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Automotive Engine Matching

1.1 Introduction

Internal combustion engines have been the primary power source for automotive vehicles since the beginning of the automotive industry. Although automobiles powered by electric motors have entered the automotive market and are likely to grow in market share, the vast majority of vehicles will still be powered by internal combustion engines in the foreseeable future. This is partly due to the bottleneck in the development of key technologies for electric vehicles, such as battery energy density, durability and charging time, and the lack of infrastructure and facilities necessary for the daily use of electric vehicles. On the other hand, proven crude oil reserves can still fuel internal combustion engines for decades to come.

Modern internal combustion engines are sophisticated systems that integrate synergistically mechanical, electrical, and electronic subsystems. Engine technologies are subjects of study in great breadth and depth in the areas of combustion, heat transfer, mechanical design and manufacturing, material engineering, and electronic control [1,2,3,4]. However, this book does not cover engines themselves and is concerned only with how the engine outputs are transmitted to the driving wheels. Readers interested in engine topics are directed to the books referenced here or other related books. The engine outputs, in terms of power and torque, fuel economy, and emissions, are considered as given throughout the text of this chapter and indeed the whole book. Note that engine mapping data are highly proprietary and is usually not available in the public domain. Figures and plots pertaining to engine data in this book are mainly for illustration purposes and may not show the precise data of production engines.

The main topic of this chapter is the matching between the engine outputs and vehicle performance through the selection of transmission ratios. The chapter specifically covers: output characteristics of internal combustion engines; vehicle road loads and acceleration; driving force (or traction) and power requirements; vehicle performance dynamics and fuel economy; and transmission ratio selection. These topics are interconnected and are described in sequential order.

Although the chapter concerns automotive engine matching, as the title indicates, the formulation and related analysis of road loads, performance dynamics, and powertrain kinematics are applicable for all ground vehicles driven by wheels. The equations derived in this chapter will be referenced throughout the book wherever needed by the text.

Automotive Power Transmission Systems, First Edition. Yi Zhang and Chris Mi.

1.2 Output Characteristics of Internal Combustion Engines

The output of an internal combustion engine depends on its design, control, and calibration. Although computer simulation can be used to analyse engine output, engine mapping is the only experimental approach to obtain reliable engine output data. For a given production engine, its output data are provided in terms of power and torque, as well as specific fuel consumption and emissions.

1.2.1 Engine Output Power and Torque

The operation status of an internal combustion engine is defined by its crankshaft rotational speed and the output torque from its crankshaft. The output torque and power depend on the throttle opening and the engine speed, i.e. the crankshaft rotational speed in RPM. It should be noted that the output torque and output power are not independent since power is the product of torque and angular velocity. The torque map of a typical IC engine is shown in Figure 1.1, where the two horizontal axes are respectively the throttle opening as a percentage of the wide open throttle (WOT) or as a degree of throttle angle and the engine speed in RPM. The vertical axis shows the engine output torque in foot pounds in the imperial standard or in newton-meters in the international standard (SI). Without considering the engine transient behavior, the engine static output torque can be found from Figure 1.1, usually by numerical interpolation, for a given set of engine speed and throttle opening. This is the torque as a load at which the engine reaches dynamic equilibrium at the specified engine speed and the throttle opening.

In practice, the engine output torque is often plotted as a curve against the engine speed for specific throttle openings as shown in Figure 1.2, where the throttle opening for each torque curve is represented as a percentage of the wide open throttle angle. Clearly, the engine output torque is a function of engine RPM for a given throttle opening and there is a torque vs RPM curve for each throttle opening. Figures 1.1 and 1.2 provide the same output torque data and are just drawn for convenience of reference.

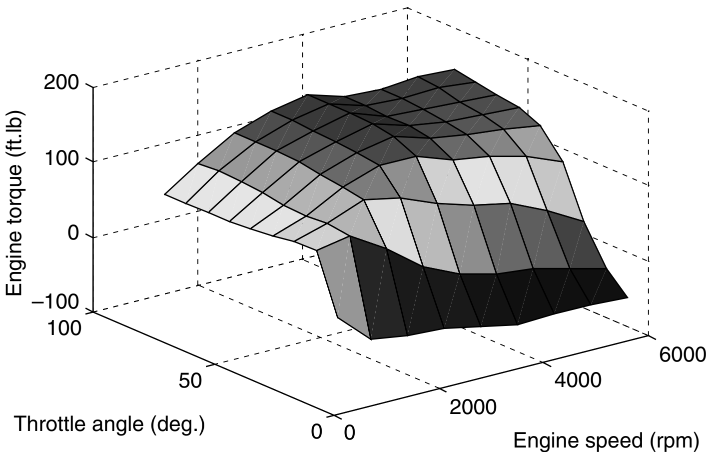

Figure 1.1 Engine output torque map.

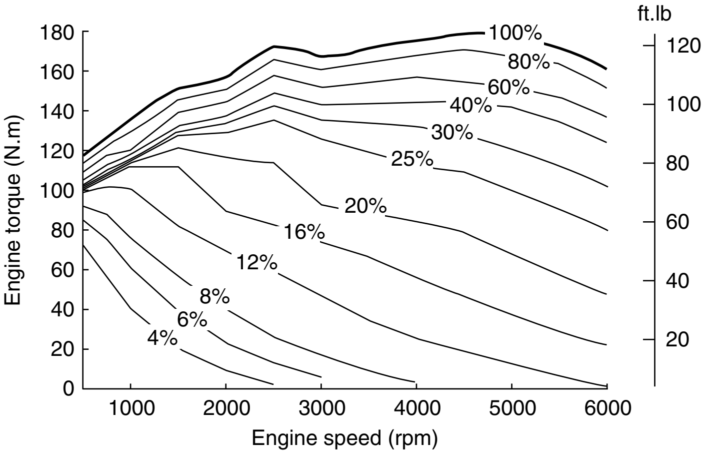

Figure 1.2 Engine torque curves for various throttle openings.

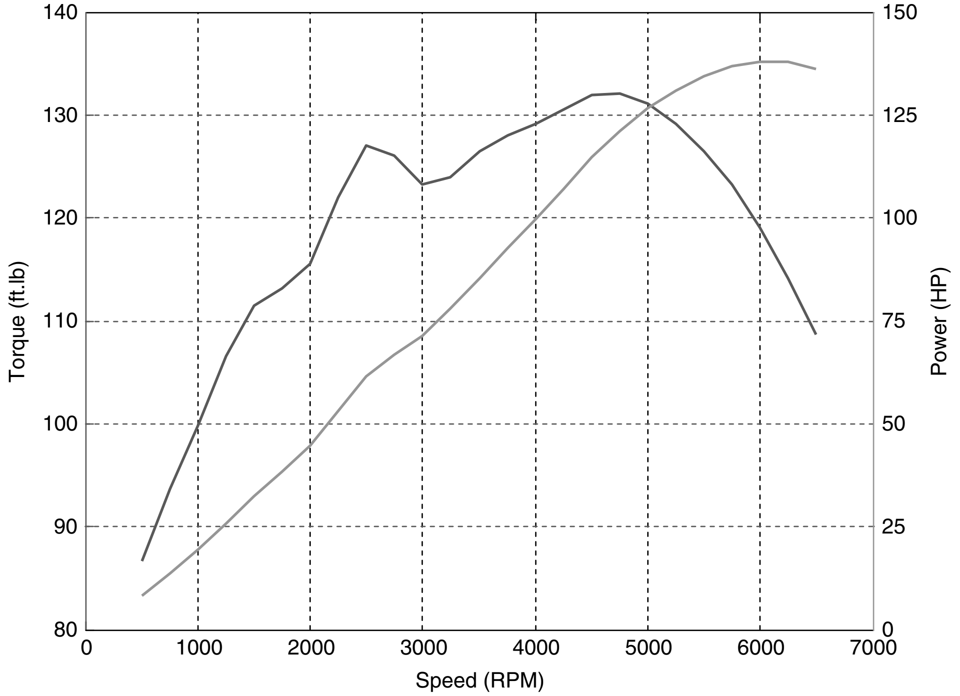

Figure 1.3 Engine torque and power at wide open throttle.

Apparently, the engine capacity torque output or the power output is achieved at the wide open throttle (WOT), as shown in Figure 1.3. It should be noted that the maximum engine torque and the maximum engine power occur only at two separate RPM values on the WOT torque and power curves. The two popularly referred engine performance

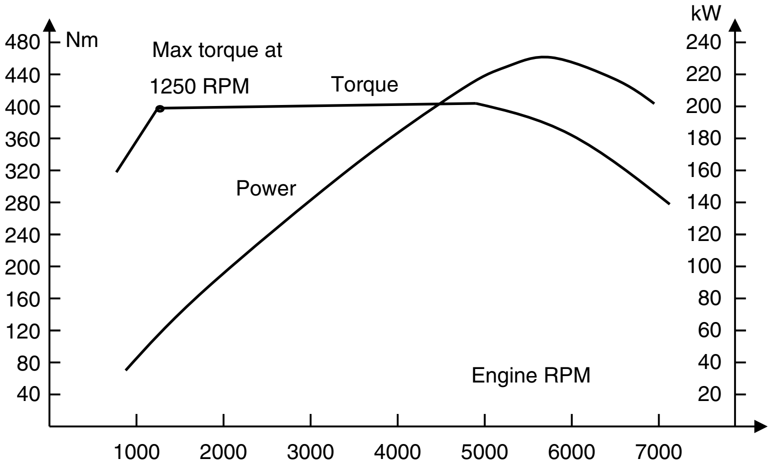

Figure 1.4 Typical torque curve of turbo engines.

specifications, engine power and engine torque, are actually the peak values for the power and torque on the WOT output plot. As can be observed in Figure 1.3, internal combustion engines provide stable power output within a range of engine rotational speed, defined by the so-called idle RPM and redline RPM. Below the idle, the engine does not run stably but stalls, without being able to provide any usable output. On the other side, running the engine beyond the redline speed may cause excessive damage to the engine.

