

## CHAPTER 1

# Principles of food engineering

## 1.1 Introduction

### 1.1.1 The Peculiarities of food engineering

Food engineering is based to a great extent on the results of chemical engineering. However, the differences in overall structure between chemicals and foods, that is, the fact that the majority of foods are of cellular structure, result in at least three important differences in the operations of food engineering – the same is valid for biochemical engineering.

- 1 Chemical engineering applies the Gibbs theory of multicomponent chemical systems, the principal relationships of which are based on chemical equilibrium, for example, the Gibbs phase rule. Although the supposition of equilibrium is only an approximation, it frequently works and provides good results. In the case of cellular substances, however, the *conditions of equilibrium do not apply in general*, because the cell walls function as semipermeable membranes, which make equilibrium practically possible only in aqueous media and for long-lasting processes. Consequently, the Gibbs phase rule cannot be a basis for determining the degrees of freedom of food engineering systems in general. For further details, see Section 1.3.2.
- 2 Another problem is that cellular substances prove to be chemically very complex after their cellular structure has been destroyed. In the Gibbs theory, the number of components in a multicomponent system is limited and well defined, not infinite. *The number of components in a food system can be practically infinite or hard to define; in addition, this number depends on the operational conditions.* Certainly, we can choose a limited set of components for the purpose of a study – and this is the usual way – but this choice will not guarantee that exclusively those components will participate in the operation considered.

Therefore, interpretation of the degrees of freedom in food engineering systems causes difficulties and is often impossible, because the number and types of participants (chemical compounds, cell fragments, crystalline substances, etc.) in food operations are hard to estimate: many chemical and physical changes may take place simultaneously, and a small change in the conditions (temperature, pH, etc.) may generate other types of chemical or physical changes. If we compare this situation with a complicated heterogeneous catalytic chemical process with many components, it is evident that

in food engineering we struggle with complex tasks that are not easier, only different.

Evidently, comminution plays a decisive role in connection with these peculiarities. However, in the absence of comminution, these two peculiarities – the existence of *intact cell wall* as barriers to equilibrium and the very high number of operational participants – may appear together as well; for example, in the roasting of cocoa beans, the development of flavours takes place inside *unbroken* cells. In such cases, cytological aspects (depot fat, mitochondria, etc.) become dominant because the cell itself works as a small chemical plant, the heat and mass transfer of which cannot be influenced by traditional (e.g. fluid-mechanical) means. This problem is characteristic of biochemical engineering.

- 3 The third peculiarity, which is a consequence of the cellular structure, is that the operational *participants* in food engineering may be not only chemical compounds, chemical radicals and other molecular groups but also *fragments of comminuted cells*.

In the case of chemical compounds/radicals, although the set of these participants can be infinitely diverse, the blocks from which they are built are well defined (atoms), the set of atoms is limited and the rules according to the participants are built are clear and well defined.

In the case of cellular fragments, none of this can be said. They can, admittedly, be classified; however, any such classification must be fitted to a given task without any possibility of application to a broader range of technological problems. This is a natural consequence of the fact that the fragments generated by comminution, in their infinite diversity, do not manifest such conspicuous qualitative characteristics as chemicals; nevertheless, they can be distinguished because slight differences in their properties, which occur by accident because of their microstructure, may become important.

This situation may be understood as the difference between discrete and continuous properties of substances: while chemical systems consist of atoms and combinations of them, to which stoichiometry can be applied, the systems of food engineering cannot be built up from such well-defined elements. This stoichiometry means that well-defined amounts by mass (atomic masses or molecular masses) may be multiplied by integers in order to get the mass fluxes in a reaction. However, in the recipes that are used for describing the compositions of foods, the mass fluxes are treated as continuous variables, contrary to the idea of stoichiometry.

### 1.1.2 The hierarchical and semi-hierarchical structure of materials

Although foods also consist of atoms in the final analysis, it is characteristic of food engineering that it does not go to an elementary decomposition of the entire raw material; however, a certain part of the raw material will be chemically

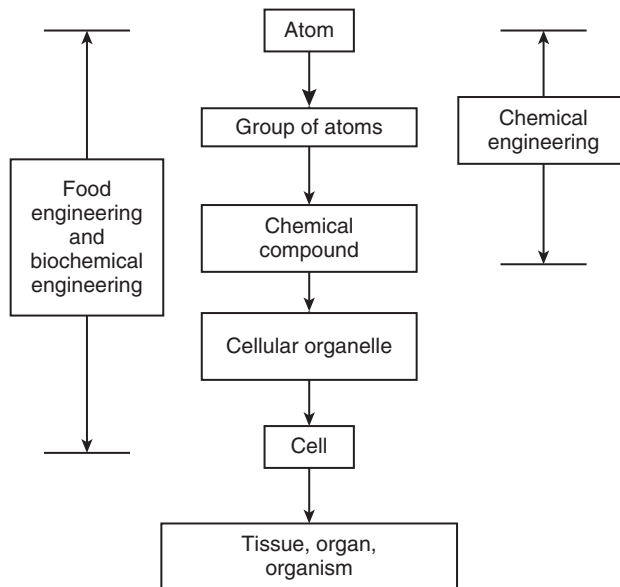
modified, and another part will be modified at the level of cells (by comminution). The structures of materials are hierarchical, where the levels of the hierarchy are joined by the *containing relation*, which is reflexive, associative and transitive (but not commutative):  $A \rightarrow B$  means that B contains A, that is,  $\rightarrow$  is the symbol for the containing relation. The meaning of the reflexive, associative and transitive properties is:

- Reflexive: A contains itself.
- Associative: if  $A \rightarrow (B \rightarrow C)$ , then  $(A \rightarrow B) \rightarrow C$ .
- Transitive: if  $A \rightarrow B \rightarrow C$ , then  $A \rightarrow C$  (the property is inheritable).

