1

INVESTIGATION OF FORCES, FORCE SYSTEMS, LOADING, AND REACTIONS

Loads deriving from the tasks of a structure produce forces. The tasks of the structure involve the transmission of the load forces to the supports for the structure. The external loads and support forces produce a resistance from the structure in terms of internal forces that resist changes in the shape of the structure. This chapter treats the basic properties and actions of forces.

1.1 PROPERTIES OF FORCES

Force is a fundamental concept of mechanics but does not yield to simple definition. An accepted concept is that a *force* is an effort that tends to change the form or the state of motion of a physical object. Mechanical force was defined by Isaac Newton as being a product of mass and acceleration; that is, $F = ma$. Gravitational attraction is a form of acceleration, and thus weight—a force we experience—is defined as $W = mg$ with g being the acceleration of gravity. Physical objects have weight, but more precisely they have mass, and will thus have *different* weights when they experience different gravitational effects, for example, on the surface of Earth or on the surface of the moon.

In U.S., aka imperial, units gravity force is quantified as the weight of the body. Gravity forces are thus measured in pounds (lb) or in some other unit such as tons (T) or kips (one kilopound, or 1000 pounds). In the SI (International System, aka metric) system force is measured in a more scientific manner related to the mass of objects, the mass of an object being a constant, whereas weight is proportional to the precise value of the acceleration of gravity, which varies from place to place. Force in metric units is measured in newtons (N) or in kilonewtons (kN) or in meganewtons (mN), whereas weight is measured in grams (g) or in kilograms (kg).

In structural engineering work, forces are described as *loads*. Loads derive from various sources, including gravity, and are dealt with in terms of their application to a given structure. Thus, the gravity load on a beam begins with the weight of the beam itself and goes on to include the weight of everything else supported by the beam.

Vectors

A quantity that involves magnitude, line of action (e.g., vertical), and sense (up, down, etc.) is a vector quantity, whereas a scalar quantity involves only magnitude and sense. Force, velocity, and acceleration are vector quantities, while energy, time, and temperature are scalar quantities. A vector can be represented by a straight line, leading to the possibility of constructed graphical solutions in some cases, a situation that will be demonstrated later. Mathematically, a scalar quantity can be represented completely as $+50$ or -50, while a vector must somehow have its line of action represented as well (50 vertical, horizontal, etc.).

Properties of Forces

In order to completely identify a force, it is necessary to establish the following:

Magnitude, or the amount of the force, measured in weight units such as pounds or kips.

Line of Action of the force, which refers to the orientation of its path, usually described by the angle that the line of action makes with some reference, such as the horizontal.

Sense of the force, which refers to the manner in which it acts along its line of action (e.g., up or down, right or left). Sense is usually expressed algebraically in terms of the sign of the force, either plus or minus.

Forces can be represented graphically in terms of these three properties by the use of an arrow, as shown in Figure 1.1*a*. Drawn to some scale, the length of the arrow represents the magnitude of the force.

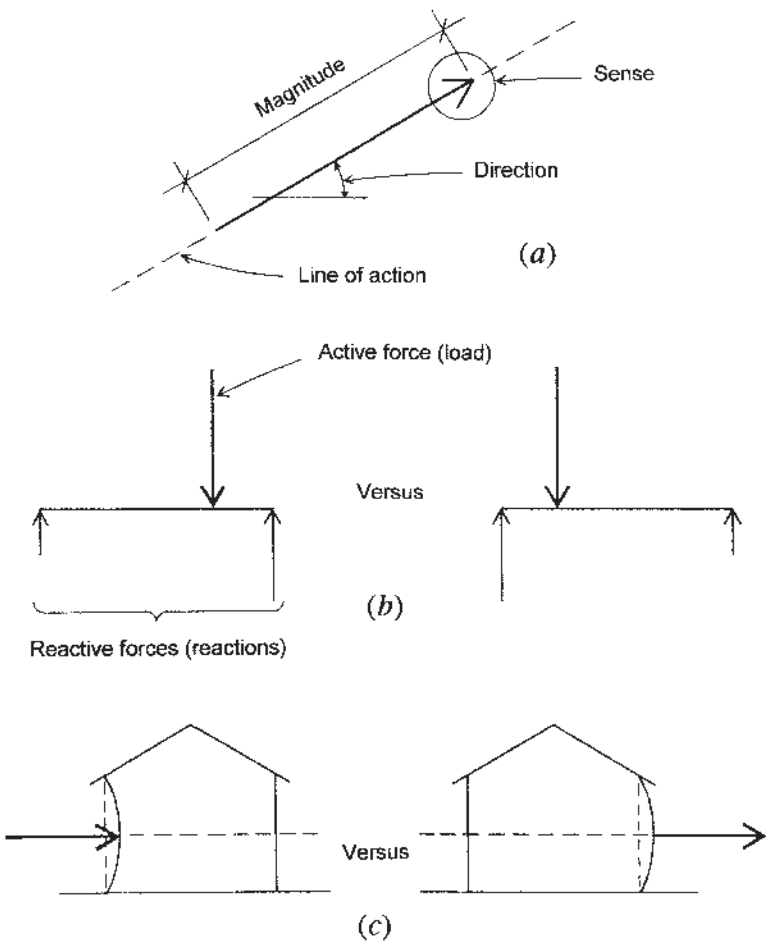

Figure 1.1 Representation of forces and force actions.

The angle of inclination of the arrow represents the direction of the force. The location of the arrowhead represents the sense of the force. This form of representation can be more than merely symbolic, since actual mathematical manipulations may be performed using the vector representation that the force arrows constitute. In the work in this book arrows are used in a symbolic way for visual reference when performing algebraic computations and in a truly representative way when performing graphical analyses.

In addition to the basic properties of magnitude, line of action, and sense, some other concerns that may be significant for certain investigations are:

The *position of the line of action* of the force with respect to the lines of action of other forces or to some object on which the force operates, as shown in Figure 1.1*b*. For the beam, shifting of the location of the load (active force) affects changes in the forces at the supports (reactions).

The *point of application* of the force along its line of action may be of concern in analyzing for the specific effect of the force on a structure, as shown in Figure 1.1*c*.

When forces are not resisted, they tend to produce motion. An inherent aspect of static forces is that they exist in a state of *static equilibrium*, that is, with no motion occurring. In order for static equilibrium to exist, it is necessary to have a balanced system of forces. An important consideration in the analysis of static forces is the nature of the geometric arrangement of forces in a given set of forces that constitute a single system. The usual technique for classifying force systems involves consideration of whether the forces in the system are:

Coplanar. All acting in a single plane, such as the plane of a vertical wall.

Parallel. All having the same direction though not along the same line of action.

Concurrent. All having their lines of action intersect at a common point.

Using these three considerations, the possible variations are given in Table 1.1 and illustrated in Figure 1.2.

TABLE 1.1 Classification of Force Systems[a]

	Qualifications		
System Variation	Coplanar	Parallel	Concurrent
1	Yes	Yes	Yes
2	Yes	Yes	No
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	No
6	No	No	Yes
7	No	No	No

[a]See Figure 1.2.

