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Geometrical Optics

1.1 Characteristics of Lasers

This book is about optical calculation methods and the principles for applying these methods to actual optical devices. Most of these devices use lasers, so we will begin by briefly examining the characteristics of lasers. The following exposition will be especially beneficial for readers whose understanding of lasers is rather limited.

The term LASER is an acronym for “Light Amplification by Stimulated Emission of Radiation.” Lasers have special characteristics that distinguish them from most light-emitting devices: a narrow, low-divergence beam (sharp *directivity*), and a very narrow wavelength spectrum (*monochromaticity*). These features of lasers make them ideally suited for the generation of high intensity beams.

The history of lasers goes back about half a century to 1960, when the first working laser was demonstrated. Lasers are now widely used in a variety of fields, including optical storage (e.g., CD drives and DVD drives), fiber-optic communication, manufacturing (especially for cutting, bending, welding, and marking materials), scientific measurement, and medicine. This diversity of application is due to the following four properties of lasers, which give them a commercial and scientific edge over other light-emitting devices.

1. *Monochromaticity*: Radiation that has a very narrow frequency band (or wavelength band) is said to possess the property of *monochromaticity*. Because the band is so narrow, the radiation can be regarded as having a single frequency (or alternatively, a single wavelength). Laser light typically has a very narrow frequency band (or wavelength band), which makes it an ideal example of the property of monochromaticity. Sunlight, by contrast, has a very broad band spectrum, with multiple frequencies and wavelengths.
2. *Beam directivity*: The term *directivity*, when used in relation to lasers, refers to the directional properties of the electromagnetic radiation they emit. A laser beam exhibits a very sharp (narrow) directivity, allowing it to propagate in a straight line with almost no expansion. By contrast, normal light sources such as flashlights and car headlights have broader directivity than lasers, so their beams cannot travel as far.
3. *Coherence*: If split laser beams which were emitted from the same source are superimposed, a fringe pattern will appear. This fringe pattern is never observed in an isolated beam. We refer to this phenomenon as *interference* caused by the wave characteristics of light. If these two beams traveling onto the same plane are superimposed while they are *in phase* (with their crests and troughs lined up), the resultant beam will appear brighter, but if these waves are superimposed with a 180° phase difference



(i.e., if crests and troughs are superimposed), the beam will appear dark and the resultant amplitude will be zero. A laser can easily generate interference patterns because of its *coherence* (uniformity of phase). Sunlight cannot readily generate interference patterns, due to its incoherence: because its coherence time and coherence length are very short, and the phases of the superimposed waves will not come into phase very easily. Sunlight will only exhibit coherence over a very short interval both of time and space.

4. *High concentration of energy (high intensity)*: A sheet of paper can be burnt simply by focusing sunlight on it, using a convex lens. A laser is much more concentrated: it can even weld two pieces of steel together. This is not merely due to the high power of the laser, but also because of the extremely high *intensity* of the laser beam, where the light energy is narrowly focused. It is relatively easy to concentrate a laser beam on a small target. A high intensity beam can be generated very easily using a laser.

All of these characteristics of lasers can best be summed up by the word “coherent.” A laser is both *temporally coherent* and *spatially coherent*. What this means is that a laser has a uniform phase over time at an arbitrary point in space, and it also has a uniform phase in space at an arbitrary point in time. Thus a laser has a uniform phase both in time and in space. A laser radiates a single-wavelength (more precisely, a very narrow wavelength spectrum) beam with a constant phase and it can propagate in a specific direction. An electric light, by contrast, radiates a multitude of different wavelengths at various phases and in all directions. Thus it is both spatially and temporally incoherent.

1.2 The Three Fundamental Characteristics of Light Which Form the Basis of Geometrical Optics

To understand geometrical optics, which is the most basic form of optics, we need to first study the fundamental characteristics of light – including light emitted by lasers. The following properties of light are confirmed by everyday experience:

1. Light rays travel in straight lines in a uniform medium.
2. Light rays are independent of one another.
3. Light rays can be reflected and refracted: they change their direction at a boundary between different media, in accordance with the laws of reflection and refraction.

The whole science of geometrical optics can be derived from these three characteristics of light.

1.2.1 Light Rays Travel in Straight Lines

Many common optical phenomena attest to the fact that light rays travel in straight lines. For instance, when the sun is shining outdoors, a tree casts a shadow whose shape is identical with its own. Without using any lenses, we can construct a pinhole camera that can capture the image of an object, simply by making a pinhole in one of the walls of a black box as shown in Figure 1.1.

1.2.2 Light Rays Act Independently of One Another

Figure 1.2 illustrates the independent action of light rays using the example of three spotlights whose light is of different colors: red, blue, and green. When we irradiate the same area on a white sheet of paper with

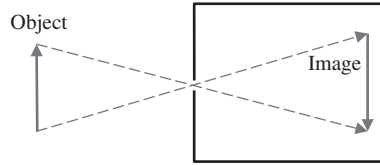


Figure 1.1 Pinhole camera

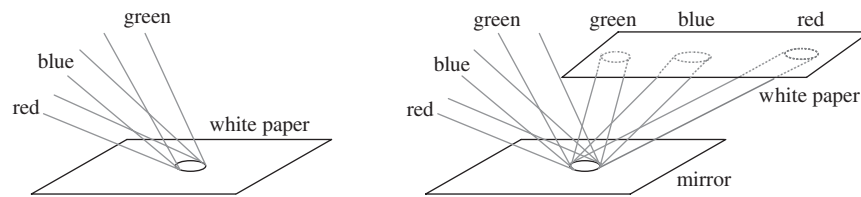


Figure 1.2 Independent action of light rays

these three spotlights, we perceive the light as “white.” However, if we replace the paper with a mirror, and then project the reflected light onto another sheet of paper, the three separate beams of light reappear in their original colors of red, blue, and green. The fact that these three beams reappear in their original colors demonstrates that light rays coming from different sources act independently of one another after being reflected by the mirror. Likewise, the fact that the light from the three spotlights appears white when they are all focused on the same area of paper can be explained in terms of the constituent light wavelengths (red, blue, and green) reaching our eyes and acting on our retinas independently. It is the superposition of waves which causes us to perceive them as white.

1.2.3 Reflection of Light Rays

As shown in Figure 1.3a, when a light ray is reflected by a mirror at the point of incidence O , we can define the normal as an imaginary line through point O perpendicular to the mirror. The reflected ray will lie in the same plane as the incident ray and the normal to the mirror surface, and the angle of reflection will be the same as the angle of incidence. We are all familiar with this phenomenon from everyday experience. It can be described by the following equation:

$$\text{Angle of incidence } \theta_i = \text{angle of reflection } \theta_r \quad (1.1)$$

1.2.4 Refraction of Light Rays

An object lying in a tub of water appears to be at a shallower depth than it actually is. This phenomenon can be explained by the refraction of light at the interface between the water and the air. As shown in Figure 1.3b, when a ray is refracted at a boundary plane between different media, the relationship between the angle of incidence and angle of refraction can be described by the following equation (Snell’s law):

$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad (1.2)$$

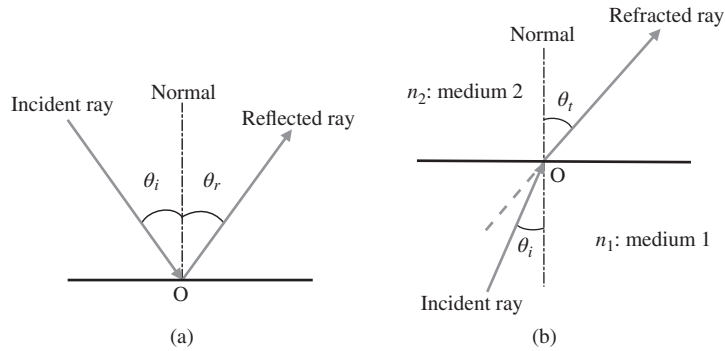


Figure 1.3 (a) Reflection of a light ray and (b) refraction of a light ray

where

n_1 = Refractive index in medium 1
 n_2 = Refractive index in medium 2
 θ_i = Angle of incidence
 θ_t = Angle of refraction.

1.3 Fermat's Principle

The law of rectilinear propagation and the law of reflection and refraction of light rays can both be derived from Fermat's principle, as explained below [1].

- (i) The velocity of light is inversely proportional to the refractive index of the medium in which light propagates. (In other words, light travels more slowly in a medium having a higher refractive index.)
 The velocity of light in a vacuum is a constant, c :

$$c = 2.99792458 \times 10^8 \text{ m/s} \quad (1.3)$$

The velocity of light v in a medium having refractive index n is:

$$v = c/n \quad (1.4)$$

In optical calculations, the optical path length L is defined as follows:

$$L = nL_0 \quad (1.5)$$

where L_0 = physical length traversed by light in the medium. (L_0 is defined as the optical path length in a vacuum, and the refractive index in a vacuum, $n_0 = 1$.)

