1

Numerical Computation

OBJECTIVES

When you have completed this chapter, you should be able to:

- **Identify** number types and systems, using principles of significant digits and rounding.
- **Solve** numerical operations using addition, subtraction, multiplication, division, reciprocals, exponents, and roots.
- **Follow** the correct order of operations to solve multi-operational problems.
- **Convert** numbers to and from scientific and engineering notation format and solve calculations with them in numerical operations.
- **Convert** units of measurement using conversion factors.
- **Solve** equations and formulas through substitution.
- **Transform** numbers into percent values and calculate percentages.

1–1 The Number Types

In mathematics, you will have trouble understanding the material unless you clearly understand the language (or meaning) of the words that are being used. So, before we start our calculator practice, let's review some definitions in Table 1-1.

TABLE 1-1 Number Types

Number Type	Definition	Example	Explanation
Integers or Signed Numbers	Whole numbers (including zero and negative whole numbers)	..., −3, −2, −1, 0, 1, 2, 3….	A *positive* (+) *number* is greater than zero (right side of zero on number line). A *negative* (−) *number* is less than zero (left side of zero on number line). It is a good idea when working with signed numbers to write either the (+) or (−) in front of the number to avoid confusion. However, the (+) sign is usually omitted because it is implied.

(continued)

TABLE 1-1 Number Types (continued)

Number Type	Definition	Example	Explanation
Rational Numbers	Includes integers and all other numbers that can be expressed as the *quotient* of two integers.	$\frac{1}{2}, -\frac{3}{5}, \frac{57}{23}, -\frac{98}{99}, 7$	A *quotient* is a number formed by dividing one number by another number.
Irrational Numbers	Numbers that *cannot* be expressed as the quotient of two integers.	$\sqrt{2}, \sqrt[3]{5}, \sqrt{7}, \pi$	These are numbers that, written as decimals, are nonterminating and nonrepeating.
Real Numbers	Rational and irrational numbers combined together into one set of numbers.	Any number other than a complex number	Rational and irrational numbers are two separate number types until we put them together into a single set of “real” numbers.
Complex Numbers	Any *imaginary* number (non-real) or a combination of a real number and an imaginary number	Any imaginary number like $\sqrt{-4}$, and any combination like $-4+\sqrt{-9}$	When we square a real number, even a negative one, we get a positive number. So, taking the square root of a negative number doesn’t make sense as a real number, and the result is an *imaginary number*.

Note that all of the numbers we will work with in Chapter 1 are real numbers. (Complex numbers are covered in Chapter 21.)

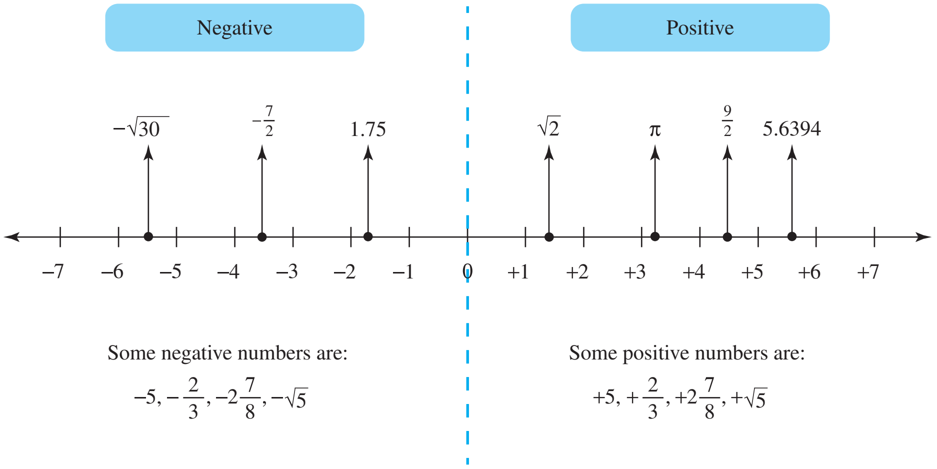

FIGURE 1-1

Positional Number Systems

A *positional number system* is one which the position of a digit determines its value. Our decimal system is positional. Table 1-2 shows the values of positions in a decimal number.

Place Value

Each position in a number has a *place value* equal to the base of the number system raised to the power of the position number. The place values in the decimal number system, as well as the place names, are shown in Table 1-2.

TABLE 1-2 Decimal Position Values

Number	Power of 10	Name
10 000	10^4	Ten Thousands
1 000	10^3	Thousands
100	10^2	Hundreds
10	10^1	Tens
1	10^0	Ones
0	0	Zero
0.1	10^{-1}	Tenths
0.01	10^{-2}	Hundredths
0.001	10^{-3}	Thousandths
0.0001	10^{-4}	Ten Thousandths

The Opposite of a Number

The *opposite* of a number n is the number which, when added to n, gives a sum of zero.

◆◆◆ **Example 1:** The opposite of 2 is -2, because $2 + (-2) = 0$. The opposite of -6 is $+6$. ◆◆◆

Geometrically, the opposite $-n$ of a number n lies on the opposite side of the zero point of the number line from n, and at an equal distance from the zero point (see Fig. 1-1). The opposite of a number is also called the *additive inverse* of that number.

Symbols of Equality and Inequality

Several symbols are used to show the relative positions of two quantities a and b on the number line.

$a = b$ means that a *equals* b and that a and b occupy the same position on the number line.

$a \neq b$ means that a and b are *not equal* and have different locations on the number line.

$a > b$ means that a is greater than b and a lies to the right of b on the number line.

$a < b$ means that a is less than b and a lies to the left of b on the number line.

$a \approx b$ means that a is approximately equal to b and that a and b are near each other on the number line.

Absolute Value

To get a good understanding of what *absolute value* means, let's look at a number line:

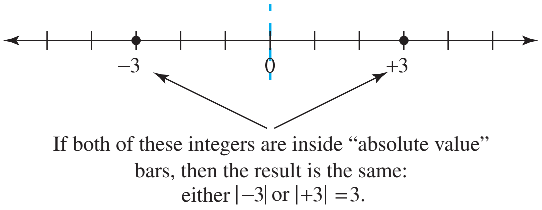

FIGURE 1-2

The *absolute value* or *magnitude* of a number n is defined as its distance from zero on a number line, regardless of its algebraic sign. Therefore, absolute value is *never* negative.
The absolute value of n is written between bars: $|n|$.

COMMON ERROR

Absolute value bars $|\ |$ around signed numbers are often confused with brackets. They do *not* work like parentheses, brackets, or braces with negative signs.

Remember: $-|-4| = -4$, *not* $+4$, as you would normally simplify $-(-4)$.

◆◆◆ **Example 2:** Find the value of $-|3 - 8|$.

Solution:

	Instruction	Looks Like
Step 1	Simplify the expression inside the absolute value bars: $3 - 8 = -5$.	$-\|-5\|$
Step 2	Convert the absolute value bars around $\|-5\|$ to a positive sign, and replace the absolute value bars with parentheses.	$-(+5)$
Step 3	Remove the parentheses and associate the negative sign to the number.	-5

We can see that the opposite of an absolute value is a negative number (or zero). ◆◆◆

◆◆◆ **Example 3:** Find each absolute value or combination of absolute values.

(a) $|5| = 5$

(b) $|-9| = (+9)$ or 9

(c) $|3 - 7| = |-4| = (+4)$ or 4

(d) $-|-4 + 7| = -|+3| = -(+3) = -3$

(e) $-|7 - 21| - |19 - 13| = -|-14| - |+6| = -(+14) - (+6) = -14 - 6 = -20$ ◆◆◆

Approximate Numbers

Most of the numbers we deal with in the fields of science and technology are *approximate*, meaning that their value is somewhat uncertain.

◆◆◆ **Example 4:**

(a) All numbers that represent *measured* quantities are approximate. A certain shaft, for example, is approximately 1.75 cm in diameter.

(b) Many *fractions* can be expressed only approximately in decimal form. Thus, $\frac{2}{3}$ is approximately equal to 0.6667.

(c) *Irrational numbers* can be written only approximately in decimal form. The number $\sqrt{3}$ is approximately equal to 1.732. ◆◆◆

Exact Numbers

Exact numbers are those that have no uncertainty.

◆◆◆ **Example 5:**

(a) There are exactly 24 hours in a day; no more, no less.

(b) An automobile has exactly four wheels.

(c) Exact numbers are usually integers, but not always. For example, an inch is *exactly* 25.4 mm, by definition.

(d) On the other hand, not all integers represent exact amounts. For example, a certain town has a population of *approximately* 3500 people. ◆◆◆

Significant Digits

Zeros are frequently used in decimal numbers as "placeholder zeros" in order to determine the location of the decimal point. These zeros are *not significant*. The remaining numbers (including any zeros in between them) are called *significant digits* (or sometimes significant figures).

◆◆◆ **Example 6:** Table 1-3 outlines some rules to follow when working with significant digits, along with examples of each rule.

TABLE 1-3 Rules for Significant Digits

Rule	Examples	
All nonzero digits are significant.	497.3	has 4 significant digits
	37.8	has 3 significant digits
	18	has 2 significant digits
Zero digits to the left of the first nonzero digit are not significant and are known as "placeholder zeros" to locate the decimal point.	0.0003	has 1 significant digit.
	0.0583	has 3 significant digits.
	0.000 583	has 3 significant digits.
Zero digits located between nonzero digits are significant. This is sometimes called the "bookend case" because the zero is "bookended" by nonzero digits.	39.05	has 4 significant digits.
	803	has 3 significant digits.
	2.0008	has 5 significant digits.
Zeros to the right of the decimal point after a number are significant. They are not needed to locate the decimal point, but show that those digits are zeros because they have been measured and are important.	8.50	has 3 significant digits.
	1.4900	has 5 significant digits.
	2.0	has 2 significant digits.
Zeros at the end of a number with an "implied decimal" are usually not significant. If the decimal point is present, these zeros are significant. An overbar is also sometimes used to show the last significant zero.	10 000	has 1 significant digit.
	10 000.	has 5 significant digits.
	$10\,\bar{0}00$	has 3 significant digits.

◆◆◆

Accuracy and Precision

The *accuracy* of a number is written as the number of significant digits in the number. The *precision* of a number is written as the position of the rightmost significant digit.

◆◆◆ **Example 7:**

Number	Accurate to:	Precise to:
1.255	4 significant digits	thousandths
23 800	3 significant digits	hundreds
0.002	1 significant digit	thousandths
3.600	4 significant digits	thousandths

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Rounding

When asked to round your answer after a computation, it is important to follow some basic rules. *Rounding down* means that when the first discarded digit is less than 5, the last retained digit does not change. *Rounding up* means that when the first discarded digit is 5 or more, the last retained digit is increased by 1. There are different rules for rounding when dealing with statistics or accounting, which will not apply in this textbook.

◆◆◆ Example 8:

Number	Rounded to Two Decimal Places	Rounded to Two Significant Digits
8.3654…	$8.36 \mid \cancel{54}\ldots$ 8.37 (rounded up)	$8.3 \mid \cancel{654}\ldots$ 8.4 (rounded up)
8.3456…	8.35	8.3
7.365 01…	7.37	7.4
14.364 999…	14.36	14
142.764 999…	142.76	140
3.141 592…	3.14	3.1

Note that "…" to the right of a decimal number shows that more digits follow to the right, which are not shown. ◆◆◆

◆◆◆ Example 9:

Number	Rounded to Three Decimal Places	Rounded to Three Significant Digits
4.3654	4.365	4.37
4.3656	4.366	4.37
4.365 501	4.366	4.37
4.365 499	4.365	4.37
1.764 999	1.765	1.76
1.764 499	1.764	1.76
−8.3499	−8.350	−8.35
−8.3599	−8.360	−8.36

◆◆◆

Exercise 1 ◆ The Number Types

Equality and Inequality Signs

Insert the proper equality or inequality sign (= , ≈, >, or <) between each pair of numbers.

1. 7 and 10
2. 9 and −2
3. −3 and 4
4. −3 and −5
5. $\frac{3}{4}$ and 0.75
6. $\frac{2}{3}$ and 0.667

Absolute Value

Evaluate the expression.

7. $-|9-23|-|-7+3|$
8. $-|7+45|-|-8-34|$
9. $|12-5+8|-|-6|+|15|$
10. $|13-6+9|-|-8|+|13|$
11. $-|3-9|-|5-11|+|21+4|$
12. $-|4-8|+|-5+11|-|-12-6|$

Significant Digits

State the number of significant digits in each approximate number.

13. 78.3
14. 925.3
15. 9274
16. 29 471
17. 4.008
18. 5.0004
19. 9400
20. 36 000
21. 20 000.
22. $80\overline{0}\,000$
23. 5000.0
24. 60 000.0
25. 0.9972
26. 0.875 32
27. 1.0000
28. 63.0000

Round each number to two decimal places.

29. 38.468
30. 1.996
31. 96.835 001
32. 55.8650
33. 398.372
34. 2.9573
35. 2985.339
36. 278.382

Round each number to one decimal place.

37. 13.98
38. 745.62
39. 5.6501
40. 0.482
41. 398.36
42. 34.927
43. 9839.2857
44. 0.847

Round each number to the nearest hundred.

