to the parameter 800. The equals sign between them, in cell D13, signifies the nature of the constraint relationship to someone who is looking at the spreadsheet and trying to understand its logic. Equation 1.4 collects all the terms involving decision variables on the left-hand side (in cell C13) and places the constant term on the right-hand side (in cell E13). This is a standard form for expressing a constraint in a spreadsheet model. The spreadsheet itself displays, but does not actually enforce, this constraint. The enforcement task is left to the optimization software. Once the constraint is met, the corresponding decisions are called *feasible*.

This is a good place to include a reminder about the software that accompanies this book. The software contains important files and programs. In terms of files, the book's website[1] contains all of the spreadsheets shown in the figures. Figures 1.1 and 1.2, for example, can be found in the file that contains the spreadsheets for Chapter 1. Those files should be loaded, or else built from scratch, before continuing with the text. As we proceed through the chapters, the reader is welcome to load each file that appears in a figure, for hands-on examination.

1.3 A HIERARCHY FOR ANALYSIS

Before we proceed, some background on the development of models in organizations may be useful. Think about the person who builds a model as an *analyst*, someone who provides support to a decision maker or *client*. (In some cases, the analyst and the client are the same.) The development, testing, and application of a model constitute support for the decision maker—a service to the client. The application phase of this process includes some standard stages of model use.

When a model is built as an aid to decision making, the first stage often involves building a prototype, or a series of prototypes, leading to a model that the analyst and the client accept as a usable decision-support tool. That model provides quantitative analysis of a base-case scenario. In Example 1.1, suppose we set a tentative unit price of \$80. This price might be called a *base case*, in the sense that it represents a tentative decision. As we have seen, this price leads to demand of 400 and profit contribution of \$16,000.

After establishing a base case, it is usually appropriate to investigate the answers to a number of "what-if" questions. We ask, what if we change a numerical input or a decision in the model—what impact would that change have? Suppose, for example, that the marginal effect of price on demand (the slope of the demand curve)

[1] The URL for the book's website is http:// faculty.tuck.dartmouth.edu/optimization-modeling/.

were −4 instead of −5. What difference would this make? Retracing our algebraic steps, or revising the spreadsheet in Figure 1.1, we can determine that the profit contribution would be $19,200.

Systematic investigations of this kind are called *sensitivity analyses*. They explore how sensitive the results and conclusions are to changes in assumptions. Typically, we start by varying one assumption at a time and tracing the impact. Then, we might try varying two or more assumptions, but such probing can quickly become difficult to follow. Therefore, most sensitivity analyses are performed one assumption at a time. Sometimes, it is useful to explore the what-if question in reverse. That is, we might ask, for the result to attain a given outcome level, what would the numerical input have to be? For example, starting with the base-case model, we might ask, what unit price would generate a profit contribution of $17,000? We can answer this question algebraically, by setting $z = 17{,}000$ in Equation 1.3 and solving for x, or, with the spreadsheet model, we can invoke Excel's Goal Seek tool to discover that the price would have to be about $86. (Actually, this is one of two prices that would deliver a profit contribution of $17,000.)

Sensitivity analyses are helpful in determining the robustness of the results and any risks that might be present. They can also reveal how to achieve improvement from better choices in decision making. However, locating improvements this way is something of a trial-and-error process, which is inefficient. Faster and more reliable ways of locating improvements are available. Moreover, with trial-and-error approaches, we seldom know how far improvements can potentially reach, so a best outcome could exist that we never detect.

From this perspective, optimization can be viewed as a sophisticated form of sensitivity analysis that seeks the best values for the decisions and the best value for the performance measure. Optimization takes us beyond mere improvement; we look for the very best outcome in our model, the maximum possible benefit or the minimum possible cost. If we have constraints in our model, then optimization also tells us which of those conditions ultimately limit what we want to accomplish. Optimization can also reveal what we might gain if we can find a way to overcome those constraints and proceed beyond the limitations they impose.