- (ii) The path taken by a ray of light between any two points in a system is always the path that can be traversed in the least time (or the path that has the minimum optical path length) (Fermat's Principle)

A more accurate and complete statement of Fermat's principle is that a ray of light traveling from one point to another point must traverse an optical path length which corresponds to a stationary point, which means that it can either be a minimum, a maximum, or a point of inflection.

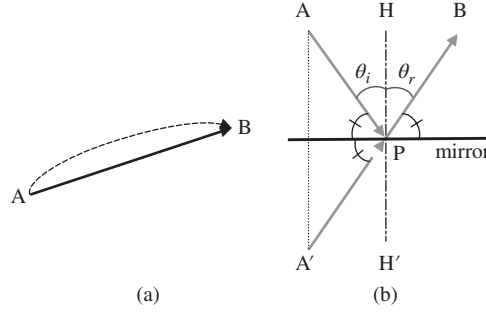


Figure 1.4 (a) Rectilinear propagation and (b) law of reflection

1.3.1 Rectilinear Propagation

Fermat's principle entails that a ray of light must travel along a rectilinear path in a uniform medium.

As shown in Figure 1.4a, a ray of light traveling from A to B in a uniform medium takes the path that can be traversed in the least time, that is, the straight line AB.

1.3.2 Reflection

When a ray of light is reflected, it will be reflected at an angle that minimizes its optical path length.

Figure 1.4b illustrates a ray of light which proceeds from point A, is reflected by the mirror at P, and reaches B. Now let us assume that the starting point is A' instead of A. (A' is the "reflection" of A in the mirror.) Clearly, the length of A'PB is equal to APB. It is also obvious that the optical path length A'PB is minimized when A'PB is rectilinear. When this is the case, the angle of reflection θ_r will equal the angle of incidence θ_i .

$$\text{Angle of incidence } \theta_i = \text{angle of reflection } \theta_r \quad (1.1)$$

1.3.3 Refraction

A ray of light traveling from one medium to another will be refracted at an angle that minimizes the optical path length.

Suppose that a ray of light proceeds from point A(0,a) inside a medium with a refractive index of n_1 , then passes through the boundary plane at point B(x,0), and finally reaches point C(d,c), inside a medium with a refractive index n_2 , as shown in Figure 1.5a.

The optical path length ABC will be:

$$\begin{aligned} L &= n_1 AB + n_2 BC \\ &= n_1 \sqrt{x^2 + a^2} + n_2 \sqrt{(d-x)^2 + c^2} \end{aligned} \quad (1.6)$$

The derivative of L with respect to x will be:

$$\frac{dL}{dx} = \frac{n_1 x}{\sqrt{x^2 + a^2}} - \frac{n_2 (d-x)}{\sqrt{(d-x)^2 + c^2}} \quad (1.7)$$

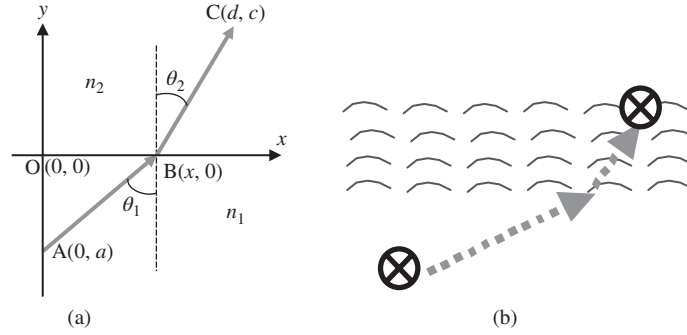


Figure 1.5 (a) Angle of refraction and (b) emergency path of a lifesaver

When $dL/dx = 0$, the optical path length L is minimized. That is,

$$\frac{dL}{dx} = \frac{n_1 x}{\sqrt{x^2 + a^2}} - \frac{n_2 (d - x)}{\sqrt{(d - x)^2 + c^2}} = 0 \quad (1.8)$$

From Figure 1.5a, the following equations can be derived:

$$\sin \theta_1 = \frac{x}{\sqrt{x^2 + a^2}} \quad (1.9)$$

$$\sin \theta_2 = \frac{d - x}{\sqrt{(d - x)^2 + c^2}} \quad (1.10)$$

Combining Equation (1.8), Equation (1.9) and Equation (1.10) yields:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (1.2)$$

Equation (1.2) is the conditional equation of refraction.

Path Swum by a Lifesaver in An Emergency Rescue

A lifesaver at a beach will swim in a path that minimizes the time to reach the victim in an emergency rescue (Figure 1.5b).

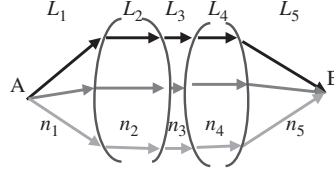
This is identical to the behavior of a ray of light which is described by Fermat's principle.

1.3.4 An Ideal Imaging System Using Lenses

Let us imagine an ideal imaging system using multiple lenses. A ray of light proceeds from the point object A, passes through the lens system, and reaches imaging point B, as in Figure 1.6. If the system is an ideal imaging system, these three routes must have the same optical path length (Fermat's principle).

In an ideal imaging system, the optical path length from the object to the image will have a constant value of ΣL :

$$\sum L = n_1 L_1 + n_2 L_2 + n_3 L_3 + n_4 L_4 + n_5 L_5 = \text{constant} \quad (1.11)$$



Note **Figure 1.6** Optical path length in an ideal imaging system

The refractive index of a medium is not always the same: it varies slightly, depending on the wavelength of the light traveling through it. Thus lenses will exhibit some degree of chromatic aberration. Spherical lenses also exhibit various monochromatic aberrations, which will be discussed in Section 1.6. In actual imaging systems, multiple lens systems are generally used in order to minimize or eliminate these aberrations.

1.4 Principle of Reversibility

Since Fermat's principle only relates to the path and not the direction traveled, all of the foregoing equations describing light rays apply equally to light traveling along the same path in a backwards direction.

If there is a ray of light which starts from point A, passes through an optical system, and reaches point B, it follows that the ray starting at B and reaching A must travel along exactly the same path. This insight, which has immense practical value, is referred to as the "Principle of reversibility."

1.5 Paraxial Theory Using Thin Lenses

We will now examine several methods of imaging using lenses [1]. First, using the concept of a minimal optical path length, we will discuss how rays can create an image of an object.

1.5.1 Equation of a Spherical Lens Surface

The equation of a spherical lens surface S whose center of curvature lies on the z -axis and whose surface touches the origin as in Figure 1.7, can be expressed by:

$$x^2 + y^2 + (z - r)^2 = r^2 \quad (1.12)$$

where r is the curvature radius of the lens surface.

Equation (1.12) can be rewritten as:

$$z = \frac{1}{2r}(x^2 + y^2 + z^2) \quad (1.13)$$

When we use a thin lens, we can neglect z^2 because $z^2 \ll x^2 + y^2$ in thin lenses. Hence

$$z = \frac{1}{2r}(x^2 + y^2) \quad (1.14)$$

$$z = \frac{1}{2}c(x^2 + y^2) \quad (1.15a)$$

$$c \equiv \frac{1}{r} \text{ (curvature)} \quad (1.15b)$$

In Equation (1.15a), since the surface of the lens bulges toward the incident rays coming from the light source, it is convex and has a positive curvature. In a two-dimensional space, Equation (1.15a)

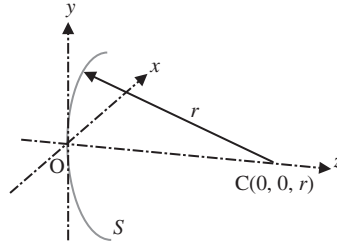


Figure 1.7 Spherical lens surface

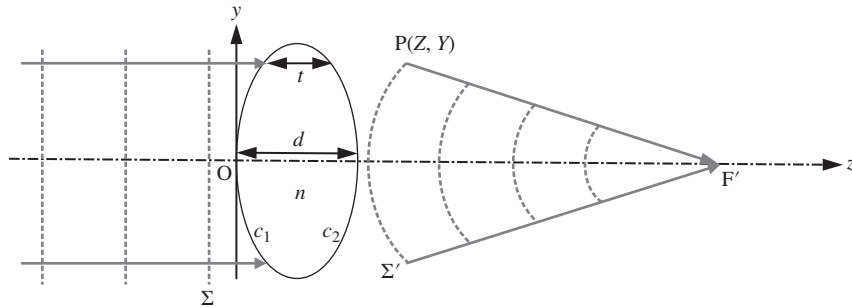


Figure 1.8 Refraction of a light ray by a convex lens

simplifies to:

$$z = \frac{1}{2}cy^2 \quad (1.16)$$

Equation (1.16) describes a convex lens with a curvature of c , whose thickness decreases as the vertical height of the lens increases.

