

CHAPTER 3

Static and Dynamic Replication

- Exploring replication.
- Exact static replication for European options.
- Approximate static replication for exotic options.
- Dynamic replication and continuous delta-hedging.
- What should you pay for convexity?
- Implied volatility is a parameter; realized volatility is a statistic.
- Hedging an option means betting on volatility.

EXACT STATIC REPLICATION

We begin this chapter by examining how we can employ static replication to re-create a wide range of payoffs using puts, calls, their underliers, and riskless bonds as ingredients.

Put-Call Parity

A vanilla European call option at expiration has the value:

$$C(S_T, T) = \max[S_T - K, 0] \quad (3.1)$$

where S_T is the price of the underlying stock at expiration, K is the strike price, and T is the time at expiration.

Similarly, the value at expiration of a European put with strike price of K is:

$$P(S_T, T) = \max[K - S_T, 0] \quad (3.2)$$

As shown in Table 3.1, if we buy a European call and sell a European put with the same strike price, we are guaranteed a payoff of $(S_T - K)$ at expiration, no matter what the final value of the stock price is.

TABLE 3.1 Payoffs of European Calls and Put Positions at Expiration

	$S_T \leq K$	$S_T \geq K$
$C(S_T, T)$	0	$S_T - K$
$P(S_T, T)$	$K - S_T$	0
$-P(S_T, T)$	$S_T - K$	0
$C(S_T, T) - P(S_T, T)$	$S_T - K$	$S_T - K$

Assume the stock pays no future dividends. At a time t , prior to expiration, if we purchase a share of the underlying stock at the prevailing price S_t , and sell $Ke^{-r(T-t)}$ of riskless bonds, then at T we will also have a portfolio worth $(S_T - K)$. By the law of one price, the two portfolios—the first long a European call and short a European put at the same strike price, the second long the stock and short the riskless bond—must have the same current price.

$$C(S, t) - P(S, t) = S - Ke^{-r(T-t)} \quad (3.3)$$

This equivalence is known as put-call parity. Rearranging Equation 3.3, it is clear that we can always replicate a call by means of a portfolio containing a put with the same strike and expiration, the underlying stock, and a position in a riskless bond. Similarly, we can replicate a put by a call with the same strike and expiration, the underlying stock, and a position in the riskless bond. Thus,

$$C(S, t) = P(S, t) + S - Ke^{-r(T-t)} \quad (3.4a)$$

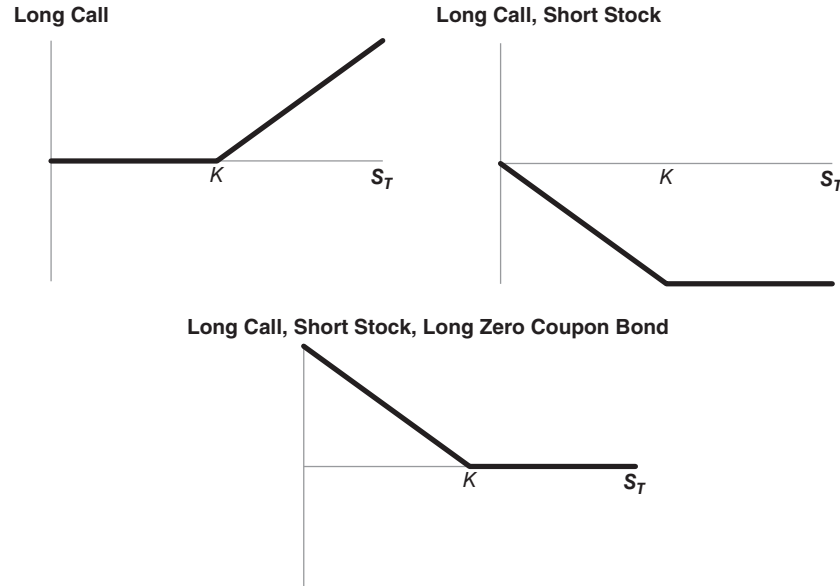
$$P(S, t) = C(S, t) - S + Ke^{-r(T-t)} \quad (3.4b)$$

Figure 3.1 shows graphically how the payoff profile at expiration of a call can be transformed into a put. This result is strictly true only for vanilla European options on non-dividend-paying underliers, though the relationship can be easily extended to the case where future dividends are known.

Replicating a Collar

A collar is a popular instrument for portfolio managers who have made some gains by time t during the year, and are willing to forgo some upside in order to gain protection on the downside for the remainder of the year (until time T). The payoff at expiration of a collar at time T with break points at L and U on a stock with terminal price S_T is shown in Figure 3.2.

Assuming we own the stock S , we can create the collar by buying a put with a strike price of L and selling a call with a strike price of U , where

**FIGURE 3.1** Put-Call Parity

$L < S < U$ and both options have the same expiration date T . The put will limit our losses if the price of the stock falls below L , and the call will cap our profits if the stock rises above U . We can write the value of a collar at time t as

$$\text{Collar} = S + P_L(S, t) - C_U(S, t) \quad (3.5)$$

where the subscripts L and U indicate the strike prices of the options.