1.4 OPTIMIZATION SOFTWARE

Optimization procedures find the best values of the decision variables in a given model. In the case of Excel, the optimization software is known as *Solver*, which is a standard tool available on the Data ribbon. (The generic term *solver* often refers to optimization software, whether or not it is implemented in a spreadsheet.) Optimization tools have been available on computers for several decades and predate the widespread use of electronic spreadsheets. Before spreadsheets became popular, optimization was available as stand-alone software. It relied on an algebraic approach and was often accessible only by technical experts. Decision makers and even their analysts had to rely on those experts to build and solve optimization models. Spreadsheets, if they were used at all, were limited to small examples. Now, however,

TABLE 1.1 Advantages of Spreadsheet and Algebraic Solution Approaches

Spreadsheet Approaches	Algebraic Approaches
Several algorithms available in one place	Large problem sizes accommodated
Integration of raw data and model data	Concise model specification
Flexibility in layout and design	Standardized model description
Ease of communication with nonspecialists	Enhancements possible for special cases
Intrinsic credibility	

the spreadsheet allows decision makers to develop their own models, without having to learn specialized software, and to find optimal solutions for those models using Solver. Two trends account for the popularity of spreadsheet optimization. First, familiarity with spreadsheets has become almost ubiquitous, at least in the business world. The spreadsheet has come to represent a common language for analysis. Second, the software packages available for spreadsheet-based optimization now include some of the most powerful tools available. The spreadsheet platform need not be an impediment to solving practical optimization problems.

Spreadsheet-based optimization has several advantages. The spreadsheet allows model inputs to be documented clearly and systematically. Moreover, if it is necessary to convert raw data into other forms for the purposes of setting up a model, the required calculations can be performed and documented conveniently in the same spreadsheet, or at least on another sheet in the same workbook. This allows integration between raw data and model data. Without this integration, errors or omissions are more likely, and maintenance becomes more difficult. Another advantage is algorithmic flexibility: The spreadsheet has the ability to call on several different optimization procedures, but the process of preparing the model is mostly the same no matter which procedure is applied. Finally, spreadsheet models have a certain amount of intrinsic credibility because spreadsheets are now so widely used for other purposes. Although spreadsheets can contain errors (and often do), there is at least some comfort in knowing that logic and discipline must be applied in the building of a spreadsheet.

Table 1.1 summarizes and compares the advantages of spreadsheet and algebraic software approaches to optimization problems. The main advantage of algebraic approaches is the efficiency with which models can be specified. With spreadsheets, the elements of a model are represented explicitly. Thus, if the model requires a hundred variables, then the model builder must designate a hundred cells to hold their respective values. Algebraic codes use a different method. If a model contains a hundred variables, the code might refer to $x(k)$, with a specification that k may take on values from 1 to 100, but $x(k)$ need not be represented explicitly for each of the hundred values.

A second advantage of algebraic approaches is the fact that they can sometimes be tailored to a particular application. For example, the very large crew-scheduling applications used by airlines exhibit a special structure. To exploit this structure in the solution procedure, algebraic codes are sometimes enhanced with specialized subroutines that add solution efficiencies when solving a crew-scheduling problem.

A disadvantage of using spreadsheets is that they are not always transparent. As noted earlier, the analyst has a lot of flexibility in the layout and organization of a spreadsheet, but this flexibility, taken too far, may detract from effective communication. In this book, we try to promote better communication by suggesting standard forms for particular types of models. By using some standardization, we make it easier to understand and debug someone else's model. Algebraic codes usually have very detailed specifications for model format, so once we're familiar with the specifications, we should be able to read and understand anyone else's model.

In brief, commercially available algebraic solvers represent an alternative to spreadsheet-based optimization. In this book, our focus on a spreadsheet approach allows the novice to learn basic concepts of mathematical programming, practice building optimization models, obtain solutions readily, and interpret and apply the results of the analysis. All these skills can be developed in the accessible world of spreadsheets. Moreover, these skills provide a solid foundation for using algebraic solvers at some later date, when and if the situation demands it.

1.5 USING SOLVER

Excel's Solver is an add-in that comes with Excel. An icon for Solver typically appears in the Data ribbon in the Analysis group. If the icon is not visible, it is possible to activate Solver by following the steps given below.