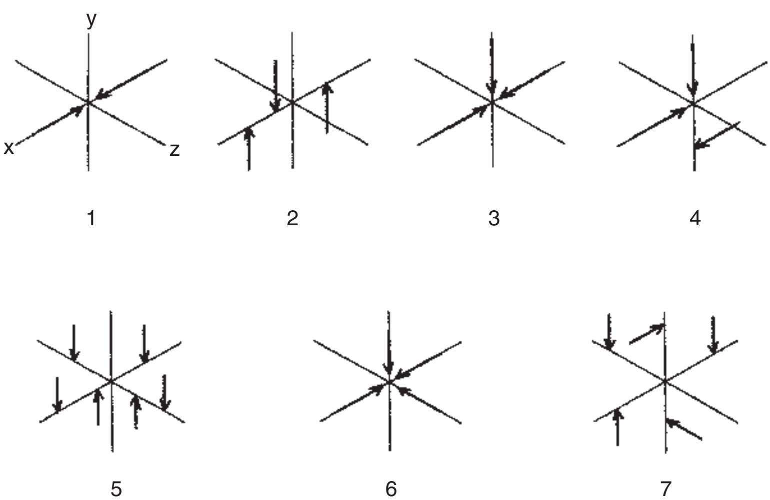

Figure 1.2 Types of force systems.

It is necessary to qualify a set of forces in the manner just illustrated before proceeding with any analysis, whether it is to be performed algebraically or graphically.

1.2 STATIC EQUILIBRIUM

As stated previously, an object is in *equilibrium* when it either is at rest or has uniform motion. When a system of forces acting on an

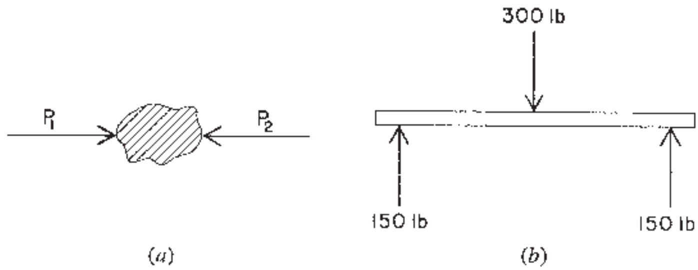

Figure 1.3 Equilibrium of forces.

object produces no motion, the system of forces is said to be in *static equilibrium*.

A simple example of equilibrium is illustrated in Figure 1.3*a*. Two equal, opposite, and parallel forces, having the same line of action, P_1 and P_2, act on a body. If the two forces balance each other, the body does not move and the system of forces is in equilibrium. These two forces are *concurrent*. If the lines of action of a system of forces have a point in common, the forces are concurrent.

Another example of forces in equilibrium is illustrated in Figure 1.3*b*. A vertical downward force of 300 lb acts at the midpoint in the length of a beam. The two upward vertical forces of 150 lb each (the reactions) act at the ends of the beam. The system of three forces is in equilibrium. The forces are parallel and, not having a point in common, are *nonconcurrent*.

1.3 FORCE COMPONENTS AND COMBINATIONS

Individual forces may interact and be combined with other forces in various situations. Conversely, a single force may have more than one effect on an object, such as a vertical action and a horizontal action simultaneously. This section considers both of these issues: adding up of forces (combination) and breaking down of single forces into components (resolution).

Resultant of Forces

The *resultant* of a system of forces is the simplest system (usually a single force) that has the same effect as the various forces in the system acting simultaneously. The lines of action of any system of two coplanar nonparallel forces must have a point in common, and the resultant of the two forces will pass through this common point. The resultant of two coplanar, nonparallel forces may be found graphically by constructing a *parallelogram of forces*.

To construct a parallelogram of two forces, the forces are drawn at any scale (so many pounds to the inch) with both forces pointing toward or both forces pointing away from the point of intersection of their lines of action. A parallelogram is then produced with the two forces as adjacent sides. The diagonal of the parallelogram passing through the common point is the resultant in magnitude, line of action, and sense, the direction of the resultant being similar to that of the given forces, toward or away from the point in common. In Figure 1.4*a*, P_1 and P_2 represent two nonparallel forces whose lines of action intersect at point O. The parallelogram is drawn, and the diagonal R is the resultant of the given system. In this illustration note that the two forces point *away* from the point in common, and hence the resultant also has its sense away from point O. It is a force upward to the right. Notice that the resultant of forces P_1 and P_2 shown in Figure 1.4*b* is R; its sense is toward the point in common.

Forces may be considered to act at any points on their lines of action. In Figure 1.4*c* the lines of action of the two forces P_1 and

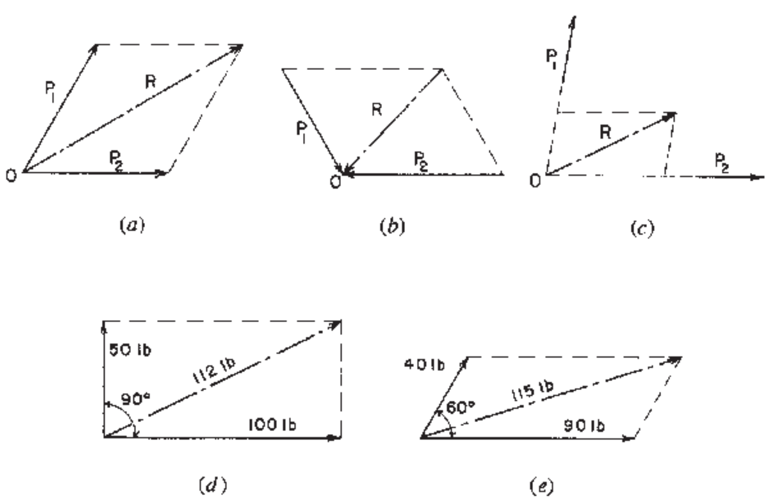

Figure 1.4 Consideration of the resultant of a set of forces.

P_2 are extended until they meet at point O. At this point the parallelogram of forces is constructed, and R, the diagonal, is the resultant of forces P_1 and P_2. In determining the magnitude of the resultant, the scale used is the same scale used in drawing the given system of forces.

Example 1. A vertical force of 50 lb and a horizontal force of 100 lb, as shown in Figure 1.4*d*, have an angle of 90° between their lines of action. Determine the resultant.

Solution: The two forces are laid off at a convenient scale from their point of intersection, the parallelogram is drawn, and the diagonal is the resultant. Its magnitude scales approximately 112 lb, its sense is upward to the right, and its line of action passes through the point of intersection of the lines of action of the two given forces. By use of a protractor it is found that the angle between the resultant and the force of 100 lb is approximately 26.5°.

Example 2. The angle between two forces of 40 and 90 lb, as shown in Figure 1.4*e*, is 60°. Determine the resultant.

Solution: The forces are laid off to scale from their point of intersection, the parallelogram of forces is constructed, and the resultant is found to be a force of approximately 115 lb, its sense is upward to the right, and its line of action passes through the common point of the two given forces. The angle between the resultant and the force of 90 lb is approximately 17.5°.

Attention is called to the fact that these two problems have been solved graphically by the construction of diagrams. Mathematics might have been employed. For many practical problems, carefully constructed graphical solutions give sufficiently accurate answers and frequently require far less time. Do not make diagrams too small as greater accuracy is obtained by using larger parallelograms of forces.

Problems 1.3.A–F By constructing the parallelogram of forces, determine the resultants for the pairs of forces shown in Figures 1.5*a–f*.

