CHAPTER 1

The Time Value of Money

The chief value of money lies in the fact that one lives in a world in which it is overestimated.

H.L. Mencken (1707–1754)

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Chapter 1 outlines the foundation for successful investing irrespective of the asset class under consideration. Namely, the chapter introduces the reader to the concept of the time value of money. Understanding the time value of money is essential for successful investing for the following reasons:

- It allows the investor to measure the value of one asset or portfolio of cash flows relative to another.
- The investor, using the time value of money, can estimate expected holding period returns.

1.1 PRESENT VALUE

Congratulations! You have won $1 million in the "Chances Are Slim Lottery." (They were very slim, but you played anyway, and got lucky.) You now have a choice, either take a lump sum payment of $500,000 or monthly payments of $2,777.78 per month over the next 30 years.

This is a classic time-value-of-money problem. The time value of money postulates that it is preferable to receive a dollar today rather than the same dollar in the future. This is because the dollar received today will have a certain purchasing power over the dollar received in the future.

For example, the dollar received today may be invested in an interest bearing account that will grow over time. Alternatively, one may spend the dollar received today rather than deferring consumption. Either way, one is better off taking the dollar today than at a later date. The winning lottery

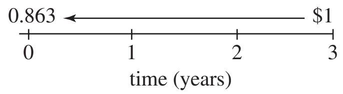

FIGURE 1.1 Present Value of a Single Payment

problem implies an interest rate: What can be earned on the $500,000 taken today as well as an inflation outlook, or what is the purchasing power of a dollar received at some point in the future?

To answer the winning lottery question we first must understand the concept of present value. The *present value* (PV) equation (1.1) allows a person to determine the value today of a dollar received at some point in the future:

$$\text{Present Value of \$1} = \frac{1}{(1+i)^n} \tag{1.1}$$

where i = Interest rate
n = Number of periods

Equation 1.1 states, that given a discount or interest rate (i) and time or number of discounting periods (n) one can determine the value today of a sum received at some point in the future.

Figure 1.1 illustrates the concept. Assuming (i) = 5% annually and (n) = 3 years, the present value of $1, or its value today, is $0.863.

So, if a person were able to invest in a "risk-free" interest bearing account earning 5% interest annually over a three-year period, she would be indifferent, in a purely economic sense, between receiving $0.86 today or a dollar three years from now. Which would you prefer? Your answer is the **time value of money**.

1.2 FUTURE VALUE

Notice that the present value problem implies a *future value* condition. Specifically, the future value is stated as $1. So, it appears that the future value of a dollar is the "tails" side of the present value coin. The future value of a dollar invested in an interest-bearing account is also determined by both the time invested (n) as well as the interest rate earned (i).

$$\text{Future Value of \$1} = (1+i)^n \tag{1.2}$$

where i = Interest rate
n = Number of periods

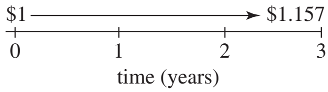

FIGURE 1.2 Future Value of a Single Sum Today

Equation 1.2 reverses the present value equation and states that given an interest rate (i) and an investment horizon or number of periods (n) the value, at a point in the future, of a sum invested today can be determined.

Figure 1.2 illustrates the concept. Assuming (i) = 5% and (n) = 3 years, the future value of $1 is $1.157.

Together equations 1.1 and 1.2 provide some intuition to solve the winning lottery problem. However, neither equation 1.1 nor 1.2 alone provides the discount rate or (i) used to equate the present value of the annuity payment of $2,777.78 to the $500,000 lump sum offered in the above winning lottery problem.

1.3 PRESENT VALUE OF AN ANNUITY

Before solving the annuity problem, it is important to recognize the difference between an *annuity* and an *annuity due*. An annuity is a group of payments or cash flows to be received at a specific interval (monthly, quarterly, annually) over a specified period of time. An annuity due specifies the payments to be made at the beginning of a period while a regular annuity specifies that the payments are made at the end of a period.

Figure 1.3 illustrates the concept of an annuity. One can simply think of an annuity as a series of payments that are discounted given a rate (i) and a period (n) over which the payments are scheduled to be made. The present value of the annuity is the sum of the present value of the payments. In the example presented in Figure 1.3, the present value of the annuity is $2.72.