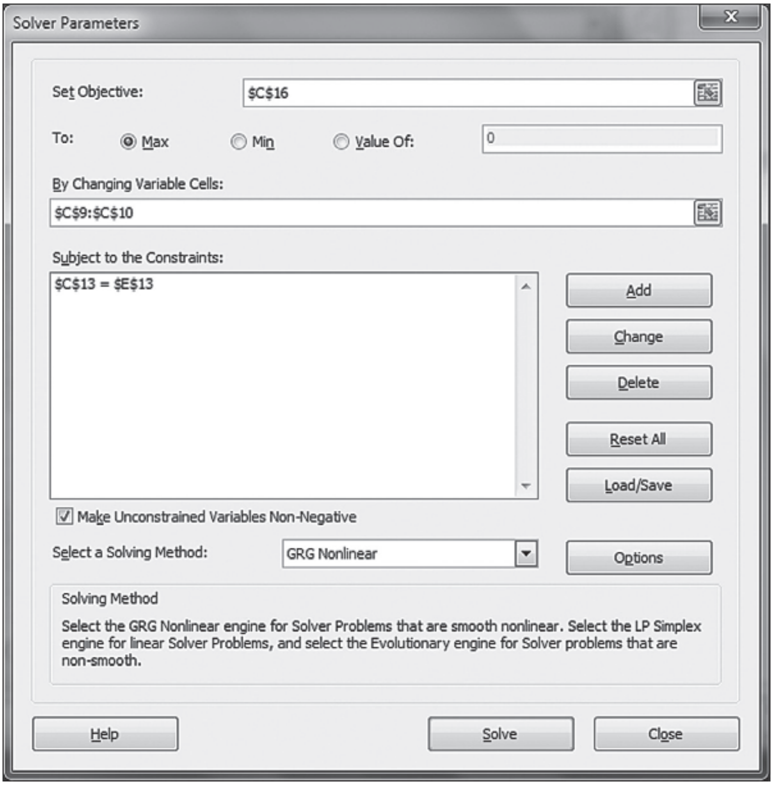
- On the File tab, select Options and then Add-ins.
- At the bottom of the window, set the drop-down menu to Manage Excel Add-ins. Then click Go … .
- In the Add-ins window, check the box for Solver Add-in and click OK.

Purchasers of this book have the option to download a Windows-based software package called **Analytic Solver Platform for Education** (**ASPE**). ASPE was developed by the same team that created Excel's Solver, and it will accommodate all models built with Excel's Solver. However, ASPE is a more powerful version of Excel's Solver and relies on a different user interface. More information on ASPE can be found in Appendix 1.

In order to illustrate the use of Solver, we return to Example 1.1. The optimization problem is to find a unit price that maximizes quarterly profit contribution. An algebraic statement of the problem is as follows:

$$\text{Maximize}\, z = (x - 40)y \;\; \text{(objective)}$$
$$\text{subject to}\, y + 5x = 800 \;\; \text{(constraint)}$$

This form of the model corresponds to Figure 1.2, which contains two decision variables (*x* and *y*, or price and demand) and one constraint on the decision variables. The spreadsheet model in Figure 1.2 is ready for optimization.



Set Objective: $C$16

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells:
$C$9:$C$10

Subject to the Constraints:
$C$13 = $E$13

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

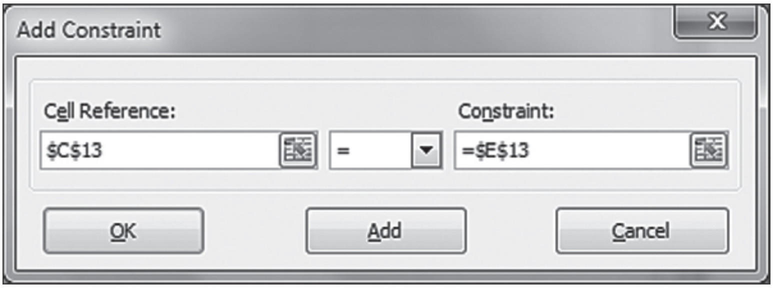
FIGURE 1.3 Solver Parameters window.