45. 28 583
46. 7550
47. 3 845 240
48. 274 837

Round each number to three significant digits.

49. 9.284
50. 2857
51. 0.048 25
52. 483 982
53. 0.083 75
54. 29.555
55. 29.450 01
56. 8372

1–2 Numerical Operations

A *numerical operation* can be described as an action or process used to solve a numerical problem. The most basic numerical operations are addition, subtraction, multiplication, and division. Exponents and roots are also numerical operations. Let's review the rules of these operations.

Addition and Subtraction

TABLE 1-4 Addition and Subtraction

Numerical Operation	Rule	Example	Explanation
Addition	Signs	$a + (-b) = a - b$	When adding a negative number $-b$ to a number a, subtract b from a.
	Commutative Law	$a + b = b + a$	You can add numbers in any order.
	Associative Law	$a + (b + c) = (a + b) + c$	You can group numbers to be added in different ways.
	Approximate Numbers	32.4 cm + 5.825 cm = 38.2 cm (not ≈ 38.225 cm)	When adding approximate numbers, keep as many decimal places in your answer as the number having the *fewest* decimal places or significant digits in the question. Do not use the ≈ symbol.
Subtraction	Signs	$a - (-b) = a + b$	When subtracting a negative number $-b$ from a number a, you add b to a.
	Approximate Numbers	79.434 m − 8.9954 m = 70.439 m (not ≈ 70.4386 m)	When subtracting approximate numbers, keep as many decimal places in your answer as the number having the *fewest* decimal places or significant digits in the question. Do not use the ≈ symbol.

Adding Signed Numbers

◆◆◆ **Example 10:**

(a) $7 + (-2) = 7 - 2 = 5$

(b) $-8 + (-3) = -8 - 3 = -11$

(c) $9.92\text{ m} + (-15.36\text{ m}) = 9.92\text{ m} - 15.36\text{ m} = -5.44\text{ m}$

◆◆◆

Subtracting Signed Numbers

◆◆◆ **Example 11:**

(a) $15 - (-3) = 15 + 3 = 18$

(b) $-5 - (-9) = -5 + 9 = 4$

(c) $-25.62 - (-5.15) = -25.62 + 5.15 = -20.47$

◆◆◆

Subtracting Negative Numbers by Calculator

COMMON ERROR

Be careful when calculating signed numbers. Calculators often have two separate buttons with the 'negative' sign. The – button is for the operation of subtraction and the (–) or +/– button on most calculators is for changing signs on signed numbers.

Addition and Subtraction of Approximate Numbers

It is a good practice to keep all the digits given throughout the calculation process, until your final answer is needed. Generally, for multiple calculation problems, your calculator will keep all the digits for you if you use the ANS button.

Don't round given numbers or calculations *before* computing your final calculation. Round the final calculated answer as the problem requires.

◆◆◆ **Example 12:**

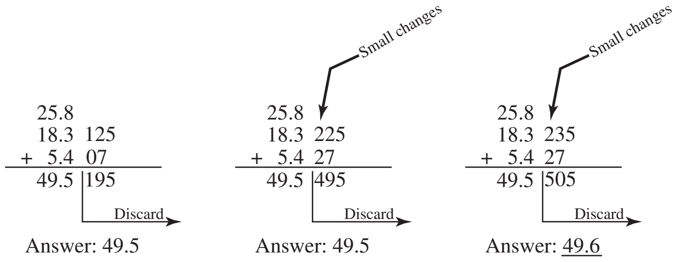


FIGURE 1-3

◆◆◆

◆◆◆ **Example 13:** A sports stadium contains about 3500 people. It starts to rain and 372 people leave. How many are left in the stadium?

Solution: Subtracting, we obtain

$$3500 - 372 = 3128$$

We round our answer to 3100 people, because the word "about" tells you that 3500 likely has only two significant digits.

◆◆◆

Adding and Subtracting Exact and Approximate Numbers

When combining an exact number with an approximate number, the approximate number will limit the accuracy of the result. You will need to round the answer to the number of decimal places found in the approximate number, even if the exact number has fewer decimal places.

◆◆◆ **Example 14:** Express 2 h and 35.8 min in minutes.

Solution: We must add an exact number, 120 min, and an approximate number, 35.8 min:

$$120 \text{ min} + 35.8 \text{ min} = 155.8 \text{ min}$$

Since 120 is an exact number, we do not round our answer to the nearest 10-min interval; instead we retain as many decimal places as in the approximate number. So our answer is 155.8 minutes.

◆◆◆

Multiplication and Division

The numbers we multiply to get a *product* are called *factors*. For example,

$$\text{Factors} \rightarrow \underset{\uparrow}{3} \times \underset{\uparrow}{5} = \underset{\uparrow}{15} \leftarrow \text{Product}$$

◆◆◆ **Example 15:** Use a calculator to multiply 183 by 27.

Solution: You should get $183 \times 27 = 4941$. ◆◆◆

TABLE 1-5 Multiplication and Division

Numerical Operation	Rule	Example	Explanation
Multiplication	Signs	$(+a)(+b) = (-a)(-b) = +ab$ $(+a)(-b) = (-a)(+b) = -ab$ $(+a)(-b)(+c)(-d) = +abcd$ $(-a)(-b)(-c) = -abc$	If a and b have the *same sign*, the product is signed +. If a and b have *different signs*, the product is signed −. When multiplying a string of numbers together, if an *even* number of factors is negative, the answer will be *positive*. If an *odd* number of factors are negative, the answer will be *negative*.
	Commutative Law	$ab = ba$	The *order* of multiplication is not important.
	Associative Law	$a(bc) = (ab)c = (ac)b = abc$	We can *group* the numbers to be multiplied any way we want.
	Distributive Law	$a(b + c) = ab + ac$	You can multiply a factor *either* by adding the group of numbers in the brackets first *or* by multiplying each factor separately, then adding the products together.
	Approximate Numbers	$\underset{\substack{\uparrow \\ \text{five} \\ \text{digits}}}{123.56} \times \underset{\substack{\uparrow \\ \text{three} \\ \text{digits}}}{2.21} = \underset{\substack{\uparrow \\ \text{keep} \\ \text{three} \\ \text{digits}}}{273}\ (273.0676)$	When multiplying two or more approximate numbers, round the answer to as many significant digits as the factor having the *fewest* significant digits in the question. Do not use the ≈ symbol.
	Exact Numbers	If a certain car tire weighs 32.2 kg when mounted, how much will four of these tires weigh? Solution: 32.2(4) = 128.8 kg Since 4 is an *exact* number, we keep as many significant digits as contained in 32.2 kg and round our answer to 129 kg.	When using *exact* numbers in a computation, treat them as if they had *more* significant digits than any of the approximate numbers in that computation.

(continued)

TABLE 1-5 Multiplication and Division (continued)

Numerical Operation	Rule	Example	Explanation
Division	Signs	$\frac{+a}{+b} = \frac{-a}{-b} = \frac{a}{b}$ and $\frac{+a}{-b} = \frac{-a}{+b} = -\frac{a}{b}$	The quotient is *positive* when dividend and divisor have the *same* sign. The quotient is *negative* when dividend and divisor have *opposite* signs.
	Zero in the Divisor	You will get an error on your calculator.	Division by zero is *not defined* and is an illegal operation in math.
	Zero in the Dividend	$\frac{0}{243} = 0$	Zero divided by any quantity (except zero) is zero.
	Approximate Numbers	846.2 ÷ 4.75 = 178.147 368 4 Since 4.75 has three significant digits, we round our quotient to 178.	When dividing one *approximate* number by another, round the quotient to as many digits as there are in the dividend or divisor with the *fewest* significant digits.
	Reciprocals	Reciprocal of $n = \frac{1}{n}$	This is the reciprocal rule.
		$\frac{\cancel{n}}{1} \times \frac{1}{\cancel{n}} = 1$	A *reciprocal* is a nonzero number "turned upside down" which when multiplied with the original number is equal to 1.

Note that $\frac{a}{b}$ is also referred to as a *fraction* or as the *ratio* of a to b (discussed further in Chapters 4 and 14).

Multiplication Quick Review

Remember to think of multiplication as repeated addition. For example, to multiply 3 by 4 means to add four 3s (or three 4s):

$$3 \times 4 = 3 + 3 + 3 + 3$$

or

$$3 \times 4 = 4 + 4 + 4$$

Multiplying Signed Numbers

◆◆◆ **Example 16:**

(a) $2(-3) = -6$

(b) $(-2)3 = -6$

(c) $(-2)(-3) = 6$

(d) $2(-3)(-1)(-2) = -12$

(e) $2(-3)(-1)(2) = 12$

◆◆◆

Line-Entry Multiplication

◆◆◆ **Example 17:** Use your calculator to multiply –96 by –83.

Solution:

(–) 9 6 × (–) 8 3 = 7968

$$(-96)\times(-83)=7968$$

◆◆◆

Division Quick Review

The *dividend*, when divided by the *divisor,* gives us the *quotient.* We can show this in two different ways, as follows.

◆◆◆ **Example 18:** Use your calculator to divide 861 by 123.

Solution: 861 (dividend) ÷ 123 (divisor) = 7 (quotient)

◆◆◆

◆◆◆ **Example 19:** When we divide 2 by 3, we get 0.666 666 666…., a *repeating decimal.* So we need to choose how many significant digits we want to keep, and then round our answer. If we round to three significant digits the answer becomes:

$$\frac{2\,(\text{dividend})}{3\,(\text{divisor})}\approx 0.667\,(\text{quotient})$$

Here it is appropriate to use the ≈ (“approximately equal to”) symbol because the answer is a repeating decimal. It is a good idea to check with your instructor to find out how many significant digits are expected.

◆◆◆

Dividing Signed Numbers

◆◆◆ **Example 20:**

(a) $85.4 \div (-2.5386) = -33.6$

(b) $-467.387 \div 3.22 = -145$

(c) $-4.8881 \div (-1.23456) = 3.9594$

◆◆◆

COMMON ERROR

Don’t confuse significant digits with decimal places. The number 274.56 has five significant digits and two decimal places. We use decimal places to determine rounding when adding or subtracting. We use significant digits to determine rounding when multiplying, dividing, raising to a power, or taking roots.

Reciprocals

Remember the *reciprocal rule*:

$$\text{Reciprocal of } n = \frac{1}{n}$$

◆◆◆ **Example 21:**

(a) The reciprocal of 10 is $\frac{1}{10}$.

(b) The reciprocal of $\frac{1}{2}$ is 2.

(c) The reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$.

◆◆◆

The reciprocal key on your calculator may look like this x^{-1} or this 1/x. Both give you the same results.

On some calculators, the reciprocal key might be a second function key; on other calculators, it might be available only from the calculator’s menu.

Be sure to identify the reciprocal key on your calculator.

◆◆◆ **Example 22:** Use your calculator to verify these statements. Assume the same number of significant digits in the answer as in the given number.

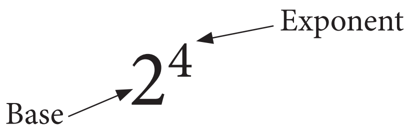
(a) The reciprocal of 6.38 is 0.157.

(b) The reciprocal of –2.754 is –0.3631.

◆◆◆

Exponents Defined

Let's look at the following expression:

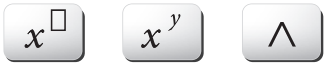


The number 2 is called the *base,* and the number 4 is called the *exponent.* The expression is read "two to the fourth power." Its value is:

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

CALCULATOR TIP

The exponent key on your calculator may look like one of these:



◆◆◆ **Example 23:** Use your calculator to verify that $(3.85)^3 = 57.1$ (rounded to three significant digits, or 3 s.d.).

Solution: The answer on your calculator will be 57.066625, which we then round to three significant digits.

◆◆◆

Exponents and Roots

Sometimes, we have an exponent to calculate that has a negative base. We may also need to find roots, which are like fractional exponents. However, there are rules that still allow us to calculate exponents in these cases.

TABLE 1-6 Exponents and Roots

Numerical Operation	Rule Name	Example	Explanation
Exponents	Negative Base	$(-1)^{24} = 1$	A negative base raised to an *even* power gives a *positive* answer.
		$(-1)^{25} = -1$	A negative base raised to an *odd* power gives a *negative* answer.
Roots	Principal Root	$\sqrt{4} = +2$, not ± 2	The *principal root* of a *positive* number means the *positive* answer.
	Odd Root	$\sqrt[3]{-8} = -2$ because $(-2)(-2)(-2) = -8$	When we take the *odd* root of a *negative* number, we get a principal root that is *negative*.

Negative Base Exponents

◆◆◆ **Example 24:**

(a) $(-2)^2 = (-2)(-2) = 4$

(b) $(-2)^3 = (-2)(-2)(-2) = -8$

◆◆◆

CALCULATOR TIP

Some older calculators are not able to work with a negative base, even though this is a valid operation. Make sure your calculator will perform this operation.

If your calculator won't complete this operation, simply enter the base as a *positive* number, find the power, and determine the sign by inspection.

◆◆◆ **Example 25:** Find $(-1.45)^5$.

Solution: If your calculator works with negative bases, key in:

(–) 1 . 4 5 x^n 5 =

-1.45^5

–6.4097...