Components of a Force

In addition to combining forces to obtain their resultant, it is often helpful to replace a single force by its *components*. The components of

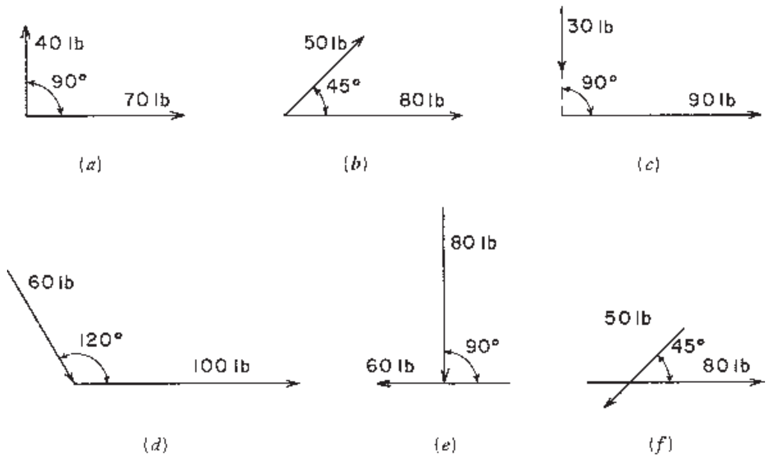

Figure 1.5 Reference for Problem 1.3, Part 1.

a force are the two or more forces that, acting together, have the same effect as the given force. In Figure 1.4*d*, if we are given the force of 112 lb, its vertical component is 50 lb and its horizontal component is 100 lb. That is, the 112-lb force has been *resolved* into its vertical and horizontal components. Any force may be considered as the resultant of its components.

Combined Resultants

The resultant of more than two nonparallel forces may be obtained by finding the resultants of pairs of forces and finally the resultant of the resultants.

Example 3. Find the resultant of the concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.6.

Solution: By constructing a parallelogram of forces, the resultant of P_1 and P_2 is found to be R_1. Similarly, the resultant of P_3 and P_4 is R_2. Finally, the resultant of R_1 and R_2 is R, the resultant of the four given forces.

Problems 1.3.G–I Using graphical methods, find the resultant of the systems of concurrent forces shown in Figure 1.7.

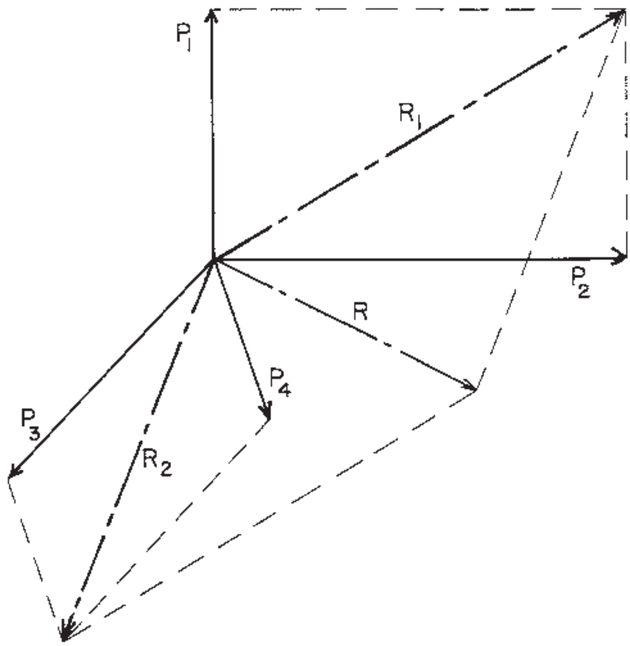

Figure 1.6 Finding a resultant by successive pairs.

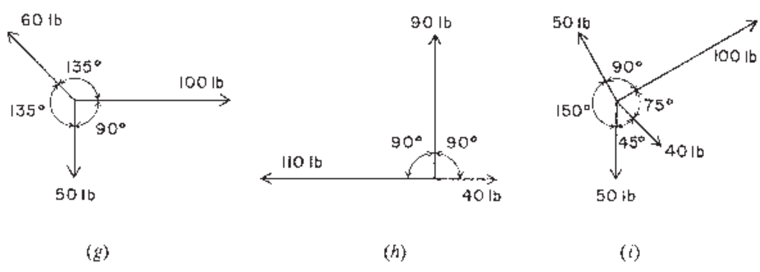

Figure 1.7 Reference for Problem 1.3, Part 2.

Equilibrant

The force, not a part of the system of forces, required to maintain a system of forces in equilibrium is called the *equilibrant* of the system. Suppose that we are required to investigate the system of two forces,

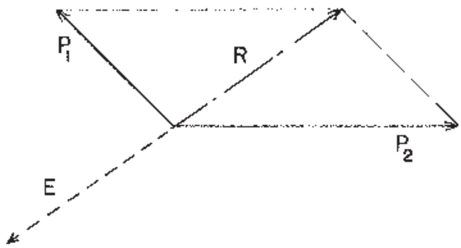

Figure 1.8 Finding an equilibrant.

P_1 and P_2, as shown in Figure 1.8. The parallelogram of forces is constructed, and the resultant is found to be R. The system is not in equilibrium. The force required to maintain equilibrium is force E, shown by the dashed line. The equilibrant, E, is the same as the resultant in magnitude and line of action but is opposite in sense. The three forces, P_1 and P_2 and E, constitute a system in equilibrium.

If two forces are in equilibrium, they must be equal in magnitude and opposite in sense and have the same direction and line of action. Either of the two forces may be said to be the equilibrant of the other. The resultant of a system of forces in equilibrium is zero.

1.4 GRAPHICAL ANALYSIS OF CONCURRENT FORCE SYSTEMS

Force Polygon

The resultant of a system of concurrent forces may be found by constructing a *force polygon*. To draw the force polygon, begin with a point and lay off, at a convenient scale, a line parallel to one of the forces, with its length equal to the force in magnitude, and having the same sense. From the termination of this line draw similarly another line corresponding to one of the remaining forces and continue in the same manner until all the forces in the given system are accounted for. If the polygon does not close, the system of forces is not in equilibrium, and the line required to close the polygon *drawn from the starting point* is the resultant in magnitude and direction. If the forces in the given system are concurrent, the line of action of the resultant passes through the point they have in common.

If the force polygon for a system of concurrent forces closes, the system is in equilibrium and the resultant is zero.

Example 4. Let it be required to find the resultant of the four concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.9*a*. This diagram is called the *space diagram*; it shows the relative positions of the forces in a given system.

Solution: Beginning with some point such as O, shown in Figure 1.9*b*, draw the upward force P_1. At the upper extremity of the line representing P_1, draw P_2, continuing in a like manner with P_3 and P_4. The polygon does not close; therefore, the system is not in equilibrium. The resultant R, shown by the dot-and-dash line, is the resultant of the given system. Note that its sense is *from* the starting point O, downward to the right. The line of action of the resultant of the given system shown in Figure 1.9*a* has its line of action passing through the point they have in common, its magnitude and direction having been found in the force polygon.

In drawing the force polygon, the forces may be taken in any sequence. In Figure 1.9*c* a different sequence is taken, but the resultant R is found to have the same magnitude and direction as previously found in Figure 1.9*b*.