To start, we click on the Solver icon in the Data ribbon. This step opens the Solver Parameters window, shown in Figure 1.3. (The location of the cursor is reflected in the first data-entry window.) The Solver Parameters window allows us to specify our model in a way that's consistent with the following sentence:

Set objective C16 to a max[imum] by changing variable cells C9:C10 subject to the constraint C13 = E13.

Three data-entry windows in Figure 1.3 allow us to make the specification. In the Set Objective window, we point to C16 or enter C16, the address of the objective function; and on the next line, we select the button for Max (or confirm that it is already selected as the default). In the Changing Variable Cells window, we point to the two-cell range C9:C10. Then, to specify the constraint, we click the Add button, which opens the Add Constraint window. Figure 1.4 shows this window as it looks when properly filled out, with the drop-down menu in the center to specify that the constraint is an equation.

In nearly all of the models we will encounter, negative values of the decision variables make no practical sense, so we typically want to require variables to be nonnegative. The simplest way to impose this requirement is to check the box for making



Cell Reference: $C$13

=

Constraint: =$E$13

FIGURE 1.4 Add Constraint window.

unconstrained variables nonnegative. (The reference to "unconstrained" variables allows us to impose more stringent constraints if we wish. In our example, we might require the unit price to be at least 40 to ensure that profits will not be negative. With such a constraint elsewhere in the model, it would be unnecessary to impose a nonnegative requirement on cell C9.)

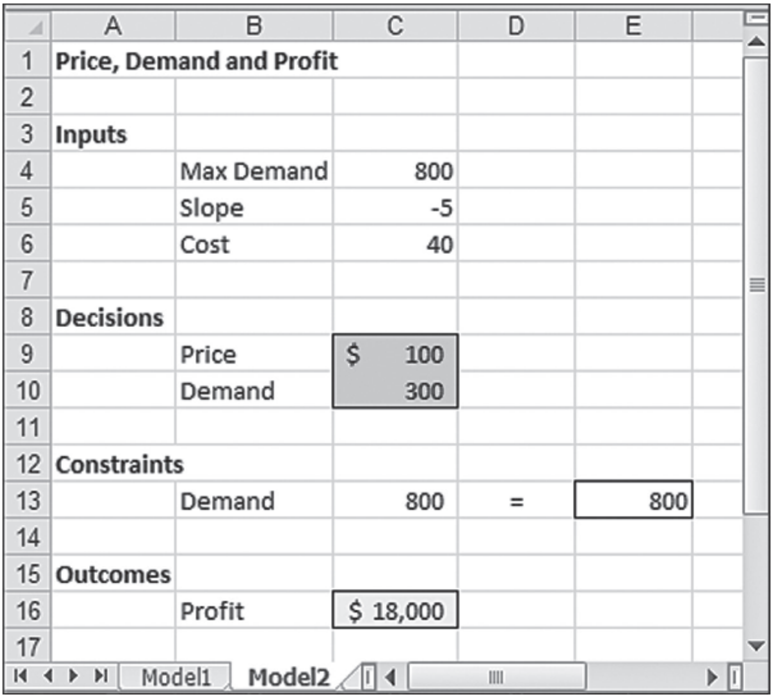
When specifying constraints, one of our design guidelines for Solver models is to reference a cell containing a *formula* in the Cell Reference box and to reference a cell containing a *number* in the Constraint box. The use of cell references keeps the key parameters visible on the spreadsheet, rather than in the less accessible windows of Solver's interface. The principle at work here is to communicate as much as possible about the model using the spreadsheet itself. Ideally, another person would not have to examine the Solver Parameters window to understand the model. (Although Solver permits us to enter numerical values directly into the Constraint box, this form is less effective for communication and complicates sensitivity analysis. It would be reasonable only in special cases where the model structure is obvious from the spreadsheet and where we expect to perform no sensitivity analyses for the corresponding parameter.)

Finally, we specify a solving method for the optimization. In this case, the default choice (GRG Nonlinear) is appropriate, so nothing else is needed. The specification is complete, and pressing Solve invokes the optimization procedure. (Alternatively, pressing Close saves the specification on the spreadsheet but does not run the optimization procedure.)