The popularity of collars with investors in the stock market forces derivatives dealers to be short puts and long calls. These market forces tend to push up the price dealers charge for puts they sell and lower the price of calls they buy, and is one of the reasons for the observed volatility smile in index options markets.

Equation 3.5 is not the only way to decompose the collar into options. Moving through the payoff in Figure 3.2 from left to right, we can see that the payoff is equivalent to a long position in a riskless bond with a notional value of L , a long position in a call with a strike of L , and a short position in a call with strike of U . In this way (or, more formally, by using the put-call parity relationship of Equation 3.3 and substituting it into Equation 3.5), we can write the value of the collar at time t as

$$\text{Collar} = Le^{-r(T-t)} + C_L(S, t) - C_U(S, t) \quad (3.6)$$

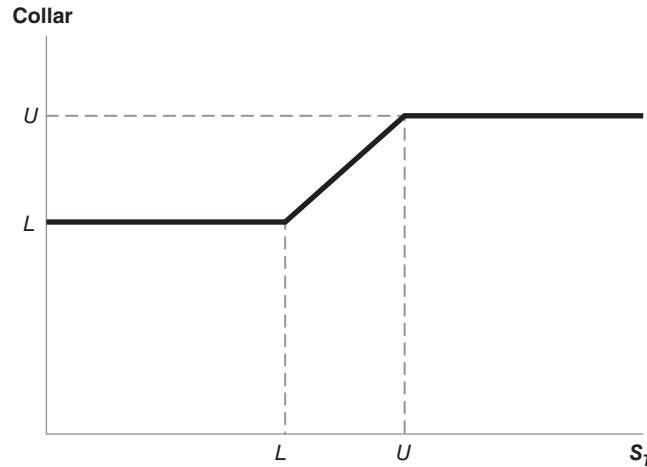


FIGURE 3.2 Payoff at Expiration of a Collar

where r is the riskless interest rate. Alternatively, moving through the payoff in Figure 3.2 from right to left, we can see that instead of using two calls we could also replicate the payoff of a collar using a long position in a riskless bond with notional value U and two puts. This is left as an exercise at the end of the chapter.

Generalized Payoffs

One can use combinations of options to replicate arbitrary payoffs at a fixed expiration. To see how, suppose you can approximate the payoff of a derivative at some future expiration time T by a piecewise-linear function of the terminal stock price S_T that is defined by its y -axis intercept I and the slopes λ_i of each successive linear piece, as shown in Figure 3.3.

It is not difficult to see that this function is the payoff of a portfolio consisting of riskless bonds with face value I and present value $Ie^{-r(T-t)}$, plus some stock (which you can think of, if you like, as a call with zero strike) and a further series of calls $C(K_i)$ with successively higher strikes K_i . The portfolio's value at an earlier time t is therefore

$$V(t) = Ie^{-r(T-t)} + \lambda_0 S_t + (\lambda_1 - \lambda_0)C(K_0) + (\lambda_2 - \lambda_1)C(K_1) + \dots \quad (3.7)$$

where S_t and $C(K_i)$ are the values of the stock and options, respectively, at time t , and we have for simplicity assumed that the stock pays no dividends.

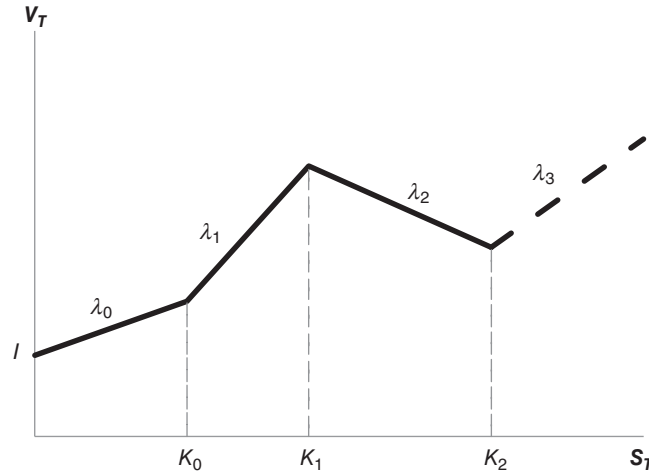


FIGURE 3.3 A General Payoff Function

The value of this generalized payoff can therefore be expressed in terms of the market value of the bonds, the stock, and the calls.

You can check the formula by seeing what happens at time T . For example, if the stock price at expiration ends up between K_1 and K_2 , then all of the calls beyond $C(K_1)$ would expire worthless, and the payoff of the portfolio would be

$$\begin{aligned} V(T) &= I + \lambda_0 S_T + (\lambda_1 - \lambda_0)(S_T - K_0) + (\lambda_2 - \lambda_1)(S_T - K_1) \\ &= I + \lambda_0 K_0 + \lambda_1(K_1 - K_0) + \lambda_2(S_T - K_1) \end{aligned} \quad (3.8)$$

where S_T is the value of the stock at expiration. This expression is consistent with the payoff function displayed in Figure 3.3.