The shape of the torque curve in the operation range defined by the idle and redline is characteristic of the IC engine, depending on its design, fuel injection method, control, and calibration. As an example, the torque curve in Figure 1.3 has a local peak at around 2500 RPM and the maximum engine torque occurs around 4800 RPM. The engine power increases from the idle point almost linearly up to a peak at around 6100 RPM.

In general, the torque curves for naturally aspirated engines can be categorized as rising and buffalo shaped [5], while for turbo-charged engines, the torque curves are flat from a certain low RPM up to a relatively high RPM, as shown in Figure 1.4. Using turbo technology, the maximum output torque can be increased by more than 50% for the same engine displacement. To make things better, this maximum torque becomes available at a much lower RPM in comparison to naturally aspirated engines and stays flat up to a high RPM. This provides the vehicle with much better acceleration performance, especially at low vehicle speed. Turbo engines with small displacements provide outputs in torque and power equivalent to those of naturally aspirated engines of much larger displacements but consumes less fuel. Because of these advantages, vehicles powered by turbo engines are increasingly popular and represent the trend in the automotive industry.

1.2.2 Engine Fuel Map

Engine fuel efficiency is a top performance specification in today's automotive industry. The fuel consumption data of internal combustion engines are indispensable for the design, operation and control of vehicle powertrain systems. These data are experimentally obtained by intensive engine mapping and are usually provided for a production

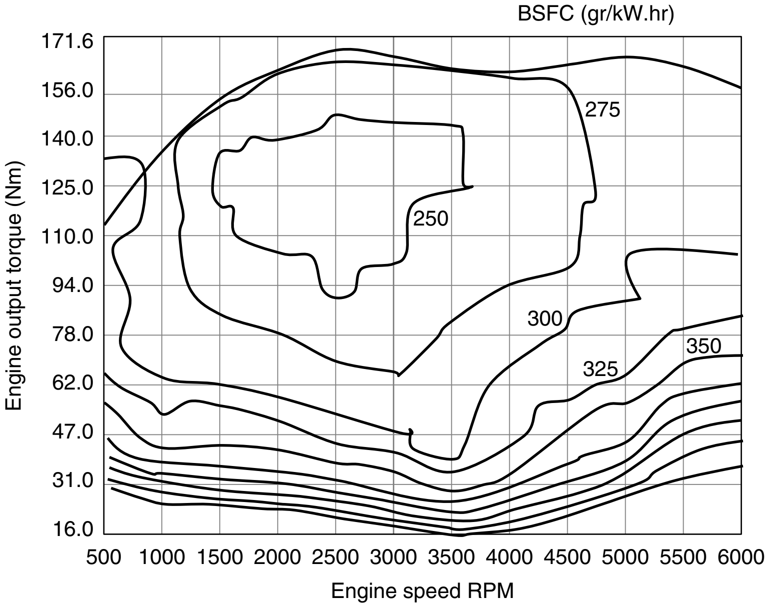

Figure 1.5 Engine specific fuel consumption map.

engine as a fuel map, which indicates in contour plots the specific fuel consumption of the engine at a given operation status, as shown in Figure 1.5. The specific fuel consumption is the amount of fuel that the engine needs to burn in order to do one horsepower hour of work, either in litres, grams, or pounds of weight.

The horizontal and vertical axes in Figure 1.5 are respectively the engine RPM and the engine output torque that define the engine operation status. The numbers by the contours are the specific fuel consumption in gram per kilowatt-hour (grams of fuel the engine consumes for it to do one kilowatt-hour of work). For example, if the engine runs stably at 4000 RPM with an output torque of 94 Nm, the specific fuel consumption is 275 gr/kW.hr. At the operation status defined by the RPM and the torque, the engine power is 39.37 kW. If the engine runs at the status continuously for one hour, the fuel consumed by the engine will then be 10.82 kg. The engine fuel consumption along a contour is the same even though the operation status is different, so the hourly fuel consumption of the engine is also 10.82 kg if it runs stably at 4740 RPM with an output torque of 125 Nm. Apparently, the engine will be more fuel efficient if it runs along the contour with 250 gr/kW.hr. It can also be observed that the most fuel efficient operation status is near the point with 2500 RPM and 125 Nm. When a vehicle is driven on a road at constant speed, the engine operation status depends on the road load, the vehicle speed, and the transmission gear ratio, the last determining the engine RPM and torque at a given vehicle speed and thus largely affects the vehicle fuel economy.

1.2.3 Engine Emission Map

When internal combustion engines generate power to propel ground vehicles, unwanted pollutants, harmful to the environment and to human health, are emitted in the process of combustion. These pollutants include CO, CO_2, NO_X, and other harmful gasses or particulate matter. The standard on emission control is increasingly stringent in the automotive industry due to environmental and human health concerns. Engine technologies, especially the technologies for combustion control and after-combustion treatment, are the key to minimizing the emission of pollutants. Transmissions also contribute to lowering vehicle emission levels by keeping the engine running in more efficient and less polluting operating ranges.

Engine emission maps are even more difficult to obtain than fuel maps because the quantity of a pollutant under various operation conditions is hard to measure. Computer simulation can be used to analyse engine emissions, but reliable emission data can only be obtained experimentally through extensive tests. Engine emission maps are provided for a given engine in formats similar to engine fuel maps. The specific quantity of a particular pollutant emitted by the engine is interpolated from the emission contour for a given engine operation status. Using the emission maps of the engine, the amount of emission of a pollutant can be simulated for a specified drive range.

1.3 Road Load, Driving Force, and Acceleration

Various forces are applied to a vehicle when it travels on a road surface. These forces include gravity, wheel–road contacting forces, road load, and driving force, which is also termed traction. Road load is against the motion of the vehicle, while traction force, or driving force, propels the motion of the vehicle. The driving force of a vehicle originates from the engine via the transmission and is fundamentally limited by the road traction limit. The total road load is the resultant of three separate road loads: rolling resistance, grade load, and air resistance. Figure 1.6 is the free body diagram of a vehicle of weight W that is being accelerated uphill.

In the free body diagram, v and a are the vehicle speed and acceleration respectively. R_A is the air drag or air resistance. The air drag is a distributed load, but for simplicity, it is assumed to be a point load acting at height h_A. R_F and R_R are the rolling resistance from

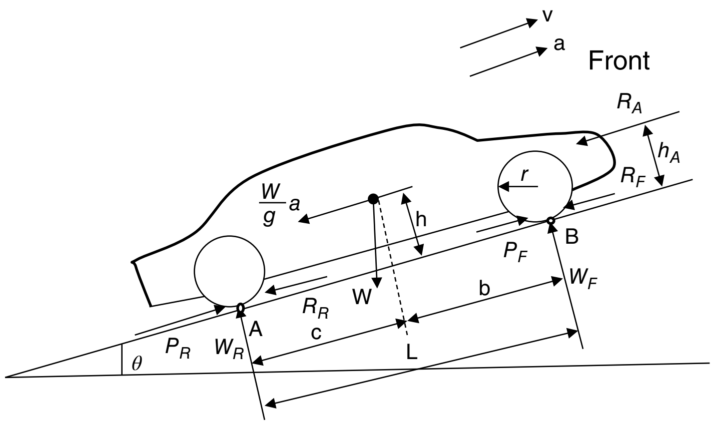

Figure 1.6 Free body diagram of a vehicle accelerated uphill.

the front and rear wheels respectively. P_F and P_R are the driving force from the front and rear wheels respectively. W_F and W_R are the axle loads, which are respectively the contact force between the front wheels and the road surface and between the rear wheels and the road surface. A and B denote the points of contact between the wheels and the road surface. θ is the grade angle of the slope and r is the rolling radius of the tire. The height of the center of gravity and the height of the air resistance are respectively denoted as h and h_A. For passenger cars, these two heights are assumed to be the same. The vehicle wheelbase is L and the longitudinal position of the gravity center is determined by b and c, which is the distance from the gravity center to the front axle and rear axle respectively. Unless otherwise stated, the US customary unit system will be used in the equations, where forces are in pounds, linear dimensions are in feet, speed is in ft/s, and acceleration is in ft/s^2. It should be noted that the inertia force $\gamma {}^{W}/_{g}\, a$ in the free body diagram is in the opposite direction to the acceleration, based on the D'Alembert's principle. γ is the equivalent mass factor that is introduced to account for the mass moments of inertia of all rotational components in the powertrain, including transmission input and output shafts, gears in the power flow path, drive shaft, differential, wheels, etc. The value of γ can be accurately determined based on the total vehicle equivalent kinetic energy as follows,

$$\frac{1}{2}\gamma\frac{W}{g}v^2 = \frac{1}{2}\frac{W}{g}v^2 + \sum_{i=1}^{n}\frac{1}{2}J_i{\omega_i}^2 \tag{1.1}$$

In the equation above, J_i is the mass moment of inertia of each rotational component and n is the total number of rotational components in the powertrain. The equivalent mass factor is then determined as,

$$\gamma = 1 + \sum_{i=1}^{n}\frac{gJ_i}{W}\left(\frac{\omega_i}{v}\right)^2 \tag{1.2}$$

For a given vehicle, the ratio $\frac{\omega_i}{v}$ is a constant for each rotational component that depends on the transmission gear ratios. Empirical formula and tables are available for the approximation of the equivalent mass factor [6]. For passenger cars, the value of γ is small and can be considered to be equal to one for vehicle acceleration analysis and transmission ratio selections.