In summary, our model specification is the following:

Objective:	C16 (maximize)
Variables:	C9:C10
Constraint:	C13 = E13

When we invoke the GRG Nonlinear procedure, Solver searches for the optimal price and ultimately places it in cell C9, as shown in Figure 1.5.



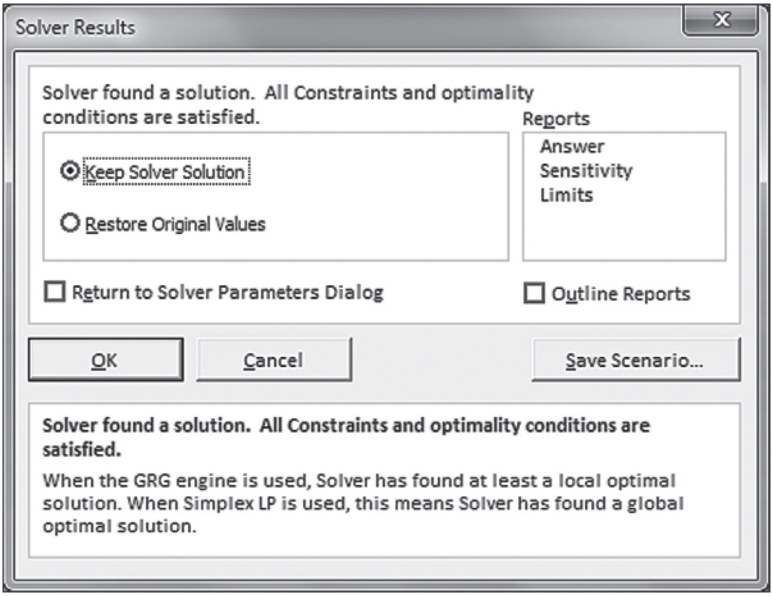
	A	B	C	D	E
1	**Price, Demand and Profit**				
2					
3	**Inputs**				
4		Max Demand	800		
5		Slope	-5		
6		Cost	40		
7					
8	**Decisions**				
9		Price	$ 100		
10		Demand	300		
11					
12	**Constraints**				
13		Demand	800	=	800
14					
15	**Outcomes**				
16		Profit	$ 18,000		
17					

FIGURE 1.5 Optimal solution produced by Solver.

The result of the optimization run is summarized in the Solver Results window, shown in Figure 1.6, which opens when the optimization run completes. The message at the top of the window states, “Solver found a solution. All Constraints and optimality conditions are satisfied.” This **optimality message**, which is elaborated at the bottom of the window, tells us that no problems arose during the optimization and Solver was able to find an optimal solution. The profit-maximizing unit price is $100, yielding an optimal profit of $18,000. No other price can achieve more than this level. Thus, if we are confident that the demand curve continues to hold, the profit-maximizing decision would be to set the unit price at $100.

Finally, the Solver Results window allows us to select a button to preserve the solution on the spreadsheet (as in Fig. 1.5) or to restore the values that were in the spreadsheet before the optimization run.

We have used Example 1.1 to introduce Solver and its user interface. This interface offers us several options that are not a concern in this problem. In later chapters, we cover many of these settings and discuss when they become relevant. We also discuss the variations that can occur in optimization runs. For example, depending on the initial values of the decision variables, the nonlinear solver may



Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution

○ Restore Original Values

□ Return to Solver Parameters Dialog

□ Outline Reports

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

FIGURE 1.6 Solver Results window.

generate the following message: "Solver has converged to the current solution. All constraints are satisfied." This **convergence message** indicates that Solver has not been able to confirm optimality. Usually, this condition occurs because of numerical issues in the solution algorithm, and the resolution is to rerun Solver from the point where convergence occurred. Normally, one or two iterations are sufficient to produce the optimality message. We discuss Solver's result messages in more detail later.

Using Solver, we can minimize an objective function instead of maximizing it. We simply select the button for Min rather than Max. (A third option allows us to specify a target value and find a set of variables that achieves the target value. This is not an optimization tool, and we will not pursue this particular capability.)