This method is a reliable replication mechanism, provided you can buy or sell the options you need. It gives you the value of the generalized payoff in terms of its ingredients and what it costs to acquire them in the market, which is much better than any theoretical model that makes assumptions about the future behavior of stocks and volatilities.

In conclusion, we note a useful principle to be used when constructing replicating portfolios: For ingredients, use the securities that most closely resemble the target security, preferably liquid securities whose prices are readily available. Even if they are less complex, avoid using securities that require you to make theoretical assumptions about their future behavior.

SAMPLE PROBLEM**Question:**

The payoff of a structured product is a piecewise-linear function of an underlying stock, S . The payoff has the following break points:

- $S = \$0$: payoff = \$10
- $S = \$10$: payoff = \$20
- $S = \$20$: payoff = \$40

How would you replicate the payoff of the structured product using only riskless bonds, the stock, and calls on the stock? Assume the riskless rate is 0%.

Answer:

We need to buy \$10 of riskless bonds, one share of the underlying stock, and one call option with a strike price of \$10.

Because the riskless rate is 0%, we do not need to worry about the $e^{-r(T-t)}$ term in Equation 3.7. The slope between the first two break points is $(\$20 - \$10)/(\$10 - \$0) = 1$. The slope between the second and third is $(\$40 - \$20)/(\$20 - \$10) = 2$. The change in slope between them is therefore $2 - 1 = 1$.

We can check our answer: At $S = \$0$, the bonds are worth \$10, the stock is worth \$0, and the call is worth \$0, or \$10 total. At $S = \$10$, the bonds are worth \$10, the stock is worth \$10, and the call is worth \$0, or \$20 total. At $S = \$20$, the bonds are worth \$10, the stock is worth \$20, and the call is worth \$10, or \$40 total. Our portfolio passes through all of the break points.

Approximate Static Hedge for a European Down-and-Out Call

It is often more useful to have an approximate static hedge that uses easily priced securities than to have a nominally perfect dynamic hedge that uses securities whose stochastic behavior is not well known.

Consider as an example an exotic option, in particular a European down-and-out call with expiration of T on a stock with current price S and dividend yield d . We denote the strike level by K and the level of the out

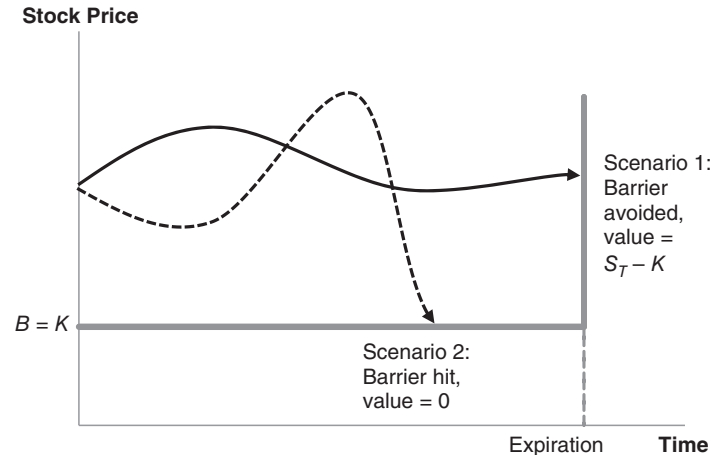


FIGURE 3.4 A Down-and-Out European Call Option with $B = K$

barrier by B . We assume in this particular example that B and K are equal and that there is no cash rebate when the barrier is hit.

There are two classes of scenarios for the stock price paths between t and T : scenarios of type 1 in which the barrier is avoided and the option finishes in-the-money; and scenarios of type 2 in which the barrier is hit before expiration and the option expires worthless. These are shown in Figure 3.4.

In scenarios of type 1, the call pays out $S_T - K$, where S_T is the unknown value of the stock price at expiration. This is the same as the payoff of a forward contract with delivery price K . At time t this forward has a theoretical value, $F = S e^{-d(T-t)} - K e^{-r(T-t)}$, where d is the continuously paid dividend yield of the stock. For scenarios of type 1, you can replicate the down-and-out call under all stock price paths with a long position in the forward.

For scenarios of type 2, where the stock price hits the barrier at any time t' before expiration, the down-and-out call immediately expires with zero value according to the terms of the contract. Notice, though, that the forward F that replicates the barrier-avoiding scenarios of type 1 is worth $K e^{-d(T-t')} - K e^{-r(T-t')}$ at time t' between t and T at which the barrier is struck. This value is equal to zero for all times t' only if $r = d$. So, if the riskless interest rate equals the dividend yield (that is, the stock forward price is equal to the current stock price), a forward with delivery price K will exactly replicate a down-and-out call with barrier and strike at the identical level K , no matter what path the stock price takes, so their prices must be equal.¹ When r is

¹ In late 1993, for example, the S&P 500 dividend yield was close in value to the short-term interest rate, so this hedge might have been applicable to short-term down-and-out S&P 500 options.

close to but not exactly equal to d , valuing the down-and-out option using this method is likely more reliable than relying on dynamic replication that makes many unconfirmed assumptions about the stochastic behavior of the stock price S and its volatility.