Bow's Notation

Thus far, forces have been identified by the symbols P_1, P_2, and so on. A system of identifying forces, known as Bow's notation, affords many advantages. In this system letters are placed in the

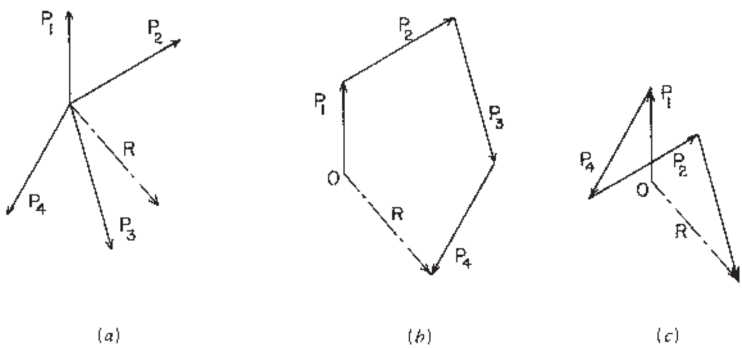

Figure 1.9 Finding a resultant by continuous vector addition of forces.

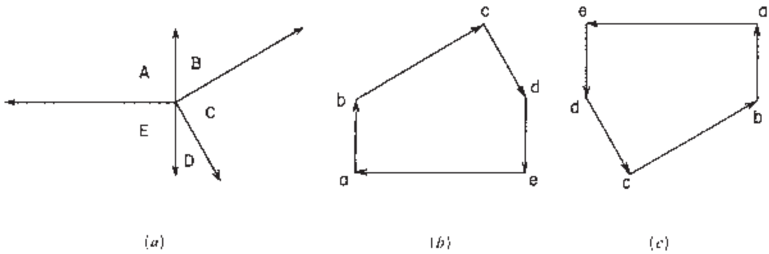

Figure 1.10 Construction of a force polygon.

space diagram on each side of a force and a force is identified by two letters. The sequence in which the letters are read is important. Figure 1.10*a* shows the space diagram of five concurrent forces. Reading about the point in common *in a clockwise manner* the forces are *AB, BC, CD, DE*, and *EA*. When a force in the force polygon is represented by a line, a letter is placed at each end of the line. As an example, the vertical upward force in Figure 1.10*a* is read *AB* (note that this is read clockwise about the common point); in the force polygon (Figure 1.10*b*) the letter *a* is placed at the bottom of the line representing the force *AB* and the letter *b* is at the top. Use capital letters to identify the forces in the space diagrams and lowercase letters in the force polygon. From point *b* in the force polygon draw force *bc*, then *cd*, and continue with *de* and *ea*. Since the force polygon closes, the five concurrent forces are in equilibrium.

In reading forces, a clockwise manner is used in all the following discussions. It is important that this method of identifying forces be thoroughly understood. To make this clear, suppose that a force polygon is drawn for the five forces shown in Figure 1.10*a*, reading the forces in sequence in a counterclockwise manner. This will produce the force polygon shown in Figure 1.10*c*. Either method may be used, but for consistency the method of reading clockwise is used here.

Use of the Force Polygon

Two ropes are attached to a ceiling and their lower ends are connected to a ring, making the arrangement shown in Figure 1.11*a*. A weight of 100 lb is suspended from the ring. Obviously, the force in the rope *AB* is 100 lb, but the magnitudes of the forces in ropes *BC* and *CA* are unknown.

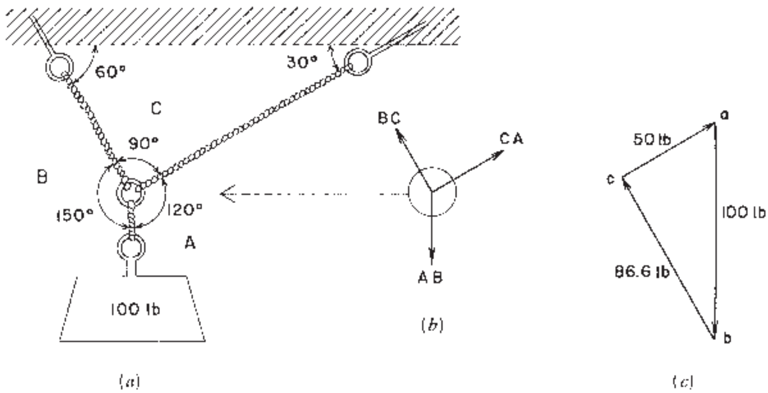

Figure 1.11 Solution of a concentric force system.

The forces in the ropes *AB*, *BC*, and *CA* constitute a concurrent force system in equilibrium (Figure 1.11*b*). The magnitude of only one of the forces is known—it is 100 lb in rope *AB*. Since the three concurrent forces are in equilibrium, their force polygon must close, and this fact makes it possible to find their magnitudes. Now, at a convenient scale, draw the line *ab* (Figure 1.11*c*) representing the downward force *AB*, 100 lb. The line *ab* is one side of the force polygon. From point *b* draw a line parallel to rope *BC*; point *c* will be at some location on this line. Next, draw a line through point *a* parallel to rope *CA*; point *c* will be at some position on this line. Since point *c* is also on the line through *b* parallel to *BC*, the intersection of the two lines determines point *c*. The force polygon for the three forces is now completed; it is *abca*, and the lengths of the sides of the polygon represent the magnitudes of the forces in ropes *BC* and *CA*, 86.6 and 50 lb, respectively.

Particular attention is called to the fact that the lengths of the ropes in Figure 1.11*a* are not an indication of the magnitudes of the forces within the ropes; the magnitudes are determined by the lengths of the corresponding sides of the force polygon (Figure 1.11*c*). Figure 1.11*a* merely determines the geometric layout for the structure.

Problems 1.4.A–D Find the sense (tension or compression) and magnitude of the internal force in the member indicated in Figure 1.12 using graphical methods.

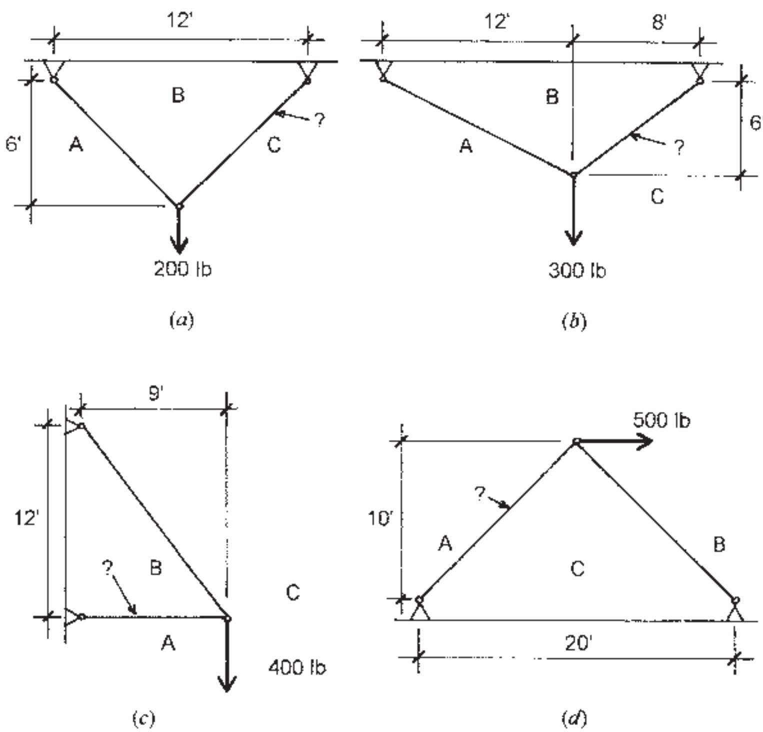

Figure 1.12 Reference for Problem 1.4.