Otherwise, key in:

1 . 4 5 x^n 5 =

1.45^5

6.4097...

Since we know that a negative number raised to an odd power is negative, we write:

$$(-1.45)^5 = -6.41 \, (3\,\text{s.d.})$$

◆◆◆

◆◆◆ **Example 26:** Use your calculator to verify that

$$(3.85)^{-3} = 0.0175 \, (3\,\text{s.d.})$$

Solution: On your calculator, key in:

3 . 8 5 x^n (–) 3 =

3.85^{-3}

0.0175233...

◆◆◆

Important: Raising a *positive number* to a *negative power* does *not* result in a *negative number*.

Fractional Exponents

We will see in Chapter 11 that *fractional exponents* are another way of writing radicals. Right now, let's just practice entering fractional exponents on the calculator. The line entry keys are the same as before; however, we need to make sure that the fractional exponent is enclosed in parentheses, as shown in the following example.

◆◆◆ **Example 27:** Use your calculator to verify that $8^{\frac{2}{3}} = 4$.

Solution: On your calculator, key in

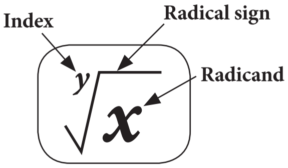
$x^{\square}$ 8 ▶ (2 ÷ 3) ▶ =

4

◆◆◆

Roots

If $z^y = x$ then $z = \sqrt[y]{x}$.



This reads "the *y*th root of *x* equals *z*". The symbol $\sqrt{\ }$ is a *radical sign*, *x* is the *radicand,* and *y* is the *index* of the radical. In the following calculator line entry, we can see that the cube root of 64 is equal to 4.

Principal Roots

Example 28:

(a) $\sqrt{4} = 2$ because $2^2 = 4$

(b) $\sqrt[3]{8} = 2$ because $2^3 = 8$

(c) $\sqrt[4]{81} = 3$ because $3^4 = 81$

◆◆◆

Example 29: Use your calculator to verify that

$$\sqrt[5]{28.4} = 1.952\ 826\ 537\ldots$$

which we round to 1.95.

◆◆◆

Odd Roots of Negative Numbers by Calculator

Remember from Table 1-6 that when we take the *odd* root of a *negative* number, we get a *negative* answer that is *not* imaginary. However, just as with exponents, some calculators do not accept a negative radicand. Let's outsmart our calculators and take the odd roots of negative numbers anyway!

Example 30: Find $\sqrt[5]{-875}$.

Solution: We know that an odd root of a negative number is real and negative, so we begin by taking the fifth root of +875. Using your calculator the line entry looks like this:

3.876 159 242

and we just put a negative sign in front of our answer: −3.876 159 242 which is rounded to −3.88 (3 s.d.).

◆◆◆

Exercise 2 ◆ Numerical Operations

Addition and Subtraction

Combine as indicated.

1. −955
+212
−347

2. 8275
−2163
−874

3. −748
−212
−156

Add each column of figures.

4. $99.84
24.96
6.24
1.56
12.48
0.98
3.12

5. 96 256
6 016
376
141
188
1 504
752

6. 98 304
6 144
384
24 576
3 072
144
49 152

Combine as indicated.

7. −576 + (−553)
8. −207 + (−819)
9. 1123 − (−704)
10. 818 − (−207) + 318

Combine each set of approximate numbers as indicated. Round your answer.

11. 39.75 + 27.4
12. 296.44 + 296.997
13. 385.28 − 692.8
14. 0.000 583 + 0.000 8372 − 0.001 73

Word Problems Involving Addition and Subtraction

15. Mt. Blanc is 15 572 ft. high, and Pike's Peak is about 14 000 ft. high. What is the difference in their heights?

16. Ontario contains 1 068 582 km^2, and Quebec 1 540 681 km^2. How much more area does Quebec contain than Ontario?

17. A man willed $125,435 to his wife and two children. To one child he gave $44 675, to the other $26 380, and to his wife the remainder. What was his wife's share?

18. A circular pipe has an inside radius *r* of 10.6 cm and a wall thickness of 2.125 cm. It is surrounded by insulation having a thickness of 4.8 cm (Fig. 1-4). What is the outside diameter *D* of the insulation?

19. A batch of concrete is made by mixing 267 kg of aggregate, 125 kg of sand, 75.5 kg of cement, and 25.25 kg of water. Find the total mass of the mixture.

20. Three resistors, having resistances of 27.3 ohms (Ω), 4.0155 Ω, and 9.75 Ω, are wired in series. What is the total resistance? (See Fig. 1-5 and Eq. A63 from Appendix A, which states that the total series resistance is the sum of the individual resistances.)

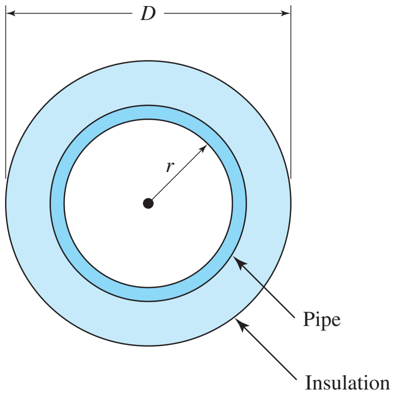


FIGURE 1-4

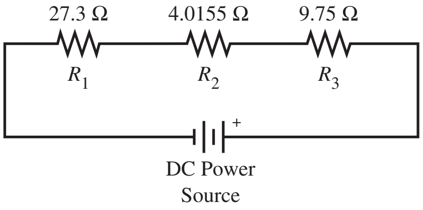


FIGURE 1-5

Multiplication

Multiply each set of approximate numbers and retain the correct number of digits in your answer.

21. 3967 × 1.84
22. 4.900 × 59.3
23. 93.9 × 0.005 5908
24. 4.97 × 9.27 × 5.78
25. 69.0 × (–258)
26. –385 × (–2.2978)
27. 2.86 × (4.88 × 2.97) × 0.553
28. (5.93 × 7.28) × (8.26 × 1.38)

Word Problems Involving Multiplication

29. What is the cost of 52.5 tonnes (t) of cement at $99.25 per tonne?

30. If 68 t of rail is needed for 1 km of track, how many tonnes will be required for 762 km, and what will be its cost at $1,425 per tonne?

31. Three barges carry 26 t of gravel each, and a fourth carries 35 t. What is the value, to the nearest dollar, of the whole shipment, at $22.75 per tonne?

32. Two cars start from the same place and travel in opposite directions, one at a rate of 45 km/h, the other at 55 km/h. How far apart will they be at the end of 6.0 h?

33. What will be the cost of installing a telephone line 274 km long, at $5,723 per kilometre?

34. The current to a projection lamp is measured at 4.7 A when the line voltage is 115.45 V. Using Eq. A65 (power = voltage × current), find the power dissipated in the lamp.

35. A gear in a certain machine rotates at a speed of 1808 rpm (revolutions per minute). How many revolutions will it make in 9.500 min?

36. How much will 1000 washers weigh if each weighs 2.375 g?

37. One inch equals exactly 2.54 cm. Convert 385.84 in. to centimetres.

38. If there are 360 degrees per revolution, how many degrees are there in 4.863 revolutions?

Division

Divide, and then round your answer to the proper number of digits.

39. 947 ÷ 5.82
40. 0.492 ÷ 0.004 78
41. –99.4 ÷ 286.5
42. –4.8 ÷ (–2.557)
43. 5836 ÷ 8264
44. 5.284 ÷ 3.827
45. 94 840 ÷ 1.338 76
46. 3.449 ÷ (–6.837)

Word Problems Involving Division

47. A stretch of roadway 1858.54 m long is to be divided into 5 equal sections. Find the length of each section.

48. At the rate of 24.5 km in 8.25 h, how fast is this person walking?

49. Canadian energy consumption for a given year is 1.73 billion barrels of oil per year. If there are 365 days in a year, what is the consumption per day, in millions of barrels? Round to two decimal places.

50. If 867 shares of pipeline stock are valued at $84,099, what is the value of each share?

Reciprocals

Find the reciprocal of each number, retaining the proper number of digits in your answer.

51. 693
52. 0.006 30
53. –396 000
54. –0.005 73
55. 938.4
56. 1.007

Word Problems Involving Reciprocals

57. Find the equivalent resistance of a 475 Ω resistor (R_1) and a 928 Ω resistor (R_2) when they are connected in a parallel circuit. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

58. When an object is placed 126 cm in front of a thin lens having a focal length f in centimetres, the image will be formed 245 cm from the lens. Find f, given that the distances are related by this equation:

$$\frac{1}{f} = \frac{1}{126} + \frac{1}{245}$$

59. The sine of an angle θ (written $\sin\theta$) is equal to the reciprocal of the cosecant of θ ($\csc\theta$). Find $\sin\theta$ if $\csc\theta = 3.58$.

60. If two straight lines are perpendicular, the slope of one line is the negative reciprocal of the slope of the other. If the slope of a line is –2.55, find the slope of a line perpendicular to that line.

Powers

Evaluate each power without using a calculator. Do not round your answers.

61. 2^3
62. 5^3
63. $(-2)^3$
64. 9^2
65. 1^3
66. $(-1)^2$
67. 3^2
68. 1^8
69. $(-1)^3$

Evaluate each power of 10.

70. 10^2
71. 10^1
72. 10^0
73. 10^5
74. 10^{-2}
75. 10^{-1}
76. 10^4
77. 10^{-3}
78. 10^{-5}

Evaluate each expression, retaining the correct number of digits in your answer.

79. $(8.55)^3$
80. $(-77.2)^2$
81. $(8.28)^{-2}$
82. $(0.0772)^{0.426}$
83. $(5.28)^{-2.15}$
84. $(35.2)^{1/2}$
85. $(462)^{2/3}$

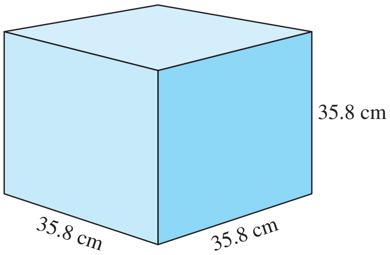


FIGURE 1-6

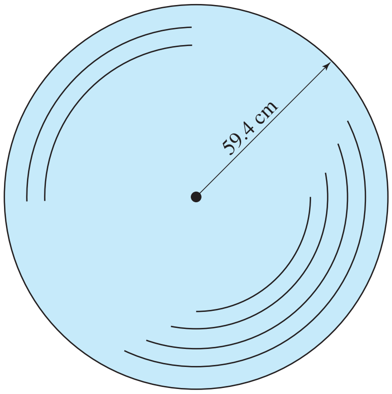


FIGURE 1-7

Word Problems Involving Powers

86. The distance travelled, in feet, by a falling body that starts from rest is equal to $16t^2$, where t is the elapsed time in seconds. In 5.448 s, the distance fallen is $16(5.448)^2$ ft. Evaluate this quantity. (Treat 16 as an approximate number.)

87. The power dissipated in a resistance R through which a current I is flowing is equal to I^2R. Therefore the power dissipated in a 365 Ω resistor carrying a current of 0.5855 A is $(0.5855)^2(365)$ W. Evaluate this power.

88. The volume of a cube of side 35.8 cm (Fig. 1-6) is $(35.8)^3$ cm^3. Evaluate this volume.

89. The volume of a sphere with radius 59.4 cm (Fig. 1-7) is $\frac{4}{3}\pi(59.4)^3$ cm^3. Find this volume.

Roots

Find each principal root without using your calculator.

90. $\sqrt{25}$
91. $\sqrt[3]{27}$
92. $\sqrt[3]{-27}$
93. $\sqrt[5]{-32}$

Evaluate each radical by calculator, retaining the proper number of digits in your answer:

94. $\sqrt{49.2}$

95. $\sqrt{1.863}$

96. $\sqrt[3]{88.3}$

97. $\sqrt[5]{-18.4}$

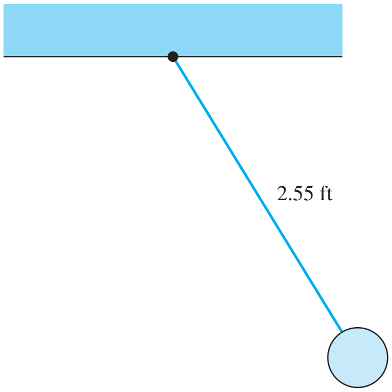


FIGURE 1-8

Applications of Roots

98. The period T (time for one swing), in seconds, of a simple pendulum 2.55 ft. long (Fig. 1-8) is

$$T = 2\pi\sqrt{\frac{2.55}{32.0}}\ \text{s}$$

Evaluate T.

99. The magnitude Z of the impedance in a circuit having a resistance of 3540 Ω and a reactance of 2750 Ω is

$$Z = \sqrt{(3540)^2 + (2750)^2}\ \Omega$$

Find Z.

100. The geometric mean B between 3.75 and 9.83 is

$$B = \sqrt{(3.75)(9.83)}$$

Evaluate B.

1–3 Order of Operations

Follow this order for completing math calculations when there is more than one operation to be completed:

Remember **BEDMAS**: Brackets (Parentheses), Exponents, Division, Multiplication, Addition, Subtraction.