1.3.1 Axle Loads

The forces in the free body diagram (Figure 1.6) form a system of equilibrium, and three scalar equations can be written based on the condition of equilibrium. As shown below, the first two equations are based on the conditions that the sum of moments made by all forces about point A and point B must be equal to zero. The third equation is that the sum of all forces, including the inertia force, must be equal to zero in the direction of vehicle motion.

$$\begin{aligned} \sum M_A &= 0 \\ \sum M_B &= 0 \\ \sum F &= 0 \end{aligned} \tag{1.3}$$

These equations can be arranged to express the axle loads and the inertia force as follows:

$$W_F = \frac{1}{L}\left(Wc\cos\theta - R_A h_A - \gamma\frac{W}{g}ah - Wh\sin\theta\right) \tag{1.4}$$

$$W_R = \frac{1}{L}\left(Wb\cos\theta + R_A h_A + \gamma\frac{W}{g}ah + Wh\sin\theta\right) \tag{1.5}$$

$$\gamma\frac{W}{g}a = P_F + P_R - R_F - R_R - R_A - W\sin\theta \tag{1.6}$$

The first two equations determine the dynamic axle weights for the vehicle. During acceleration, there is a weight transfer equal to the magnitude of the inertia force from the front axle to the rear axle, as shown in Eqs (1.4) and (1.5). The static axle weights on level ground are obtained from the equations by making the slope angle θ, the air drag R_A, and the acceleration a equal to zero. It should be noted that tractions are available from both front and rear wheels only for a four wheel drive vehicle. P_R is zero for front wheel drive and P_F is equal to zero for rear wheel drive. Total driving force and rolling resistance from both front wheels and rear wheels are:

$$\begin{aligned} P &= P_F + P_R \\ R &= R_F + R_R \end{aligned} \tag{1.7}$$

The rolling resistance depends on many factors, such as tire material, texture, tread, inflation, speed, etc. Accurate calculation of the rolling resistance is very difficult, indeed impractical. For simplicity, it is common practice in the automotive industry to calculate the rolling resistance by:

$$R = fW\cos\theta \approx fW \tag{1.8}$$

where f is the rolling resistance coefficient and is approximately equal to 0.02. By rearranging Eqs (1.4–1.7) with the assumption that $h \approx h_A$, the axle loads can be solved in the following form:

$$P - fW = \gamma\frac{W}{g}a + R_A + W\sin\theta \tag{1.9}$$

$$W_F = \frac{Wc}{L} - \frac{h}{L}(P - fW) \tag{1.10}$$

$$W_R = \frac{Wb}{L} + \frac{h}{L}(P - fW) \tag{1.11}$$

where $\frac{c}{L}$ and $\frac{b}{L}$ are the weight distribution factors. The term $\frac{h}{L}(P - fW)$ is the dynamic weight transfer. Eqs (1.10) and (1.11) represent the dynamic axle weights in terms of the static axle weights and the weight transfer. The dynamic axle weight on the driving axle determines the maximum traction available for the vehicle under a given road condition.

1.3.2 Road Loads

There are three kinds of road loads that are against vehicle motion when the vehicle travels on a road surface: rolling resistance, air drag, and grade load, as shown in Figure 1.6.

The rolling resistance is calculated by Eq. (1.8). The grade load is the component of gravity on the slope direction and is equal to $W \sin\theta$. At level ground, only rolling resistance and air drag exist. At high vehicle speed, the air drag becomes more significant than the rolling resistance.

There are two causes for the generation of air resistance: friction between the air and the vehicle body surface; and air turbulence formed around the vehicle body [6]. The latter is the main cause of air drag for ground vehicles. Factors affecting the magnitude of the air drag include the shape and finish of vehicle body, the vehicle frontal projected area, air density and atmospheric condition, and most importantly, the vehicle's speed. It is very challenging to exactly determine the air drag by analytical means. In the standard of the Society of Automotive Engineers (SAE), the air resistance or air drag is calculated by the following formulation [6]:

$$R_A = 0.26 C_D A \left(\frac{v}{10}\right)^2 \tag{1.12}$$

where C_D is the unit less air drag coefficient that mainly depends on vehicle body shape and body surface smoothness. The air drag coefficient can be determined with high accuracy by wind tunnel testing. Modern passenger cars with streamlined body can have an air drag coefficient as low as 0.26. A is the vehicle frontal projected area in ft^2 that mainly depends on the vehicle size. This is the area of the vehicle body that confronts the air flow in the direction perpendicular to vehicle motion. To determine this area, a flat board can be held perpendicular to the road surface behind the parked vehicle and a flashlight is then used to beam the vehicle body horizontally in front of the vehicle. The area of the shadow casted on the board is the frontal projected area. As shown in the formulation, the air drag is proportional to the square of the vehicle speed v relative to the wind. With the speed v in mph, the formulation determines the air drag as a force in pounds. For the analysis and calculations of vehicle dynamics, the vehicle speed and acceleration are often in ft/s and ft/s^2, then the formulation for air drag will be used in the following form:

$$R_A = 0.00118 C_D A v^2 \tag{1.13}$$

The equation above is directly transformed from Eq. (1.12) by considering that one mph is equal to 1.467 ft/s. The resultant road is the sum of the rolling resistance, grade load and air drag, as expressed below,

$$R = 0.00118 C_D A v^2 + fW + W \sin\theta \tag{1.14}$$

1.3.3 Powertrain Kinematics and Traction

There are various layouts for vehicle powertrain systems. In this section, the powertrain of a rear wheel drive vehicle with a manual transmission (MT), in the layout shown in Figure 1.7, is used as the example for demonstration. The analysis on powertrain kinematics and related equations derived in the example are applicable to all other powertrain layouts.

The engine output torque and output angular velocity are denoted as T_e and ω_e respectively, the transmission output torque and angular velocity are denoted as T_t and ω_t, the

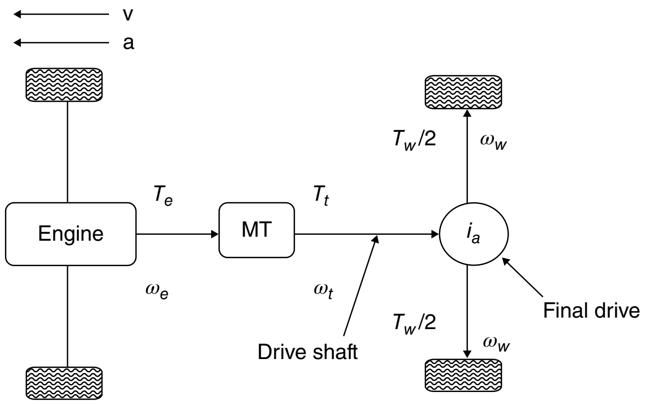

Figure 1.7 Layout of RWD manual transmission powertrain.

angular velocity of the driving wheel is denoted as ω_w, and the torque on each of two wheels on the driving axle is denoted as $\frac{T_w}{2}$. As shown in Figure 1.7, the engine output is transmitted through the transmission to the drive shaft, which then transmits the transmission output to the final drive. The final drive assembly contains a pair of spiral bevel or hypoid gears that multiplies the transmission output torque by the final drive ratio i_a and transmits the rotation of the drive shaft to the wheels between the two perpendicular axes. The outputs of the engine and the transmission are related by the following equation:

$$T_t = \eta_t i_t T_e \tag{1.15}$$

$$\omega_t = \frac{\omega_e}{i_t} \tag{1.16}$$

where i_t is the transmission ratio defined as the division of the input angular velocity by the output angular velocity and η_t is the transmission efficiency. The transmission ratio is a stepped variable for manual transmissions. For a five speed transmission, the ratio varies between the highest value in the first gear and the lowest value in the fifth gear. Note that the accurate determination of the transmission efficiency is experimental by nature, and the efficiency is assumed to be a constant value throughout the text of this book.