1.5 ALGEBRAIC ANALYSIS OF NONCONCURRENT FORCE SYSTEMS

The simplest analysis for nonconcurrent force systems is not graphical, as it was for concurrent force systems, but rather algebraic. Usually, the forces in the system are resolved into vertical and horizontal components, and the components are added; the result is the components of the resultant, which can be used to determine its actual magnitude, its line of action, and its sense.

Example 5. Find the resultant of the two forces in Figure 1.13*a*.

Solution: The two forces are parallel, so they can simply be added to find the resultant, with a magnitude of 15 lb, a line of action parallel to the forces, and a sense the same as the forces, as shown in

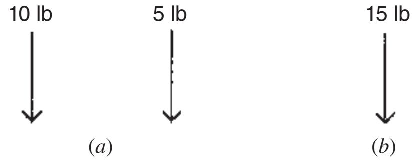

Figure 1.13 Reference for Example 5.

Figure 1.13*b*. Consideration for the location of the lines of action of the forces and their resultant requires something more than simple force addition; this is discussed in the following section.

Moments

The term moment of a force is commonly used in engineering problems. It is fairly easy to visualize a length of 3 ft, an area of 26 in.^{2}, or a force of 100 lb. A moment, however, is less easily understood; it is a force multiplied by a distance. A moment is the *tendency of a force to cause rotation about a given point or axis*. The magnitude of the moment of a force about a given point is the magnitude of the force (pounds, kips, etc.) multiplied by the distance (feet, inches, etc.) from the force to the point of rotation. The point is called the center of moments, and the distance, which is called the *lever arm* or *moment arm*, is measured by a line drawn through the center of moments perpendicular to the line of action of the force. Moments are expressed in compound units such as foot-pounds and inch-pounds or kip-feet and kip-inches. In summary,

$$\text{Moment of force} = \text{magnitude of force} \times \text{moment arm}$$

Consider the horizontal force of 100 lb shown in Figure 1.14*a*. If point *A* is the center of moments, the lever arm of the force is 5 ft. Then the moment of the 100-lb force with respect to point *A* is $100 \times 5 = 500$ ft-lb. In this illustration the force tends to cause a *clockwise* rotation (shown by the dashed-line arrow) about point *A* and is called a positive moment. If point *B* is the center of moments, the moment arm of the force is 3 ft. Therefore, the moment of the 100-lb force about point *B* is $100 \times 3 = 300$ ft-lb. With respect to point *B*, the force tends to cause *counterclockwise* rotation; it is called a negative moment. It is important to remember that you can never consider the moment of a force without having in mind the particular point or axis about which it tends to cause rotation.

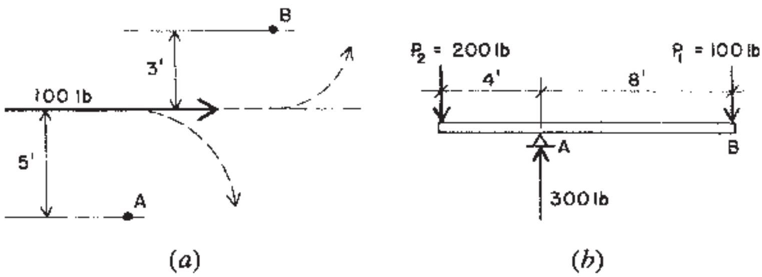

Figure 1.14 Development of moments.

Figure 1.14*b* represents two forces acting on a bar that is supported at point *A*. The moment of force P_1 about point *A* is $100 \times 8 = 800$ ft-lb, and it is clockwise or positive. The moment of force P_2 about point *A* is $200 \times 4 = 800$ ft-lb. The two moment values are the same, but P_2 tends to produce a counterclockwise, or negative, moment about point *A*. The positive and negative moments are equal in magnitude and are in *equilibrium*; that is, there is no motion. Another way of stating this is to say that the sum of the positive and negative moments about point *A* is zero, or

$$\sum M_A = 0$$

Stated more generally, if a system of forces is in equilibrium, the algebraic sum of the moments is zero. This is one of the laws of equilibrium.

In Figure 1.14*b* point *A* was taken as the center of the moments, but the fundamental law holds for any point that might be selected. For example, if point *B* is taken as the center of moments, the moment of the upward supporting force of 300 lb acting at *A* is clockwise (positive) and that of P_2 is counterclockwise (negative). Then

$$(300 \times 8) - (200 \times 12) = 2400 - 2400 = 0$$

Note that the moment of force P_1 about point *B* is $100 \times 0 = 0$; it is therefore omitted in writing the equation. The reader should be satisfied that the sum of the moments is zero also when the center of moments is taken at the left end of the bar under the point of application of P_2.

Example 6. Find the location of the line of action of the resultant for the forces in Figure 1.13 if the two forces are 30 ft apart.

Solution: In Example 5 the magnitude of the resultant was determined to be 15 lb, but the location of its line of action was not determined. Solution of this problem requires the use of moments of the forces. A procedure for this solution is as follows.

Consider the layout of the forces and their resultant as shown in Figure 1.15. A condition for the resultant is that it must be capable of completely replacing the forces; this is true for force summation and also for any moment summation. To use this relationship, we consider an arbitrary reference point (*P* in Figure 1.15). Moments of the forces about this point are

$$M_p = (10 \text{ lb} \times 5 \text{ ft}) + (5 \text{ lb} \times 35 \text{ ft})$$
$$= 50 \text{ ft-lb} + 175 \text{ ft-lb} = 225 \text{ ft-lb}$$

As shown in Figure 1.15, the distance of the resultant from *P* is defined as x. Its moment about *P* is thus Rx, and equating this to the moment of the forces we determine

$$x = \frac{M_P}{R} = \frac{225}{15} = 15 \text{ ft}$$

As stated, the location of *P* is arbitrary, but the true location of the line of action of the resultant is a constant. The reader should try a different location for *P* to verify this.

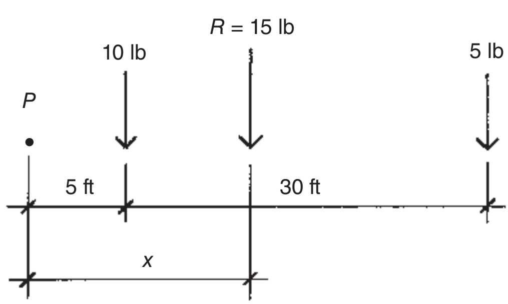

Figure 1.15 Reference for Example 6.