One important caution: If and when the stock hits the barrier, you must be able to sell the forward to close out the replication. If you don't, the target down-and-out option will have knocked out, but the replicating portfolio will still continue evolving, resulting in subsequent losses or gains.

A SIMPLIFIED EXPLANATION OF DYNAMIC REPLICATION

Options theory is based on the insight that, in an idealized and simplified world, options are not an independent asset. Because of this, we can use dynamic replication, using simpler securities to mimic the payoff of options. How closely the actual world matches the hypothetical simplified one determines how well the theory works in practice.

To begin with, for pedagogic simplicity, assume that the expected rate of return of a stock is zero. An investor who is long the stock makes money if it goes up, and loses money if it goes down. The profit and loss (P&L) is linear in the price of the stock. Figure 3.5 shows our binomial model for a share of stock with current price S and volatility σ . The change in the value

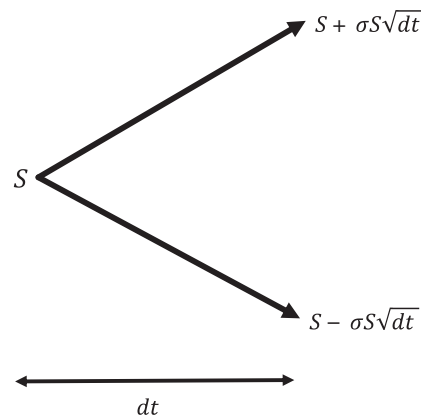


FIGURE 3.5 Binomial Model of Underlying Stock Price, $\mu = 0$

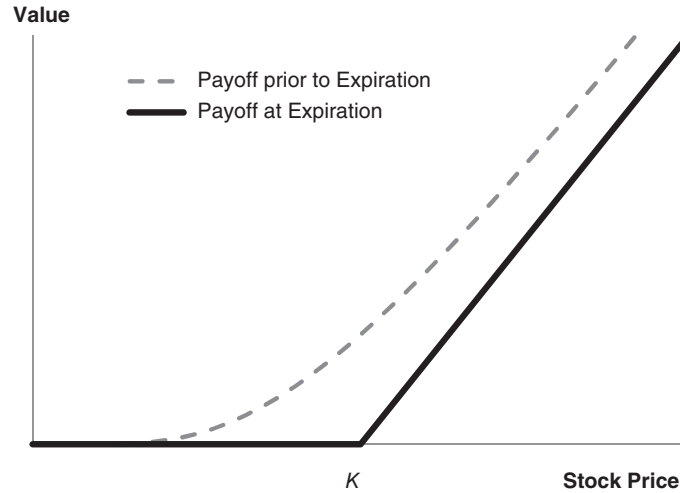


FIGURE 3.6 The Payoff of a Vanilla Call Option at Expiration

of the stock over dt is $dS = \pm \sigma S \sqrt{dt}$, so that $dS^2 = \sigma^2 S^2 dt$, irrespective of whether the stock moves up or down.

Now consider an option on the stock. The solid line in Figure 3.6 displays the payoff of a vanilla call option at expiration, and the dashed line represents its value at some earlier time, both plotted as a function of the underlying stock price. The graph of the payoff is kinked, and the value at an earlier time is more smoothly curved. Both lines have convexity, a quintessential quality of options. As a consequence of the convexity, the option increases more in value if the stock moves above the strike than if it moves the same amount below the strike. Convexity is a valuable quality in a security, and the fundamental question of options valuation is: What should you pay for convexity?

We can answer this by using the principle of replication and the law of one price, as originally discovered by Black and Scholes, and Merton. We can specify the change in the price $C(S, t)$ of a vanilla call when the underlying stock, whose price is S at time t , changes by a small amount dS during time dt , by using a Taylor series expansion of the call price:

$$C(S + dS, t + dt) = C(S, t) + \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} dS^2 + \dots \quad (3.9)$$

We have terminated the Taylor series at the dS^2 term because, from Figure 3.5, the size of the squared change in S in our binomial model is

proportional to dt . For small dt , terms involving dt^2 or $dSdt$ and any higher-order terms will be extremely small and considered negligible.