TABLE 1-7 Order of Operations

Order	Operation
1	Parentheses (brackets)
2	Exponents and roots (in any order)
3	Multiplication and division (from left to right)
4	Addition and subtraction (from left to right)

Our first group of calculations will be with integers only, followed by some problems that will require rounding. First, let's look at a problem containing both addition and multiplication.

◆◆◆ **Example 31:** Evaluate $7 + 3 \times 4$.

Solution: According to our rules for the order of operations, we do the multiplication before the addition.

$$7 + 3 \times 4 = 7 + 12 = 19$$

◆◆◆

◆◆◆ **Example 32:** Evaluate 5×3^2.

Solution: We do the exponent operation before multiplying:

$$5 \times 3^2 = 5 \times 9 = 45$$

◆◆◆

Parentheses

When an expression contains a parenthesis, solve the expression inside the parenthesis first and then move on to the expression as a whole.

◆◆◆ **Example 33:** Evaluate $(7 + 3) \times 4$.

Solution:

$$(7 + 3) \times 4 = 10 \times 4 = 40$$

◆◆◆

MATH SHOW OFF

If the sum or difference of more than one number is to be raised to an exponent, those numbers must be enclosed in parentheses.

Let's look at an example.

◆◆◆ **Example 34:** Evaluate $(5 + 2)^2$.

Solution: We combine the numbers inside the parenthesis before squaring.

$$(5 + 2)^2 = 7^2 = 49$$

◆◆◆

◆◆◆ **Example 35:** Evaluate $(2 + 6)(7 + 9)$.

Solution: Do the math inside the parentheses before multiplying.

$$(2 + 6)(7 + 9) = 8 \times 16 = 128$$

(It's the same answer if you use the "FOIL" or "First, Outer, Inner, Last" rule. Try it!) ◆◆◆

Important: Be careful to treat the numerator and denominator of a fraction like parentheses.

◆◆◆ **Example 36:** Evaluate $\dfrac{8 + 4}{9 - 3}$.

Solution: We will treat the parts of the fraction the same as parentheses, so written as a single line, we break down the problem like this:

$$(8 + 4) \div (9 - 3) = 12 \div 6 = 2$$

◆◆◆

Order of Operations with Approximate Numbers

Approximate numbers follow the same rules for order of operations, but we need to be careful when rounding, making sure we follow the rules we learned earlier in this chapter.

◆◆◆ **Example 37:** Evaluate this expression involving approximate numbers.

$$\left(\frac{118.8 + 4.23}{\sqrt{136}}\right)^3$$

Solution: Using a calculator, our answer is 1174.153047… . The question now becomes "How many significant digits do we keep?" Remembering back to our rules on significant digits with approximate numbers, let's break it down:

$$\left(\frac{118.8\,(\text{four significant digits, one decimal}) + 4.23\,(\text{three significant digits, two decimal places})}{\sqrt{136}\,(\text{three significant digits})}\right)^3$$

When we add the numerator, we are allowed to keep one decimal place, so we get an answer with four significant digits:

$$\left(\frac{123.0\,(\text{four significant digits, one decimal})}{\sqrt{136}\;(\text{three significant digits})}\right)^3$$

The denominator has just three significant digits, so we round our answer to three significant digits: 1170. ◆◆◆

◆◆◆ **Example 38:** A rectangular courtyard (Fig. 1-9) with side lengths of 21.8 ft. and 33.74 ft. has a diagonal measurement x in feet. We will use the Pythagorean Theorem to solve for x.

$$x = \sqrt{a^2 + b^2}$$
$$x = \sqrt{(21.8)^2 + (33.74)^2}$$

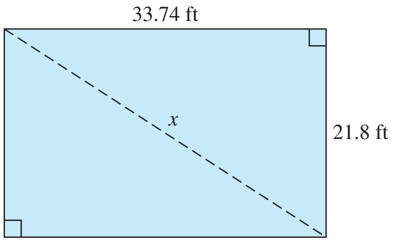


FIGURE 1-9

Solution: Using a calculator we get $x = 40.169\,983\ldots$. We round our answer to three significant digits because 21.8 has only three significant digits, so the diagonal measurement is 40.2 ft. ◆◆◆

Exercise 3 ◆ Order of Operations

Combined Operations with Exact Numbers

Perform each computation by calculator.

1. $(37)(28) + (36)(64)$
2. $(22)(53) - (586)(4) + (47)(59)$
3. $(63 + 36)(37 - 97)$
4. $(89 - 74 + 95)(87 - 49)$
5. $\dfrac{219}{73} + \dfrac{194}{97}$
6. $\dfrac{228}{38} - \dfrac{78}{26} + \dfrac{364}{91}$
7. $\dfrac{647 + 688}{337 + 108}$
8. $\dfrac{809 - 463 + 1858}{958 - 364 + 508}$
9. $(5 + 6)^2$
10. $(422 + 113 - 533)^4$
11. $(423 - 420)^3$
12. $\left(\dfrac{853 - 229}{874 - 562}\right)^2$
13. $\left(\dfrac{141}{47}\right)^3$
14. $\sqrt{434 + 466}$
15. $\sqrt{(8)(72)}$
16. $\sqrt[3]{657 + 553 - 1085}$
17. $\sqrt[4]{(27)(768)}$
18. $\sqrt{\dfrac{2404}{601}}$
19. $\sqrt[4]{\dfrac{1136}{71}}$
20. $\sqrt[4]{625} + \sqrt{961} - \sqrt[3]{216}$

Order of Operations with Approximate Numbers

Perform each computation, keeping the proper number of digits in your answer.

21. $(7.37)(3.28) + (8.36)(2.64)$
22. $(522)(9.53) - (586)(4.70) + (847)(7.59)$
23. $(63.5 + 83.6)(8.37 - 1.72)$
24. $\dfrac{583}{473} + \dfrac{946}{907}$
25. $\dfrac{6.73}{8.38} - \dfrac{5.97}{8.06} + \dfrac{8.63}{1.91}$
26. $\dfrac{809 - 463 + 744}{758 - 964 + 508}$
27. $(5.37 + 2.36)^2$
28. $(4.25 + 4.36 - 5.24)^4$
29. $(6.423 + 1.05)^2$
30. $\left(\dfrac{45.3 - 8.34}{8.74 - 5.62}\right)^{2.5}$
31. $\left(\dfrac{8.90}{4.75}\right)^2$
32. $\sqrt{4.34 + 4.66}$
33. $\sqrt[3]{657 + 553 - 842}$
34. $\sqrt{(28.1)(5.94)}$
35. $\sqrt[5]{(9.06)(4.86)(7.93)}$
36. $\sqrt{\dfrac{653}{601}}$
37. $\sqrt[4]{\dfrac{4.50}{7.81}}$
38. $\sqrt{9.74} + \sqrt{12.5}$
39. $\sqrt[4]{528} + \sqrt{94.2} - \sqrt[3]{284}$
40. $\sqrt[3]{653} \times \sqrt{55.3}$

1–4 Scientific and Engineering Notation

Evaluating Powers of 10

We did some work with powers (exponents) in Section 1–2 and saw, for example, that 2^3 means

$$2^3 = 2 \times 2 \times 2 = 8$$

Here, the power 3 tells us how many 2s are to be multiplied to give the product. For powers of 10, the power tells us how many 10s are to be multiplied to give the product.

◆◆◆ Example 39:

(a) $10^2 = 10 \times 10 = 100$

(b) $10^3 = 10 \times 10 \times 10 = 1000$

Negative powers are calculated by $x^{-a} = \dfrac{1}{x^a}$ (Eq. 35 in Appendix A).

◆◆◆

◆◆◆ Example 40:

(a) $10^{-2} = \dfrac{1}{10^2} = \dfrac{1}{100} = 0.001$

(b) $10^{-5} = \dfrac{1}{10^5} = \dfrac{1}{100\,000} = 0.000\,01$

Some powers of 10 are summarized in the following table:

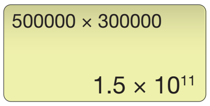
TABLE 1-8 Powers of Ten

Positive Powers	Negative Powers
$1\ 000\ 000 = 10^6$	$0.1 = 10^{-1}$
$100\ 000 = 10^5$	$0.01 = 10^{-2}$
$10\ 000 = 10^4$	$0.001 = 10^{-3}$
$1000 = 10^2$	$0.0001 = 10^{-4}$
$100 = 10^3$	$0.000\ 01 = 10^{-5}$
$10 = 10^1$	$0.000\ 001 = 10^{-6}$
$1 = 10^0$	$0.000\ 000\ 1 = 10^{-7}$

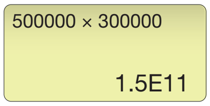
◆◆◆

Scientific and Engineering Notation

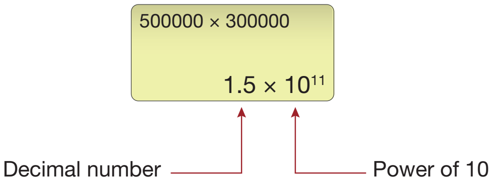
When we multiply two large numbers 500 000 × 300 000 on our calculator, we get a display like this:


500000 × 300000
1.5 × 10^{11}

or


500000 × 300000
1.5E11

This is because our answer (150 000 000 000) is too large to fit on the calculator screen, so the calculator automatically defaults to scientific notation. The calculator screen here shows two numbers: a decimal number, 1.5, and an integer, 11. Our answer is equal to the decimal number multiplied by 10 raised to the power of the integer.


500000 × 300000
1.5 × 10^{11}

Scientific notation refers to a number with an absolute value between 1 and 10 which is multiplied by a power of 10.

1. There is always only a single nonzero digit to the left of the decimal point.
2. The exponent can be any integer.

A good way to remember this is "one digit dot."

Engineering notation is similar to scientific notation, but with two differences.

1. There can be one, two, or three digits to the left of the decimal point.
2. The exponent is a multiple of 3.

A good way to remember this is "multiples of three."

TABLE 1-9 Scientific and Engineering Notation

Notation Style	Converting to Notation	Converting from Notation
Scientific	Rewrite the number with a single positive digit to the left of the decimal point and discard any nonsignificant zeros. Multiply this number by the power of 10 that makes it equal to the original number. $2700 = 2.7 \times 1000 = 2.7 \times 10^3$	Reverse the process: $4.82 \times 10^5 = 4.82 \times 100\,000 = 482\,000$
	When converting a number of absolute value less than 1, the power is negative. Note that the negative exponent has nothing to do with the sign of the original number. $0.000\,009\,50 = 9.50 \times 0.000\,001 = 9.50 \times 10^{-6}$	Reverse the process: $8.25 \times 10^{-3} = 8.25 \times 0.001 = 0.008\,25$
	A negative number is converted to scientific notation in the exact same way as a positive number. When using a calculator, treat the numbers as positive and then add the negative (–) sign to the answer. $-34\,720 = -3.472 \times 10\,000 = -3.472 \times 10^4$	Reverse the process: $-3.5 \times 10^{-2} = -3.5 \times 0.01 = -0.035$
Engineering	Conversion is easier when the digits of the decimal number are grouped into sets of three, either by commas or by spaces (the SI convention used throughout this textbook). $548\,000 = 548 \times 10^3$ $21\,840 = 21.84 \times 10^3$ $7\,256\,000 = 7.256 \times 10^6$	Reverse the process: $725 \times 10^{-6} = 0.000\,725$ $28.72 \times 10^{-12} = 0.000\,000\,000\,028\,72$ $8.14 \times 10^9 = 8\,140\,000\,000$
	When we convert a number of absolute value less than 1, it helps to separate the digits following the decimal point into groups of three. $0.87217 = 0.872\,17 = 872.17 \times 10^{-3}$ $0.000736492 = 0.000\,736\,492 = 736.492 \times 10^{-6}$ $0.0000000472 = 0.000\,000\,047\,2 = 47.2 \times 10^{-9}$	

Note that in this textbook we will mostly be following SI conventions for writing numerical values. We'll learn more about this topic in the next section.

Addition and Subtraction

If two or more numbers to be added or subtracted have the *same* power of 10, we combine the numbers and keep the same power of 10.

◆◆◆ Example 41:

(a) $(2 \times 10^5) + (3 \times 10^5) = 5 \times 10^5$

(b) $(8 \times 10^3) - (5 \times 10^3) + (3 \times 10^3) = 6 \times 10^3$ ◆◆◆

When the powers of 10 are *different*, we need to make them equal before we can combine them. Shift the decimal point one place to the left to increase the exponent by 1 or once to the right to decrease the exponent by 1.

◆◆◆ Example 42:

(a) $(1.5 \times 10^4) + (3 \times 10^3) = (1.5 \times 10^4) + (0.3 \times 10^4) = 1.8 \times 10^4$

(b) $(1.25 \times 10^5) - (2 \times 10^4) + (4 \times 10^3) = (1.25 \times 10^5) - (0.2 \times 10^5) + (0.04 \times 10^5) = 1.09 \times 10^5$

◆◆◆

Multiplication

Powers of 10 are multiplied by *adding* their exponents.

◆◆◆ Example 43:

(a) $10^3 \times 10^4 = 10^{3+4} = 10^7$

(b) $10^{-2} \times 10^5 = 10^{-2+5} = 10^3$

When multiplying two numbers in scientific or engineering notation, multiply the decimal parts and the powers of 10 *separately*.