The transitive property is particularly important: if  $A = \text{atom}$ ,  $B = \text{organelle}$  and  $C = \text{cell}$  (considered as levels), then the transitive relation means that if an organelle (at level B) contains an atom (at level A) and if a cell (at level C) contains this organelle (at level B), then that cell (at level C) contains the atom in question (at level A) as well.

The hierarchical structure of materials is illustrated in Figure 1.1. For the sake of completeness, Figure 1.1 includes the hierarchical levels of tissue, organs and organisms, which are of interest when one is choosing ripened fruit, meat from a carcass and so on. In a sense, the level of the organism is the boundary of the field of food (and biochemical) engineering.

This hierarchical structure is characteristic of cellular materials only when they are in an intact, unbroken state. Comminution may disrupt this structure; for example, if cellular fragments are dispersed in an aqueous solution and these fragments may themselves contain aqueous solutions as natural ingredients,



**Figure 1.1** Hierarchical structure of materials.

then these relations can be represented by

$$A_1 \rightarrow C \rightarrow A_2$$

where  $A_1$  represents the natural ingredients of a cell (an aqueous solution),  $C$  represents the cellular material and  $A_2$  represents the aqueous solution in which the cellular material is dispersed. Evidently, in this case, the hierarchical levels are mixed, although they still exist to some extent. Therefore, for such cases of bulk materials, the term *semi-hierarchical structure* seems more appropriate.

If we allow that the degrees of freedom cannot be regarded as the primary point of view, a more important, in fact crucial, question is whether the set of chemical and/or physical changes that occur in an operation can be defined at all. The answer is difficult, and one must take into consideration the fact that an exact determination of this set is not possible in the majority of cases. Instead, an approximate procedure must be followed that defines the decisive changes and, moreover, the number and types of participants. In the most favourable cases, this procedure provides the result (i.e. product) needed.

## 1.2 The Damköhler equations

### 1.2.1 The application of the Damköhler equations in food engineering: conservative substantial fragments

In spite of the differences discussed earlier, the Damköhler equations, which describe the conservation of the fluxes of mass, component, heat and momentum, can provide a mathematical framework from the field of chemical engineering that can be applied to the tasks in food engineering (and biochemical engineering), with a limitation referring to the flux of component.

The essence of this limitation is that the entire set of components cannot be defined in any given cases. This limitation has to be taken into account by defining both the chemical components studied and their important reactions. The conservation law of component fluxes does hold *approximately* for this partial system. The correctness of the approximation may be improved if this partial set approaches the entire set of components. For example, if we consider the backing of biscuit dough, it is impossible to define all the chemical reactions taking place and all the components participating in them; therefore, the conservation equations for the components cannot be exact, because of the disturbing effect of by-reactions. However, what counts as a by-reaction? This uncertainty is the source of inaccuracy.

The conservation equations for mass, heat and momentum flux can be used without any restriction for studying the physical (and mechanical) operations since their concern is bulk materials. In Appendix 5, the concepts *conservative elements* and *conservative substantial fragments* are discussed in detail. In food

engineering, the concept conservative substantial fragment can substitute the concept of conservative elements (Mohos, 1982) which latter are essential in the chemistry. The epithet *conservative* practically means here that the Damköhler conservation equations hold also for these fragments. For example, diffusion of humidity (water) in cellular substances can be regarded if the other fraction of the substances were unchanged, that is, for the water content the Fick equation [see the Damköhler equation (1.4)] were exactly hold. However, it has to be mentioned that the conservation of these fragments are determined by the technological (physical and chemical) conditions. The situation is the same as in the case of atoms: in certain conditions also the atoms are splitting.

### 1.2.2 The Damköhler equations in chemical engineering

This chapter principally follows the ideas of Benedek and László (1964). Some further important publications (although not a comprehensive list) that are relevant are Charm (1971), Pawlowski (1971), Schümmer (1972), Meenakshi Sundaram and Nath (1974), Loncin and Merson (1979), Stephan and Mitrovic (1984), Zlokarnik (1985), Mahiout and Vogelpohl (1986), Hallström *et al.* (1988), Stichlmair (1991), VDI-Wärmeatlas (1991), Zogg (1993), Chopey (1994), Stieß (1995), Perry (1998), Hall (1999), Sandler (1999), McCabe *et al.* (2001), Zlokarnik (2006) and Dobre and Marcano (2007).

According to Damköhler, chemical–technological systems can be described by equations of the following type:

$$\text{convection} + \text{conduction} + \text{transfer} + \text{source} = \text{local change} \quad (1.1)$$

In detail,

$$\text{div}[\Gamma \mathbf{v}] - \text{div}[\delta \text{grad } \Gamma] + \omega \varepsilon \Delta \Gamma + G = -\frac{\partial \Gamma}{\partial t} \quad (1.2)$$

where  $\mathbf{v}$  is the linear velocity (in units of m/s);  $\Gamma$  is a symbol for mass, a component, heat or momentum;  $\delta$  is the generalized coefficient of convection ( $\text{m}^2/\text{s}$ );  $\omega$  is the transfer surface area per unit volume ( $\text{m}^2/\text{m}^3$ );  $\varepsilon$  is the generalized coefficient of transfer;  $G$  is the flux of source and  $t$  is the time (s). Such equations can be set up for fluxes of mass, components, heat and momentum.

The Damköhler equations play a role in chemical and food engineering similar to that of the Maxwell equations in electrodynamics. The application of the Damköhler equations to food-technological systems is presented in Chapter 2. Let us consider these equations one by one.