The partial derivatives in Equation 3.9 are so frequently used that practitioners denote them by the following Greek letters:

$$\Theta = \frac{\partial C}{\partial t} \quad (3.10a)$$

$$\Delta = \frac{\partial C}{\partial S} \quad (3.10b)$$

$$\Gamma = \frac{\partial^2 C}{\partial S^2} \quad (3.10c)$$

For the remainder of the book, we will refer to an option's theta, delta, or gamma when discussing these partial derivatives. We can then write Equation 3.9 more succinctly as

$$C(S + dS, t + dt) = C(S, t) + \Theta dt + \Delta dS + \frac{1}{2} \Gamma dS^2 \quad (3.11)$$

How would the value of a call option change in our binomial model when the underlying price changes as in Figure 3.5? In Figure 3.7, we use Equation 3.11 to calculate the corresponding change in the value of the call due to the stock price changes in Figure 3.5.

Except for the $\pm \Delta \sigma S \sqrt{dt}$ terms, the payoffs are the same whether the stock moves up or down. If we could somehow eliminate this Δ term, we would have a guaranteed (i.e., riskless) payoff an instant later, and, based on

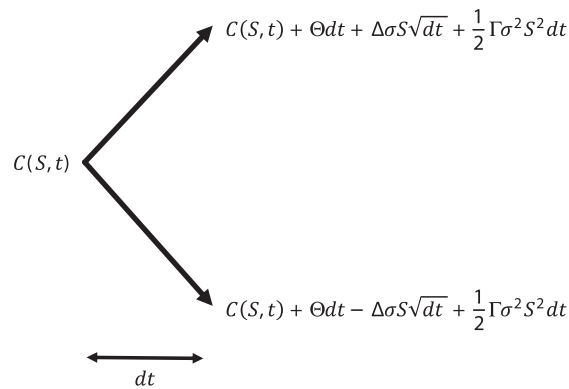
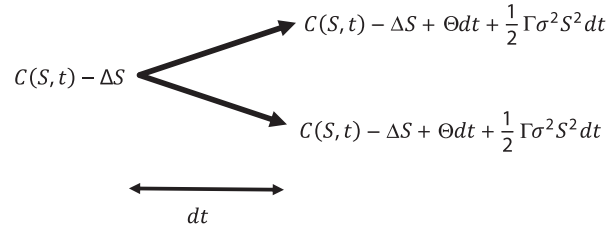


FIGURE 3.7 Binomial Model of the Value of a Call Option, $\mu = 0$

**FIGURE 3.8** Delta-Hedged Call Option, $\mu = 0$

the law of one price, we know that all riskless payoffs should earn the riskless rate of return. Requiring that the return on this instantaneously riskless portfolio be equal to the riskless rate would then lead to the Black-Scholes-Merton (BSM) option pricing formula.

In our binomial framework, in order to cancel out the $\pm\Delta\sigma S\sqrt{dt}$ terms that distinguish the up-payoff from the down-payoff in Figure 3.7, we need to short Δ shares of the underlying stock S . The binomial evolution of the long-call/short-stock portfolio, which is called a *delta-hedged portfolio*, is shown in Figure 3.8.

Since the delta-hedged portfolio in Figure 3.8 has the same value whether the stock moves up or down, it is riskless.

Call the initial value of the delta-hedged portfolio $V = C(S, t) - \Delta S$. Figure 3.8 shows that the change in value of the hedged position, V , is given by

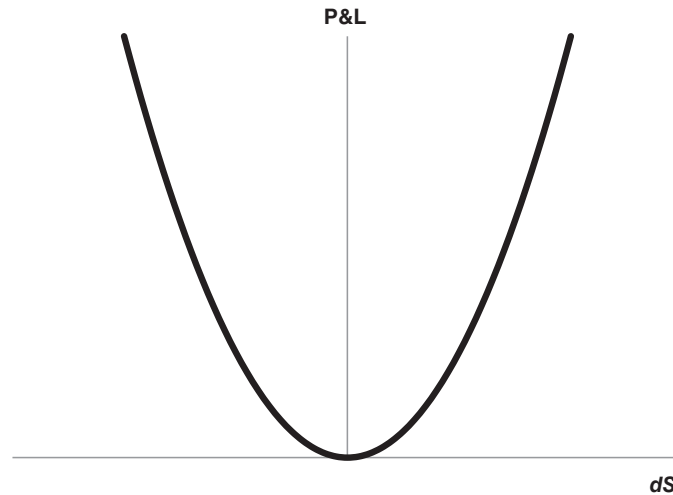
$$dV(S, t) = \Theta dt + \frac{1}{2} \Gamma \sigma^2 S^2 dt \quad (3.12)$$

or, equivalently,

$$dV(S, t) = \Theta dt + \frac{1}{2} \Gamma dS^2 \quad (3.13)$$

The second term in Equation 3.13 is quadratic in dS , and describes a parabola. It is much smaller than the linear change in value of V , proportional to dS , which has been removed by the delta hedge. If Γ is positive, then we say that the option position displays positive convexity or is convex in dS . To get the benefit of pure curvature, you must delta-hedge away the linear part of the change in the call option's value due to dS , which would otherwise swamp the small but significant change, proportional to dS^2 , that arises from the curvature.

Figure 3.9 shows the change in value (the P&L) of a hedged option with positive convexity, for a small change, dS , in the stock price.