◆◆◆

◆◆◆ Example 44:

(a) $(2 \times 10^5)(3 \times 10^2) = (2 \times 3)(10^5 \times 10^2) = 6 \times 10^{5+2} = 6 \times 10^7$

(b) $(4.0 \times 10^6)(5.0 \times 10^{-2}) = (4.0 \times 5.0)(10^6 \times 10^{-2}) = 20 \times 10^4 = (2.0 \times 10^1) \times 10^4 = 2.0 \times 10^5$

◆◆◆

Division

Powers of 10 are divided by *subtracting* the exponent of the divisor from the exponent of the dividend.

◆◆◆ Example 45:

(a) $\dfrac{10^5}{10^3} = 10^{5-3} = 10^2$

(b) $\dfrac{10^{-4}}{10^{-2}} = 10^{-4-(-2)} = 10^{-2}$

◆◆◆

When dividing two numbers in scientific or engineering notation, divide the decimal parts and the powers of 10 *separately*.

◆◆◆ Example 46:

(a) $\dfrac{8 \times 10^5}{4 \times 10^2} = \dfrac{8}{4} \times \dfrac{10^5}{10^2} = 2 \times 10^{5-2} = 2 \times 10^3$

(b) $\dfrac{12 \times 10^3}{4 \times 10^5} = \dfrac{12}{4} \times \dfrac{10^3}{10^5} = 3 \times 10^{3-5} = 3 \times 10^{-2}$

◆◆◆

Exercise 4 ◆ Scientific and Engineering Notation

Powers of 10

Write each number as a power of 10.

1. 100
2. 1 000 000
3. 0.0001
4. 0.001

Write each power of 10 as a decimal number.

5. 10^5
6. 10^{-2}
7. 10^{-5}
8. 10^{-1}

Scientific and Engineering Notation

Write each number in scientific notation and in engineering notation.

9. 186 000
10. 0.000 054 6
11. 25 742
12. 1.257 42
13. $80\overline{0}0$
14. 16 000
15. 98.3×10^3
16. 87.34×10^3
17. 0.0775×10^{-2}
18. $0.008\,71 \times 10^{-2}$

Convert each number from scientific or engineering notation to decimal notation.

19. 2.85×10^3
20. 1.75×10^{-5}
21. 9×10^4
22. 9.00×10^4
23. 3.667×10^{-3}
24. 76.3×10^3
25. 248.2×10^{-6}
26. 7×10^6
27. 30.00×10^6
28. 942.56×10^{-9}

Multiplication and Division

Multiply the following powers of 10.

29. $10^5 \cdot 10^2$
30. $10^4 \cdot 10^{-3}$
31. $10^{-5} \cdot 10^{-4}$
32. $10^{-2} \cdot 10^5$

Divide the following powers of 10.

33. $10^8 \div 10^5$
34. $10^4 \div 10^6$
35. $10^5 \div 10^{-2}$
36. $10^{-2} \div 10^{-4}$

Multiply without using a calculator.

37. $(3.0 \times 10^3)(5.0 \times 10^2)$
38. $(2 \times 10^{-2})(4 \times 10^{-5})$

Divide without using a calculator.

39. $(8 \times 10^4) \div (2 \times 10^2)$
40. $(9 \times 10^4) \div (3 \times 10^{-2})$

Addition and Subtraction

Combine without using a calculator. Give your answers in decimal notation.

41. $(3.0 \times 10^4) + (2.1 \times 10^5)$
42. $(75.0 \times 10^2) + 32\bar{0}0$
43. $(1.557 \times 10^2) + (9.000 \times 10^{-1})$
44. $(7.2 \times 10^4) + (1.1 \times 10^4)$

Scientific Notation and Engineering Notation on a Calculator

Perform each computation, and give your answers in both scientific and engineering notation. Combine the powers of 10 by hand or with your calculator. Round to the appropriate number of digits.

45. $(1.58 \times 10^2)(9.82 \times 10^3)$
46. $(9.83 \times 10^5) \div (2.77 \times 10^3)$
47. $(3.87 \times 10^{-2})(5.44 \times 10^5)$
48. $(2.74 \times 10^3) \div (9.13 \times 10^5)$
49. $(7.72 \times 10^8) \div (3.75 \times 10^{-9})$
50. Three resistors, having resistances of $4.98 \times 10^5\ \Omega$, $2.47 \times 10^4\ \Omega$, and $9.27 \times 10^6\ \Omega$, are wired in series (Fig. 1-10). Find the total resistance, using the equation for resistors in series, Eq. A63: $R = R_1 + R_2 + R_3 + \dots$.
51. Find the equivalent resistance if the three resistors of Problem 50 are wired in parallel (Fig. 1-11). Use the equation for resistors in parallel, Eq. A64: $\left(\frac{1}{R}\right) = \left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \dots,$ to find the resistance in parallel.
52. A wire 4.75×10^3 cm long is seen to stretch, when loaded, by 9.55×10^{-2} cm. Find the strain in the wire, using the equation $\varepsilon = \frac{e}{L}$, where L is the original length and e is the extension or increase in length. (Strain is a unitless quantity or pure number.)
53. How many hours will it take a rocket travelling at a rate of 3.2×10^6 miles per hour (mi./h) to travel the 3.8×10^8 miles (mi.) from the Earth to the Moon? Use the basic physics equation $D = Rt$, or distance = rate × time. (Remember to manipulate the equation to calculate for time.)

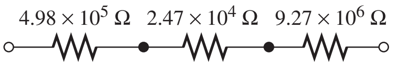


FIGURE 1-10

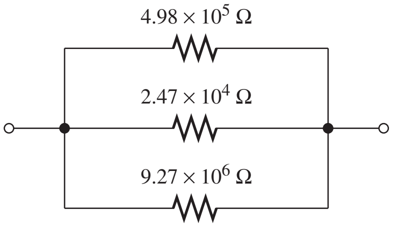


FIGURE 1-11

1–5 Units of Measurement

Systems of Units

A *unit* is a standard of measurement of a particular quantity, such as the metre, inch, second, kilogram, or dollar. There are two main systems of units in use, the *metric system* or *SI* (metres, kilograms, litres, etc.) and the *imperial system* (feet, pounds, gallons, etc.; called the *standard* system in the United States). The SI is the *International System of Units,* or *Système International d'Unités.* Any physical quantity can be expressed as a certain number of *units.* For example, length can be measured in metres, inches, nautical miles, angstroms, feet, and so on. This section will focus on converting from one metric unit to another metric unit, from one imperial unit to another imperial unit, and from the metric system to the imperial system and vice versa.

Symbols and Abbreviations

Units have symbols (or abbreviations, in the case of non-SI units), so we don't usually write the full name of the unit when giving a measurement. The symbol for metres is "m" and the symbol for ohms is "Ω". The symbols or abbreviations for most of the units used in this text can be found in a table of conversion factors in Appendix B.

Conversion Factors

To convert one unit to another unit we need to multiply by a number called a *conversion factor*.

◆◆◆ **Example 47:** Convert 654.5 feet (ft.) to metres (m).

Solution: From Appendix B we find the relationship between feet and metres:

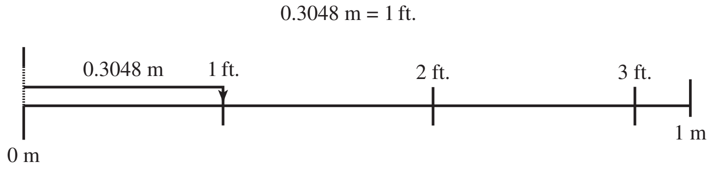


FIGURE 1-12

If we divide both sides of this equation by 1 ft. we get our conversion factor: $\frac{(0.3048\,\text{m})}{1\,\text{ft.}} = 1$. This is also called a unity ratio (a fraction that is equal to 1).

To do this particular conversion, we want to *multiply* our original number by the conversion factor. This will *cancel* the units we want to eliminate. We set up our equation like this:

$$654.5\,\text{ft.} = 654.5\,\cancel{\text{ft.}} \times \frac{(0.3048\,\text{m})}{(1\,\cancel{\text{ft.}})} = 199.5\,\text{m}$$

Our answer is rounded to four significant digits, because all the numbers used in the calculation have at least four significant digits (0.3048 m is an exact number). ◆◆◆

Significant Digits: Try to use a conversion factor that is an exact number or one that contains, if possible, at least one more significant digit than your original number. Your answer should be rounded to as many significant digits as in the original number.

In Example 47, if the measurements were in metres, so that we needed to *divide* both sides by 0.3048 m, we would have used a different conversion factor, which would look like this:

$$\frac{1\,\text{ft.}}{0.3048\,\text{m}} = 1$$

Conversion Factors: The relationship between two units of measurement gives us two conversion factors. This is called a *unity ratio* (a fraction that is equal to 1). Since both conversion factors equal 1, we can multiply any quantity by a conversion factor without changing the value of the quantity. How do you know which conversion factor to use? **Rule:** Multiply by the conversion factor that will cancel the units you want to eliminate.

◆◆◆ **Example 48:** Convert 134 acres to hectares (ha).

Solution: From Appendix B, we find the relationship between acres and hectares:

1 ha = 2.471 acres (not an exact number in this case)

To do this conversion, we want to multiply our original number by the conversion factor that will cancel the units we want to eliminate, so we set up our equation like this:

$$134\,\text{acres} = 134\,\cancel{\text{acres}} \times \frac{(1\,\text{ha})}{(2.471\,\cancel{\text{acres}})} = 54.2\,\text{ha}$$

The conversion factor used here is not an exact number, but it does have more significant digits than the original number. We have rounded our answer to three significant digits, the same as in the original number. ◆◆◆

Using More Than One Conversion Factor

Sometimes you may not be able to find a single conversion factor linking the units you want to convert. You may have to use more than one.

◆◆◆ **Example 49:** Convert 7375 yards (yd.) to nautical miles (M).

Solution: Appendix B does not have a conversion factor between nautical miles and yards, but we see that

$$1\text{ M} = 1852\text{ m and } 0.9144\text{ m} = 1\text{ yd.}$$

So

$$7375\text{ yd.} = 7375\,\cancel{\text{yd}}.\times\frac{0.9144\,\cancel{\text{m}}}{1\,\cancel{\text{yd.}}}\times\frac{1\text{ M}}{1852\,\cancel{\text{m}}} = 3.641\text{ M}$$

◆◆◆

Metric Units

The SI is based on the metric system of weights and measures that was developed in France in 1793, and has since been adopted by most countries. It is the preferred system for scientific work in North America.

The usual base SI units are shown in Table: 1-10:

TABLE 1-10 Base SI Units

Measurement	Base SI Unit	Examples of Derived or Related Units
Length	metre (m)	centimetre (cm) millimetre (mm)
Mass	kilogram (kg)	gram (g) tonne (t)
Time	second (s)	minutes (min) hours (h)
Temperature	kelvin (K)	degrees Fahrenheit (°F) degrees Celsius (°C)
Amount of substance	mole (mol)	grams per mole (g/mol)
Current	ampère or amp (A)	milliamp (mA)
Luminosity	candela (cd)	watts per steradian (W/sr)
Volume	cubic metre (m^3)	litre (L) millilitre (mL)
Area	square metre (m^2)	hectare (ha)

Metric Prefixes

Converting between metric units is made easy because larger and smaller units are related to the base units by factors of 10. These larger or smaller units are indicated by placing a *prefix* (a letter or group of letters placed at the beginning of a word to modify the meaning of that word) before the base unit. SI prefixes are given in Table 1-11.

◆◆◆ **Example 50:**

(a) A *kilometre* (km) is a thousand metres because *kilo* means one thousand (10^3).

$$1\text{ km} = 1000\text{ m}$$

(b) A *centimetre* (cm) is one-hundredth of a metre, because *centi* means one hundredth.

$$1\text{ cm} = \frac{1}{100}\text{ m} = 0.01\text{ m}$$

(c) A *millimetre* (mm) is one-thousandth of a metre, because *milli* means one thousandth.

$$1\text{ mm} = \frac{1}{1000}\text{ m} = 0.001\text{ m}$$

◆◆◆

TABLE 1-11 Metric Prefixes

Amount	Multiple or Submultiples	Prefix	Symbol	Pronunciation	Meaning
1 000 000 000 000	10^{12}	tera	T	ter′a	One trillion times
1 000 000 000	10^{9}	giga	G	gi′ga	One billion times
1 000 000	10^{6}	mega	M	meg′a	One million times
1000	10^{3}	kilo	k	kil′o	One thousand times
100	10^{2}	hecto	h	hek′to	One hundred times
10	10	deka	da	dek′a	Ten times
0.1	10^{-1}	deci	d	des′i	One-tenth of
0.01	10^{-2}	centi	c	sen′ti	One-hundredth of
0.001	10^{-3}	milli	m	mil′i	One-thousandth of
0.000 001	10^{-6}	micro	μ	mī′kro	One-millionth of
0.000 000 001	10^{-9}	nano	n	nan′o	One-billionth of
0.000 000 000 001	10^{-12}	pico	p	pē′co	One-trillionth of
0.000 000 000 000 001	10^{-15}	femto	f	fem′to	One-quadrillionth of
0.000 000 000 000 000 001	10^{-18}	atto	a	at′to	One-quintillionth of

Converting Between Metric Units

Converting from one metric unit to another is usually a matter of multiplying or dividing by a power of 10. Most of the time, the names of the units will tell how they are related, so we do not even have to look them up.