*Flux of mass:*

$$\text{div}[\rho \mathbf{v}] - [D \text{grad } \rho] + \omega \beta' \Delta \rho + G = -\frac{\partial \rho}{\partial t} \quad (1.3)$$

where  $\mathbf{v}$  is the linear velocity (m/s),  $\rho$  is the density ( $\text{kg}/\text{m}^3$ ),  $\beta'$  is the mass transfer coefficient (m/s),  $D$  is the self-diffusion coefficient ( $\text{m}^2/\text{s}$ ) and  $G$  is the source of mass flux ( $\text{kg}/\text{m}^3 \text{ s}$ ).

Flux of a component:

$$\text{div}[c_i \mathbf{v}] - \text{div}[D \text{grad } c_i] + \omega \beta \Delta c_i + v_i r = -\partial c_i / \partial t$$

Fick's 2nd law (1.4)

where  $c_i$  is the concentration of the  $i$ th component ( $\text{mol}/\text{m}^3$ ),  $D$  is the diffusion coefficient ( $\text{m}^2/\text{s}$ ),  $\beta$  is the component transfer coefficient ( $\text{m}/\text{s}$ ),  $v_i$  is the degree of reaction for the  $i$ th component and  $r$  is the velocity of reaction [ $(\text{mol}/(\text{m}^3 \text{ s}))$ ].

Flux of heat:

$$\text{div}[\rho c_p T \mathbf{v}] - \text{div}[\lambda \text{grad } T] + \omega \alpha \Delta T + v_i r \Delta H = -\partial(\rho c_p T) / \partial t$$

Fourier's 2nd law Newton's law of cooling (1.5)

where  $c_p$  is the specific heat ( $p$  is constant) [ $\text{J}/(\text{kg K})$ ],  $T$  is the temperature (K),  $\lambda$  is the thermal conductivity ( $\text{W}/\text{m K}$ ),  $\Delta H$  is the heat of reaction ( $\text{J}/\text{mol}$ ) and  $\alpha$  is the heat transfer coefficient [ $\text{J}/(\text{m}^2 \text{ s K})$ ].

The flux of momentum is described by the Navier–Stokes law,

$$\text{Div}\{\rho \mathbf{v} \cdot \mathbf{v}\} - \text{Div}\{\eta \text{Grad } \mathbf{v}\} + \omega \gamma \Delta \mathbf{v} + \text{grad } p = -\frac{\partial[\rho \mathbf{v}]}{\partial t} \tag{1.6}$$

where Div is tensor divergence, Grad is tensor gradient,  $\cdot$  is the symbol for a dyadic product,  $\eta$  is dynamic viscosity [ $\text{kg}/(\text{m s})$ ],  $\gamma = (f' \rho \mathbf{v} / 2)$  is coefficient of momentum transfer [ $\text{kg}/(\text{m}^2 \text{ s})$ ],  $f'$  is frictional (or Darcy -) coefficient [dimensionless], and  $p$  is pressure [ $\text{kg}/(\text{m s}^2)$ ].

Equations (1.3)–(1.6) are called the *Damköhler equation system*.

In general, the Damköhler equations cannot be solved by analytical means. In some simpler cases, described later, however, there are analytical solutions. For further details, see Grassmann (1967), Charm (1971), Loncin and Merson (1979), Hallström *et al.* (1988) and Banks (1994).

### 1.3 Investigation of the Damköhler equations by means of similarity theory

#### 1.3.1 Dimensionless numbers

Let us suppose that a set of Damköhler equations called *Form 1* are valid for a technological system called *System 1*, and a set of equations *Form 2* are valid for *System 2*. It is known from experience that if similar phenomena take place in the two systems, then this similarity of phenomena can be expressed by a relationship denoted by  $\sim$ , as in *Form 1*  $\sim$  *Form 2*. Similarity theory deals with the description of this relationship.

The simplest characteristics of this similarity are the ratios of two geometric sizes, two concentrations and so on. These are called *simplex values*.

### 1.3.1.1 Complex values

The first perception of such a relationship is probably connected with the name of Reynolds, who made the observation, in relation to the flow of fluids, that System 1 and System 2 are similar if *the ratios of momentum convection to momentum conduction in these systems are equal to each other*.

Let us consider Eqn (1.1),

$$\text{convection} + \text{conduction} + \text{transfer} + \text{source} = \text{local change} \quad (1.1)$$

for *momentum flux*. Since the terms for convection, conduction and so forth on the left-hand side evidently have the same dimensions in the equation, their ratios are dimensionless. One of the most important dimensionless quantities is the ratio of momentum convection to momentum conduction, which is called the *Reynolds number*, denoted by *Re*.  $Re = Dv\rho/\eta$ , where *D* is a geometric quantity characteristic of the system and *v* is a linear velocity,

$$v = \frac{Q}{R^2\pi} \quad (1.7)$$

where *Q* is volumetric flow rate (m<sup>3</sup>/s) and *R* is radius of tube (m).

For conduits of non-circular cross section, the definition of the *equivalent diameter*  $D_e$  is

$$D_e = \frac{\text{area of stream cross section}}{\text{wetted perimeter}} \quad (1.8)$$

The value of  $D_e$  for a tube is  $4D^2\pi/4D\pi = D$  (the inner diameter of the tube), and for a conduit of square section, it is  $4a^2/4a = a$  (the side of the square). For heat transfer, the total length of the heat-transferring perimeter is calculated instead of the wetted perimeter (e.g. in the case of part of a tube).