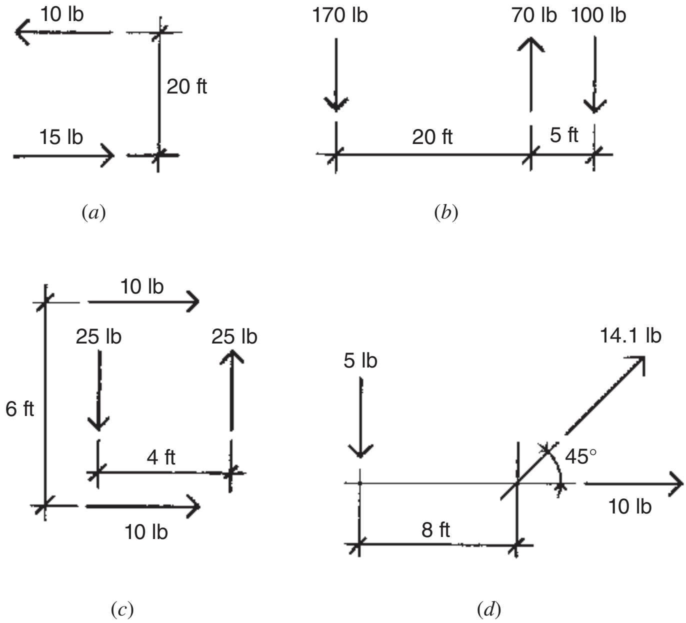

Figure 1.16 Reference for Problem 1.5.

Problems 1.5.A–D Find the resultant for the force systems in Figure 1.16. Find the magnitude, line of action, and sense of the resultant.

1.6 LAWS OF EQUILIBRIUM

When an object is acted on by a number of forces, each force tends to move the object. If the forces are of such magnitude and position that their combined effect produces no motion of the object, the forces are said to be in *equilibrium* (Section 1.2). The three fundamental laws of static equilibrium for a general set of coplanar forces are:

1. The algebraic sum of all the vertical forces equals zero.
2. The algebraic sum of all the horizontal forces equals zero.
3. The algebraic sum of the moments of all the forces about any point equals zero.

These laws, sometimes called the conditions for equilibrium, may be expressed as follows (the symbol $\sum$ indicates a summation, i.e., an algebraic addition of all similar terms involved in the problem):

$$\sum V = 0 \qquad \sum H = 0 \qquad \sum M = 0$$

The law of moments, $\sum M = 0$, was presented in the preceding discussion.

The expression $\sum V = 0$ is another way of saying that *the sum of the downward forces equals the sum of the upward forces*. Thus, the bar of Figure 1.14*b* satisfies $\sum \mathrm{V} = 0$ because the upward force of 300 lb equals the sum of P_1 and P_2.

Moments of Forces

Figure 1.17*a* shows two downward forces of 100 and 200 lb acting on a beam. The beam has a length of 8 ft between the supports; the supporting forces, which are called reactions, are 175 and 125 lb. The four forces are parallel and for equilibrium, therefore, the two laws, $\sum V = 0$ and $\sum M = 0$, apply.

First, because the forces are in equilibrium, the sum of the downward forces must equal the sum of the upward forces. The sum of the downward forces, the loads, is $100 + 200 = 300$ lb; the sum of the upward forces, the reactions, is $175 + 125 = 300$ lb. Thus, the force summation is zero.

Second, because the forces are in equilibrium, the sum of the moments of the forces tending to cause clockwise rotation (positive moments) must equal the sum of the moments of the forces tending to produce counterclockwise rotation (negative moments) about any center of moments. Considering an equation of moments about point A at the right-hand support, the force tending to cause clockwise rotation (shown by the curved arrow) about this point is 175 lb; its moment is $175 \times 8 = 1400$ ft-lb. The forces tending to cause counterclockwise rotation about the same point are 100 and 200 lb, and their moments are 100×6 and 200×4 ft-lb. Thus, if $\sum M_A = 0$, then

$$175 \times 8 = (100 \times 6) + (200 \times 4)$$

$$1400 = 600 + 800$$

$$1400 \text{ ft-lb} = 1400 \text{ ft-lb}$$

which is true.

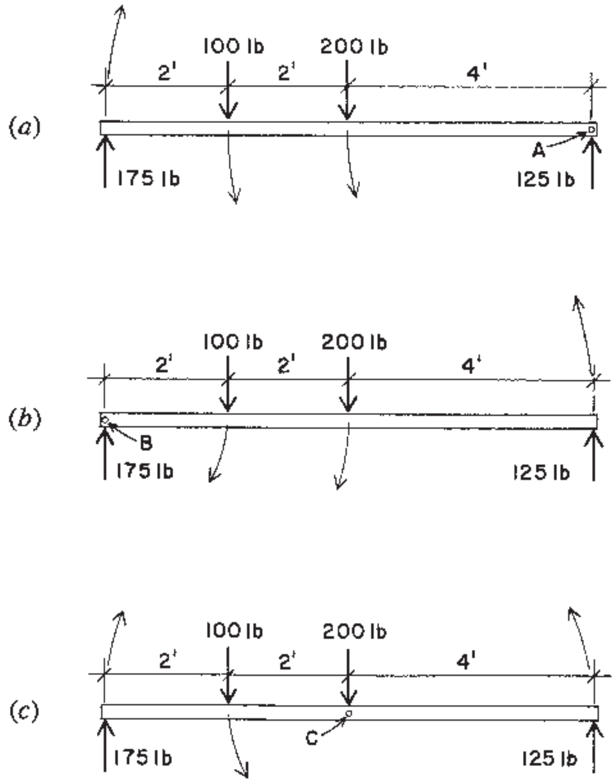

Figure 1.17 Summation of moments about selected points.

The upward force of 125 lb is omitted from the above equation because its lever arm about point A is 0 ft, and consequently its moment is zero. A force passing through the center of moments does not cause rotation about that point.

Now select point B at the left support as the center of moments (see Figure 1.17*b*). By the same reasoning, if $\sum M_B = 0$, then

$$(100 \times 2) + (200 \times 4) = 125 \times 8$$

$$200 + 800 = 1000$$

$$1000 \text{ ft-lb} = 1000 \text{ ft-lb}$$

Again the law holds. In this case, the force of 175 lb has a lever arm of 0 ft about the center of moments and its moment is zero.

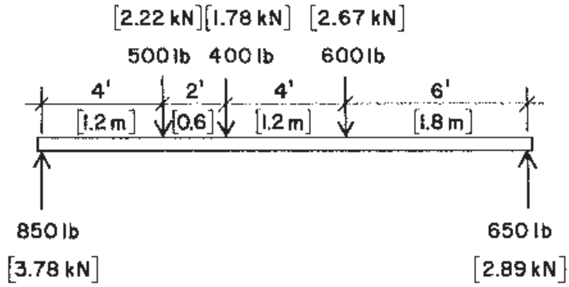

Figure 1.18 Reference for Problem 1.6.A.

The reader should verify this case by selecting any other point, such as point C in Figure 1.17*c* as the center of moments, and confirming that the sum of the moments is zero for this point.

Problem 1.6.A Figure 1.18 represents a beam in equilibrium with three loads and two reactions. Select five different centers of moments and write the equation of moments for each, showing that the sum of the clockwise moments equals the sum of the counterclockwise moments.

1.7 LOADS AND REACTIVE FORCES

Structural members, such as beams, are acted on by external forces that consist of the loads and the reaction forces developed by the beam's supports. The two types of loads that commonly occur on beams are called *concentrated* and *distributed*. A concentrated load is assumed to act at a definite point; such a load is that caused when one beam supports another beam. A distributed load is one that acts over a considerable length of the beam; such a load is caused by a floor deck supported directly by a beam. If the distributed load exerts a force of equal magnitude for each unit of length of the beam, it is known as a *uniformly distributed load*. The weight of a beam is a uniformly distributed load that extends over the entire length of the beam. However, some uniformly distributed loadings supported by the beam may extend over only a portion of the beam length.