The transmission output is further transmitted by the final drive to the driving wheels. The torque on the two driving wheels and the angular velocity of the driving wheels are related to the engine output torque and engine angular velocity by the following equations:

$$T_w = \eta_a i_a T_t = \eta_a \eta_t i_a i_t T_e \tag{1.17}$$

$$\omega_w = \frac{\omega_t}{i_a} = \frac{\omega_e}{i_a i_t} \tag{1.18}$$

where i_a is the final drive ratio and η_a is the final drive efficiency. When a vehicle travels on road, there is always a small amount of slippage between the tire and the road surface. The amount of tire spillage is determined by the tire slip rate defined as:

$$\delta = \frac{\omega r - v}{\omega r} \tag{1.19}$$

Under normal driving conditions, the slip rate is very small ($\delta \le 0.02$) and can be neglected for the selection of gear ratios in engine–transmission matching. If the slippage is not considered, then vehicle motion and driving wheel rotation are related as,

$$\begin{aligned} \omega_w &= \frac{v}{r} \\ \alpha_w &= \frac{a}{r} \end{aligned} \tag{1.20}$$

where α_w is the angular acceleration of the driving wheels. Based on Eqs (1.17), (1.18), and (1.20), the engine output torque, the engine angular velocity, the torque on the driving wheels, and the vehicle speed are related by the following equations,

$$\begin{cases} T_w = \eta i_a i_t T_e \\ \omega_e = i_a i_t \dfrac{v}{r} \end{cases} \tag{1.21}$$

where η is the overall powertrain efficiency. The second of these equations relates the engine angular velocity and the vehicle speed without considering tire slippage. Knowing the torque on the driving wheels, the driving force that originates from the engine and propels the vehicle is then determined by the following equation for the rear wheel drive (RWD) vehicle in the example:

$$P = P_R = \frac{\eta i_a i_t T_e}{r} \tag{1.22}$$

The maximum value of the driving force for ground vehicles driven by wheels is limited by road–tire contact conditions and is commonly termed as the traction limit. For the RWD vehicle in question, the traction is limited by the following inequality:

$$P = P_R \le \mu W_R \tag{1.23}$$

where μ is the so-called traction coefficient, which depends on the road condition and the tire properties. On a standard highway surface (road surface with skid number 81), the value of the traction coefficient is equal to 0.81. The traction force or driving force available from the powertrain cannot exceed the traction limit, or the driving wheels will slip excessively. W_R is the dynamic rear axle load, which is determined by Eq. (1.11). It is emphasized here that Eqs (1.15–1.23) are derived for the RWD vehicle in the example, and are applicable for all other types of powertrain systems as mentioned previously. With the driving force determined by Eq. (1.22), the vehicle acceleration can then be determined from Eq. (1.16) by:

$$a = \frac{P - (fW + W\sin\theta + 0.00118 C_D A v^2)}{\gamma \frac{W}{g}} \tag{1.24}$$

Eq. (1.9), or Eq. (1.24), is actually the equation of motion of the vehicle when it runs on a straight path. The vehicle acceleration can be calculated by Eq. (1.24) at any given vehicle speed if the engine throttle opening and the road condition are provided. The equivalent mass factor γ in Eq. (1.24) is approximately equal to one for passenger vehicles. The following example demonstrates how the equation series is used for the calculation of vehicle acceleration and fuel economy.

Example 1.1 A manual transmission used for a vehicle with data given below has six forward speeds and the gear ratios are: 1st gear (3.72), 2nd gear (2.31), 3rd gear (1.51), 4th gear (1.07), 5th gear (0.81), 6th gear (0.63). The engine WOT output is as given in Figure 1.3 and the fuel map is as given in Figure 1.5. The vehicle has the following data:

Front axle weight: 1750 lb	Rear axle weight: 1550 lb
Center of gravity height: 17 in.	Wheelbase: 104 in.
Air drag coefficient: 0.32	Frontal projected area: 22.90 sq.ft
Tire radius: 11.70 in.	Roll resistance coefficient: 0.018
Powertrain efficiency: 0.94	Traction coefficient: 1.0
Max. power @6000 RPM: 138 HP	Max. torque @4500 RPM: 132 ft.lb

a) The engine RPM drops by 662 (RPM) when a 4–5 upshift is made at a vehicle speed of 45 mph. Determine the final drive ratio of the vehicle.
b) Determine the engine torque and the engine power when the vehicle is cruising at a constant speed of 65 mph on level ground in the 6th and 5th gears respectively.
c) Determine the fuel economy in mpg or litres per 100 km for the conditions in (b).
d) Determine the maximum acceleration that the vehicle can achieve in 4th gear at a speed of 65 mph on a 2% slope.

Solution:

a)
$$v = 45\,\text{mph} = 1.46(45) = 65.7\,\text{ft/s}$$
$$\omega = \frac{v}{r} = \frac{65.7}{11.70/12} = 67.38\,\text{rad/s} = 643.5\,\text{RPM}$$

The engine RPM drops by 662 in a 4–5 upshift, so $i_4 i_a \omega_w - i_5 i_a \omega_w = 662$, and:

$$i_a = \frac{662}{\omega_w(i_4 - i_5)} = \frac{662}{643.5(1.07 - 0.81)} = 3.96$$

b)
$$v = 65\,\text{mph} = 65(1.46) = 94.9\,\text{ft/s} = 104\,\text{km/h}$$
$$R = fW + 0.00118 c_D A v^2 = 0.018(1750 + 1550) + 0.00118(.32)(22.9)(94.9)^2 = 137.3\,\text{lb}$$
$$\omega_w = \frac{v}{r} = \frac{94.9}{11.7/12} = 97.33\,\text{rad/s}$$

Since the vehicle speed is constant, engine power is the same for both gears and is equal to:

$$\frac{Rv}{\eta} = \frac{137.3(94.9)}{.94} = 13861.6\,\text{ft.lb/s} = \frac{13861.6}{550} = 25.2\,\text{HP} = 18.8\,\text{kW}$$

Engine torque depends on the gear ratio and is calculated respectively for the 6th and 5th gears,

$$\text{6th gear}: T_e^{(6)} = \frac{Rr}{\eta i_6 i_a} = \frac{137.3(11.7/12)}{.94(.63)(3.96)} = 57.1\,\text{ft.lb} = 76.5\,\text{Nm}$$
$$\text{5th gear}: T_e^{(5)} = \frac{i_6}{i_5} T_e^{(6)} = \frac{.63}{.81}(57.1) = 44.4\,\text{ft.lb} = 59.5\,\text{Nm}$$

c) The angular velocity of the engine for the condition in question (b) is respectively,

$$\omega_e^6 = i_6 i_a \omega_w = (.63)(3.96)(97.33) = 242.8\ \text{rad/s} = 2319\,\text{RPM}$$
$$\omega_e^5 = i_5 i_a \omega_w = (.81)(3.96)(97.33) = 312.2\,\text{rad/s} = 2981\,\text{RPM}$$

So the engine operation status is defined respectively as (2319, 76.5) and (2981, 59.5). From the fuel map (Figure 1.5), the specific fuel consumption is 275 gr/(kW.hr) and 285 gr/(kW.hr) respectively. The fuel consumed per hour can then be calculated as:

$$\text{6th gear}: 18.8(275) = 5170\,\text{gram} = 6.89\,\text{litres};$$
$$\text{5th gear}: 18.8(285) = 5358\,\text{gram} = 7.14\,\text{litres}$$

Fuel consumption in litres per 100 km:

$$\text{6th gear}: \frac{100(6.89)}{104} = 6.63; \text{5th gear}: \frac{100(7.14)}{104} = 6.87$$

Fuel consumption in mpg:

$$\text{6th gear}: \frac{65}{5.17/3.03} = 38\,\text{mpg}; \text{5th gear} = \frac{65}{5.36/3.03} = 36.7\,\text{mpg}$$

d) $$\omega_e = i_4 i_a \omega_w = 1.07(3.96)(97.73) = 412.41\,{}^{\text{rad}}/_{\text{s}} = 3938\,\text{RPM}$$

At 3938 RPM, the WOT engine torque T_e is found from Figure 1.3 and is equal to 129 ft.lb.

$$P = \frac{\eta i_4 i_a T_e}{r} = \frac{0.94(1.07)(3.96)(129)}{11.7/12} = 526.98\,\text{lb}$$
$$R = fw + 0.00118 c_D A v^2 + 0.02\,\text{W} = 137.366 + 203.3\,\text{lb}$$
$$a = \frac{P - R}{W/g} = \frac{526.98 - 203.3}{3300/32.2} = 3.16\,\text{ft/s}^2$$

1.3.4 Driving Condition Diagram

As can be observed from Eqs (1.21) and (1.22), the driving force available from the engine at a given throttle position can be calculated for a given vehicle speed. This is because the engine angular velocity is related to the vehicle speed via the transmission and final drive ratios, and the engine output torque is a function of engine speed at a given throttle. Apparently, the capacity propulsive force available from the engine is obtained when the engine is operating at wide open throttle (WOT) and this capacity driving force is a function of vehicle speed as determined by Eqs (1.21) and (1.22). Since the transmission has stepped gear ratios, each gear of the transmission provides a function or relationship between the engine torque and the vehicle speed. Similarly, the road load is also a function of vehicle speed, as shown in Eq. (1.14). Therefore, the net force for acceleration at engine capacity can be found by subtracting the road load from the driving force available under WOT condition at any vehicle speed. The driving force and road load can be plotted over the speed range of the vehicle in the so-called driving condition diagram, as shown in Figure 1.8. In this figure, the horizontal axis is for the vehicle speed and the vertical axis is for the driving force and road load. In the US customary unit, the driving force and road load are in pounds, and the vehicle speed is in ft/s, while in International standard, the traction (i.e. driving force) and road load are in N and the vehicle speed is in m/s.