**FIGURE 3.9** P&L with Positive Convexity**SAMPLE PROBLEM****Question:**

Yesterday, XYZ stock closed at \$100. At the close, a call option with a delta of 0.50, gamma of 0.02, and theta of -3.65 was worth \$5.00. Today, XYZ was up 10%. Using Equation 3.11, estimate the final price of the call option today. *Note:* By convention, theta is quoted in dollars per year; assume 365 days in a year.

Answer:

The change in the stock price, dS , is $\$100 \times 0.10 = \10 ; dt is $1/365$ years. Using Equation 3.11, we can estimate the final call price as

$$\begin{aligned}
 C(S + dS, t + dt) &= C(S, t) + \Theta dt + \Delta dS + \frac{1}{2} \Gamma dS^2 \\
 &= \$5 + \frac{-\$3.65}{\text{year}} \frac{1}{365} \text{year} + 0.5 \cdot \$10 + \frac{1}{2} 0.02 \cdot 10^2 \\
 &= \$5 - \$0.01 + \$5 + \$1 \\
 &= \$10.99
 \end{aligned}$$

Note, in this case, which is not atypical, most of the change in the value of the option is due to the delta term. You can often get good estimates for changes in the values of portfolios using Taylor series in this way.

What Should You Pay for Convexity?

In our binomial model, the delta-hedged option position is riskless over an infinitesimal time dt , and should therefore, according to the law of one price, earn the riskless rate of return. If we continue with the additional assumption, convenient but not necessary, that the riskless rate is zero, then our delta-hedged position should earn zero profit, so there should be no change in the value of the position after a time dt passes. From Equation 3.12, therefore,

$$\begin{aligned} dV &= \Theta dt + \frac{1}{2} \Gamma \sigma^2 S^2 dt = 0 \\ \Theta + \frac{1}{2} \Gamma \sigma^2 S^2 &= 0 \end{aligned} \quad (3.14)$$

For a long option position, when rates are zero, the amount Θdt that the option loses from time decay must be precisely offset by the gain $(1/2) \Gamma \sigma^2 S^2 dt$ that results from convexity as the stock price moves by $\pm \sigma S \sqrt{dt}$.

Written out in full, Equation 3.14 is the BSM equation for zero interest rates:

$$\frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = 0 \quad (3.15)$$

When the riskless rate r is nonzero, a riskless position worth V must earn interest $rVdt$. As we will show in a subsequent chapter, because of this, the BSM equation for nonzero rates involves two additional terms and can be written as:

$$\frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC \quad (3.16)$$

The solution is a function $C(S, t, K, T, \sigma, r)$ where K is the strike of the call option, σ is the volatility of the stock, and r is the riskless interest rate. We will display and discuss this classic Black-Scholes-Merton (BSM) formula in the next chapter.

In our binomial model, the option delta (the number of shares necessary to cancel the term linear in dS in the P&L) is fixed over our short time step, dt . Over the life of the option, as the price changes and the time to expiration

decreases, the delta of the option changes as well. Like the call price, Δ is also a function of S , t , K , T , σ , and r . The BSM equation assumes that we can instantaneously and continuously re hedge our portfolio at every instant of time using the formula for Δ . True dynamic replication in the continuous time limit is equivalent to shrinking the time step in our binomial model to an infinitesimally small interval, and making sure we re hedge at the end of each period after the underlying stock has moved up or down.

The Distinction between Implied Volatility and Realized Volatility

In the BSM formula, S , t , K , T , and r are all known at the moment the option is priced. But where do we get our value of the volatility σ ?

If you look back at Figure 3.5, you will see that σ determines the size of the next up- or down-move of the stock price S . It is a variable whose value will become known only after the move. Before that, it's a sort of guess or expectation. We can look back at the size of previous up- or down-moves in the stock to get a statistical estimate of past volatility, but future volatility is truly unknown. When Black and Scholes first started making use of their formula, they used past volatility for σ in their formula.

Over time, most people have come to use the model differently. They first obtain the price of a particular option from the market. Then they force the model to fit this market price by tuning the value of σ until the model price matches the market price. That value of σ that matches the model to the market is called the implied volatility. It's the value that the *unknown future* stock volatility has to assume in order that the model will have valued the option correctly in advance. The implied volatility is the constraint that the model wants to impose on future stock evolution. Given the implied volatility, one can then use the model to calculate the appropriate hedge ratio Δ to use in dynamic replication.

In finance we refer to backed-out estimates of the future values of parameters obtained by forcing a market price to fit a model as implied values. Implied values are predictions, but they are predictions based on currently observed market prices. The implied volatility can be fruitfully regarded as the market's expected value of future volatility. When time passes, we get to see what the value should have been. We refer to the values that we observe after time has passed as realized values. Thus, the initial value of the parameter σ that fits the model to the market price is a *parameter* called the *implied volatility*. The statistical standard deviation of returns per unit of time that can be measured after the stock has moved between t and T is a *statistic* called the *realized volatility*.