◆◆◆ **Example 51:** Convert 72 925 metres (m) to kilometres (km).

Solution: A *kilo*metre is a thousand metres.

$$\frac{1\text{ km}}{1000\text{ m}} = 1$$

So, using the same process as in Example 49,

$$72\,925\text{ m} = 72\,925\ \cancel{\text{m}} \times \frac{1\text{ km}}{1000\ \cancel{\text{m}}} = 72.925\text{ km}$$

◆◆◆

For more unusual metric units, simply look up the conversion factor in a table. One of the simplifying features of the SI is that each quantity is generally assigned only one base unit.

Although you may still encounter them, the use of older metric units such as the *dyne* (a unit of force) and *are* (a unit of area) is discouraged. Note also that nonapproved units do not have assigned symbols.

◆◆◆ **Example 52:** Convert 2.75×10^5 dynes to newtons (N).

Solution: We cannot tell from their names how these two units are related to each other. However, from Appendix B we find that

$$1\text{ N} = 10^5\text{ dyne}$$

Converting in the usual way, we obtain

$$2.75 \times 10^5\text{ dyne} = 2.75 \times 10^5\ \cancel{\text{dyne}} \times \frac{(1\text{ N})}{\left(10^5\ \cancel{\text{dyne}}\right)} = 2.75\text{ N}$$

◆◆◆

Converting from One Imperial Unit to Another

You should be aware that there is a difference in some units between Canadian (imperial) and U.S. (standard) measures—including the gallon (gal.), quart (qt.), pint (pt.), and fluid ounce (fl. oz.). For example, 1 fl. oz. (U.S.) is about 1.041 fl. oz. (imperial); 1 pt. (U.S.) = 16 fl. oz. (U.S.), while 1 pt. (imperial) = 20 fl. oz. (imperial).

◆◆◆ **Example 53:** Convert 2.84 cubic feet (cu. ft.) to U.S. gallons (gal.).

Solution: From Appendix B we find

$$1 \text{ cu. ft.} = 7.481 \text{ gal. (U.S.)}$$

Converting gives

$$2.84 \text{ cu. ft.} = 2.84 \cancel{\text{cu. ft.}} \times \frac{(7.481 \text{ gal})}{(1 \cancel{\text{cu. ft.}})} = 21.2 \text{ gal.}$$

rounded to three significant digits. ◆◆◆

Converting Areas and Volumes

Length may be given in, say, centimetres (cm), *area* in square centimetres (cm^2), and *volume* in cubic centimetres (cm^3). To obtain a conversion factor for area or volume, simply square or cube the conversion factor for length.

If we take the equation

$$1 \text{ in.} = 2.54 \text{ cm}$$

and square both sides, we get

$$(1 \text{ in.})^2 = (2.54 \text{ cm})^2$$

or

$$1 \text{ sq. in.} = 6.4516 \text{ cm}^2$$

This gives us a conversion between square centimetres and square inches.

◆◆◆ **Example 54:** Convert 864 square yards to acres.

Solution: Appendix B has no conversion for square yards. However,

$$1 \text{ yd.} = 3 \text{ ft. and } 1 \text{ acre} = 43\,560 \text{ sq. ft.}$$

Squaring 1 yd. yields

$$1 \text{ sq. yd.} = (3 \text{ ft.})^2 = 9 \text{ sq. ft.}$$

So

$$864 \text{ sq. yd.} = 864 \cancel{\text{sq. yd.}} \times \frac{9 \cancel{\text{sq. ft.}}}{1 \cancel{\text{sq. yd.}}} \times \frac{1 \text{ acre}}{43\,560 \cancel{\text{sq. ft.}}} = 0.179 \text{ acre}$$

◆◆◆

Converting Rates to Other Units

A *rate* is the amount of one quantity expressed *per unit of some other quantity.* Some rates, with typical units, are

rate of travel (km/h or mi./h)
application rate (kg/ha)
flow rate (m^3/s or gal./min)
unit price ($/kg or cents/100 mL)

Each rate contains *two* units of measure: kilometres per hour, for example, has kilometres in the numerator and hours in the denominator. It may be necessary to convert either or both of those units to other units. Sometimes a single conversion factor can be found (such as 1 m/s = 3.6 km/h exactly), but more often you will have to convert each unit using a separate conversion factor.

◆◆◆ **Example 55:** A certain chemical is to be added to a pool at the rate of 1.75 lb. per cubic foot of water (lb./cu. ft.). Convert this to grams of chemical per litre of water.

Solution: We write the original quantity as a fraction and multiply by the appropriate factors, also written as fractions. When 1 lb. is used as a unit of mass, we use this conversion factor:

$$1 \text{ lb.} = 453.6 \text{ g}$$

Therefore,

$$1.75 \text{ lb./cu. ft.} = \frac{1.75\ \cancel{\text{lb}}}{1\ \cancel{\text{cu. ft.}}} \times \frac{453.6 \text{ g}}{1\ \cancel{\text{lb.}}} \times \frac{(1\ \cancel{\text{cu.ft.}})}{(28.32 \text{ L})} = 28.0 \text{ g/L}$$

◆◆◆

Exercise 5 ◆ Units of Measurement

Convert each imperial unit. Look in Appendix B for any conversion factors you need.

1. 152 in. to feet
2. 0.153 mi. to yards
3. 762.0 ft. to inches
4. 627 ft. to yards
5. 29 tons to pounds
6. 88.90 lb. to ounces
7. 89 600 lb. to tons
8. 8552 ounces (oz.) to pounds

Convert each metric or SI unit. Write your answer in scientific notation if the numerical value is greater than 1000 or less than 0.1.

9. 48 300 m^2 to hectares
10. 364 000 m to kilometres
11. 0.000 473 volts (V) to millivolts
12. 735 900 g to kilograms
13. 7.68×10^{-5} kilowatts (kW) to watts
14. $6.2 \times 10^9\ \Omega$ to megohms
15. 825×10^4 N to kilonewtons
16. 9348 picofarads (pF) to microfarads
17. 84 398 ns to milliseconds

Convert each area or volume.

18. 2840 sq. yd. to acres
19. 7.360 cu. ft. to cubic inches
20. 8.220 gal. (U.S.) to cubic feet

Convert between imperial (or standard) and metric or SI units.

21. 364.0 m to feet
22. 6.83 in. to millimetres
23. 7.35 lb. to kilograms
24. 2.55 horsepower (hp) to kilowatts
25. 4.66 gal. (imp.) to litres
26. 1.28×10^3 N to pounds-force
27. 3.94 yd. to metres
28. 834 cm^3 to U.S. gallons

Convert each area or volume.

29. 24.8 sq. ft. to square metres
30. 3.72 m^2 to square feet
31. 0.982 km^2 to acres
32. 5.93 acres to square metres
33. 4.83 m^3 to cubic yards
34. 73.8 cu. yd. to cubic metres
35. 267 mm^3 to cubic inches
36. 112 L to imperial gallons

Convert each time rate unit.

37. 4.86 ft./s to miles per hour
38. 777 gal./min (U.S.) to cubic metres per hour
39. 66.2 mi./h to kilometres per hour
40. 52.0 knots (nautical miles per hour, or kn) to miles per minute
41. 953 births per year to births per week

Convert each unit price.

42. \$1.25 per gram to dollars per kilogram
43. \$800 per acre to cents per square metre
44. \$3.54 per pound to cents per ounce
45. 238 cents per pound to dollars per tonne

Applications of Unit Conversion

46. The jet fuel tank in Fig. 1-13 has a volume of 15.7 cu. ft. How many litres of jet fuel will it hold?
47. The surface area of a certain lake, shown in Fig. 1-14, is 7361 sq. yd. Convert this to square metres.

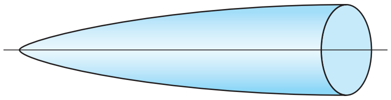


FIGURE 1-13

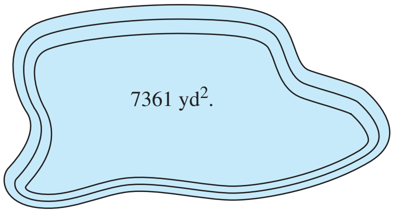


FIGURE 1-14

48. A solar collector, shown in Fig. 1-15, has an area of 8834 sq. in. Convert this to square metres.

49. The volume of a balloon is 8360 cu. ft. Convert this to cubic centimetres.

50. You are driving through Buffalo, New York and need to buy gas for your 2014 Honda Civic. The volume of the gas tank is 50 L. The price of gas is $3.88 per U.S. gallon and you have $40 U.S. to spend on gas. How much gas can you buy, in U.S. gallons and in litres?

51. An airplane is cruising at a speed of 785 mi./h. Convert this speed to kilometres per hour.

52. Algonquin Park in Ontario has an area of 765 345 ha. Convert this area to square kilometres and also to square miles. Use any built-in unit conversion features that your calculator may have.

53. A can of beer holds 900 mL. Express this volume in imperial fluid ounces. Use any built in unit conversion features that your calculator may have.

54. A fully grown male black bear weighs 560 lb. Convert this weight to kilograms. Use the conversion factor: 1 kg = 2.205 lb.

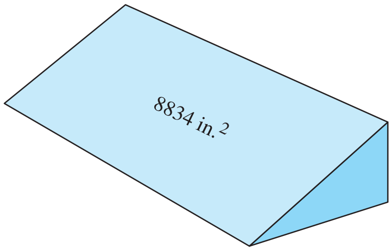


FIGURE 1-15

1–6 Substituting into Equations and Formulas

Substituting into Equations

We get an *equation* when two expressions are set equal to each other.

◆◆◆ **Example 56:** $x = 5a - 2b + 3c$ is an equation that enables us to find x if we know a, b, and c.

We will study equations in detail later, but for now we will simply substitute into equations and use a calculator to compute the result. To *substitute into an equation* means to replace the letter quantities in the equation by their given numerical values and perform the computation indicated.

◆◆◆ **Example 57:** Substitute the values $a = 5$, $b = 3$, and $c = 6$ into the equation and calculate the result.

$$x = \frac{3a + b}{c}$$

Solution: Substituting, we obtain

$$x = \frac{3(5) + 3}{6} = \frac{18}{6} = 3$$

◆◆◆

When substituting approximate numbers, be sure to round your answer to the appropriate number of digits. We usually treat any integers in the equation as exact numbers.

Substituting into Formulas

A *formula* is an equation expressing some general mathematical or physical fact, such as the formula for the area of a circle of radius r. (114 is the number of the formula in Appendix A.)

Area of a Circle	$A = \pi r^2$	**114**

We substitute into formulas just as we substitute into equations, except that we now carry *units* along with the numerical values. You will often need conversion factors to make the units cancel properly, so that the answer will be in the desired units.

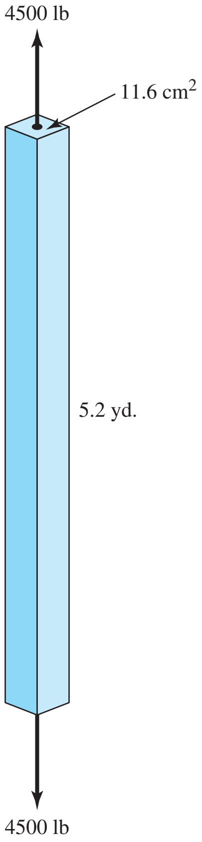


FIGURE 1-16

◆◆◆ **Example 58:** A tensile load of 4500 lb. is applied to a bar that is 5.2 yd. long and has a cross sectional area of 11.6 cm^2 (Fig. 1-16). The elongation is 0.38 mm. Using Eq. A54 from Appendix A, find the modulus of elasticity E in pounds per square inch.

Solution: Substituting the values *with units* into Eq. A54, we obtain

$$E = \frac{PL}{ae} = \frac{4500\text{ lb.} \times 5.2\text{ yd.}}{11.6\text{ cm}^2 \times 0.38\text{ mm}}$$

Notice that we have a length (5.2 yd.) in the numerator and a length (0.38 mm) in the denominator. To make these units cancel, we use the conversion factors

$$25.4\text{ mm} = 1\text{ in.} \quad \text{and} \quad 36\text{ in.} = 1\text{ yd.}$$

Also, our answer is to have square inches in the denominator, not square centimetres. So we use another conversion factor.

$$6.4516\text{ cm}^2 = 1\text{ sq. in.}$$

$$E = \frac{4500\text{ lb.} \times 5.2\ \cancel{\text{yd.}}}{11.6\ \cancel{\text{cm}^2} \times 0.38\ \cancel{\text{mm}}} \times \frac{25.4\ \cancel{\text{mm}}}{\cancel{\text{in.}}} \times \frac{36\ \cancel{\text{in.}}}{\cancel{\text{yd.}}} \times \frac{6.4516\ \cancel{\text{cm}^2}}{\text{sq. in.}}$$

$$= 31\,000\,000\text{ lb./sq. in.} = 3.1 \times 10^7\text{ lb./sq. in. (2 s.d.)}$$

◆◆◆

COMMON ERROR

Students often forget to include units when substituting into a formula. When this happens, the result is often that the units do not cancel properly.