When an optimization model contains several decision variables, we can enter them one at a time, separated by commas. More conveniently, we can arrange the spreadsheet so that all the variables appear in adjacent cells, as in Figure 1.2, and reference their cell range with just one entry in the Solver Parameters window. Because most optimization problems have several decision variables, we save time by placing them in adjacent cells. This layout also makes the information in the Solver Parameters window easier to interpret when someone else is trying to audit our work or if we are reviewing it after not having seen it for a long time. However, exceptions to this design guideline sometimes occur. Certain applications sometimes lead us to use nonadjacent locations for convenience in laying out the decision variable cells (Box 1.2).

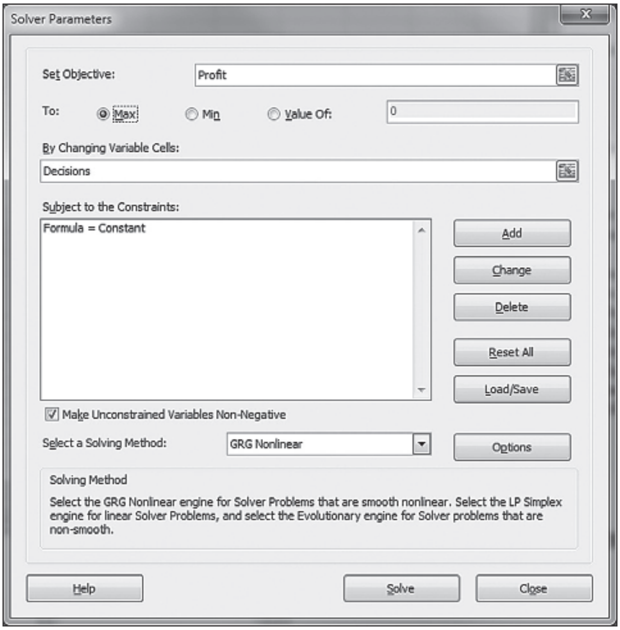
BOX 1.2 *Excel Mini-Lesson: Using Range Names with Solver*

Excel offers the opportunity to refer to a cell range using a custom name. The range name can be entered in the Name box (located just above the heading for column A) after selecting the desired range of cells. A one-cell range can be named in the same manner.

To show the effect of named ranges, we return to the model of Figure 1.2 and name the following cells:

Cells	Name
C9:C10	Decisions
C13	Formula
E13	Constant
C16	Profit

Then, the Solver Parameters window describes the model with range names instead of cell references, as shown in Figure 1.7. When a new user examines the model, this form is likely to be more meaningful than the use of literal cell references because the range names provide both description and documentation. Thus, range names are valuable for situations in which communicating the model to other audiences is an important consideration. When Solver is applied in an organizational setting, the use of range names is normally desirable. In the remainder of this book, however, we will continue to rely on cell references because they relate the information in the Solver Parameters window directly to the contents of the spreadsheet display.



Solver Parameters

Set Objective: Profit

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells: Decisions

Subject to the Constraints:
Formula = Constant

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

FIGURE 1.7 Solver Parameters window for the model with range names.

SUMMARY

Many types of applications invite the use of Excel's Solver. In one sense, then, this book is about using Solver to obtain solutions to optimization problems. Because Solver is a spreadsheet tool, the book builds skill and confidence in applying spreadsheet-based methods.

In another sense, this book is about the problem types that Solver can handle, but the information on how to run Solver is incidental. The transcendent theme is the building of optimization models. If Solver weren't around to produce solutions, then some other software would perform the computational task. The more basic skill is creating the model in the first place and recognizing its potential role in decision support. Because a variety of problem types is covered, this book provides insight into the kinds of applications that can be addressed with optimization techniques.

Thus far, we have introduced six design guidelines for spreadsheet optimization models:

1. Separate inputs from decisions and decisions from outputs.
2. Create distinct modules for decision variables, objective function, and constraints.
3. Display parameters explicitly on the spreadsheet, rather than in formulas.
4. Enter parameters in the spreadsheet, rather than in the Add Constraints window.
5. Place decision variables in adjacent cells.
6. Highlight important cells, such as the decision variables and the objective.

Subsequent chapters introduce additional features of good spreadsheet design. This is not a claim that each example spreadsheet is the only possible way of designing a model, or even that it's the best way. A model should be easy to recognize, debug, use routinely, and pass on to others. A key feature of a good spreadsheet model is its ability to communicate clearly.