1.5.2 Wave Front Radii of Incident Rays versus Rays Refracted by a Convex Lens

A collimated light beam (whose rays are parallel) is incident onto a convex lens having a refractive index of n , a thickness of d in the center, and front and back surfaces whose curvatures are c_1 ($\equiv 1/r_1$) and c_2 ($\equiv 1/r_2$), respectively. Suppose that the front surface of the lens touches the origin, as shown in Figure 1.8. The wave front surface (defined as a set of points whose optical path lengths from a point light source all have the same value) of the incident rays will lie on a plane Σ , as the light beam is collimated. Using the concept of optical path length, we can illustrate how the shape of the wave front changes, after passing through the lens.

Suppose the ray at a height Y travels a distance Z parallel to the z -axis, while the ray going along the z -axis travels through the center of a lens whose thickness is d . After passing through the lens, the wave front will be a surface consisting of points having the same optical path length from the light source.



From Equation (1.16), the thickness t of the lens at height Y will be:

$$t = d - \frac{1}{2}(c_1 - c_2) Y^2 \quad (1.17)$$

The optical path length of the ray L_0 which travels through the center of the lens will be:

$$L_0 = nd \quad (1.18)$$

The optical path length of the ray L_Y which travels through the lens at a vertical height of Y will be:

$$L_Y = nt + Z - t \quad (1.19)$$

The condition for both rays having the same optical path length is that $L_0 = L_Y$. Thus

$$nd = n \left\{ d - \frac{1}{2}(c_1 - c_2) Y^2 \right\} + Z - d + \frac{1}{2}(c_1 - c_2) Y^2 \quad (1.20)$$

In other words,

$$Z = d + \frac{1}{2}(n - 1)(c_1 - c_2)Y^2 \quad (1.21)$$

Equation (1.21) represents a sphere with a curvature of $(n - 1)(c_1 - c_2)$, which includes the rear surface of the lens on the axis. From the preceding discussion, it follows that the surface of the wave front, after passing through the lens, will be transformed into a sphere whose equation is represented by Equation (1.21).

Note

The foregoing equations all presuppose that we can apply the thin lens approximation: light rays maintain the same vertical height while passing through a lens.

1.5.3 The Refractive Power of a Lens and the Thin Lens Equations

Let us now apply Equation (1.21) to incident rays which are not parallel but diverging. From Equation (1.21), the ray of light at a vertical height of Y will travel a distance of $1/2(n - 1)(c_1 - c_2)Y^2$ further than the ray along the z -axis, which travels through the lens at the point where it is thickest (Figure 1.9). Putting it another way, after passing through a convex lens, the wave front curvature will increase by a factor of $(n - 1)(c_1 - c_2)$. We refer to this value as the refractive power of the lens, K .

$$K = (n - 1)(c_1 - c_2) \quad (1.22)$$

As a result, a convex lens will refract the incident rays inwards. Figure 1.9 shows that a convex lens will refract a diverging beam of incident light (which has a negative wave front curvature) inwards, generating a converging beam of light (with a positive wave front curvature). The refractive power K will be the difference in wave front curvatures between the light rays behind the lens and those in front of the lens.

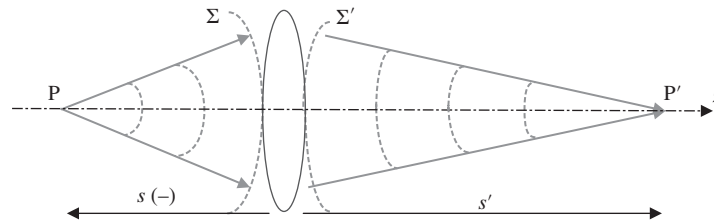


Figure 1.9 Refraction by a lens



We can define K as the (refractive) power of a lens. The point light source P situated at a distance $s(-)$ in front of the lens generates a wave front Σ (a sphere with its center at P) at the lens front. After passing through the lens, the wave front Σ' is transformed into a sphere centered at P' which is situated at a distance $s'(+)$ to the right of the lens along the z -axis. The wave front curvatures of Σ and Σ' will be $1/s$ and $1/s'$. The curvature after passing through the lens $1/s'$ should be the value added by the (refractive) power K to the initial curvature of the wave front before passing through the lens $1/s$. That is,

$$\frac{1}{s'} = \frac{1}{s} + K \quad (1.23)$$

More generally, we can replace K with the inverse of the focal length $1/f$. This gives us:

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \quad (1.24)$$

where s is the distance of the object from the lens, and s' is the distance of the image from the lens. These values will be positive when they are situated to the right of the lens and negative when they are situated to the left of the lens. f is the focal length of the lens.

Equation (1.24) is the famous Gaussian lens formula. Using this formula, we can easily calculate the distance of the image. If $s > 0$, the object is situated behind the lens, so the object will be a virtual object. If $s' < 0$, the image is situated in front of the lens, so the image will be a virtual image.

From Equation (1.22), the focal length of a thin lens can be expressed as follows:

$$\frac{1}{f} = (n - 1) (c_1 - c_2) = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (1.25)$$

If the lens is plano-convex (or plano-concave), $c_2 = 0$ ($r_2 = \infty$), and we can then write:

$$\frac{1}{f} = \frac{(n - 1)}{r} \quad (1.26)$$

1.5.4 Imaging Equations for a Lens

1.5.4.1 A Method of Drawing Rays Before and After Passing through the Lens, in Order to Obtain Their Image Point

Figure 1.10 illustrates the rays of light before and after going through the lens, where

- F = First focal point
- F' = Second focal point
- H = Primary principal point (H_1HH_2 = primary principal plane)
- H' = Secondary principal point ($H'_1H'H'_2$ = secondary principal plane)
- f = First focal length ($-$)
- f' = Second focal length ($+$)
- z = Distance of object from the first focal point ($-$)
- s = Distance of object from the primary principal point ($-$)
- z' = Distance of image from the second focal point ($+$)
- s' = Distance of image from the secondary principal point ($+$)
- A = Object
- AB = Normal to the axis (B is situated on the axis)
- A' = Image
- $A'B'$ = Normal to the axis (B' is situated on the axis).

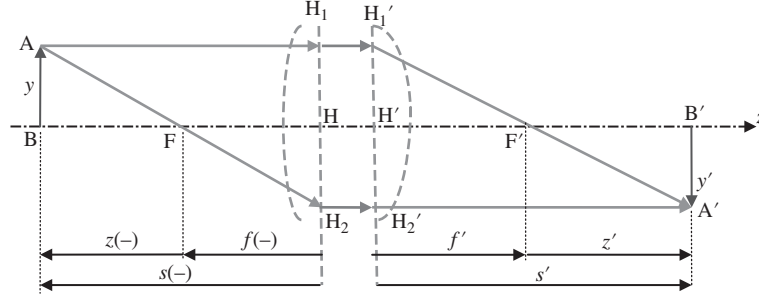


Figure 1.10 A method of drawing rays in order to obtain their image point

In Figure 1.10,

- ① A ray leaves object A, travels parallel to the axis, is refracted at the secondary principal plane H_1' and then passes through the second focal point F' .
- ② Another ray leaves A and passes through the first focal point F, is refracted at the primary principal plane H_2 , and then travels parallel to the axis.

Rays ① and ② are illustrated in Figure 1.10. Rays ① and ② meet again at A' after passing through the lens. This point will be the image point A' , that is, the lens will make an image $A'B'$ of the object AB.

1.5.4.2 Lateral Magnification β

The lateral magnification β is defined as the value of the height of the image divided by the height of the object. In Figure 1.10,

$$\Delta FAB \propto \Delta FH_2H, \quad \Delta F'H_1'H' \propto \Delta F'A'B'$$

Hence,

$$\beta = \frac{y'}{y} = -\frac{f}{z} = -\frac{z'}{f'} \quad (1.27)$$

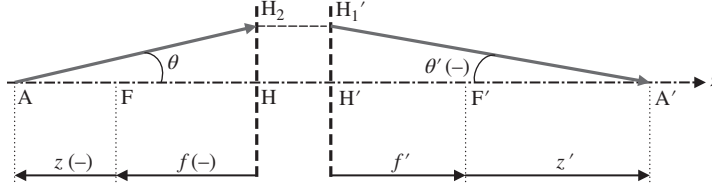
1.5.4.3 Newton's Formula

From Equation (1.27), the relation between z , z' , f , and f' can be expressed as follows:

$$zz' = ff' \quad (1.28)$$

Equation (1.28) is referred to as Newton's formula. When the lens is surrounded by air, we have $f = -f'$. The lateral magnification β can then be re-expressed as:

$$\beta = -\frac{f}{z} = -\frac{z'}{f'} = \frac{z' + f'}{z + f} = \frac{s'}{s} \quad (1.27')$$