The driving condition diagram in Figure 1.8 is for an example vehicle equipped with a five-speed manual transmission. There are five separate traction curves, one for each gear. There must be a certain amount of slippage between the engine output and transmission input at vehicle launch because the engine cannot provide output torque below the idle RPM. Each driving force curve or traction curve covers the vehicle speed range corresponding to the engine speed range from idle to redline. For example, the vehicle speeds at the starting point and end point on the first gear driving force or traction curve are determined as:

$$
\begin{aligned}
v_{1i} &= \frac{\pi\, n_e^{idle} i_a i_1 r}{30} \\
v_{1r} &= \frac{\pi\, n_e^{redline} i_a i_1 r}{30}
\end{aligned}
\tag{1.25}
$$

where v_{1i} and v_{1r} are the vehicle speed in the first gear corresponding to the engine idle speed n_e^{idle} and redline speed $n_e^{redline}$ respectively. For any other point along the first gear traction curve, we can first divide the interval $[v_{1i}, v_{1r}]$ evenly and then pick up a vehicle speed from the interval. This speed is then used to determine the engine angular velocity from Eq. (1.21). Knowing the engine angular velocity, the engine torque can then be found from the engine WOT torque output. The driving force is finally determined by using Eq. (1.22).

In the driving condition diagram, the road load is plotted against vehicle speed on different grades, starting from level ground. The road load curves on different grades are parallel parabolic curves. In the SAE standard, the steepness of a slope is expressed by the grade percentage defined as follows:

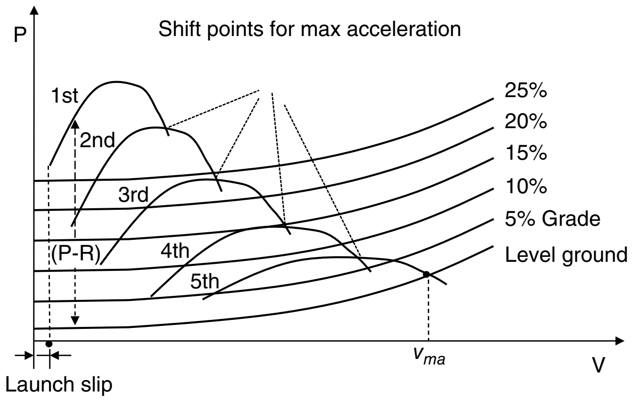

Figure 1.8 Driving condition diagram.

$$\frac{G}{100} = \tan\theta \approx \sin\theta \tag{1.26}$$

where θ is the slope angle and G is the grade number. If the slope angle θ is small, the slope percentage can be approximated as $\sin\theta$. Thus the grade load is often calculated as $\frac{G}{100}W$.

The following data are required to complete the driving condition diagram for a vehicle: engine output data, the efficiency and ratios of the transmission and final drive, vehicle data such as weight, tire rolling radius, frontal projected area and air drag coefficient. The driving condition diagram provides graphically technical data on vehicle dynamic performance. For example, performance related data in the following list can be obtained by observing the driving condition diagram:

- Net driving force available for vehicle acceleration in different gears and on different grades over the whole vehicle speed range. This net force is the difference between the traction curve and the road load curve.
- Maximum vehicle speed based on powertrain capacity. As shown in Figure 1.8, the maximum speed on level ground occurs at the intersection between the traction curve of the fifth gear and the level ground road load curve for the vehicle in the example. It is also possible to find the maximum vehicle speed on other grades and to find the gear at which the maximum speed is achieved.
- Shifting points for maximum vehicle acceleration. To reach the maximum vehicle speed from standstill in the shortest time, the driver must make gear shifts at points that give the largest net driving force for acceleration. For the driving condition diagram shown in Figure 1.8, the shift points should be at the points of intersection of the two adjacent traction curves. If the traction curves do not intersect, then the shift point for maximum acceleration should be at the redline point.
- Grade on which the vehicle can be started at a given gear. Theoretically, the vehicle can be started on a grade as long as the traction curve is above the road load curve. The higher the traction curve above the road load curve, the easier it is to start the vehicle.

The list above shows some of the vehicle performance related data that can be directly obtained from the driving condition diagram. It should be pointed out that the driving condition diagram can be stored as a computer data file for easy reference or interpolation.

1.3.5 Ideal Transmission

The ideal transmission is the one that can vary the gear ratio continuously and is therefore called continuously variable transmission (CVT). Since the transmission ratio can be varied continuously, the engine speed can be controlled at the value that optimizes a selected objective at any given vehicle speed. This optimized objective can be the best fuel economy or the maximum power available from the powertrain. If the maximum power is the objective, then the CVT gear ratio is controlled to keep the engine speed corresponding to the peak power for the whole vehicle speed range. This leads to the following equation for the traction curve of CVT:

$$Pv = \eta(Power)_{max} = C \tag{1.27}$$

where $(Power)_{max}$ is the engine maximum output power, η is the efficiency of the whole powertrain including the CVT and the final drive, and C is a constant. It should be noted here that the efficiency of the CVT itself is actually lower than that of a manual stepped ratio transmission due to the higher friction loss. The fuel economy advantage of CVT is from its capability that the engine operation status can be always controlled at the most efficient range. As shown in Figure 1.9, Eq. (1.27) represents a hyperbola on the driving condition diagram for the traction curve of a CVT powertrain. The hyperbolic traction curve of a CVT powertrain is the envelope to the piecewise traction curves of a stepped ratio transmission. If the same overall powertrain efficiency is assumed, the hyperbola will touch the piecewise traction curves at the point corresponding to the maximum engine power. Several important observations can be made from Figure 1.9:

- For a stepped ratio transmission, the maximum engine power can only be useful at one vehicle speed in each gear. There are always losses of engine power potential due to the mismatch between the vehicle speed and engine status.
- The more gears a stepped ratio transmission has, the more the piecewise traction curves and the closer these curves get to the hyperbola (the ideal CVT traction curve). Generally, more transmission gears provide better matching to the engine and make the engine output power available over a wider vehicle speed range. For example, a semi-truck may have as many as 16 gears in the transmission to fully utilize the engine power potential over a wide speed range.
- Similarly, more transmission gears provide better engine matching in terms of fuel economy because they allow the engine to run closer to the status corresponding to the most fuel efficient range.
- Figure 1.9 provides a judgment on how well the transmission matches the engine. A good match must have those piecewise traction curves closely enveloped by the hyperbola.

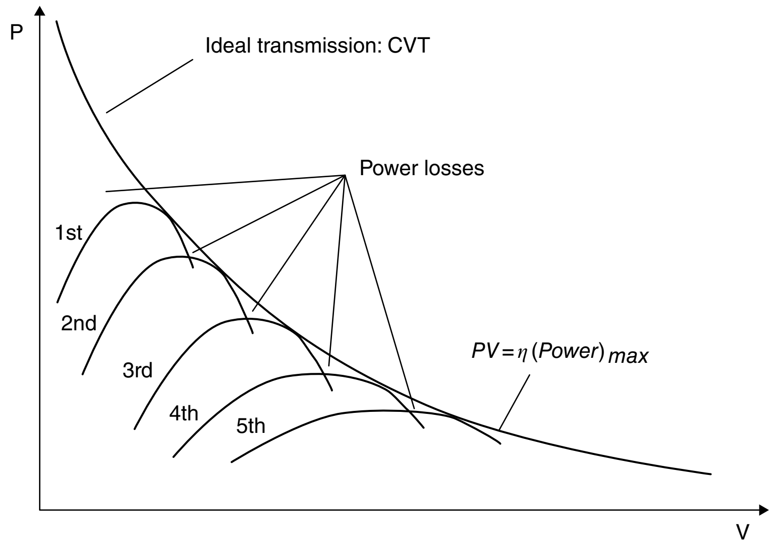

Figure 1.9 Traction curve of the ideal transmission.

A complete driving condition diagram that includes the CVT traction curve provides important technical data on vehicle performance as described previously. In addition, it tells us, to a certain degree, how the transmission ratios match the engine output.