Solver does not consist of a single procedure. Solver actually consists of four main optimization procedures, which are covered in subsequent chapters. We refer to these procedures as:

The nonlinear solver
The linear solver
The integer solver
The evolutionary solver

Chapters 2–5 deal with the linear solver, introducing many features of optimization analysis in the process. Chapters 6 and 7 deal with models that can be solved with the integer solver, and Chapter 8 deals with the nonlinear solver. The evolutionary solver, which is introduced in Chapter 9, is not properly an optimization procedure in the same sense as the others, but it applies in situations where the other solvers might fail. Each chapter is filled with illustrative examples and followed by a set of practice exercises. If readers work through the examples and the exercises, they will develop a firm grasp on how to solve practical optimization problems using spreadsheets.

EXERCISES

1.1 **Determining an Optimal Price:** A firm's marketing department has estimated the demand curve of a product as $y = 1100 - 7x$, where y represents demand and x represents the unit selling price (in dollars) for the relevant decision period. The unit cost is known to be \$24. What price maximizes net income from sales of the product?

1.2 **Pricing in Two Markets:** Global Products, Inc. has been making an electronic appliance for the domestic market. Demand for the appliance is price sensitive, and the demand curve is known to follow the linear function $D = 4000 - 5P$, where D represents annual demand and P represents selling price in the home currency, which is the frank (F). The cost of manufacturing the appliance is 100 F.

For the coming year, Global is planning to sell the same product in a foreign market, where the currency is the marc (M). From surveys, the demand curve in the foreign country is estimated to follow a different linear function, $D = 2000 - 2P$, where the price is denominated in marcs.

All production will be carried out at Global's domestic plant, with the expectation that the unit cost will remain unchanged. The exchange rate is 1.5 M/F, and Global plans to offer an equivalent price in both markets.

(a) If Global were to operate exclusively in its domestic market, what would be its profit-maximizing price and its annual profit?

(b) When Global sells in both markets at one equivalent price, what is its profit-maximizing price and its annual profit?

1.3 **Locating a Distribution Center:** Northeast Parts Supply is a wholesale distributor of components for printers, fax machines, scanners, and related equipment. Northeast stocks expensive spare parts, which dealers prefer not to hold, and offers same-day delivery on any order. The firm now serves eight dealers in the New England area and wishes to locate its distribution facility at a central point. In particular, its dealers have each been assigned a location on an x–y grid, and Northeast would like to find the best location for the distribution facility.

The eight dealers and their grid locations are shown in the following table:

Dealer	1	2	3	4	5	6	7	8
x-location	25	82	10	27	93	14	68	147
y-location	32	36	71	58	68	163	149	192

(a) Determine the location that minimizes the sum of the distances from the distribution facility to the dealers.

(b) Determine the location that minimizes the maximum distance from the distribution facility to any of the dealers.

1.4 **Siting a Warehouse:** An appliance dealer offers free delivery in any one of its five Texas cities, which will be serviced by a single warehouse. The distance from the warehouse site to a given city is considered to be a good proxy for the annual cost of providing delivery service, and the objective is to minimize the total distance from the warehouse to the five cities. The cities are treated as if they were each a single point located at a specific latitude and longitude, as listed in the table below.

City	Austin	Houston	Midland	Tyler	Waco
Latitude	30.3	30.0	32.0	32.4	31.6
Longitude	97.7	95.4	102.2	95.4	97.2

When locations are specified in terms of latitude (*Lat*) and longitude (*Lon*), a good approximation for the distance (in miles) between two locations is the following:

$$\text{Distance} = \left(x^2 + y^2\right)^{1/2}$$

where $x = 69.1\,(Lat_2 - Lat_1)$ and $y = 69.1\,(Lon_2 - Lon_1)\,[\cos(Lat_1/57.3)]$

What is the best location (latitude and longitude) for the warehouse?

1.5 **Finding a Lost Plane:** A private plane went down off the coast and sank during a bad storm although rescuers were able to save its crew. Aboard the plane was a transmitter that was able to send out a signal for 72 hours after the plane went down. When the weather cleared, searchers went out in three different boats carrying equipment that could detect the signal and estimate its distance from the transmitter. The locations of the three boats on an x–y grid and their distance estimates (in miles) are shown in the table below.