It has been shown that several different types of flow can be characterized by their Reynolds numbers:

$Re < \text{about } 2300$ : laminar flow

$Re > 2300$  to  $Re < 10\,000$ : transient flow

$Re > 10\,000$ : turbulent flow

This means, for example, that if for System 1 the Reynolds number  $Re(1)$  is 1000 and for System 2 the Reynolds number  $Re(2)$  is 1000, then the flow shows the same (laminar) properties in both systems. Moreover, *all systems in which the Reynolds numbers are the same show the same flow properties*.

In order to understand the role of the Reynolds number, let us interpret the form of Eqn (1.6) as

momentum convection + momentum conduction = local change of momentum

If  $Re = 1$ , this means for the momentum part that convection = 50% and conduction = 50%; if  $Re = 3$ , then convection = 75% and conduction = 25%; and if  $Re = 99$ , then convection = 99% and conduction = 1%.

It is difficult to overestimate the importance of Reynolds' idea of similarity, because this has become the basis of modelling. One can investigate the

phenomena first with a small model, which is relatively cheap and can be made quickly, and then the size of the model can be increased on the basis of the results. Modelling and increasing the size (scaling-up) are everyday practice in shipbuilding, in the design of chemical and food machinery, and so on.

If, for a given system,  $D$ ,  $\rho$  and  $\eta$  are constant, the type of flow depends on the linear velocity ( $v$ ) if only convection and conduction take place.

Using similar considerations, many other dimensionless numbers can be derived from the Damköhler equations; some of these are presented in Tables 1.1 and 1.2. From Table 1.1, we have the following, for example:

- In Eqn (1.4), the ratio of convection to conduction is the Peclet number for component transfer ( $Pe'$ ),

$$Pe' = \frac{\text{div}[c_i \mathbf{v}]}{\text{div}[D \text{grad } c_i]} = \frac{\mathbf{v}d}{D}$$

- In Eqn (1.6), the ratio of the momentum source to the momentum convection is the Euler number ( $Eu$ ),

$$Eu = \frac{\text{grad } p}{\text{Div}\{\rho \mathbf{v} \cdot \mathbf{v}\}} = \frac{\Delta p}{\rho v^2}$$

Another way of deriving dimensionless numbers is illustrated in Table 1.2. In the third column of this table, the ratio of transfer to conduction is represented instead of the ratio of transfer to convection, and in this way another system of dimensionless numbers (i.e. variables) is derived.

Note that:

- If the source is a force due to a stress, equal to  $\Delta p d^2$ , then the *Euler number* is obtained.
- If the source is a gravitational force, equal to  $\rho g d^3$ , then the *Fanning number* is obtained.

**Table 1.1** Derivation of dimensionless numbers.

Flux	Convection/conduction	Transfer/convection	Source/convection
Component (Eqn 1.4)	$Pe'$	$St'$	$Da(I)$
Heat (Eqn 1.5)	$Pe$	$St$	$Da(III)$
Momentum (Eqn 1.6)	$Re$	$f/2$	$Eu$ or $Fa$

**Table 1.2** Another way of deriving dimensionless numbers.

Flux	Convection/conduction	Transfer/conduction	Source/convection
Component (Eqn 1.4)	$Pe'$	$Nu'$	$Da(I)$
Heat (Eqn 1.5)	$Pe$	$Nu$	$Da(III)$
Momentum (Eqn 1.6)	$Re$	$A$ (no name)	$Eu$ or $Fa$



The dimensionless numbers in Tables 1.1 and 1.2 are as follows:

$Pe' = vd/D$ , the *Peclet number for component transfer*.

$Pe = vd/a$ , the *Peclet number for heat transfer* ( $a$  = temperature conduction coefficient or heat diffusion coefficient).

$St' = \beta/v$ , the *Stanton number for component transfer* ( $\beta$  = component transfer coefficient).

$St = \alpha/\rho c_p v$ , the *Stanton number for heat transfer* ( $\alpha$  = heat transfer coefficient).

$\gamma = f' \rho v/2$ , the *momentum transfer coefficient* ( $f'/2 = \gamma/\rho v$ ).

$Da(I) = v_i r d/c_i v$ , the *first Damköhler number*; this is the component flux produced by chemical reaction divided by the convective component flux.

$Da(III) = v_i \Delta H r d/\rho c_p v \Delta T$ , the *third Damköhler number*; this is the heat flux produced by chemical reaction divided by the convective heat flux.

$Eu = \Delta p/\rho v^2$ , the *Euler number*; this is the stress force divided by the inertial force.

$Fa = g d/v^2$ , the *Fanning number*; this is the gravitational force divided by the inertial force.

$Nu' = \beta \beta d/D$ , the *Nusselt number for component transfer* ( $D$  = diffusion coefficient).

$Nu = \alpha d/\lambda$ , the *Nusselt number for heat transfer* ( $\lambda$  = thermal conductivity).

Following van Krevelen's treatment (1956),  $3 \times 3 = 9$  independent dimensionless numbers can be derived in this way from three equations (*rows*) and four types of phenomena (*columns*, namely, convection, conduction, transfer and sources), and three rates can be produced from these numbers. With the help of such matrices of nine elements (see Tables 1.1 and 1.2), other dimensionless numbers can also be obtained, which play an important role in chemical and food engineering. For example, values of *efficiency* can be derived in this way:

$Pr = Pe/Re = \nu/a$ , the *Prandtl number*

$Sc = Pe'/Re = \nu/D$ , the *Schmidt number*

$Le = Sc/Pr = a/D$ , the *Lewis number*

### 1.3.2 Degrees of freedom of an operational unit

The number of degrees of freedom of an operational unit is a generalization of corresponding concept in the Gibbs phase rule. The question of how to determine the number of degrees of freedom of an operational unit was first put by Gilliland and Reed (1942); further references are Morse (1951), Benedek (1960) and Szolcsányi (1960).