Reactive Forces

Reactive forces are the upward forces acting at the supports that hold in equilibrium the downward forces or loads. Reactive forces are often

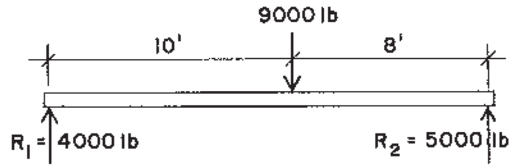

Figure 1.19 Beam reactions for a single load.

refered to as "reactions." The left and right reactions of a simple beam are usually called R_1 and R_2, respectively. Determination of reactions for simple beams is achieved with the use of equilibrium conditions for parallel force systems.

If a beam 18 ft long has a concentrated load of 9000 lb located 9 ft from the supports, it is readily seen that each upward force at the supports will be equal and will be one-half the load in magnitude, or 4500 lb. But consider, for instance, the 9000-lb load placed 10 ft from one end, as shown in Figure 1.19. What will the upward supporting forces be? This is where the principle of moments can be used. Consider a summation of moments about the right-hand support R_2. Thus,

$$\sum M = 0 = +(R_1 \times 18) - (9000 \times 8) + (R_2 \times 0)$$

$$R_1 = \frac{72{,}000}{18} = 4000 \text{ lb}$$

Then, considering the equilibrium of vertical forces,

$$\sum V = 0 = +R_1 + R_2 - 9000$$

$$R_2 = 9000 - 4000 = 5000 \text{ lb}$$

The accuracy of this solution can be verified by taking moments about the left-hand support. Thus,

$$\sum M = 0 = -(R_2 \times 18) + (9000 \times 10) + (R_1 \times 0)$$

$$R_2 = \frac{90{,}000}{18} = 5000 \text{ lb}$$

Example 7. A simple beam 20 ft long has three concentrated loads, as indicated in Figure 1.20. Find the magnitudes of the reactions.

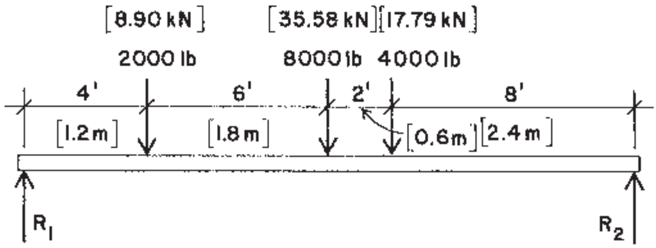

Figure 1.20 Reference for Example 7.

Solution: Using the right-hand support as the center of moments,

$$\sum M = +(R_1 \times 20) - (2000 \times 16) - (8000 \times 10) - (4000 \times 8)$$

from which

$$R_1 = \frac{32{,}000 + 80{,}000 + 32{,}000}{20} = 7200 \text{ lb}$$

From a summation of the vertical forces,

$$\sum V = 0 = +R_2 + 7200 - 2000 - 8000 - 4000$$
$$R_2 = 6800 \text{ lb}$$

With all forces determined, a summation about the left-hand support—or any point except the right-hand support—will verify the accuracy of the work.

The following example demonstrates a solution with uniformly distributed loading on a beam. A convenience in this work is to consider the total uniformly distributed load as a concentrated force placed at the center of the distributed load.

Example 8. A simple beam 16 ft long carries the loading shown in Figure 1.21*a*. Find the reactions.

Solution: The total uniformly distributed load may be considered as a single concentrated load placed at 5 ft from the right-hand support;

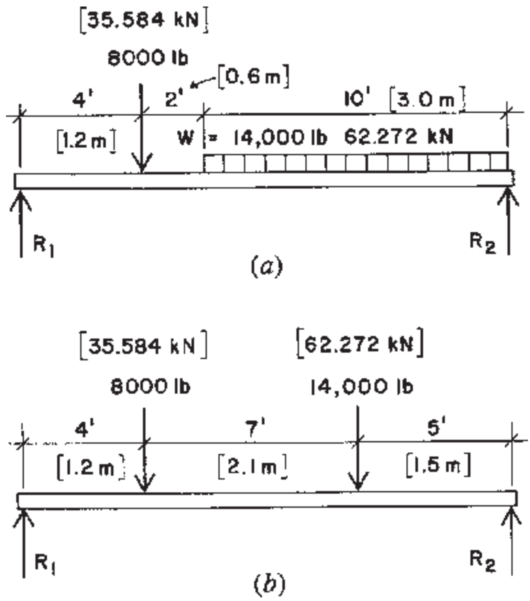

Figure 1.21 Reference for Example 8.

this loading is shown in Figure 1.21*b*. Considering moments about the right-hand support,

$$\sum M = 0 = +(R_1 \times 16) - (8000 \times 12) - (14{,}000 \times 5)$$

$$R_1 = \frac{166{,}000}{16} = 10{,}375 \text{ lb}$$

And, from a summation of vertical forces,

$$R_2 = (8000 + 14{,}000) - 10{,}375 = 11{,}625 \text{ lb}$$

Again, a summation of moments about the left-hand support will verify the accuracy of the work.

In general, any beam with only two supports, for which the supports develop only vertical reaction forces, will be statically determinate. This includes the simple span beams in the preceding examples as well as beams with overhanging ends.

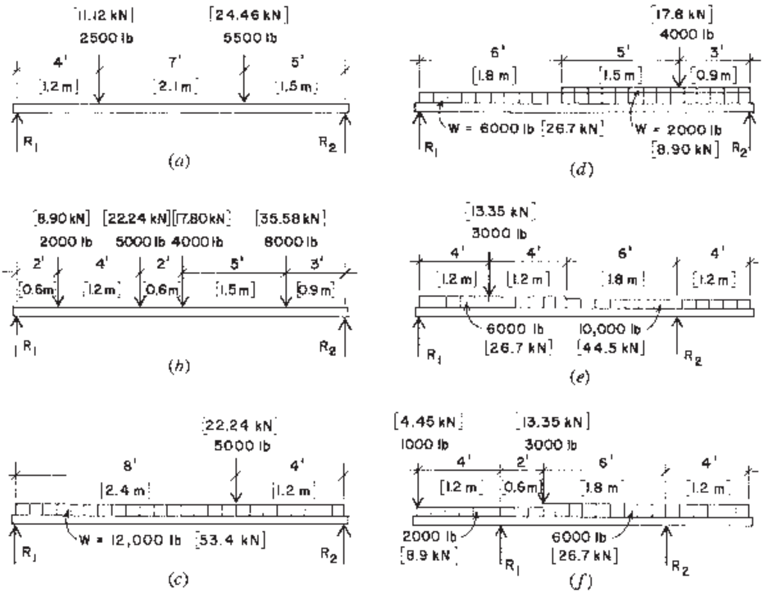

Figure 1.22 Reference for Problem 1.7.

Problems 1.7.A–F Find the reactions for the beams shown in Figure 1.22.