Figure 1.11 Angular magnification γ

1.5.4.4 Angular Magnification γ

As shown in Figure 1.11, when a lens forms an image A' of an object A situated on the optical axis, then for any arbitrary ray proceeding from A and reaching A' , we have:

$$\tan \theta = \frac{HH_1}{HA} = \frac{HH_1}{(-z) + (-f)} \quad \tan(-\theta') = \frac{H'H_1'}{H'A'} = \frac{H'H_1'}{f' + z'}$$

where θ and θ' are the angles of the incident ray and the imaging ray, respectively, with the optical axis. Since $HH_1 = H'H_1'$,

$$\gamma \equiv \frac{\tan \theta'}{\tan \theta} = \frac{f + z}{f' + z'} \quad (1.29)$$

From Newton's formula, Equation (1.28), the following equation can be derived:

$$\frac{z}{f'} = \frac{f}{z'} \quad (1.28')$$

Combining Equation (1.29) and Equation (1.28'), we obtain:

$$\gamma = \frac{f + z}{f' + z'} = \frac{z}{f'} = \frac{f}{z'} \quad (1.30)$$

1.5.4.5 Longitudinal Magnification α

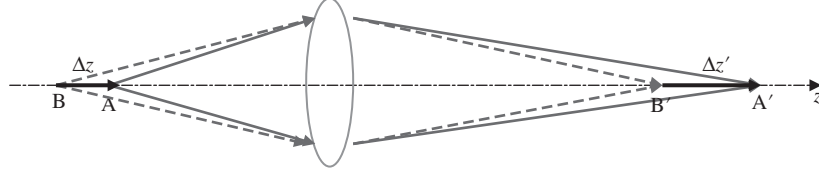
Longitudinal magnification denotes the factor by which an image increases in size, as measured along the optical axis. In Figure 1.12, the size of the object and the image along the optical axis are Δz and $\Delta z'$, respectively. Using Newton's formula, Equation (1.28), the longitudinal magnification α will be:

$$\alpha = \frac{\Delta z'}{\Delta z} = \frac{dz'}{dz} = -\frac{z'}{z} = -\frac{ff'}{z^2} \quad (1.31)$$

1.5.4.6 Relation between the Three Magnifications α , β , γ

Using Equation (1.27), Equation (1.30) and Equation (1.31), we can show that

$$\frac{\alpha}{\beta^2} = -\frac{f'}{f} \quad \beta\gamma = -\frac{f}{f'}$$

Figure 1.12 Longitudinal magnification α

When the lens is surrounded by air, we have $f = -f'$. The relation between α , β , and γ will then be:

$$\alpha = \beta^2 \quad (1.32a)$$

$$\gamma = \frac{1}{\beta} \quad (1.32b)$$

1.5.4.7 Paraxial Ray Equations with the Origin at the Principal Point

Suppose that the point object along the optical axis is O and that of its image is O' (the conjugate point of O), and the lateral magnification is β_0 . From Equation (1.27), the coordinates of O and O' will be:

$$-f/\beta_0 \text{ for O, and } -\beta_0 f' \text{ for O' .}$$

Let us postulate two more conjugate points A and A', with a lateral magnification of β . The coordinates of A and A' will then be:

$$-f/\beta \text{ for A, and } -\beta f' \text{ for A' .}$$

Now let us define s and s' as the coordinates of A and A', relative to their respective origins O and O'. If we define $s = OA$ and $s' = O'A'$, s and s' will then be:

$$s = -\frac{f}{\beta} - \left(-\frac{f}{\beta_0}\right) = -\frac{f}{\beta} + \frac{f}{\beta_0} \quad s' = -\beta f' - (-\beta_0 f') = -\beta f' + \beta_0 f'$$

β can be eliminated from the two preceding equations, as shown below:

$$\frac{f}{\beta_0 s} + \frac{\beta_0 f'}{s'} = 1$$

If we let $\beta_0 = 1$, then the above equation becomes:

$$\frac{f}{s} + \frac{f'}{s'} = 1 \quad (1.33)$$

In the special case where $\beta_0 = 1$, the objective point O and image point O' then coincide with the principal points H and H', respectively.

If the lens is surrounded by air, then $f = -f'$. Equation (1.33) can then be rewritten as Equation (1.24'):

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f'} \quad (1.24')$$



This equation is the same as the one which we previously obtained. If we rewrite f' as f ,

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \quad (1.24)$$

Once again, we have obtained the Gaussian lens formula. This time we have introduced principal points in order to allow us to treat actual lenses, which have finite thicknesses.

Figure 1.13 illustrates how the rays should be drawn, in accordance with Equation (1.24).

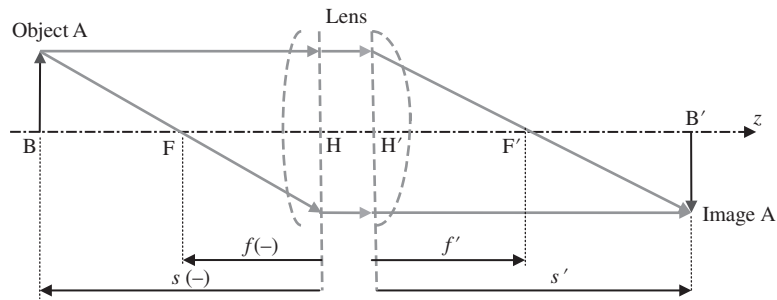


Figure 1.13 Illustration of rays, showing the lens, the object, and its image

where

- AB = Object
- A'B' = Image
- F = First focal point
- F' = Second focal point
- H = Primary principal point
- H' = Secondary principal point
- f = Focal length of the lens
- s = Distance HB of object from lens, which has a $-$ sign when s is to the left of the lens.
- s' = Distance H'B' of image from lens, which has a $+$ sign when s' is to the right of the lens.

As shown in Figure 1.13, both the distance of the object s and of its image s' will have a positive sign when they are to the right of the lens, and a negative sign when they are to the left of the lens. Likewise, the sign of the focal length f will be positive when the lens is convex and negative when it is concave.

1.5.5 Simple Lenses

For a simple lens having a refractive index of n , surface radii of r_1 and r_2 , and a central thickness of t , we can calculate the first and second focal lengths f and f' and the positions of primary and secondary principal points p and p' , using the following equations (Figure 1.14) (see Appendix A for supporting details).

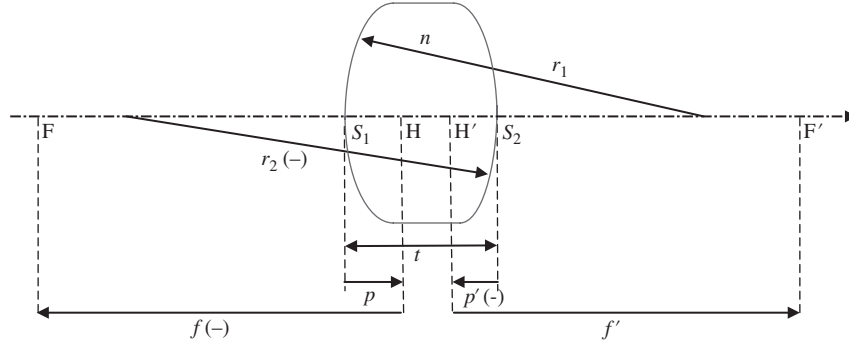


Figure 1.14 Focal length and principal point of a simple lens

$$\frac{1}{f'} = -\frac{1}{f} = (n-1) \left\{ \left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n-1)t}{nr_1r_2} \right\} \quad (1.34)$$

$$p = -\frac{r_1 t}{n(r_2 - r_1) + (n-1)t} \quad (1.35)$$

$$p' = -\frac{r_2 t}{n(r_2 - r_1) + (n-1)t} \quad (1.36)$$

where the sign of the range is positive (negative) when it is from left (right) to right (left), and $f = -f'$. These equations can be derived by treating the two parts of the lens, that is, the front surface and the back surface, as a combination.

If the curvatures of the front and back surface have the same value, we can substitute $r = r_1 = -r_2$ in the preceding equations. This yields the following result:

$$\frac{1}{f'} = -\frac{1}{f} = (n-1) \left\{ \frac{2}{r} - \frac{(n-1)t}{nr^2} \right\} \quad (1.37)$$

Given f , we can calculate the value of r using Equation (1.38).

$$r = \frac{n(n-1)f' + \sqrt{n^2(n-1)^2f'^2 - n(n-1)^2f't}}{n} \quad (1.38)$$

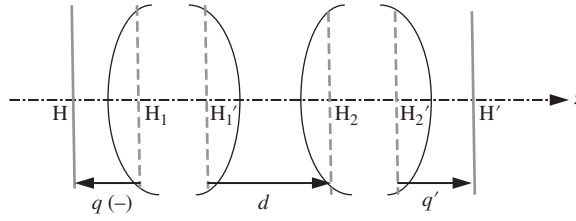


Figure 1.15 Combination of two simple lenses

1.5.6 The Focal Lengths and Principal Points of a Two-Lens Combination

Let us now consider a combination of two simple lenses. In Figure 1.15, let f_1 and f_1' denote the first and the second focal lengths of the first lens and let H_1 and H_1' denote the primary and secondary principal points, respectively. Let f_2 , f_2' , H_2 and H_2' denote the corresponding values for the second lens. The first and second focal lengths f and f' and the primary and the secondary principal points H and H' of the combination of two lenses can be calculated by applying the following equations. The results for a combination of three or more lenses can easily be obtained by iterating this process (see Appendix A for further details).

$$f = \frac{f_1 f_2}{d - f_1' + f_2} \quad (1.39)$$

$$f' = -\frac{f_1' f_2'}{d - f_1' + f_2} \quad (1.40)$$

$$q = H_1 H = \frac{f_1 d}{d - f_1' + f_2} \quad (1.41)$$

$$q' = H_2' H' = \frac{f_2' d}{d - f_1' + f_2} \quad (1.42)$$

where d is the distance between the principal points of the first and the second lenses.

$$d = H_1' H_2$$

We can calculate the focal lengths and the principal points of any arbitrary lens system by using the equations for a simple lens and for a combination of two lenses [Equation (1.34), Equation (1.35), Equation (1.36), Equation (1.39), Equation (1.40), Equation (1.41) and Equation (1.42)]. All of these equations can be derived from paraxial theory (see Appendix A).