For multiphase systems, the Gibbs classical theory, as is well known, prescribes the equality of the chemical potentials for each component in each phase in equilibrium. If  $\mu_k^f$  (where  $k = 1, 2, \dots, K$ , and  $f = 1, 2, \dots, F$ ) denotes the chemical potential of the  $k$ th component in the  $f$ th phase, then the following holds in equilibrium:

- For the  $f$ th phase, when there are  $K$  components,

$$\mu_1^f = \mu_2^f = \dots = \mu_K^f$$

that is,  $F(K - 1)$  equations.

- For the  $k$ th component, when there are  $F$  phases,

$$\mu_k^1 = \mu_k^2 = \dots = \mu_k^F$$

that is,  $K(F-1)$  equations.

In equilibrium, the additional variables which are to be fixed are  $T$  and  $p$ . Consequently, in equilibrium, the number of variables ( $\varphi$ ) which can be freely chosen is

$$\varphi = F(K-1) - K(F-1) + 2 = K - F + 2 \quad (1.9)$$

This is the Gibbs phase rule, which is essential for studying multiphase systems.

Even in the extreme case where the solubility of a component in a solvent is practically zero, the phase rule can nevertheless be applied by considering the fact that the chemical potential of this component is sufficient for equilibrium in spite of its very small concentration.

The generalization that we need in order to obtain  $\varphi$  for an operational unit is given by

$$\varphi = L - M \quad (1.10)$$

where  $\varphi$  is the number of degrees of freedom,  $L$  is the total number of variables describing the system and  $M$  is the number of independent relations between variables.

In the simplest case, that of a simple stationary operational unit with an isolated wall, if the number of input phases is  $F$  and the number of output phases is  $F'$ , then the total number of variables is

$$L = (F + F')(K + 2)$$

where  $K$  is the number of components. (To describe a homogeneous phase,  $(K+2)$  data points are needed.)

Let us now consider the constraints. There are constraints derived from the conservation laws for every component and also for energy and momentum, which means  $(K+2)$  constraints for every phase.

The number of constraints for equilibrium between two phases is  $(K+2)$ , which means  $(F'-1)(K+2)$  constraints for the output phases. Consequently, the total number of constraints is

$$M = (K+2) + (F'-1)(K+2)$$

and, finally,

$$\varphi = F(K+2) \quad (1.11)$$

However, in the case of *cellular substances*, the conditions of equilibrium typically do not apply; moreover, the number of components can usually not be determined. Therefore, the Gibbs phase rule cannot be used for food-technological systems except in special cases where exclusively chemical changes are taking place in the system studied. This uncertainty relating to the degrees of freedom is an essential characteristic of food engineering.

### 1.3.3 Polynomials as solutions of the Damköhler equations

The solution of the Damköhler equation system can be approximated by the product:

$\Pi_1^a$  can be obtained in the form

$$\prod_2^b \times \prod_3^c \times \cdots \times \prod_i^d \times \cdots \quad (1.12)$$

where  $\Pi_i$  is dimensionless numbers created from the terms of the Damköhler equations and  $a, b, c, d, \dots$  are exponents which can be positive/negative integers or fractions.

First of all, it is to be remarked that Eqn (1.12) supposes that the solution is provided by the so-called *monom* (not by *binom* as, e.g.  $\Pi_1^a$  can be obtained in the form  $\Pi_2^b \times \Pi_3^c \times \cdots \times \Pi_i^d \times \cdots$ , i.e. monom does not contain addition but multiplication operation only) – this supposition is not fulfilled in each case!

While derivation of dimensionless numbers from the Damköhler equations refers to a special circle of phenomena of transfer, which is crucial from our point of view, dimensional analysis is a general method that is not limited to chemical engineering. The principle of dimensional analysis has been first expressed likely by Buckingham, therefore, it is known as Buckingham's  $\Pi$ -theorem. This theorem is the base of Eqn (1.12) as well. According to the formulation of Loncin and Merson (1979), 'if  $n$  independent variables occur in a phenomenon and if  $n'$  fundamental units are necessary to express these variables, every relation between these  $n$  variables can be reduced to a relation between  $n-n'$  dimensionless variables.'

The principal idea represented by Eqn (1.12) is that convergent polynomial series, for example, a Taylor series, can approximate well almost any algebraic expression and thus also a solution of the Damköhler equations. But it is not unimportant how many terms are taken into account. There are algebraic expressions that cannot be approximated by a monomial, because they are not a product of terms but a sum of terms.

However, the general idea is correct, and formulae created from the dimensionless numbers  $\Pi_i$  according to Eqn (1.12) provide good approximations of monomial or binomial form. (Trinomials are practically never used.)

How can this practical tool be used? Let us consider a simple example. A warm fluid flows in a tube, which heats the environment; for example, this might be the heating system of a house. If heat radiation is negligible, the Nusselt, Reynolds and Peclet numbers for the simultaneous transfer of momentum and heat should be taken into account (see Table 1.2). Since the appropriate dimensionless numbers created from the terms of the Damköhler equations are:

$Nu$  for heat (convection/conduction)

$Re$  for momentum (convection/conduction)

$Pe$  for heat (convection/conduction) or  $Pr = Pe/Re$ , therefore, neglecting the gravitational force

We obtain the following function  $f$ :

$$Nu = f(Re, Pr) \quad (1.13)$$

which is an expression of Eqn (1.12) for the aforementioned case.

Equation (1.12) is one of the most often applied relationships in chemical and food engineering. Its usual form is

$$Nu = CRe^a \times Pr^b \quad (1.14)$$

which has the same monomial form as Eqn (1.12).