1.3.6 Power–Speed Chart

When a vehicle is driven on a road surface, the power available from the powertrain, $(Power)_A$, and the power of the road load, $(Power)_R$, are respectively determined by the following equations:

$$(Power)_A = Pv \tag{1.28}$$

$$(Power)_R = Rv \tag{1.29}$$

where the driving force P and the road load R are respectively represented by Eqs (1.22) and (1.14). Obviously, the power available from the powertrain must be higher than the road load power for acceleration, and the two powers are balanced when the vehicle cruises at constant vehicle speed. In general, vehicle power requirements depend on its operation conditions. In a power–speed chart, the power available from the powertrain and the road load power to be overcome are both plotted against the vehicle speed, as shown in Figure 1.10. In this chart, the horizontal axis is the vehicle speed, in either ft/s or m/s, depending on the units used. The vertical axis represents the power in horsepower or kilowatts available from the powertrain in each gear, the maximum engine power, and the road load power. The power available from the powertrain is plotted for each gear separately and the road load power is plotted on different grades. The maximum engine power is a horizontal line in the power–speed chart and is realized only by the ideal transmission (CVT) for all vehicle speeds. While the driving condition diagram tells the net driving force for vehicle acceleration, the power–speed chart provides a graphic quantification on the net power reserve under all vehicle operation conditions. The combination of the driving condition diagram and the power–speed chart gives a complete picture of the availability of driving force and power for acceleration or

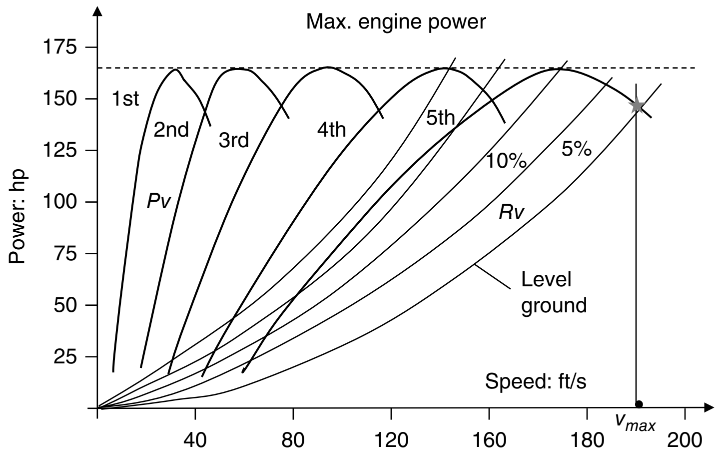

Figure 1.10 Power–speed chart.

gradability. For example, the maximum vehicle speed on level ground can also be found on the power–speed chart. As shown in Figure 1.10, the power curve for the highest gear intersects with the road load power curve on level ground. The maximum vehicle speed is achieved at this intersection because no extra power from the powertrain would be available to overcome the additional road power resulting from any further speed increase.

1.4 Selection of Gear Ratios

The principal considerations for the selection of transmission gear ratios are vehicle performance and fuel economy. The process of gear ratio selection is trial and error by nature, in which experience and data on previous vehicles play an important role. There is no closed form formula that would provide the precise values for the gear ratios that match the engine output for the best results in dynamic performance and fuel efficiency. However, approximate gear ratio values can be calculated analytically by equations derived in this section. These approximate ratios can be used as the starting values for the finalization of transmission ratios.

1.4.1 Highest Gear Ratio

The highest gear ratio is that for the top gear and has the lowest value. This ratio is usually selected such that the vehicle will achieve the maximum speed on level ground that is allowed by the maximum engine power. As discussed previously, vehicle maximum speed occurs at the intersection between the available power curve for the highest gear and the road load power curve on level ground (Figure 1.10). There exists a unique gear ratio for the highest gear such that the available power curve defined by this ratio, the line of maximum power, and the road load power curve on level ground intersect at the same point, which is point H as shown in Figure 1.11. Obviously, the vehicle speed corresponding to point H is the very maximum vehicle speed as allowed by the engine capacity power. At point H, the maximum engine power is fully matched by the highest gear ratio to balance the road load power at the maximum vehicle speed.

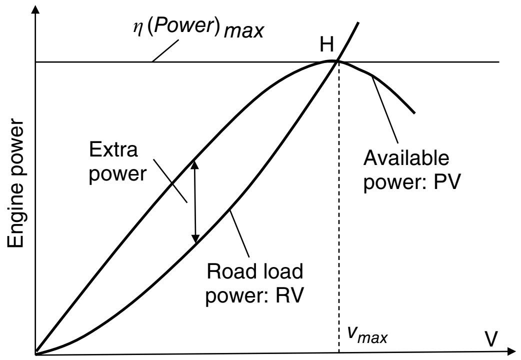

Figure 1.11 Matching maximum engine power for maximum vehicle speed.

As shown in Figure 1.3, the maximum engine power only occurs at a unique engine speed, and this speed corresponds to the maximum vehicle speed calculated as follows:

$$v_{max} = \frac{\pi n_{maxp}}{30(i_{th} i_a)} r \tag{1.30}$$

where n_{maxp} is the engine RPM at which the engine output power is the maximum, r is the tire rolling radius, and $(i_{th} i_a)$ is the overall powertrain ratio. Since the maximum engine power is balanced by the road load power, the following equation becomes apparent,

$$R v_{max} = \eta (Power)_{max} \tag{1.31}$$

where R is the road load determined by Eq. (1.14) with the slope angle θ is equal to zero, η is the overall powertrain efficiency that is assumed to be a constant, and $(Power)_{max}$ is obtained directly from the WOT engine output plot. An equation of the following form can be derived by plugging Eqs (1.14) and (1.30) into Eq. (1.31):

$$\frac{\pi n_{maxp}}{30(i_{th} i_a)} r \left[0.00118 C_D A \left(\frac{\pi n_{maxp}}{30(i_{th} i_a)} r \right)^2 + fW \right] = \eta (Power)_{max} \tag{1.32}$$

This equation contains only one unknown $(i_{th} i_a)$, which is the overall powertrain ratio. Through some simple manipulations, this equation can be transformed to a cubic equation in terms of the unknown $(i_{th} i_a)$:

$$\eta (Power)_{max} (i_{th} i_a)^3 - fW \frac{\pi n_{maxp}}{30} r (i_{th} i_a)^2 - 0.00118 C_D A \left(\frac{\pi n_{maxp}}{30} r \right)^3 = 0 \tag{1.33}$$

For a given vehicle the only unknown in the equation above is overall powertrain ratio $(i_{th} i_a)$, which can be solved by a simple iteration procedure. The transmission ratio in top gear, i_{th}, can then be easily determined if the final drive ratio i_a is given.

1.4.2 First Gear Ratio

The first gear ratio has the largest value and is designed such that the vehicle will be capable of negotiating the theoretical maximum grade that is allowed by the maximum engine output torque. When the vehicle negotiates the maximum grade in the first gear at a constant low speed, the vehicle acceleration is zero and the air drag can be neglected in the calculation of road load. Thus, the road load on the maximum grade at low vehicle speed is approximated as:

$$R = fW \cos\theta_{max} + W \sin\theta_{max} \approx fW + \frac{WG_{max}}{100} \tag{1.34}$$

The equation for the road load above uses the designation for the grade defined in Eq. (1.26). For the vehicle to be able to negotiate the maximum grade, the driving force in the first gear when the engine torque is the maximum must satisfy the following inequality,

$$\frac{T_{emax}(i_{t1} i_a)\eta}{r} \geq fW + \frac{WG_{max}}{100} \tag{1.35}$$

Therefore,

$$(i_{t1} i_a) = \beta \frac{Wr\left(f + \frac{G_{max}}{100}\right)}{\eta T_{emax}} \tag{1.36}$$

where β is a reservation factor that should be larger than one. The engine maximum output torque T_{emax} is obtained directly from the engine WOT output torque plot (Figure 1.3). The powertrain based gradability is usually designed to reach the gradability allowed by the traction limit. That is to say, the value of $\frac{G_{max}}{100}$ in Eq. (1.36) can be determined based the traction condition when the vehicle is running uphill. If rolling resistance and air drag are neglected in calculation, then the traction based gradability for a RWD vehicle is limited by the following inequality:

$$W \sin\theta_{max} \le \mu W_R = \mu \frac{W}{L}(b\cos\theta_{max} + h\sin\theta_{max}) \tag{1.37}$$

where W_R is the real axle weight determined by Eq. (1.5) when the acceleration and air resistance are zero, and μ is the traction coefficient. Solving Eq. (1.37) for $\tan\theta_{max}$ and using the definition for grade percentage in Eq. (1.26), the maximum traction based gradability for a RWD vehicle can be approximated by:

$$G_{max} = 100\tan\theta_{max} \approx (100)\frac{\mu \frac{b}{L}}{1 - \mu \frac{h}{L}} \tag{1.38}$$

Finally, the overall first gear ratio for the RWD vehicle can be found by plugging Eq. (1.38) into Eq. (1.36). The first gear ratio thus determined enables the vehicle with the maximum gradability allowed by the engine torque capacity. Note that the maximum vehicle gradability $\frac{G_{max}}{100}$ is approximately equivalent to a vehicle acceleration of $\frac{G_{max}}{100}g$ on level ground. This can be observed from Eq. (1.24) by dropping the air drag and rolling resistance in the equation. As can be observed in Eq. (1.36), the value of the first gear ratio heavily depends on the vehicle weight and the engine maximum output torque. For vehicles that have a high ratio between total vehicle weight and engine output torque, such as a heavy duty truck, there needs to be a very large first gear ratio for gradability and acceleration capability during launch.