1.6 The Five Seidel Aberrations

Paraxial theory, which we examined in the previous section, can yield accurate results for *paraxial* rays (which lie close to the optical axis and make a small angle to the optical axis). However, in real life, rays emitted from a point object travel outwards in *all* directions. For this reason, no lens can make

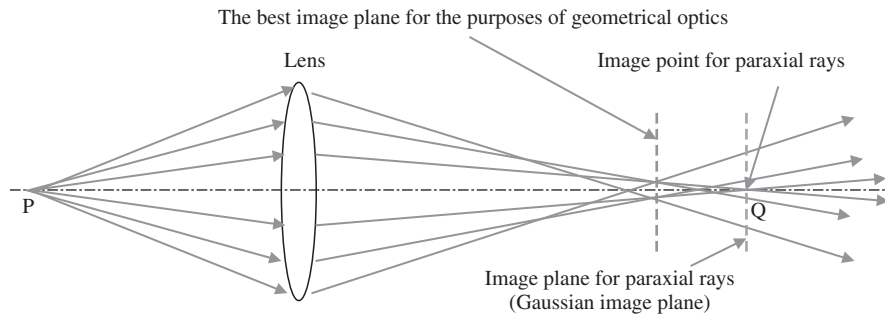


Figure 1.16 Spherical aberration

rays emitted from a point object converge exactly at a single point. *Aberration* can thus be defined as an imperfection in image formation by an optical system, caused by the inherent shortcomings of the lens. Aberrations fall into two main classes: chromatic aberrations and monochromatic aberrations. Here, we will confine our discussion of monochromatic aberrations to their significance in laser applications [2].

1.6.1 Monochromatic Aberration: A Brief Outline

Monochromatic aberrations include spherical aberration, coma aberration, astigmatism, field curvature, and distortion. These aberrations are referred to as the *five Seidel aberrations*, named after Philip Ludwig von Seidel, who first identified them in 1857.

1.6.1.1 Spherical Aberration

Spherical aberration is the only one of Seidel's five aberrations which occurs for an object lying on the optical axis. As shown in Figure 1.16, if rays from the point object P on the axis make a small angle with the optical axis, then after passing through the lens, they will converge on a point Q which also lies on the optical axis, and whose distance from the lens is equal to the distance calculated using paraxial theory, which we discussed in the previous section. However, rays passing through those parts of the lens which lie well above (or below) the optical axis will be refracted slightly further inwards than the paraxial, "ideal" rays which converge to form an image at point Q. For this reason, the image of a point object on the image plane which includes Q will not be a point, but a circle. This aberration is referred to as *spherical aberration*. The image plane on point Q is referred to as the Gaussian image plane, and the best image plane from the point of view of geometrical optics will be one which is located slightly closer to the lens than the Gaussian image plane.

1.6.1.2 Coma Aberration

Coma is an aberration which occurs for an object lying *off-axis*. In Figure 1.17, the ray from the off-axis point source P passing through the center of the entrance pupil of the lens will form a point image P' on the image plane. However, rays from P which pass through the large circle A on the entrance pupil will form a large circular image A' on the image plane. Likewise, the rays from P that pass through the small circle B on the pupil will form a small circular image B' on the image plane. Thus the image of the point source P will not be a point, but a small comet-shaped patch.

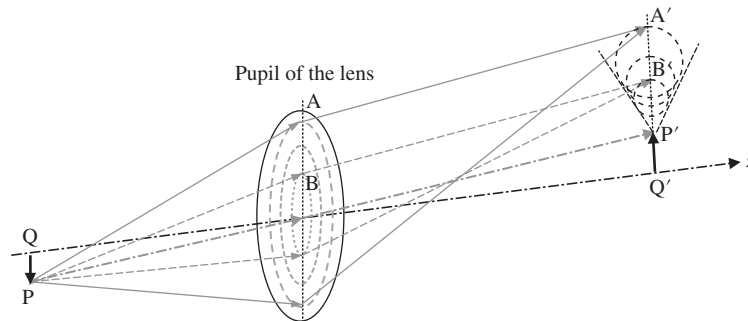


Figure 1.17 Coma aberration

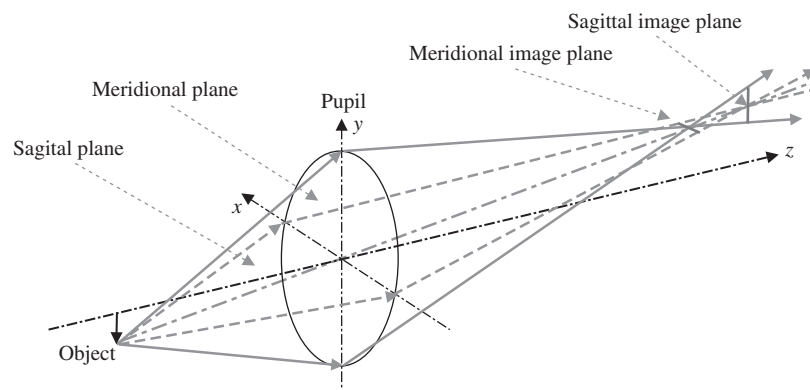


Figure 1.18 Astigmatism

1.6.1.3 Astigmatism

In Figure 1.18, we can define the meridional plane as the plane that includes the optical axis and the principal ray which passes through the center of the lens. The sagittal plane is the plane containing the principal ray which is also perpendicular to the meridional plane. Rays which proceed from a point which lies off the optical axis and then travel along the meridional plane will not converge on the same point as rays which proceed from the same off-axis point, but travel along the sagittal plane. The former will converge on the meridional image plane and the latter on the sagittal image plane. Because of this aberration, the image of the point object P will not be a point, but two images lying on different planes. This aberration is referred to as *astigmatism*.

1.6.1.4 Field Curvature

Due to aberrations in the optical system, a flat (planar) object cannot always form a perfectly flat image. Instead, the image will usually be on a curved surface. As shown in Figure 1.19, aberrations result when a flat screen is used to record the image formed by the curved image surface. This aberration is referred to as *field curvature*.

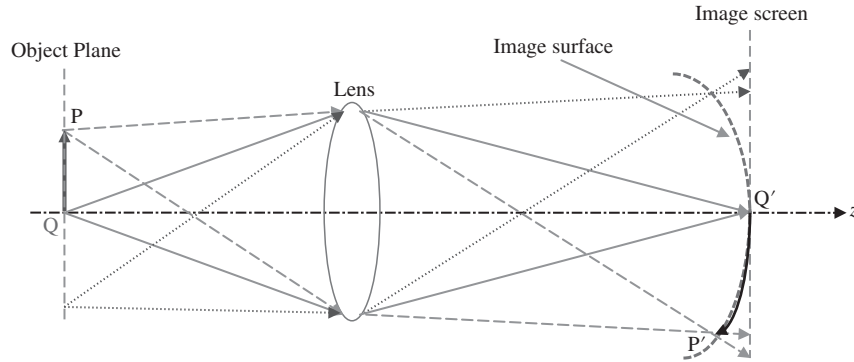


Figure 1.19 Aberration caused by field curvature

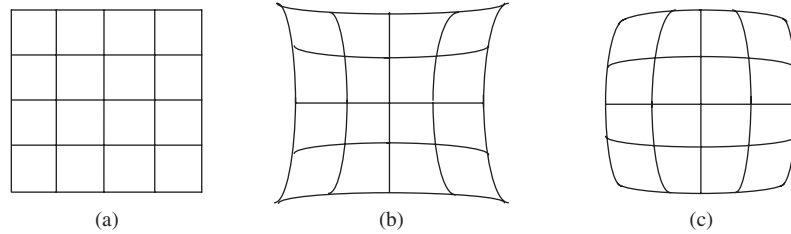


Figure 1.20 Distortion. (a) Original, (b) pin-cushion distortion, and (c) barrel distortion

1.6.1.5 Distortion

If we start with a picture such as the grid in Figure 1.20a as our object, the image formed may exhibit either pin-cushion distortion, as shown in Figure 1.20b, or barrel distortion, as in Figure 1.20c. These aberrations are collectively referred to as *distortion*.