Exercise 6 ◆ Substituting into Equations and Formulas

Substitute the given integers into each equation and calculate the result. Do not round your answer.

1. $y = 5x + 2$ $(x = 3)$
2. $y = 2m^2 - 3m + 5$ $(m = -2)$
3. $y = 2a - 3x^2$ $(x = 3,\ a = -5)$
4. $y = 3x^3 - 2x^2 + 4x - 7$ $(x = 2)$
5. $y = 2b + 3w^2 + 5z^3$ $(b = 3,\ w = -4,\ z = 2)$
6. $y = \frac{r^2}{x} - \frac{x^3}{r} + \frac{w}{x^2}$ $(x = 5,\ w = 3,\ r = -4)$

Substitute the given approximate numbers into each equation and calculate the result. Treat the constants in the equations as exact numbers. Round your answer to the appropriate number of digits.

7. $y = 7x - 5$ $(x = 2.73)$
8. $y = 2w^2 - 3x^2$ $(x = -11.5,\ w = 9.83)$
9. $y = 8 - x + 3x^2$ $(x = -8.49)$
10. $y = \sqrt[3]{8x + 7w}$ $(x = 1.255,\ w = 2.304)$
11. $y = \sqrt{x^3 - 3x}$ $(x = 4.25)$
12. $y = (w - 2x)^{1.6}$ $(x = 1.8,\ w = 7.2)$
13. Use the equation $y = a(1+nt)$, where a is the principal, n is the interest rate, and t is the time in years, to find the amount to which \$3,000 will accumulate in 5 years at a simple interest rate of 6.5%. Give your answer to the nearest dollar.
14. Using the equation $s = v_o t + \frac{at^2}{2}$, where v_o is the initial downward velocity in metres per second, a is the constant downward acceleration (9.8 m/s^2), and t is the time in seconds, find the displacement in metres, after 1.30 s, of a body thrown downward with a speed of 3.60 m/s.
15. Using the equation $c = \frac{5}{9}(f - 32)$, convert 128 °F to degrees Celsius.

1–7 Percentage

Let’s review the variety of methods used to calculate with percents.

TABLE 1-12 Methods of Calculating Percent

Numerical Operation	Explanation	Example
Percent	The word *percent* means by the hundred, or per hundred. A percent gives us the number of parts in every hundred.	If a brand of concrete mix is 12% cement by weight, then 12 kg of every 100 kg of concrete mix will be cement.
Rate	*Rate* is often used to indicate a percent or percentage rate (e.g., rejection rate, rate of inflation, growth rate).	A steel rod failure rate of 2% means that, on average, 2 steel rods out of every 100 would be expected to fail.
Percent as Fraction	Percent is another way of expressing a fraction with 100 as the denominator.	If a builder has finished 75% of a house, he has finished $\frac{75}{100}$ or $\frac{3}{4}$ of the house.
Converting Decimals to Percent	Move the decimal point *two places* to the *right* and affix a percent symbol (%).	(a) $0.75 = 75\%$ (b) $3.65 = 365\%$ (c) $0.003 = 0.3\%$ (d) $1.05 = 105\%$
Converting Fractions or Mixed Numbers to Percent	First write the fraction or mixed number as a decimal, then convert to percent (as above).	(a) $\frac{1}{4} = 0.25 = 25\%$ (b) $\frac{5}{2} = 2.5 = 250\%$ (c) $1\frac{1}{4} = 1.25 = 125\%$
Converting Percent to Decimals	Move the decimal point *two places* to the *left* and remove the percent symbol (%).	(a) $13\% = 0.13$ (b) $155\% = 1.55$ (c) $4.5\% = 0.045$ (d) $27\frac{3}{4}\% = 0.2775$
Converting Percent to a Fraction	Write the fraction with 100 in the denominator and the percent in the numerator. Remove the % sign and reduce the fraction to lowest terms. Make sure any decimal points are cleared by multiplying both numerator and denominator by powers of 10.	(a) $75\% = \frac{75}{100} = \frac{3}{4}$ (b) $87.5\% = \frac{87.5}{100} = \frac{875}{1000} = \frac{7}{8}$ (c) $125\% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

PERCENTAGE

Percentage problems always involve three variables:

1. The *percent rate*, P.
2. The *base*, B: the quantity we are taking the percent of.
3. The *amount*, A: the result when we take the percent of the base.

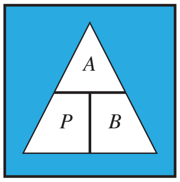
In a percentage problem, you will be given two of these three variables, and will need to calculate the third. This can be done easily if you follow this *percentage equation* and the “triangle method” shown in Fig, 1-17.

Percentage

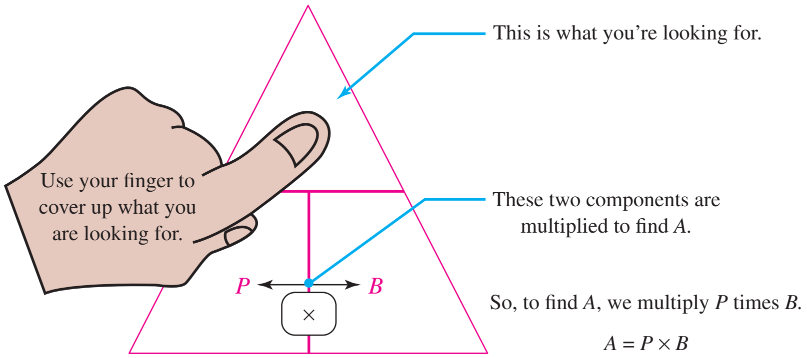
Amount = rate × base

$$A = PB$$

where P is expressed as a decimal.
You can also use the triangle to the right to help you get the right answer.



The Triangle Method



This is what you're looking for.

Use your finger to cover up what you are looking for.

These two components are multiplied to find A.

So, to find A, we multiply P times B.

$$A = P \times B$$

FIGURE 1-17

Finding the Amount When Base and Rate are Known

Let's work through the following example together using the percentage equation and the triangle method to solve for the *amount*.

◆◆◆ **Example 59:** What is 35.0% of 80.0?

Solution: In this example, 35.0% is the percent rate, which we need to convert a decimal:

$$P = 0.350$$

COMMON ERROR

Don't forget to convert the *percent rate* to a decimal before you "plug" it into the percentage equation.

What is the number 80.0? Is it the *amount* or the *base?*

MATH SHOW OFF

Look for the key phrase *percent of*. The *quantity* following this phrase is always the base.

So we look for the key phrase "percent of" or "% of":

$$\text{What is } \overset{\text{Percent rate } (P)}{\overset{\downarrow}{35 \text{ percent } (\%)}} \text{ of } \overset{\text{Base } (B)}{\overset{\downarrow}{80.0}}?$$

Since 80.0 immediately follows *percent of*,

$$B = 80.0$$

So we automatically know that we need to solve for the *amount*, because we have been given the *percent rate* and the *base*. Using the triangle method, we can see that our equation will be

$$A = PB = (0.350)(80.0) = 28.0$$

◆◆◆

◆◆◆ Example 60: Find 3.74% of 5710.

Solution: Substitute into our percentage equation with

$$P = 0.0374 \quad \text{and} \quad B = 5710$$

So,

$$A = PB = (0.0374)(5710) = 214$$

Our answer after rounding to three significant digits is 214.

◆◆◆

Finding the Base When a Percent of It is Known

We see from the percentage equation and the triangle method that the *base* equals the *amount* divided by the *percent rate* (expressed as a decimal), or $B = \frac{A}{P}$, as shown in Fig. 1-18:

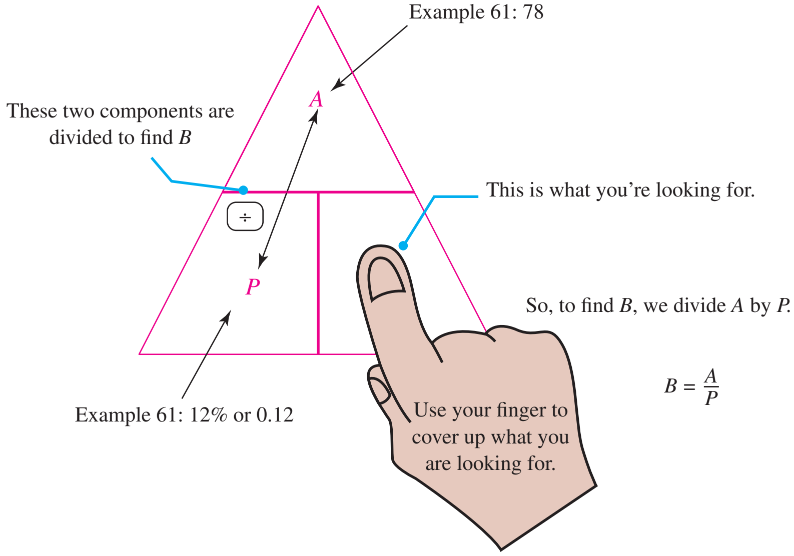


FIGURE 1-18

◆◆◆ Example 61: 12% of what number is 78?

Solution: We can see that we need to find the *base*. So let's work it out.

$$A = 78 \quad \text{and} \quad P = 0.12$$

$$B = \frac{A}{P} = \frac{78}{0.12} = 650$$

Our answer phrase becomes "12 percent of 650 is 78" or "78 is 12% of 650."

◆◆◆

◆◆◆ Example 62: 140 is 25% of what number?

Solution:

$$B = \frac{A}{P} = \frac{140}{0.25} = 560$$

140 is 25% of 560.

◆◆◆

Finding One Number as a Percent of Another Number

We see from our percentage equation and the triangle method that the *percent rate* equals the *amount* divided by the *base*, or $P = A/B$:

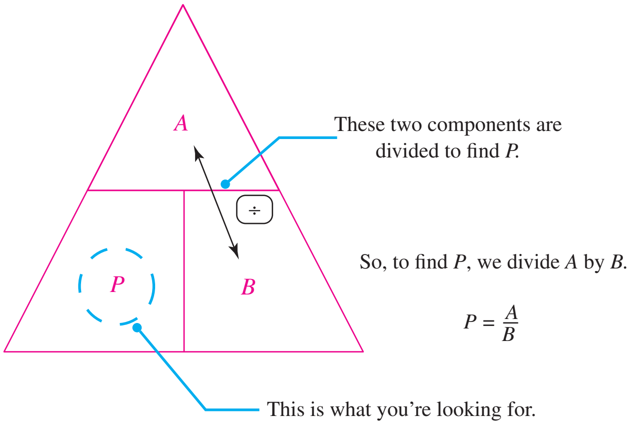


FIGURE 1-19

◆◆◆ **Example 63:** 42.0 is what percent of 405?

Solution: $A = 42.0$ and $B = 405$, so

$$P = \frac{A}{B} = \frac{42.0}{405} = 0.104 = 10.4\%$$

◆◆◆

◆◆◆ **Example 64:** What percent of 1.45 is 0.357?

Solution:

$$P = \frac{A}{B} = \frac{0.357}{1.45} = 0.246 = 24.6\%$$

◆◆◆

Percent Change, Efficiency, Error, and Concentration

TABLE 1-13 Percent Operations

Numerical Operation	Formula	Explanation
Percent Change	$\text{Percent change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$	When two numbers that are being compared involve a *change*, the *original value* is *subtracted* from the *new value* and the result *divided* by the *original value*. The price of steel rose 3% over last year. The weights of two cars differed by 20%. Production dropped 5% from last year. Percent change can be either (+) or (–), so make sure you indicate this in your answer.
Percent Efficiency	$\text{Percent efficiency} = \frac{\text{output}}{\text{input}} \times 100\%$	The *output* of any machine or device is always less than the *input* due to inevitable power losses within the device. The *efficiency* of the device acts as a measure of those losses.

TABLE 1-13 Percent Operations (continued)

Numerical Operation	Formula	Explanation
Percent Error	$\text{Percent error} = \dfrac{\text{measured value} - \text{known value}}{\text{known value}} \times 100\%$	The accuracy of measurement is also known as *percent error.* The percent error is the difference between the *measured value* and the *known value*, divided by the *known value*, expressed as a percent. Percent error can also be either (+) or (−), so make sure you indicate this in your answer.
Percent Concentration	$\text{Percent concentration of ingredient A} = \dfrac{\text{amount of A}}{\text{amount of mixture}} \times 100\%$	*Percent concentration* applies to mixtures with two or more ingredients.

Percent Change

◆◆◆ **Example 65:** The price of a chocolate bar rose from \$1.55 to \$1.75. Find the percentage change in the price.

Solution: We use \$1.55 as the original value and plug it into our equation for percent change from Table 1-13:

$$\text{Percent change} = \frac{\$1.75 - \$1.55}{\$1.55} \times 100\% = 12.9\% \text{ increase}$$

◆◆◆

A common type of problem is to *find the new value* when the original value is changed by a given percent. Let's look at how to calculate this. Rearranging the percent change equation, we get:

$$\text{New value} = \text{original value} + (\text{original value} \times \text{percent change})$$

◆◆◆ **Example 66:** Find the cost of a \$156.00 suit after the price increases by $2\frac{1}{2}\%$.