Many handbooks give instructions for determining the values of the exponents  $a$  and  $b$  and the constant  $C$ , depending upon the boundary conditions. Let us consider the physical ideas on which this approach is based.

## 1.4 Analogies

### 1.4.1 The Reynolds analogy

An analogy can be set up between mechanisms as follows:

Momentum transfer  $\leftrightarrow$  heat transfer

Momentum transfer  $\leftrightarrow$  component transfer

Component transfer  $\leftrightarrow$  heat transfer

This analogy can be translated into the mathematical formalism of the transfer processes.

From physical considerations, Reynolds expected that the momentum flux ( $J_p$ ) and the heat flux ( $J_q$ ) would be related to each other, that is, if

$$J_q = \frac{\alpha}{c_p \rho} \times A \Delta(\rho c_p T) \quad (1.15)$$

then

$$J_q = \frac{\gamma}{\rho} \times A \Delta(\rho v) \quad (1.16)$$

In other words, *the moving particles transport their heat content also*. Then he supposed that

$$\frac{\alpha}{c_p \rho} = \frac{\gamma}{\rho} \quad (1.17)$$

or, in another form,

$$\frac{f'}{2} = \frac{\alpha}{c_p \rho} = St \quad (1.18)$$

If the flux of a component is

$$J_i = \beta F \Delta c_i \quad (1.19)$$

then Reynolds' supposition can be extended to this third kind of flux as follows:

$$St = St' = \frac{f'}{2} \quad (1.20)$$

where  $St$  is the Stanton number for heat transfer ( $St = \alpha / c_p \rho$ ),  $St'$  is the Stanton number for component transfer ( $St' = \beta / \mathbf{v}$ ),  $f'/2 = \gamma / \rho \mathbf{v}$  and  $\gamma$  is the momentum transfer coefficient.

If the Reynolds analogy formulated in Eqn (1.20) is valid, then if we know one of the three coefficients  $\alpha$ ,  $\beta$  or  $\gamma$ , the other two can be calculated from this equation. This fact would very much facilitate practical work, since much experimental work would be unnecessary.

But proof of the validity of the Reynolds analogy is limited to the case of strong turbulence. In contrast to the Reynolds analogy,

$$a \neq v \neq D \quad (1.21)$$

that is,

$$Pr \neq Sc \neq Le \quad (1.22)$$

Equation (1.17) is valid only for turbulent flow of gases. In the case of gases,

$$Pr \approx 0.7-1 \quad (1.23)$$

is always valid.

### 1.4.2 The Colburn analogy

Colburn introduced a new complex dimensionless number, and this made it possible to maintain the form of the Reynolds analogy:

$$J_q = St Pr^{2/3} \quad (1.24)$$

$$J_i = St' Sc^{2/3} \quad (1.25)$$

and

$$J_p = \frac{f'}{2} \quad (1.26)$$

Finally, formally similarly to the Reynolds analogy,

$$St Pr^{2/3} = St' Sc^{2/3} = \frac{f'}{2} \quad (1.27)$$

The Colburn analogy formulated in Eqn (1.27) essentially keeps Reynolds' principal idea about the coupling of the momentum (mass) and thermal flows and gives an expression that describes the processes better. *Equation (1.27) is the basis of the majority of calculations in chemical engineering.*

In view of the essential role of Eqn (1.27), it is worth looking at its structure:

$$St = \frac{Nu}{Re Pr} = \frac{\alpha}{\rho c_p v}$$

$$St' = \frac{Nu'}{Re Sc} = \frac{\beta}{v}$$

$$\frac{f'}{2} = \frac{\gamma}{\rho v}$$

The numbers  $Pr$  and  $Sc$  are parameters of the fluid:

$$Pr = \frac{\nu}{a}$$
$$Sc = \frac{\nu}{D}$$

Additional material parameters are needed for calculations, namely,  $\alpha$ ,  $\rho$  and  $c_p$ . If  $\mathbf{v}$  is known,  $f'$  and  $\beta$  can be calculated.

This theoretical framework (see Eqns 1.13, 1.14 and 1.27) can be modified if, for example, a buoyancy force plays an important role – in such a case, the *Grashof number*, which is the ratio of the buoyancy force to the viscous force, appears in the calculation. A detailed discussion of such cases would, however, be beyond the scope of this book. A similar limitation applies to cases where the source term is related to a chemical reaction: chemical operations in general are not the subject of this book.

A more detailed discussion of these topics can be found in the references given in Section 1.2.

### 1.4.3 Similarity and analogy

Similarity and analogy are quite different concepts in chemical and food engineering, although they are more or less synonyms in common usage. Therefore, it is necessary to give definitions of these concepts, which emphasize the differences in our understanding of them in the present context.

*Similarity* refers to the properties of machines or media. Similarity means that the geometric and/or mechanical properties of two machines or streaming media can be described by the same mathematical formulae (i.e. by the same dimensionless numbers) that our picture of the flux (e.g. laminar or turbulent) is similar in two media. Similarity is the basis of scaling-up.

*Analogy* refers to transfer mechanisms. Analogy means that the mechanisms of momentum, heat and component transfer are related to each other by the way that components are transferred by momentum and, moreover, components transfer heat energy (except in the case of heat radiation). This fact explains the important role of the Reynolds number, which refers to momentum transfer.

## 1.5 Dimensional analysis

This is a simple mathematical tool for creating relationship between physical variables, keeping the rule that *the physical expressions shall be homogeneous from the viewpoint of dimension*: both sides of the equations must have the same dimension. Homogeneity also means that the equation remains unchanged if the system of the fundamental units changes (e.g. SI  $\leftrightarrow$  Anglo-Saxon system). Dimensional analysis can be very fruitful for solving complicated problems easily in various fields of physics, biology, economics and others.