1.6.2 Ray Aberration

Let us suppose that rays proceeding from a point object travel through an optical system and reach an image plane. In this process, (transverse) ray aberration denotes the distance between the rays on the ideal image plane between the ray passing through the actual optical system and the ideal optical system. In this section we will briefly discuss the magnitude of Seidel's five aberrations.

In Figure 1.21, a ray proceeds from an off-axis object and leaves the exit pupil at $Q(\rho, \varphi)$. The x - y coordinates of Q can be expressed as follows:

$$x = \rho \sin \varphi \quad y = \rho \cos \varphi$$

$P'(X, Y)$ denotes the coordinates of the ideal image point (for paraxial rays) on the ideal image plane.

Suppose that a ray starts at object P , leaves the exit pupil at Q , and creates an image at P'' . Due to aberrations in the optical system, P'' will be located some distance from the point P' where paraxial rays form their image. The distance between P' and P'' along the optical axis is referred to as *longitudinal aberration*. The ray QP'' intercepts the paraxial image plane (the $\varepsilon_x - \varepsilon_y$ plane

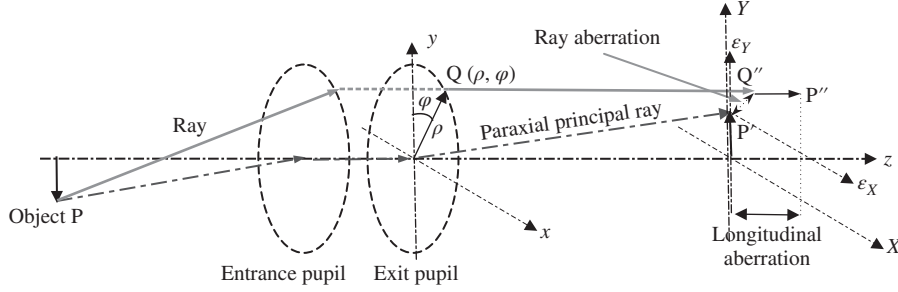


Figure 1.21 Ray aberration

in Figure 1.21) at Q'' . The distance between Q'' and P' along the $\epsilon_X - \epsilon_Y$ plane is referred to as *ray aberration*.

Ray aberration can be computed precisely, by performing a ray trace calculation, which applies the equations of refraction (Snell's law) to the boundaries of each lens surface. It is also possible to calculate ray aberration, using the following approximation:

$$\sin \theta \cong \theta - \frac{\theta^3}{3!} \quad \cos \theta \cong 1 - \frac{\theta^2}{2!}$$

We refer to the ray aberration using this approximation as a *third-order aberration*. The following are the third-order aberration results for ray aberration.

$$\epsilon_X = S_1 \rho^3 \sin \varphi + S_2 Y \rho^2 \sin 2\varphi + P Y^2 \rho \sin \varphi \quad (1.43a)$$

$$\epsilon_Y = S_1 \rho^3 \cos \varphi + S_2 Y \rho^2 (2 + \cos 2\varphi) + (S_3 + P) Y^2 \rho \cos \varphi + S_5 Y^3 \quad (1.43b)$$

We refer to these aberrations as *Seidel's five aberrations*, where

- S_1 = Spherical aberration coefficient
- S_2 = Coma coefficient
- S_3 = Astigmatism coefficient
- P = Petzval sum coefficient
- S_5 = Distortion coefficient
- ρ = Ray height at the exit pupil
- φ = Ray position angle at the exit pupil
- Y = Ideal image height on the image plane.

Equation (1.43a,b) describes the case where the image lies on the Y -axis. We have assumed this, in order to simplify the calculation results. Here, we are using lenses with spherical surfaces which are symmetrical with respect to the optical axis.

1.6.2.1 Spherical Aberration

In Equation (1.43a,b), if we set all of the coefficients to zero except for S_1 , then we get:

$$\epsilon_X^2 + \epsilon_Y^2 = (S_1 \rho^3)^2 \quad (1.44)$$



Equation (1.44) describes the equation of a circle. *Spherical aberration* distorts the image of a point object, so that it appears as a circle (Figure 1.16), whose radius is proportional to the cube of the ray height at the exit pupil. Since Equation (1.44) contains no term Y , the aberration observed will be independent of the image height. (In other words, the degree of aberration is uniform over the entire image plane.) The (\pm) sign appearing in front of the degree of spherical aberration will depend on whether the lens is convex or concave. The foregoing results imply that we can reduce spherical aberration by using a combination of lenses with convex and concave surfaces.

1.6.2.2 Coma Aberration

In Equation (1.43a,b), if we set all of the coefficients to zero except for S_2 , then we get:

$$\epsilon_X^2 + (\epsilon_Y - 2S_2 Y \rho^2)^2 = (S_2 Y \rho^2)^2 \quad (1.45)$$

Equation (1.45) describes the equation of a circle centered at $(0, 2S_2 Y \rho^2)$ and having a radius of $S_2 Y \rho^2$. Hence the aberration spot will be shaped like a comet, as in Figure 1.17. When $Y = 0$, the radius equals zero. Hence there will not appear to be any *coma aberration* for objects on the optical axis. For off-axis objects, coma aberration increases in proportion to ρ^2 (the square of the height of the ray, on exiting the pupil). The condition for there being no coma for a small object near the axis will be:

$$\frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = \beta \quad (1.46)$$

Equation (1.46) was derived by Abbe, where

- n_1, n_2 = Refractive indices of the media surrounding the object and the image, respectively
- θ_1, θ_2 = Angles of the ray to the optical axis when it leaves the object and when it reaches the image plane, respectively
- β = Lateral magnification.

Equation (1.46) is referred to as *Abbe's sine condition*. We will return to this equation in Section 1.7.

1.6.2.3 Astigmatism

In Equation (1.43a,b), if $S_3 \neq 0$ and $P = 0$, and if we set the other coefficients to zero, we get:

$$\epsilon_X = 0 \quad (1.47a)$$

$$\epsilon_Y = S_3 Y^2 \rho \cos \varphi \quad (1.47b)$$

Equation (1.47a,b) tells us that there is no aberration along the X -axis; rather there is an aberration of $\epsilon_Y = S_3 Y^2 \rho$ in a line along the Y -axis. Originally, we assumed that the image lay on the Y -axis. However, the image may have both X - and Y -coordinates. By symmetry, the aberration could also be in a line along the X -axis, on a slightly different image plane. Thus, the beam from the off-axis object will converge on the two different image planes. We refer to this phenomenon as *astigmatism* (Figure 1.18).

1.6.2.4 Aberration Caused by (Petzval) Field Curvature

In Equation (1.43a,b), if $S_3 = 0$ and $P \neq 0$, and if we set the other coefficients to zero, we get:

$$\epsilon_X = P Y^2 \rho \sin \varphi \quad (1.48a)$$

$$\epsilon_Y = P Y^2 \cos \varphi \quad (1.48b)$$



From Equation (1.48a,b) we obtain:

$$\epsilon_x^2 + \epsilon_y^2 = (PY^2\rho)^2 \quad (1.49)$$

Equation (1.49) describes the equation of a circle with a radius of $PY^2\rho$. The aberration radius will be proportional to ρ and Y^2 (Figure 1.19).

1.6.2.5 Distortion

In Equation (1.43a,b), if $S_5 \neq 0$, and if we set the other coefficients to zero, we get:

$$\epsilon_x = 0 \quad (1.50a)$$

$$\epsilon_y = S_5 Y^3 \quad (1.50b)$$

Equation (1.50a,b) entails that this aberration will be proportional to the cube of the image height Y . In other words, this aberration will equal zero at the center of the image, but will increase rapidly as we approach the periphery. Distortion can be thought as an aberration caused by lateral magnification error, and depending on the image height (Figure 1.20).

1.7 The Sine Condition

1.7.1 The Abbe Sine Condition

When an optical system has no aberration for an on-axis object which emits wide angle (diverging) rays, the necessary condition for an object situated just a little way from the on-axis object in a perpendicular direction also having no aberration is referred to as the Abbe sine condition [3]. In other words, the sine condition is a necessary condition for the absence of coma in an optical system which has no spherical aberration. This condition can be described by the following:

$$\frac{n \sin \theta}{n' \sin \theta'} = \beta = \frac{y'}{y} \quad (1.46')$$

where

β = Lateral magnification

y = Height of object

y' = Height of image

θ = Angle of the incident ray to the optical axis

θ' = Angle of the imaging ray to the optical axis

n = Refractive index of medium surrounding object

n' = Refractive index of medium surrounding image.