1.4.3 Intermediate Gear Ratios

The low and high gears define the transmission ratio range, and the gap between low and high gears must be bridged by a number of intermediate gears. These ratios affect the engine RPM range under various vehicle operation statuses. Generally, internal combustion engines have a certain RPM range within which the engine output torque is close to the maximum. The lowest specific fuel consumption usually falls within this range. For a given engine, the low and high bounds of this range can be selected from the engine output torque plot. Apparently, for the best results in acceleration performance and fuel efficiency, the intermediate gear ratios should be designed such that the engine RPM is kept within this range while the vehicle operates in different gears. As shown in Figure 1.12, the low bound and high bound of the RPM range are denoted by L and M respectively.

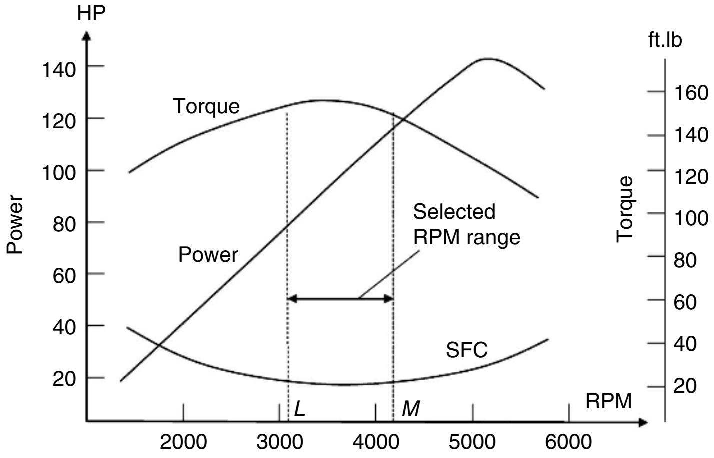

Figure 1.12 Engine RPM range.

The intermediate gear ratios are designed such that the engine RPM is maintained within the interval $[L, M]$ when the transmission shifts gears. Suppose the vehicle is being accelerated in the current gear, or a lower gear. As the vehicle speed increases, the engine speed also increases toward the high bound M. To keep the engine speed within the range, an upshift must be made when the high bound M is reached. Immediately after the upshift, the engine speed should be down to the low bound L, while the vehicle speed remains almost unchanged because the shift only lasts a short time. If the current (lower) overall gear ratio is i_L, then the vehicle speed corresponding to the high bound M is,

$$v = \frac{\pi M}{30 i_L} r \tag{1.39}$$

After the upshift, the engine speed drops to the low bound L, but the vehicle speed remains almost the same and is related to the next (higher) overall gear ratio i_H as:

$$v = \frac{\pi L}{30 i_H} r \tag{1.40}$$

These two equations result in the following relation for the current and the next overall powertrain ratios:

$$i_H = \frac{L}{M} i_L = c i_L \tag{1.41}$$

where c is a constant, since the lower and upper bounds are specified. After the upshift, the vehicle will be driven with the gear ratio i_H. As the vehicle speed increases, the engine speed will reach the high bound M again. Then an upshift is made again to keep the engine RPM within the range. By similar analysis, the gear ratio after the next upshift will be $c i_H$ or $c^2 i_L$. Thus it can be observed by deduction that the gear ratios should form a geometric progression if the engine RPM is to be kept within the speed range $[L, M]$. For example, if designed in a geometric progression, the gear ratios of a five speed manual transmission will be related by the following equation,

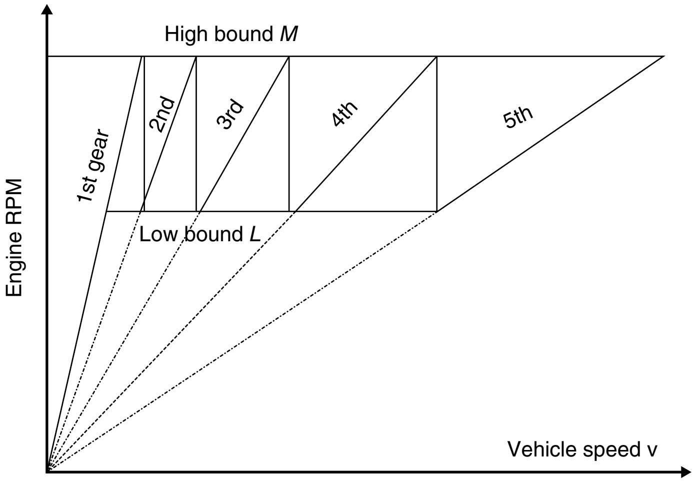

Figure 1.13 Engine RPM vs vehicle speed for gear ratios in geometric progression.

$$\frac{i_5}{i_4} = \frac{i_4}{i_3} = \frac{i_3}{i_2} = \frac{i_2}{i_1} = \frac{L}{M} = c \tag{1.42}$$

If all upshifts are made at the same engine RPM, then the engine RPM will drop by the same amount, as shown in Figure 1.13. Clearly, the engine speed is kept between the low bound and the high bound.

The number of gears needed to match the engine output largely depends on the shape of the engine output torque curve and on the type of vehicle. For an engine with sharply peaked torque curve, the ratio between the low bound and high bound $c = \frac{L}{M}$ is higher than that for an engine with a flatter torque curve. This means that more gear ratios are needed to bridge the gap between the low gear and the high gear. The higher the value of $\frac{L}{M}$, the more the transmission gears needed. In other words, the narrower the RPM range within which the engine is kept to operate, the more gear ratios are needed. If a relatively small engine is used as the power plant for a large vehicle, such as the case for a semi-truck, it is highly desirable to keep the engine RPM nearest to the peak output, thus the ratio $\frac{L}{M}$ is close to one and a large number of gears are required in the transmission.

Summarizing this analysis, the first gear ratio $i_{t1}i_a$ for the whole powertrain is designed for maximum gradability or acceleration, the highest gear ratio $i_{th}i_a$ is designed for the maximum vehicle speed allowed by the maximum engine power, and the intermediate gear ratios fall in a geometric progression such that the engine RPM will be kept within the same range for optimized engine performance in torque and fuel efficiency. The final drive ratio i_a is always a constant and is realized separately from the transmission. Therefore the first gear ratio i_{t1} and the highest gear ratio i_{th} of the transmission become known after the final drive ratio is selected. If the number of gears is pre-determined, then the intermediate gear ratios can be calculated by the following equations:

$$c = \sqrt[(n-1)]{\frac{i_{th}}{i_{t1}}} \tag{1.43}$$

$$i_{t2} = ci_{t1};\ i_{t3} = c^2 i_{t1}\ , \ldots\ldots,\ i_{t(n-1)} = c^{(n-2)} i_{t1} \tag{1.44}$$

where n is the number of gears in the transmission. If the engine RPM range is specified, that is, the value of $c = \frac{L}{M}$ is predetermined, then the number of gears can be calculated firstly by using:

$$n = 1 + \frac{\ln \frac{i_{th}}{i_{t1}}}{\ln c} \tag{1.45}$$

The answer from this equation must be rounded up to the nearest integer. Then Eqs (1.43) and (1.44) are used to calculate all the intermediate gear ratios. It can be observed from Eq. (1.45) that if the first gear ratio i_{t1} is large, as in the case of heavy duty truck, the value of n, i.e. the number of gears, will also be large. Similarly, if c gets closer to one, the number of gears will become larger, as mentioned previously.

1.4.4 Finalization of Gear Ratios

The gear ratio values calculated above may not be the final gear ratios used for the transmission, but these values serve as a good starting point in the transmission ratio selection. After these starting values are calculated, the driving condition diagram and the power–speed chart, Figures 1.8 and 1.10, can then be plotted for the vehicle to judge how good the engine–transmission match is. With the selected gear ratios, the vehicle acceleration performance and fuel economy can also be simulated under various drive ranges. Based on the simulation results, necessary modifications of the gear ratios can be made for priorities in acceleration performance or fuel economy or for the optimized trade-off between the two.

Note that, in addition to the geometric progression method described in this section, transmission gear ratios can also be designed in progressive steps [5]. When transmission gear ratios are in progressive steps, the difference between two adjacent gears becomes smaller toward the high gears. This means that the engine RPM will not drop the same amount during upshifts, as shown in Figure 1.13. Instead, the engine RPM drop decreases for upshifts in higher gears. Gear ratios for passenger car transmissions often have low gear ratios close to a geometric series and high gear ratios with the characteristics of progressive steps.

Example 1.2 A five-speed manual transmission is used for a FWD car with the following data:

Front axle weight: 1820 lb	Rear axle weight: 1700 lb
Center of gravity height: 14 in.	Wheelbase: 105 in.
Air drag coefficient: 0.31	Frontal projected area: 21.50 sq.ft
Tire radius: 12.0 in.	Roll resistance coefficient: 0.02
Powertrain efficiency: 0.94	Road adhesion coefficient: 1.0
Max. power @ 5500 RPM: 198 HP	Max. torque @3850 RPM: 199 ft.lb.