As time passes, what was once the future will become the past. One can then compare the implied volatility parameter to the realized volatility

statistic. For example, we can compare today's one-month implied volatility for a stock, extracted from option prices, to realized volatility over the next month. Similarly, in the interest rate market, one can compare implied forward interest rates to realized ones.

Is this comparison valid? Should we expect our realized statistic to match our implied parameter? If they are not expected to be the same, should you hedge with implied volatility or with what you think realized volatility might be? Though these questions do not have unambiguous answers, we will examine them in a future chapter.

Though everyone in the options world has become accustomed to this state of affairs and considers it unremarkable, it isn't. Let's compare this use of a classic financial model to the use of a classic physics model.

In mechanics, considering the motion of a projectile, one begins from its initial position and velocity and then, using Newton's laws, predicts the future trajectory. Amazingly, this works. Physics models move forward in time. In finance, when it comes to pricing options, we need first to estimate (guess?) what the future volatility of the stock will be and then to use that estimate of the future to determine the option's current price. In a sense, then, finance models go backwards in time.

In finance it is not uncommon for current values to depend on expectations about the future. Current stock prices reflect expected future earnings, life insurance premiums reflect expectation of future mortality, and fire insurance premiums reflect expectations of future fires. Future earnings, future mortality, and the probability of future fires are at present unknown, but we need to guess their future distribution in order to value these important financial products. While this backwards logic might be common in finance, the mathematical elegance and precision of the BSM model—a framework borrowed from the physics of diffusion—makes it easy to forget that we are using the model in a way that is very different from how physicists use the model.

Notation for Implied Variables

Implied variables are parameters backed out from market prices. Implied price per square foot for an apartment, for example, is the parameter in a model that matches the market price to the model price using the equation $[\text{market price}] = [\text{price per square foot}] \times [\text{area}]$. Similarly, implied volatility is the parameter that matches the model price of a call option to its market price using the BSM equation. In that sense, because they are derived from current market prices, implied variables are more closely related to the market prices than to the past or realized values of the parameters. Implied variables represent the present and the imagined future. Realized variables represent the past.

Throughout this book, we will use capital letters to represent market-derived prices. The price of a stock, bond, call, and put will typically be represented by S , B , C , and P , respectively, for example. To emphasize that implied volatility is also a market-derived parameter rather than a statistic, implied volatility will typically be represented by a capital Σ , in contrast to realized volatility, a statistic, which will be represented by a lowercase σ .

Hedging an Option Means Betting on Volatility

In accordance with our convention, we will denote the implied volatility by Σ , which in the framework of our model can be regarded as the market's anticipated value for future volatility, σ , which is unknown. If the realized volatility σ turns out to be different from what we expected, then the stock will move either more or less than we anticipated. If σ turns out to be greater than Σ , the convex delta-hedged option position $V = C - \Delta S$ in Figure 3.8 will increase in value more than anticipated, no matter which direction the stock moves. Similarly if σ is lower than anticipated, the hedged position will appreciate less.

We can quantify the gain made from convexity and the loss from time decay for a long option position. Replacing σ with Σ in Equation 3.15 to account for the fact that we anticipate a volatility of Σ , we have

$$\frac{\partial C}{\partial t} + \frac{1}{2}\Gamma\Sigma^2S^2 = 0 \quad (3.17)$$

The amount we expect to lose due to time decay during time dt is $(1/2)\Gamma\Sigma^2S^2dt$. The gain from convexity, if the stock moves an amount $dS = \pm\sigma S\sqrt{dt}$ with a realized volatility σ , is $(1/2)\Gamma\sigma^2S^2dt$. The net infinitesimal profit or loss (P&L) after time dt is then the difference between these two quantities:

$$\text{Profit} = \frac{1}{2}\Gamma S^2(\sigma^2 - \Sigma^2)dt \quad (3.18)$$

Figure 3.10 illustrates how the P&L of the hedged position varies with the realized move dS in the stock price.

As is clear from Equation 3.18 and Figure 3.10, when we delta-hedge a long option position, we are effectively making a bet on volatility. To profit, we need the realized volatility to be greater than the implied volatility. A short position profits when the opposite holds. In the next chapter we will discuss volatility and variance swaps, instruments that the market has developed to help traders bet directly on volatility.