Annuity payments may be constant from one period or payment to the next (certain cash flows[1]) or they may vary. Cash flows that vary from one

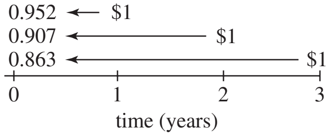

FIGURE 1.3 Present Value of Annuity Payment

[1]The cash flows are certain absent the likelihood of default or early repayment.

period to the next are generally referred to as contingent cash flows. That is, the size of the cash flow may be contingent on exogenous factors. When the annuity payments are constant the present value or future value of the annuity can be calculated using the following closed form solutions:

The present value of an "ordinary" annuity, one under which payments are made at the end of a period, can be expressed as:

$$\text{Present value of annuity} = \frac{1 - \frac{1}{(1+i)^n}}{i} \tag{1.3}$$

where i = Interest rate
n = Number of periods

To adjust for the difference in the timing of the cash flows between an annuity and an annuity due, one simply multiplies the present value of an annuity by $(1 + i)$. Thus, the equation for the present value of an annuity due is:

$$\text{Present value annuity due} = \left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right] \times [(1+i)] \tag{1.4}$$

where i = Interest rate
n = Number of periods

1.4 FUTURE VALUE OF AN ANNUITY

Of course, you could decide to reinvest (save) the proceeds from your lottery winnings. If this were the case, then the future value of the payments saved is given by equation 1.5.

$$\text{Future value of annuity} = \frac{(1+i)^n - 1}{i} \tag{1.5}$$

where i = Interest rate
n = Number of periods

1.5 SOLVING FINANCIAL QUESTIONS WITH PRESENT AND FUTURE VALUE

To solve the winning lottery problem all that is left is to determine the appropriate discount rate, or (i). The discount rate (i) is usually determined by a market quoted rate, like the 10-year Treasury yield. So, assuming the 10-year Treasury yield is 3.0%, the present value of the $2,777.78 monthly annuity is $658,859. This is considerably more than the $500,000 lump sum offered by the Chances Are Slim Lottery.

What is the discount rate used by the Chances Are Slim Lottery? We can iteratively solve for this by successively increasing or decreasing the discount rate until the present value of the annuity is equal to the lump sum amount offered. The discount rate used by the Chances Are Slim Lottery is 5.43%. This rate is equivalent to the assumed 10-year Treasury rate of 3% plus a premium or "spread" of 2.43%.

Your analysis, based on the time value of money, indicates you must believe that over the lifetime of the annuity you could earn more than 5.43%—your break-even rate—before accepting the lump-sum payment offered by the Chances Are Slim Lottery.

As you ponder the question, "Can I earn more than 5.43% if I were to take the lump sum payment and invest the proceeds myself?" you receive a call from a well-known and highly successful hedge fund investor who is interested in buying your winning lottery ticket. She indicates that she is willing to purchase your lottery ticket today for $575,000.

You quickly realize that her analysis, investing in or purchasing your winning ticket, is different from your "winning" analysis. She, unlike you, will make a substantial investment to purchase the ticket. What is her return for purchasing the ticket? Her return, or internal rate of return (IRR), is the discount rate that equates the present value of a series of cash flows received in the future to the initial outlay made to purchase those cash flows today. Figure 1.4 graphically illustrates the concept. What is the discount rate or *i* that equates three equal payments over three years to a $75 outlay today?

Returning to the lottery example, you calculate that the hedge fund investor is willing to accept a annualized return of 4.17% in order to purchase the winning lottery ticket. Her offer to purchase the ticket results in a higher lump sum payment and lower internal rate of return than the Chances Are Slim Lottery is willing to offer you.

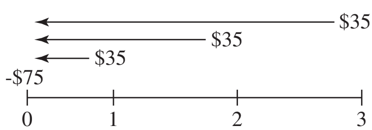

FIGURE 1.4 Internal Rate of Return

1.6 APPLICATION TO FIXED-INCOME SECURITIES

Yield to maturity YTM is the internal rate of return (IRR). It relates the initial cash outlay to purchase a bond for the future cash flow received the—principal and interest earned over the life of the bond.

CONCEPT 1.1

A bond represents both an annuity and a single principal payment. Consider a non-callable bond that pays a 5% coupon semi-annually that matures in 3 years with a principal payment of $1,000. The coupon payment represents an annuity of six equal payments of $25. The principal represents a single lump-sum payment of $1,000 at maturity. Assuming one were to pay $1,000 today for the bond the IRR or yield to maturity is 5%.

The present value of the coupon annuity is:

$$\text{Present value of annuity} = \frac{1 - \frac{1}{(1+(0.05/2))^6}}{(0.05/2)} \times \$25 = \$137.70 \tag{1.6}$$

The present value of the principal payment is:

$$\text{Present value of } \$1 = \frac{1}{(1 + (0.05/2))^6} \times \$1{,}000 = \$862.30 \tag{1.7}$$

Said another way, using a 5% discount rate, the present value of a non-callable bond representing an annuity of six equal payments of $25 and a single payment of $1,000 in six years is $1,000.

The time value of money is the basis upon which the valuation of all fixed-income securities rests. In fact, it is the foundation of all investment decisions. Using the time value of money the investor can compare the expected return of one portfolio of assets relative to another portfolio of assets.