Solution: The original value is 156.00 and the percent change once converted to a decimal is 0.025. So plugged into our equation, we get

$$\text{New value} = \$156.00 + (\$156.00 \times 0.025) = \$159.90$$

◆◆◆

Percent Efficiency

◆◆◆ **Example 67:** An electric motor consumes 865 W and has an output of 1.12 hp (Fig. 1-20). Find the efficiency of the motor (1 hp = 746 W).

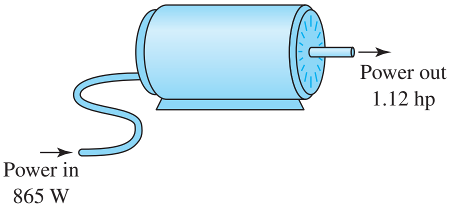


FIGURE 1-20

Solution: Since output and input must be in the same units, we need to convert either to horsepower or to watts. Converting the output to watts, we get:

$$\text{Output} = 1.12 \text{ hp}\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 835.52 \text{ W}$$

Using our percent efficiency equation,

$$\text{Percent efficiency} = \frac{835.52 \text{ W}}{865 \text{ W}} \times 100\% = 96.6\%$$

◆◆◆

Percent Error

◆◆◆ **Example 68:** A laboratory weight that is certified to be 500.0 g is placed on a scale (Fig. 1-21). The scale reading is 507.0 g. What is the percent error of the scale?

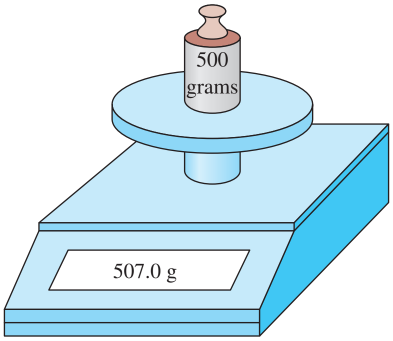


FIGURE 1-21

Solution: Using our percent error equation from Table 1-13,

$$\text{Percent error} = \frac{507.0\text{ g} - 500.0\text{ g}}{500.0\text{ g}} \times 100\% = 1.4\%\text{ high}$$

◆◆◆

Percent Concentration

◆◆◆ **Example 69:** A fuel mixture contains 18.9 L of alcohol and 84.7 L of gasoline. Find the percentage of gasoline in the mixture.

Solution: The total amount of mixture is

$$18.9\text{ L} + 84.7\text{ L} = 103.6\text{ L}$$

Using our percent concentration equation from Table 1-13,

$$\text{Percent gasoline} = \frac{84.7\text{ L}}{103.6\text{ L}} \times 100\% = 81.8\%$$

◆◆◆

COMMON ERROR

Don't forget: The denominator in the percent concentration equation must be the *total amount* of mixture, or the *sum* of all ingredients.

Exercise 7 ◆ Percentage

Conversions

Convert each decimal to a percent.

1. 3.72
2. 0.877
3. 0.0055
4. 0.563

Convert each fraction to a percent. Round to three significant digits.

5. $\frac{2}{5}$
6. $\frac{3}{4}$
7. $\frac{7}{10}$
8. $\frac{3}{7}$

Convert each percent to a decimal.

9. 23%
10. 2.97%
11. $287\frac{1}{2}\%$
12. $6\frac{1}{4}\%$

Convert each percent to a fraction.

13. 37.5%
14. $12\frac{1}{2}\%$
15. 150%
16. 3%

Finding the Amount

Find each amount.

17. 41.1% of 255 t
18. 15.3% of 326 mi.
19. 33.3% of 662 kg
20. 12.5% of 72.0 gal.
21. A resistance, now 7250 Ω, is to be increased by 15.0%. How much resistance should be added?
22. How much metal will be obtained from 375 t of ore if the metal is 10.5% of the ore?

Finding the Base

Find the amount of which

23. 86.5 is 16.7%.
24. 45.8 is 1.46%.
25. 1.22 is 1.86%.
26. 55.7 is 25.2%.
27. A man withdrew 25% of his bank deposits and spent 33% of the money withdrawn to purchase a radio worth $25. How much money did he have in the bank to begin with?
28. Solar panels provide 65% of the heat for a certain building. If $225 per year is spent on heating oil, what would have been spent if the solar panels were not used?

Finding the Percent

What percent of

29. 26.8 is 12.3?
30. 36.3 is 12.7?
31. 44.8 km is 8.27 km?
32. 844 is 428?
33. 483 t is 287 t?
34. A 50 500 L-capacity tank contains 5840 L of water (Fig. 1-22). Express the amount of water in the tank as a percentage of the total capacity.
35. In a journey of 1560 km, a person travelled 195 km by car and the rest of the distance by rail. What percent of the distance was travelled by rail?

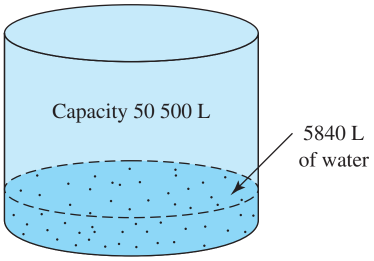


FIGURE 1-22

Percent Change

Find the percent change when a quantity changes

36. from 29.3 to 57.6.
37. from 107 to 23.75.
38. from 227 to 298.
39. from 0.774 to 0.638.
40. The temperature in a building rose from 19.0 °C to 21.0 °C during the day. Find the percent change in temperature.
41. A casting initially weighing 115 lb. has 22.0% of its material machined off. What is its final weight?

Percent Efficiency

42. A certain device (Fig. 1-23) consumes 92.5 W and delivers 62.0 W. Find its efficiency.
43. An electric motor consumes 1250 W. Find the horsepower it can deliver if it is 85.0% efficient (1 hp = 746 W).
44. A water pump requires an input of 370 W and delivers $5\bar{0}00$ kg of water per hour to a house 22 m above the pump. Find its efficiency. (Use the formula: Output power = mass rate × upward displacement, and the fact that $1\ W = 0.102\ m \cdot kg/s$)

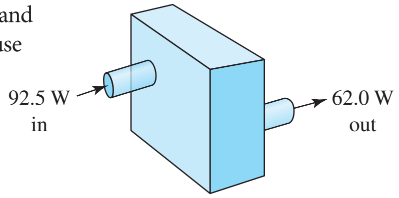


FIGURE 1-23

Percent Error

45. A certain quantity is measured at 125.0 units but is known to be actually 128.0 units. Find the percent error in the measurement.
46. A shaft is known to have a diameter of 35.000 mm. You measure it and get a reading of 34.725 mm. What is the percent error of your reading?
47. A certain capacitor has a working voltage of 125.0 V (DC) – 10%, + 150%. Between what two voltages would the actual working voltage lie?

Percent Concentration

48. A solution is made by mixing 75.0 L of alcohol with 125 L of water. Find the percent concentration of alcohol.
49. 8.0 m^3 of cement is contained in a concrete mixture that is 12% cement by volume. What is the volume of the total mixture?
50. How many litres of alcohol are contained in 455 L of a gasohol mixture that is 5.5% alcohol by volume?

CHAPTER 1 REVIEW PROBLEMS

1-1 The Number Types

1. Give the number of significant digits in each number.
(a) 9.886 (b) 1.002 (c) 0.3500 (d) $15.\overline{0}00$

2. Evaluate: $-|-4+2|-|-9-7|+5$

3. Round to two decimal places.
(a) 7.977 (b) 4.655 (c) 11.845 (d) 1.004

4. Round to three significant digits.
(a) 179.2 (b) 1.076 (c) 4.8550 (d) 45 725

5. Evaluate: $|-5|-|2-7|+|-6|$

6. Give the number of significant digits in each number.
(a) 6013.00 (b) 10 000.0 (c) 10 100 (d) 0.000 034

7. Evaluate: $-|-4|-|-31|+|+41|-|+12\,|-|+53-36+2-14|+7-2$

8. Give the numbers of significant digits and decimal places for each number.
(a) 100.1 (b) 27 382.645 (c) 0.000 386 (d) 18.370

9. Evaluate: $|-11+8-4+12-17|-21+|-8+9|$

10. Combine: $1.435-7.21+93.24-4.1116$

1-2 Numerical Operations

11. Divide: $88.25 \div 9.15$

12. Find the reciprocal of 2.89.

13. Evaluate: $(9.73)^2$

14. Evaluate: $(7.75)^{-2}$

15. Evaluate: $\sqrt{29.8}$

16. Divide: -39.2 by $-0.003\,826$

17. Evaluate: $\sqrt[5]{82.8}$

18. Find the reciprocal of -0.582.

19. Evaluate: $(45.2)^{-0.45}$

20. Combine: $4.928+2.847-2.836$

21. Multiply: $2.84(38.4)$

22. Divide: $48.3 \div 2.841$

23. Find the reciprocal of 4.82.

24. Evaluate: $(3.84)^2$

25. Evaluate: $(7.62)^{-2}$

26. Evaluate: $\sqrt{38.4}$

27. Multiply 746 by the reciprocal of 992.

28. Evaluate: $(47.3)^{-0.26}$

29. Combine: $6.128+8.3470-7.236\,12$

30. Multiply: $7.184(16.8)$

31. Divide: $78.7 \div 8.251$

32. Find the reciprocal of 0.825.

33. Evaluate: $(0.55)^2$

34. Multiply: $21.8(3.775 \times 1.07)$

1–3 Order of Operations

35. Evaluate: $(123)(2.75)-(81.2)(3.24)$

36. Evaluate: $(91.2-88.6)^2$

37. Evaluate: $\left(\dfrac{77.2-51.4}{21.6-11.3}\right)^2$

38. Combine:
$-385-(227-499)-(-102)+(-284)$

39. Evaluate: $(49.3-82.4)(2.84)$

40. Evaluate: $\dfrac{3.87\cdot(14.26-8.76)}{1.55\cdot(7-4)^2}$

41. Evaluate:
$4\times 6.2+(4.88+9.12)\cdot(2.35-1.442)$

42. Evaluate: $18.2 \div (93.221-36.874)+32$

43. Evaluate: $2+3-4\times 5+6.66\div 2$

44. Evaluate:

(a) $\dfrac{(2\times 3)^3\cdot(2+4-8)}{(2+4)\cdot(3\times 2)^2}$

(b) $\left(2.4+\dfrac{5}{3}\right)\div\dfrac{4}{3}-2.25$

1-4 Scientific and Engineering Notation

45. Combine:
$(8.34 \times 10^5) + (2.85 \times 10^6) - (5.29 \times 10^4)$

46. Divide: 8.24×10^{-3} by 1.98×10^7

47. Write in decimal notation: 5.28×10^4

1-5 Units of Measurement

48. A news report states that a new hydroelectric generating station in Quebec will produce 140 terajoules of power per year (TJ/a) and that this power, for 20 years of operation, is equivalent to 2.0 million barrels of oil. Using these figures, to how many gigajoules is each barrel of oil equivalent?

49. The average solar radiation in the continental United States is about 7.4×10^5 joules per square metre per hour ($J/(m^2 \cdot h)$). How many kilowatts would be collected by 15 acres of solar panels?

50. Convert 6930 Btu/h to kilowatts.

51. Convert 0.000 426 mA to microamperes.

52. Convert 49.3 pounds-force to newtons.

53. Convert 36.82 in. to centimetres.

54. How many centimetres are in $14\frac{3}{4}$ inches? (Answer to three decimal places.)

55. Convert 0.000 033 1 m to millimetres.

56. Convert 0.000 25 cm to millimetres. Give your answer in scientific notation.

57. Convert 4 350 000 000 mg to kilograms. Give your answer in engineering notation.

58. Convert 1 h to seconds.

59. To two significant digits, how many feet, and how many inches, are in 5.7 yd.?

1-6 Substituting into Equations and Formulas

60. Evaluate $y = 3x^2 - 2x$ when $x = -2.88$.

61. Evaluate $y = 2ab - 3bc + 4ac$ when $a = 5$, $b = 2$, and $c = -6$.

62. Evaluate $y = 2x - 3w + 5z$ when $x = 7.72$, $w = 3.14$, and $z = 2.27$.

1-7 Percentage

63. A generator has a power input of 2.50 hp and delivers 1310 W. Find its percent efficiency.

64. A train running at 25 km/h increases its speed by 12.5%. How fast is it then travelling?

65. An item rose in price from $29.35 to $31.59. Find the percent increase.

66. Find the percent concentration of alcohol if 2.0 L of alcohol is added to 57 L of gasoline.

67. A bar, known to be 2.0000 cm in diameter, is measured at 2.0064 cm. Find the percent error in the measurement.

68. What percent of 40.8 is 11.3?

69. Find 49.2% of 4827.

70. Find the percent change in a voltage that has increased from $11\overline{0}$ V to 118 V.

71. A homeowner added insulation, and her yearly fuel consumption dropped from 2210 L to 1425 L. What percent of her former oil consumption is her present consumption?

72. 8460 is what percent of 38 400?

73. The population of a certain town is 8118, which is 12.5% more than it was three years ago. What was the population then?