The lateral magnification β can be expressed as $\beta = \frac{s'/n'}{s/n} = \frac{ns'}{n's}$. Then, Equation (1.46') can then be rewritten as:

$$s \sin \theta = s' \sin \theta' \quad (1.51)$$

We can now explain the significance of Equation (1.51). In the previous section, we examined the significance of the primary and the secondary principal plane in paraxial theory. However, as θ and θ'

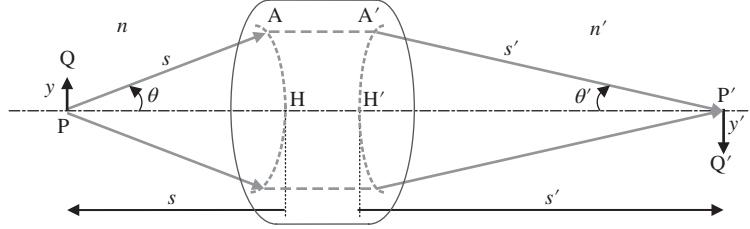


Figure 1.22 Abbe sine condition

get larger, we have to go beyond the scope of paraxial theory. Under these more general conditions, instead of using the term “principal planes,” we should speak of the primary principal surface (which is a sphere centered at the object P, on whose surface the primary principal point lies), and the secondary principal surface (which is a sphere centered at the image P', on whose surface the secondary principal point lies). If the sine condition is satisfied, then the line connecting the point A (where the incident ray proceeding from the on-axis object P intersects with the primary principal surface) to the point A' (where the imaging ray traveling to the on-axis image P' intersects with the secondary principal surface) will always run parallel to the optical axis (Figure 1.22).

1.7.2 The Sine Condition for an Off-Axis Object and Its Off-Axis Image

The sine condition described above holds for an object and its image, on the optical axis. However, a similar condition can also be satisfied by an off-axis object and its image [3]. In Figure 1.23, if there is no aberration for the object P on the axis and also no aberration for the off-axis object Q which lies on a plane which is perpendicular to the optical axis and passes through the on-axis object P, then the following equation will be satisfied.

$$\frac{n(\sin \theta - \sin \theta_0)}{n'(\sin \theta' - \sin \theta'_0)} = \beta_M \equiv \frac{\delta Y'}{\delta Y} \quad (1.52)$$

where

- θ_0 = Angle of the principal ray of the beam proceeding from the off-axis object Q
- θ'_0 = Angle of the principal ray of the imaging beam to the off-axis Image Q'
- θ = Angle of the meridional ray proceeding from the off-axis object Q
- θ' = Angle of the meridional ray of the imaging beam to the off-axis image Q'
- β_M = Lateral magnification for a small object δY on Q, with respect to the meridional ray ($=\delta Y'/\delta Y$).

If the image height Y' is small and there is no distortion, then β_M can be assumed to equal β :

$$\beta_M \cong \beta \quad (1.53)$$

We can apply the concept of the principal surface to off-axis objects, just as we did for objects lying on the optical axis. In Figure 1.24, the primary principal surface will touch the point where the principal ray

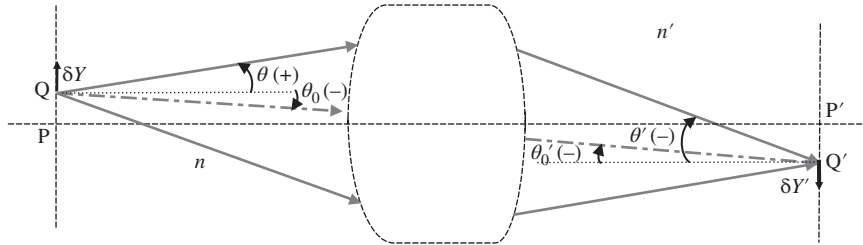


Figure 1.23 Sine condition for off-axis object and its image

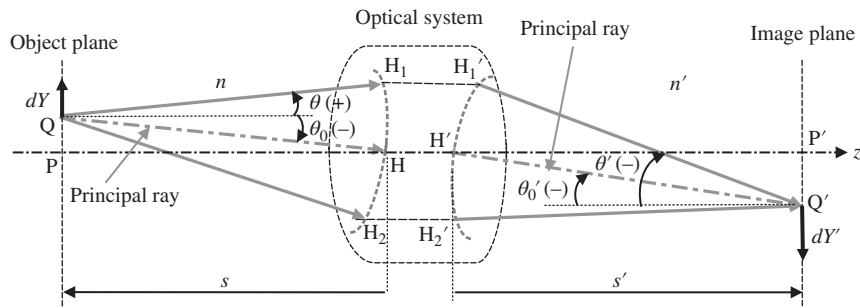


Figure 1.24 Principal surfaces for an off-axis object and its image

from the object Q intersects with the optical axis, while the secondary principal surface will touch the point where the principal ray traveling to the image Q' intersects with the optical axis. If the optical system is surrounded by air and the image height is small, it can be assumed that the primary and secondary principal surfaces lie on the primary and secondary principal points, and whose curvature centers are situated at the object Q and its image Q' , respectively. (In this case, the principal ray will pass through the principal points.)

We can understand why Equation (1.52) and Equation (1.53) will yield similar results to the Sine condition. As in Figure 1.24, if the optical system has no aberration, an arbitrary ray proceeding from the object at Q will intercept the primary principal surface at H_1 , and then intercept the secondary principal surface at H_1' . The ray H_1H_1' will always be parallel to the optical axis.

If the incident beam is collimated as shown in Figure 1.25, the primary principal surface will be a plane which is normal to the incident beam and is located at the intersection of the principal ray and the optical axis. In this case, the lines H_1H_1' and H_2H_2' in Figure 1.25 will always be parallel to the optical axis, as in Figure 1.24.

1.8 Aplanatic Lenses

A lens which is free from spherical and coma aberration is referred to as an aplanatic lens [3]. The theory of geometrical optics predicts that meniscus lenses can be used for configuring optical systems that have both a low aberration and a large NA (numerical aperture).

An aplanatic lens has one aplanatic surface (a sphere). In Figure 1.26a, if the object P and its image P' are related to each other as in Equation (1.54a) and Equation (1.54b), then spherical aberration and

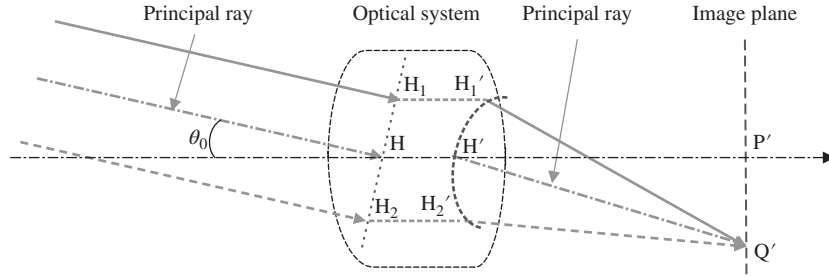


Figure 1.25 Principal surfaces for an off-axis object and its image (collimated beam incidence)

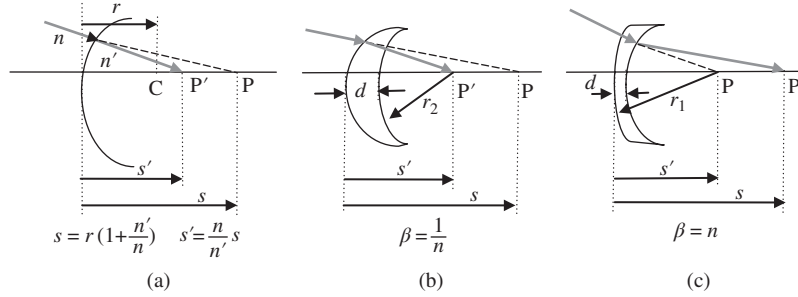


Figure 1.26 Aplanatic sphere and aplanatic lenses. (a) Aplanatic sphere, (b) convex lens, and (c) concave lens

coma aberration will both vanish.

$$s = r \left(1 + \frac{n'}{n} \right) \quad (1.54a)$$

$$s' = r \left(1 + \frac{n}{n'} \right) = \frac{n}{n'} s \quad (1.54b)$$

The lateral magnification β will then be:

$$\beta = \frac{s'/n'}{s/n} = \frac{s'}{s} \frac{n}{n'} = \left(\frac{n}{n'} \right)^2 \quad (1.54c)$$

Both convex and concave lenses can be regarded as aplanatic lenses which satisfy Equation (1.54a–c). An aplanatic lens surrounded by air can be described as follows:

- *Convex lens:* Let $n = 1$ and let $n' = n$. Then we can describe the first surface of the lens as an aplanatic sphere, while the second surface is a sphere centered at point P' , and having a radius of r_2 . Let d denote the thickness of the lens (Figure 1.26b). The radii of the first and the second surfaces will then be:

$$r_1 = \frac{s}{n+1} \quad (1.55a)$$

$$r_2 = \frac{s}{n} - d \quad (1.55b)$$

$$\beta = \frac{1}{n} \quad \beta : \text{Lateral magnification} \quad (1.55c)$$



- *Concave lens*: Let $n = n'$ and let $n' = 1$. Then we can describe the second surface as an aplanatic sphere, while the first surface is a sphere centered at P, and having a radius of r_1 . Let d denote the thickness of the lens (Figure 1.26c). The radii of the first and the second surfaces will then be:

$$r_1 = s \quad (1.56a)$$

$$r_2 = (s - d) \frac{n}{(n + 1)} \quad (1.56b)$$

$$\beta = n \quad \beta : \text{Lateral magnification} \quad (1.56c)$$

Aplanatic surfaces can reduce spherical aberration and coma to zero, no matter what the value of the NA. For this reason, they are very effective when used in optical systems with large NA, such as a microscope having both a large magnification and a large NA. However, aplanatic lenses always have a positive lateral magnification – that is, they form virtual images. For real-life applications, we need to use aplanatic lenses in combination with lenses that form actual images. We can use this combination to alter the focal length of an optical system, without increasing the aberration.