The torque output curve of the engine is given in Figure 1.14. Design the overall powertrain ratio of the highest gear for the highest vehicle speed and the ratio of the lowest gear for the maximum gradability, using a reservation factor $\beta = 1.35$.

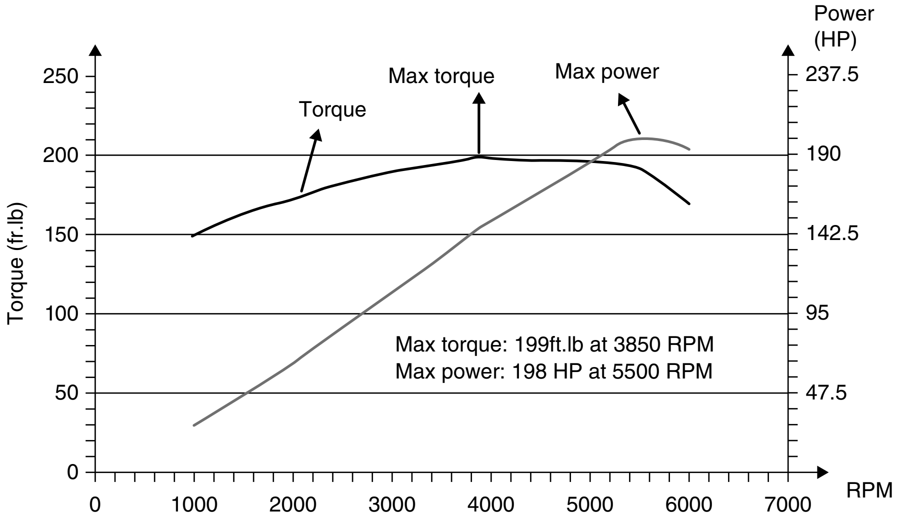

Figure 1.14 Engine WOT output for Example 1.2.

a) Determine all the gear ratios assuming they fall into a geometric progression.
b) Determine the maximum grade the vehicle can negotiate in 4th gear at a constant speed of 65 mph.
c) What will be the maximum vehicle speed if the overall powertrain ratio of 5th gear is designed as 2.15?

Note: Show how the data are taken from the plot.

Solution:

a) Fifth gear ratio:

Plugging the relevant data given in the problem into Eq. (1.33), we can obtain a cubic equation with $(i_a i_5)$ as the unknown:

$$102366(i_a i_5)^3 - 40527.2(i_a i_5)^2 - 1500368.7 = 0$$

Solving the equation above by iteration, $(i_a i_5) = 2.57$

First gear ratio:

For an FWD vehicle, the traction based maximum gradability is determined as:

$$W \sin\theta_{max} \le \mu_o \left(\frac{c}{L} W \cos\theta_{max} - \frac{h}{L} \sin\theta_{max} \right)$$

$$G_{max} = 100 \tan\theta_{max} = 100 \frac{\mu_o \frac{c}{L}}{1 + \mu_o \frac{h}{L}} = 100 \frac{1.0\left(\frac{1820}{3520}\right)}{1 + 1.0\left(\frac{14}{105}\right)} = 45.6$$

$$(i_a i_1) = \beta \left[\frac{Wr\left(f + \frac{G_{max}}{100}\right)}{\eta T_{emax}} \right] = 1.35 \left[\frac{0.02(3520) + 0.456(3520)}{199(.94)} \right] = 12.10$$

Intermediate gear ratios:

Since the gear ratios are in a geometric series, $i_5 = c^4 i_1$, so, $c = \sqrt[4]{\frac{2.57}{12.10}} = 0.679$, the other ratios are:

$$i_a i_2 = 0.679(12.10) = 8.22$$
$$i_a i_3 = 0.679(8.22) = 5.58$$
$$i_a i_4 = 0.679(5.58) = 3.79$$

b) Maximum grade:

$$v = 65\,\text{mph} = 1.467(65) = 95.36\,\text{ft/s}$$
$$R = 0.02 + (3520) + 0.00118(.31)(21.5)(95.36)^2 + \sin\theta = 140.25 + 3520\sin\theta$$
$$\omega_e = i_4 i_a \frac{v}{r} = 3.79\left(\frac{95.36}{1.0}\right) = 361.41\,\text{rad/s} = 3451\,\text{RPM}$$

From the torque plot, the engine torque T_e is found to be 192 ft.lb at 3451 RPM, so

$$P = \frac{0.94(3.79)(192)}{1.0} = 684\,\text{lb}$$

At maximum grade, traction and road load are balanced, so

$$140.25 + 3520\sin\theta_{max} = 684$$
$$G_{max} \approx 100\sin\theta_{max} = 15.2$$

c) When the vehicle reaches its maximum speed, the available power from the power-train is fully balanced by road load power, i.e. $Rv_{max} = \cong \eta(Power)$ or,

$$v_{max}\left[\left(0.02(3520) + 0.00118(0.31)(21.5)v_{max}^2\right) = 0.94(\text{Power})\right.$$
$$74.89v_{max} + 0.008367v_{max}^3 = Power$$

Note that *Power* is a function of engine RPM. The solution for v_{max} is based on iteration.

First iteration: assuming $v_{max} = \dfrac{3.14(5500)(1.0)}{30(2.57)} = 223.99\,\text{ft/s}$, then,

$$\text{Road load power}: 74.89(223.99) + 0.008367\left(223.99^3\right) = 110802\,\text{ft.lb/s} = 201\,\text{HP}$$
$$\text{RPM} = 2.15\left(\frac{30}{\pi}\right)\frac{v}{r} = 2.15\left(\frac{30}{\pi}\right)\left(\frac{223.99}{1.0}\right) = 4598.7$$

At 4598 RPM, the engine power is about 165 HP and is below 201 HP. So $v_{max} < 223.99\,\text{ft/s}$

Second iteration: assuming $v_{max} = 190\,\text{ft/s}$, then

Road load power: $74.87(190) + 0.008367(190)^3 = 129\,\text{HP}$

$$\text{RPM} = 2.15\left(\frac{190}{1.0}\right)\frac{30}{\pi} = 3900$$

At 3900 RPM, the engine power is 152 HP and is higher than 129 HP. So $v_{max} > 190\,\text{ft/s}$

Third iteration: assuming $v_{max} = 210\,\text{ft/s}$, then

Road load power: $74.89(210) + 0.008367(210)^3 = 168\,\text{HP}$

$$\text{RPM} = 2.15\left(\frac{210}{1.0}\right)\frac{30}{\pi} = 4311$$

At 4311 RPM, the engine power is about 162 HP, so $v_{max} \approx 210\,\text{ft/s} = 143\,\text{mph}$

References

1 Stone, R.: *Introduction to Internal Combustion Engines*, Second Edition, Society of Automotive Engineers, 1993. ISBN 1-56091-390-8.

2 Taylor, C.F.: *The Internal Combustion Engine in Theory and Practice, Vol. 1: Thermodynamics, Fluid Flow Performance*, Second Edition, MIT Press, 1985. ISBN 13:978-0262700269.

3 Taylor, C.F.: *The Internal Combustion Engine in Theory and Practice, Vol. 2: Combustion, Fuels Design*, Second Edition, MIT Press, 1985. ISBN 13:978-0262700276.

4 Isermann, R.: *Engine Modelling and Control*, Springer-Verlag, 2014, ISBN: 978-3-642-39934-3.

5 Lechner, G.; Naunheimer, H.: *Automotive Transmissions: Fundamentals, Selection, Design and Application*, Springer, 1999. ISBN 3-540-65903.

6 Gillespie, T.D.: *Fundamental Vehicle Dynamics*, Society of Automotive Engineers, 1992. ISBN 978-1-56091-199-9.

Problem

A FWD vehicle has the following data:

Front axle weight: 1750 lb	Rear axle weight: 1200 lb
Center of gravity height: 15 in.	Wheelbase: 105 in.
Air drag coefficient: 0.30	Frontal projected area: 22.0 square feet
Tire radius: 11.40 in.	Roll resistance coefficient: 0.02
Max. power @6000 RPM: 138 HP	Max. torque @4500 RPM: 132 ft.lb
Powertrain efficiency: 0.96	Final drive ratio: 3.143

A six-speed manual transmission is used for the vehicle and the gear ratios from 1st to 4th gears are: 1st gear (3.92), 2nd gear (2.76), 3rd gear (1.85), 4th gear (1.35). The engine WOT output plot is as given in Figure 1.3.

a) The vehicle runs in the 5th gear at a speed of 55 mph with the engine speed at 2450 RPM. The driver then makes a 5–6 upshift and the engine RPM drops by 500 RPM immediately after the shift. Determine the 5th and the 6th gear ratios.

b) Determine the engine torque and work done by the engine when the vehicle cruises for 1.5 miles at a constant speed of 65 mph on level ground in the 6th and 5th gears respectively.
c) The driver floors the gas pedal and simultaneously makes a 6–5 downshift when the vehicle runs on a 3% slope at a speed of 65 mph. Determine the vehicle acceleration immediately after the 6–5 downshift.
d) What is the steepest percentage slope the vehicle can negotiate at a speed of 70 mph?