Searcher	1	2	3
x-location	25	35	70
y-location	60	40	10
Estimate	29.3	34.7	15.5

The estimates are known to be unreliable, but the information may sufficient to locate the sunken plane, at least approximately. What is its x–y-location?

1.6 **Collecting Credit Card Debt:** A bank offers a credit card that can be used in various locations. The bank's analysts believe that the percentage P of accounts receivable collected by t months after credit is issued increases at a decreasing rate. Historical data suggest the following function:

$$P = 0.9[1 - \exp(-0.6t)]$$

The average credit issued in any 1 month is \$125 million, and historical experience suggests that for new credit issued in any month, collection efforts cost \$1 million/month.

(a) Determine the number of months that collection efforts should be continued if the objective is to maximize the net collections (dollars collected minus collection costs). Allow for fractional months.

(b) Under the optimal policy in (**a**), what percentage of accounts receivable should be collected?

1.7 **Allocating Plant Output:** A firm owns five manufacturing plants that are responsible for the quarterly production of 50,000 pounds of an industrial solvent. The production process exhibits diseconomies of scale. At plant *p*, the cost of making *x* thousand pounds of the solvent is approximated by the quadratic function $f(x) = (1/c_p)x^2$. The parameters c_p are plant dependent, as shown in the following table.

p	1	2	3	4	5
c_p	3	6	4	8	5

How should production be allocated among the five plants in order to minimize the total cost of meeting the volume requirement?

1.8 **Determining Production Lot Sizes:** Four products are routed through a machining center that is notorious for its delays. Each product has had stable demand for some time, so that average weekly demand is predictable over a 3–6 months' time frame. However, in the short run, demand fluctuates a great deal, and the load at the machining center varies considerably. The production control system dictates the lot size for each of the products. These quantities are shown, along with other relevant information, in the following table.

Product no.	Demand (weekly)	Setup (hours)	Run time (hours/1000)	Lot size
1	100	3	30	100
2	500	15	45	500
3	50	6	75	100
4	250	24	150	1500

With the current lot sizes, the machining center is running at a utilization of about 76%, but long lead times, sometimes over 2 weeks, have discouraged production planners from increasing its load. (A week contains 120 productive hours.) In the past, lead times spiraled out of control when utilization grew to around 80%.

A lead time model for this problem has been constructed on a spreadsheet.[2] The model permits the user to select lot sizes and thereby influence the average lead time through the bottleneck work center. The lead time prediction is based on advanced modeling techniques, but the details of the model are not of primary importance.

What is the shortest possible lead time, and what lot sizes achieve this value?

1.9 **Resolving a Construction Dilemma:** A library building is about to undergo some renovations that will improve its structural integrity. As part of the process, a number of steel beams will be carried through the existing bookcases from a broad, open area around the entry point. The central aisle between the bookcases is 10 ft wide, while the side aisles (which run perpendicular to the central aisle) are 6 ft wide. The renovation will require that steel beams be carried through the stacks, down the main aisle and turning into the smaller aisles.

What is the longest steel beam that can be carried horizontally through this space to a construction point along the outer walls?

1.10 **Selecting the Number of Warehouses:** The customers of a particular company are located throughout an area comprised of S square miles, and they are serviced from k warehouses. On average, the distance in miles between a warehouse and a customer is given by the formula $(S/k)^{0.5}$. The annual capital cost of building a warehouse is \$40,000 and the annual operating cost of running a warehouse is \$60,000. Annual shipping costs average \$1 per mile per customer.

Suppose that the current market size is 250,000 customers, spread out over an area of 500 sq. miles. What is the optimal number of warehouses for the firm to operate?

REFERENCES

1. McFedries, P. Excel 2013 Simplified. John Wiley and Sons, 2013.
2. Walkenbach, J. Excel 2013 Bible. John Wiley and Sons, 2013.

[2] The lead time model is available in the Datasets workbook, at the book's website: (http://faculty.tuck.dartmouth.edu/optimization-modeling).