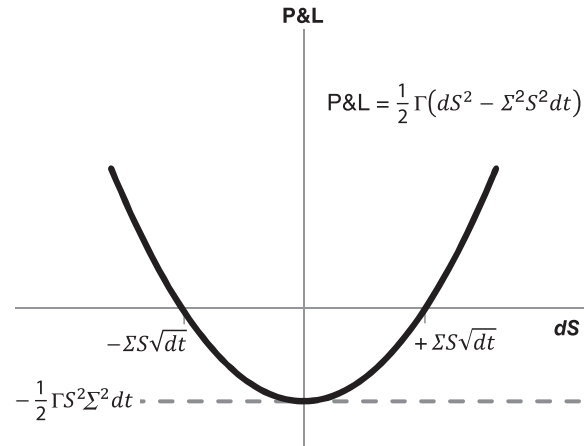


FIGURE 3.10 P&L from Implied versus Realized Volatility

END-OF-CHAPTER PROBLEMS

- 3-1. How could you replicate a collar without using any call options? Assume the underlying stock pays no dividends.
- 3-2. Figure 3.11 shows the payoff from a butterfly position $B(S, t)$ on an underlying stock, S . The break points are at x and y coordinates

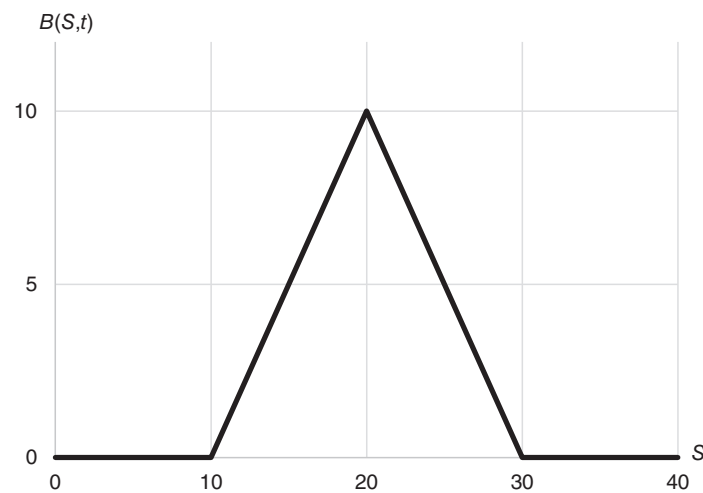


FIGURE 3.11 Payoff of Butterfly at Expiration

- (10, 0), (20, 10), and (30, 0). Replicate this payoff using riskless bonds, calls, and the underlying stock, as necessary.
- 3-3. Your firm owns 100 puts. Each put has a delta of -0.40 , gamma of 0.04 , and theta of -7.3 . The underlying price is \$100. How many shares should you buy or short in order to delta-hedge this position? After you have delta-hedged the position, how much would you expect to make if, by the end of the next day, the stock moved up 1%? Down 1%? Assume 365 days per year and a riskless rate of 0%.
- 3-4. Using the same information from the previous question, what would happen if the stock moved up 4%?
- 3-5. With the price of GOOG at \$500 per share, your firm owns 100 European call options on GOOG with a strike price of \$550, and has shorted \$10,000 worth of stock in order to delta-hedge the position correctly. Assume that interest rates are zero and that GOOG pays no dividends. If, instead of 100 calls, your firm had purchased 100 European puts at the same strike price and with the same time to expiration, how much GOOG stock would have been needed to delta-hedge the position? When interest rates are zero, what is the relationship between put and call deltas for options with the same strike and same time to expiration?
- 3-6. Figure 3.12 shows the payoff function for an option strategy at expiration in four months. Determine the value of this option strategy. The

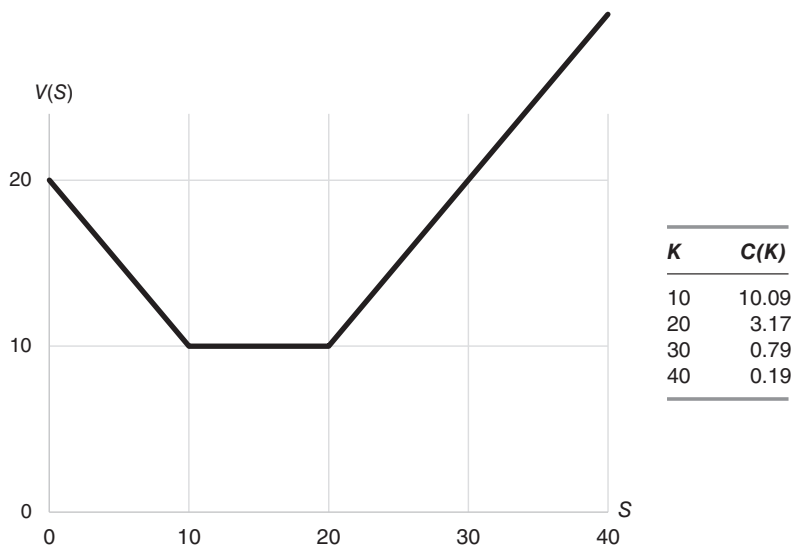


FIGURE 3.12 Option Payoff at Expiration

accompanying table provides prices for four-month calls at various strike prices. Assume the riskless rate is 0%. The current price of the underlier is \$20.

- 3-7. Replicate the payoff function from the previous problem taking into account that out-of-the-money options tend to be more liquid. Assume that you can easily buy and sell four-month calls with a strike of 20, but calls with a strike of 10 are unavailable and only puts with a strike of 10 can be traded. How can you replicate the payoff function now? What is the value?