Dimensional analysis contracts physical variables into dimensionless groups, which will be the new variables; by so doing, the number of variables will be decreased. The lesser the number of variables, the greater the advantage: for example, if instead of 6 variables only 3 variables are to be studied experimentally, supposed that 5 points of every variable are to be measured, then instead of  $5^6 = 15\,625$  only  $5^3 = 125$  points are to be measured in the labour experiments.

There is a developed theory of dimensional analysis which abundantly applies the results of linear algebra and computerization (see Barenblatt, 1987; Huntley, 1952; Zlokarnik, 1991). Instead of discussing these classical methods based on solutions of linear equation system, we represent here the Szirtes method (Szirtes, 1998; 2006) by examples in a rather simplified and slightly modified form, which is very easy and can be generally used.

Szirtes exhaustively details the cases as well for which the approaches of dimensional analysis must be cautiously used. Two considerations of him are mentioned here:

- 1 The Buckingham  $\Pi$ -theorem relates to *products* of dimensionless  $\pi$ -numbers, that is, monoms, which do not contain the algebraic operation addition (+). If a formula contains addition (i.e. it is binom, trinom, etc.), its transformation into a dimensionless formula by dimensional analysis either needs some special considerations or impossible.
- 2 The obtained dimensionless formula needs *experimental checking* in every case, since the dimensional correctness is only a necessary but not a sufficient condition.

### Example 1.1 Heat transfer by fluid in tube

The choice of variables is done according to physical considerations:

$\alpha$  : heat transfer coefficient ( $\text{kg s}^{-3}$ )

$w$  : velocity of fluid ( $\text{m s}^{-1}$ )

$q$  : heat capacity ( $\text{kg m}^{-1} \text{s}^{-2}$ )

$\nu$  : kinematic viscosity of fluid ( $\text{m}^2 \text{s}^{-1}$ )

$d$ : diameter of tube (m)

$\lambda$  : coefficient of thermal conductivity ( $\text{kg m s}^{-3}$ )

$\Delta t$  : temperature different between the fluid and the tube wall (K)

Units	Variables						
	$\alpha$	$w$	$q$	$\nu$	$d$	$\lambda$	$\Delta t$
$M$ (kg)	1	0	1	0	0	1	0
$L$ (m)	0	1	-1	2	1	1	0
$T$ (s)	-3	-1	-2	-1	0	-3	0
$t$ (K)	-1	0	-1	0	0	-1	1

We obtained the so-called *dimension matrix* of  $(4 \times 7)$  size. In dimension matrix, a non-singular quadratic matrix has to be chosen (i.e. its determinant is not zero),

which is shown by bold numbers here, denoted by **A**. The sequence of variables has to be written in such a way that this quadratic matrix should be on the right side. The residue of the dimension matrix on the left side is denoted by **B**. That is, the dimension matrix has 

<b>B</b>	<b>A</b>
----------	----------

 the following form:

In the next step, this dimension matrix of (4×7) size has to be completed to a quadratic matrix as follows.

<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
1	0	1	0	0	1	0
0	1	-1	-2	1	1	0
-3	-1	-2	-1	0	-3	0
-1	0	-1	0	0	-1	1

The completed quadratic matrix has the following form (I: unit matrix; 0: zero matrix):

<b>I</b>	<b>0</b>
<b>B</b>	<b>A</b>

The next step is to calculate  $\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}$  the inverse of the completed quadratic matrix.

Variables	$\pi_1$	$\pi_2$	$\pi_3$				
$\alpha$	<b>1</b>	<b>0</b>	<b>0</b>	0	0	0	0
$w$	<b>0</b>	<b>1</b>	<b>0</b>	0	0	0	0
$q$	<b>0</b>	<b>0</b>	<b>1</b>	0	0	0	0
$v$	<b>0</b>	<b>-1</b>	<b>1</b>	-3	0	-1	0
$d$	<b>1</b>	<b>1</b>	<b>0</b>	5	1	2	0
$\lambda$	<b>-1</b>	<b>0</b>	<b>-1</b>	1	0	0	0
$\Delta t$	<b>0</b>	<b>0</b>	<b>0</b>	1	0	0	1

The structure of the inversed matrix is as follows:

<b>I</b>	<b>0</b>
$-\mathbf{A}^{-1} \times \mathbf{B}$	$\mathbf{A}^{-1}$

From the inverse matrix  $\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{A}^{-1} \times \mathbf{B} & \mathbf{A}^{-1} \end{bmatrix}$ , the values of the dimensionless numbers can be directly obtained:

$$\pi_1 = \alpha d \lambda^{-1} = \text{Nusselt number}$$

$$\pi_2 = w d v^{-1} = \text{Reynolds number}$$

$$\pi_3 = q v \lambda^{-1} = \text{Prandtl number}$$

That is, the classical formula is obtained:  $Nu = \text{constant} \times Re^a Pr^b$



Evidently, the choice of variables is contingent: in this case, the temperature difference ( $\Delta t$ ) turned out to be a surplus! This uncertainty can be, on the one hand, an inadequacy of dimensional analysis but sometimes also an advantage as the aforementioned example shows, since it can be seen from the calculation – before any experiment! – that  $\Delta t$  can be neglected. The lesson of this example is that only six variables have to be accounted instead of seven; in addition, the experiments need only three variables ( $No$ ,  $Re$ ,  $Pr$ ), and the constants can be determined by linearization.

**Example 1.2** By the Szirtes method, let us calculate the flow rate ( $V$ ) in a tube of  $D$  diameter if the pressure difference is  $\Delta p$  and the viscosity of fluid is  $\eta$  (the solution is the well-known Hagen–Poiseuille equation). This example is derived from Szirtes (1998, 2006).