1.9 Reflection and Transmission

Reflectance and transmittance refer to the ratios of reflected and transmitted light to the total amount of incident light respectively, at the boundary between different media [2, 4]. Reflectance and transmittance values depend on the polarization angle. In this section, we will briefly examine how reflectance and transmittance vary in relation to the direction of polarization. A polarized ray can be thought of as being composed of two orthogonal linearly polarized elements: one parallel to the plane which contains the incident, reflected and refracted rays (p), and the other, perpendicular to that (s).

In Figure 1.27, A_i , A_r , and A_t represent the amplitudes of the incident wave, the reflected wave, and the transmitted wave, respectively. Each amplitude can be broken down into “ p ” and “ s ” components, where “ p ” denotes the polarization component which is parallel to the plane containing the incident, reflected, and refracted rays (i.e., parallel to the page), while “ s ” denotes the polarization component which is vertical to the plane described above (i.e., perpendicular to the page).

1.9.1 Angles of Reflection and Refraction

In Figure 1.27, the relationship between the angle of incidence θ_i , the angle of reflection θ_r , and the angle of transmission θ_t will be:

$$\theta_r = \pi - \theta_i \quad (1.1')$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad (1.2)$$

where

n_1 = Refractive index of the medium through which the incident light propagates

n_2 = Refractive index of the medium through which the transmitted light propagates.

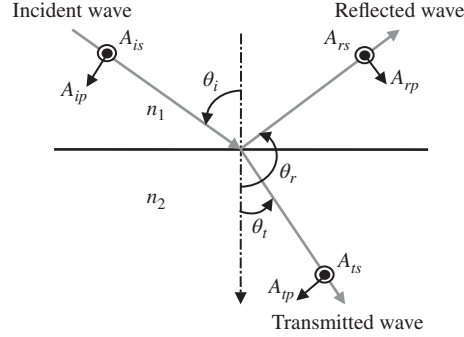


Figure 1.27 Reflection and refraction of light

1.9.2 Amplitude Reflection and Transmission Coefficients

In Figure 1.27, the amplitude reflection coefficients r_p , r_s and the amplitude transmission coefficients t_p , t_s will be as follows:

$$r_p \equiv \frac{A_{rp}}{A_{ip}} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \quad (1.57)$$

$$r_s \equiv \frac{A_{rs}}{A_{is}} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (1.58)$$

$$t_p \equiv \frac{A_{tp}}{A_{ip}} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)} = \frac{2n_1 \cos \theta_i}{n_2 \cos \theta_i + n_1 \cos \theta_t} \quad (1.59)$$

$$t_s \equiv \frac{A_{ts}}{A_{is}} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (1.60)$$

1.9.3 Reflectance and Transmittance

The reflectance R and transmittance T are as follows:

$$R = \frac{|A_r|^2}{|A_i|^2} = |r|^2 \quad (1.61)$$

$$T = \frac{n_2 \cos \theta_t |A_t|^2}{n_1 \cos \theta_i |A_i|^2} = \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} |t|^2 \quad (1.62)$$

R can be simply expressed as the square of r ; however, as we can see, the expression of T is not quite that simple. Another expression for T is:

$$T = 1 - R \quad (1.63)$$

The above equations are derived from the boundary condition. The theoretical explanation has been omitted here.

Calculations for r , t , R , and T in the case where a ray of light traveling through the air is incident upon a glass surface (assume that $n_1 = 1$, $n_2 = 1.5$) are shown in Table 1.1.

Table 1.1 Calculations for r , t , R , and T

θ_i degrees	$n_1 = 1$						$n_2 = 1.5$					
	θ_i degrees	r_p	r_s	t_p	t_s	R_p	R_s	T_p	T_s			
0	0	0.2	-0.2	0.8	0.8	0.04	0.04	0.96	0.96			
5	3.3309802	0.1989814	-0.201018	0.7993209	0.7989818	0.0395936	0.0404083	0.9604064	0.9595917			
10	6.6477771	0.1958864	-0.204107	0.7972576	0.7958934	0.0383715	0.0416595	0.9616285	0.9583405			
15	9.9358837	0.190595	-0.209368	0.79373	0.7906317	0.0363265	0.0438351	0.9636735	0.9561649			
20	13.180142	0.1828976	-0.216981	0.7885984	0.7830186	0.0334515	0.0470809	0.9665485	0.9529191			
25	16.36441	0.1724799	-0.227208	0.7816533	0.7727919	0.0297493	0.0516235	0.9702507	0.9483765			
30	19.471221	0.1588998	-0.240408	0.7725999	0.7595918	0.0252491	0.0577961	0.9747509	0.9422039			
35	22.48145	0.1415526	-0.257058	0.7610351	0.7429421	0.0200371	0.0660788	0.9799629	0.9339212			
40	25.373994	0.1196225	-0.277773	0.746415	0.7222272	0.0143095	0.0771577	0.9856905	0.9228423			
45	28.125506	0.0920134	-0.303337	0.7280089	0.696663	0.0084665	0.0920134	0.9915335	0.9079866			
50	30.710221	0.0572497	-0.334736	0.7048332	0.6652637	0.0032775	0.1120484	0.9967225	0.8879516			
56.31	33.690098	-7.09E-07	-0.384616	0.6666662	0.615384	5.031E-13	0.1479295	1	0.8520705			
60	35.26439	-0.042449	-0.420204	0.6383672	0.5797959	0.0018019	0.1765715	0.9981981	0.8234285			
65	37.17167	-0.113857	-0.477559	0.5907621	0.5224406	0.0129634	0.228063	0.9870366	0.771937			
70	38.789556	-0.206132	-0.547352	0.5292453	0.4526476	0.0424904	0.2995947	0.9575096	0.7004053			
75	40.087037	-0.326749	-0.631946	0.4488338	0.3680538	0.1067651	0.399356	0.8932349	0.600644			
80	41.036411	-0.486635	-0.73389	0.3422432	0.2661097	0.2368138	0.5385949	0.7631862	0.4614051			
85	41.615601	-0.70232	-0.855772	0.1984531	0.1442281	0.4932538	0.7323455	0.5067462	0.2676545			
90	41.810315	-1	-1	0	0	1	1	0	0			

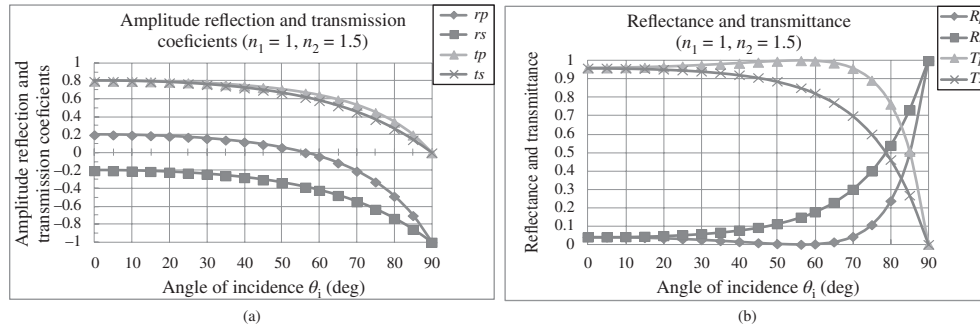


Figure 1.28 (a) Calculation results for r and t and (b) calculation results for R and T

The graphs in Figure 1.28 show that the amplitude reflection coefficients r_p , r_s , and the reflectances R_p , R_s of glass ($n = 1.5$) when surrounded by air will be as follows:

when $\theta_i = 0$, $R_p = 0.04$, $R_s = 0.04$, $r_p = 0.2$, $r_s = -0.2$ and
 when $\theta_i = 56.31^\circ$, $R_p = 0$, $R_s = 0.148$, $r_p = 0$, $r_s = 0.385$.

The results calculated above mean that when light is incident vertically on the glass, the reflectance will be 4%, and the amplitude reflection coefficient will be 20%.

When the angle of incidence is 56.31° , there will be no reflection for light rays polarized parallel to the incident plane, that is, there will be perfect (100%) transmission. This angle is referred to as Brewster's angle.

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