The completed dimension matrix and its inverse.

	$V$	$\Delta p$	$\eta$	$D$
	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>
$M(\text{kg})$	0	1	1	0
$L(\text{m})$	3	-2	-1	1
$T(\text{s})$	-1	-2	-1	0

	$\pi_1$			
$V$	<b>1</b>	0	0	0
$\Delta p$	-1	-1	0	-1
$\eta$	<b>1</b>	2	0	1
$D$	-4	0	1	-1

Solution:  $\pi_1 = (V\eta/\Delta p D^4)$ , that is,  $V = \text{constants} \times \Delta p D^4/\eta$

*Remark:* In case of more or other variables (e.g. the length or/and cross section of tube), the solution is too complicated. Also this example demonstrates that albeit dimensional analysis is very many-sided but not omnipotent.

## 1.6 System theoretical approaches to food engineering

A strong tendency in food process engineering is the growing attention paid to the relations between processes, products, emerging technologies, heat treatments and food safety. Research tools like mathematical modelling, especially

computer fluid dynamics, and sophisticated methods of product characterization are the most intensively developing fields (see Bimbenet *et al.*, 2007).

The models currently used in food process engineering simplify too much both the food system description and the mechanisms and rate equations of changes: The food system is supposed to be homogeneous and continuous. In this way, thermodynamic and kinetic equations deduced for ideal gas or liquids, in conditions close to equilibrium are applied to cellular solid foods, in conditions far away from the equilibrium. However, it is necessary to develop advanced concepts and methodologies in food process engineering. The new models for food and processes development must incorporate information about all these aspects (thermodynamic, structural, chemist and biochemist and even mechanics). Only in this way, they would be able to calculate and predict the real changes in the whole quality of food product in line with the process progression.

In the spirit of such ideas the system theory of chemical engineering developed by Blickle and Seitz (1975), Blickle (1978) was adapted to food engineering by Mohos (1982). For the mathematical details and examples, see Appendix 5.

Fito *et al.* (2007) present a comprehensive model of food engineering called systematic approach to food process engineering (*SAFES*) in the sense of food process engineering for product quality. The *SAFES* methodology (Fito *et al.*, 2007) recognizes the complexity of food system and allows coordinating the information about food structure, composition, quality, thermodynamic and so on in adequate tools to develop real food and processes models. This brief review is not capable of replacing the original article, which can be found in the Internet; therefore, it is limited to itemize the main ideas of it.

- *Food product engineering*: modelling of food and biological systems by studying the structure of food system as the structure-properties ensemble (e.g. levels of complexity in matter condensation).
- The *SAFES* defines a *simplified space of the structured phases and components*, moreover, of *aggregation states* in order to describe the material structure.
- It defines the descriptive matrix, a *mathematical tool to describe the food system* by the help of:
  - The state variables: the share-out of matter among components and structured phases
  - Mass and volume balances inside the product
  - The energy inside the system: the Gibbs free energy
  - Equilibrium and driving forces
  - Transport mechanisms and rate equations
- Food process engineering consists of modelling of food operations and processes:
  - Definition of unit operation and stage of change
  - Mass balances and transformed matrices: matrix of changes
  - To construct the process matrix

## 1.7 Food safety and quality assurance

When studying the principles of food engineering, the concepts food safety and quality assurance (QA) must not be omitted although a detailed discussion of them exceeds the possibilities of this work. Therefore, this presentation is limited to a sketch and provides the appropriate essential references, which can be found in the Internet.

In the food industry, QA systems such as the Hygiene Code (FAO/WHO, 2009; Codex Alimentarius Committee, 1969), the Hazard Analysis Critical Control Points (HACCP) and the International Organization for Standardization (ISO) 9000 series are applied to ensure food safety and food quality to prevent liability claims and to build and maintain the trust of consumers.

Quality is defined by the ISO as 'the totality of features and characteristics of a product that bear on its ability to satisfy stated or implied needs.' Safety differs from many other quality attributes since it is a quality attribute that is difficult to observe. A product can appear to be of high quality, that is, well coloured, appetizing, flavourful and so on, and yet be unsafe because it is contaminated with undetected pathogenic organisms, toxic chemicals or physical hazards. On the other hand, a product that seems to lack many of the visible quality attributes can be safe.

Safety or QA program should focus on the prevention of problems, not simply curing them. Safety and QA should be ongoing processes incorporating activities beginning with selecting and preparing the soil and proceeding through to consumption of the product. Both safety and QA should focus on the prevention of problems, not simply curing them since, once safety or quality is reduced, it is virtually impossible to go back and improve it for that item. It is possible, however, to assure that the same problem does not affect future products (Silva *et al.*, 2002).

HACCP aims to assure the production of safe food products by using a systematic approach (i.e. a plan of steps) to the identification, evaluation and control of the steps in food manufacturing that are critical to food safety (Leaper, 1997). HACCP focuses on technological aspects of the primary process.

CAOBISCO (2011) provides a Guide of Good Hygiene Practices that can be regarded a competent document in this field.

The ISO 9000 series aims to achieve uniformity in products and/or services, preventing technical barriers to free trade throughout the world. ISO consists of a checklist to assure managerial aspects. It requires the establishment of procedures for all activities and handling, which must be followed by ensuring clear assignment of responsibilities and authority (Hoogland *et al.*, 1998). See further ISO (1984, 1990, 1994) documents.

For studying further references concerning food safety, QA and food quality, see Lásztity (2008), Carpenter *et al.* (2000), Defence Fuel & Food Services (2013), Food Safety Authority of Ireland (2011) and Martin (1997). References to the

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