1.8 LOAD SOURCES

Structural tasks are defined primarily in terms of the loading conditions imposed on the structure. There are many potential sources of load for building structures. Designers must consider all the potential sources and the logical combinations with which they may occur. Building codes currently stipulate both the load sources and the form of combinations to be used for design. The following loads are listed in the 2013 edition of *Minimum Design Loads for Buildings and Other Structures* [of the American Society of Civil Engineers (ASCE), Ref. 1], hereinafter referred to as ASCE 2013:

D = Dead load
E = Earthquake-induced force
L = Live load, except roof load
Lr = Roof live load

S = Snow load
W = Load due to wind pressure

Additional special loads are listed, but these are the commonly occurring loads. The following is a description of some of these loads.

Dead Loads

Dead load consists of the weight of the materials of which the building is constructed, such as walls, partitions, columns, framing, floors, roofs, and ceilings. In the design of a beam or column, the dead load used must include an allowance for the weight of the structural member itself. Table 1.2, which lists the weights of many construction materials, may be used in the computation of dead loads. Dead loads are due to gravity and they result in downward vertical forces.

Dead load is generally a permanent load once the building construction is completed unless remodeling or rearrangement of the construction occurs. Because of this permanent, long-time character, the dead load requires certain considerations in design, such as the following:

1. Dead load is always included in design loading combinations, except for investigations of singular effects, such as deflections due to only live load.
2. Its long-time character has some special effects causing permanent sag and requiring reduction of design stresses in wood structures, development of long-term, continuing settlements in some soils, and producing creep effects in concrete structures.
3. Dead load contributes some unique responses, such as the stabilizing effects that resist uplift and overturn due to wind forces.

Although weights of materials can be reasonably accurately determined, the complexity of most building construction makes the computation of dead loads possible only on an approximate basis. This adds to other factors to make design for structural behaviors a very approximate science. As in other cases, this should not be used as an excuse for sloppiness in the computational work, but it should be recognized as a fact to temper concern for high accuracy in design computations.

TABLE 1.2 Weight of Building Construction

	psf[a]
Roofs	
3-ply ready roofing (roll, composition)	1
3-ply felt and gravel	5.5
5-ply felt and gravel	6.5
Shingles: Wood	2
Asphalt	2–3
Clay tile	9–12
Concrete tile	6–10
Slate, 3 in.	10
Insulation: Fiber glass batts	0.5
Foam plastic, rigid panels	1.5
Foamed concrete, mineral aggregate	2.5/in.
Wood rafters: 2 × 6 at 24 in.	1.0
2 × 8 at 24 in.	1.4
2 × 10 at 24 in.	1.7
2 × 12 at 24 in.	2.1
Steel deck, painted: 22 gage	1.6
20 gage	2.0
Skylights: Steel frame with glass	6–10
Aluminum frame with plastic	3–6
Plywood or softwood board sheathing	3.0/in.
Ceilings	
Suspended steel channels	1
Lath: Steel mesh	0.5
Gypsum board, 1/2 in.	2
Fiber tile	1
Drywall, gypsum board, 1/2 in.	2.5
Plaster: Gypsum	5
Cement	8.5
Suspended lighting and heating, ventilation, and air conditioning (HVAC), average	3
Floors	
Hardwood, 1/2 in.	2.5
Vinyl tile	1.5
Ceramic tile: 3/4 in.	10
Thin-set	5
Fiberboard underlay, 0.625 in.	3
Carpet and pad, average	3
Timber deck	2.5/in.
Steel deck, stone concrete fill, average	35–40
Concrete slab deck, stone aggregate	12.5/in.
Lightweight concrete fill	8.0/in.

TABLE 1.2 (*Continued*)

	psf[a]
Wood joists: 2 × 8 at 16 in.	2.1
2 × 10 at 16 in.	2.6
2 × 12 at 16 in.	3.2
Walls	
2 × 4 studs at 16 in., average	2
Steel studs at 16 in., average	4
Lath, plaster—see *Ceilings*	
Drywall, gypsum board, $^1/_2$ in.	2.5
Stucco, on paper and wire backup	10
Windows, average, frame + glazing:	
Small pane, wood or metal frame	5
Large pane, wood or metal frame	8
Increase for double glazing	2–3
Curtain wall, manufactured units	10–15
Brick veneer, 4 in., mortar joints	40
$^1/_2$ in., mastic-adhered	10
Concrete block:	
Lightweight, unreinforced, 4 in.	20
6 in.	25
8 in.	30
Heavy, reinforced, grouted, 6 in.	45
8 in.	60
12 in.	85

[a] Average weight per square foot of surface, except as noted. Values given as /in. (per in.) are to be multiplied by actual thickness of material.

Building Code Requirements

Structural design of buildings is most directly controlled by building codes, which are the general basis for the granting of building permits—the legal permission required for construction. Building codes (and the permit-granting process) are administered by some unit of government: city, county, or state. Most building codes, however, are based on some model code.

Model codes are more similar than they are different and are in turn largely derived from the same basic data and standard reference sources, including many industry standards. In the several model codes and many city, county, and state codes, however, there are some items that reflect particular regional concerns. With respect to control of structures, all codes have materials (all essentially the same) that relate to the following issues:

1. *Minimum Required Live Loads*. All building codes have tables that provide required values to be used for live loads. Tables 1.3 and 1.4 contain some loads as specified in ASCE 2013 (Ref. 1).
2. *Wind Loads*. These are highly regional in character with respect to concern for local windstorm conditions. Model codes provide data with variability on the basis of geographic zones.
3. *Seismic (Earthquake) Effects*. These are also regional with predominant concerns in the western states. These data, including recommended investigations, are subject to quite frequent modification, as the area of study responds to ongoing research and experience.
4. *Load Duration*. Loads or design stresses are often modified on the basis of the time span of the load, varying from the life of

TABLE 1.3 Minimum Floor Live Loads

Building Occupancy or Use	Uniformly Distributed Load (psf)	Concentrated Load (lb)
Apartments and Hotels		
Private rooms and corridors serving them	40	
Public rooms and corridors serving them	100	
Dwellings, One and Two Family		
Uninhabitable attics without storage	10	
Uninhabitable attics with storage	20	
Habitable attics and sleeping rooms	30	
All other areas except stairs and balconies	40	
Office Buildings		
Offices	50	2000
Lobbies and first-floor corridors	100	2000
Corridors above first floor	80	2000
Stores		
Retail		
First floor	100	1000
Upper floors	75	1000
Wholesale, all floors	125	1000

Source: ASCE 2013 (Ref 1), used with permission of the publisher, American Society of Civil Engineers.

TABLE 1.4 Live-Load Element Factor, K_{LL}

Element	K_{LL}
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified above	1

Source: ASCE 2013 (Ref. 1), used with permission of the publisher, American Society of Civil Engineers.

the structure for dead load to a few seconds for a wind gust or a single major seismic shock. Safety factors are frequently adjusted on this basis. Some applications are illustrated in the work in the design examples.

5. *Load Combinations*. These were formerly mostly left to the discretion of designers but are now quite commonly stipulated in codes, mostly because of the increasing use of ultimate strength design and the use of factored loads.
6. *Design Data for Types of Structures*. These deal with basic materials (wood, steel, concrete, masonry, etc.), specific structures (rigid frames, towers, balconies, pole structures, etc.), and special problems (foundations, retaining walls, stairs, etc.). Industrywide standards and common practices are generally recognized, but local codes may reflect particular local experience or attitudes. Minimal structural safety is the general basis, and some specified limits may result in questionably adequate performances (bouncy floors, cracked plaster, etc.).
7. *Fire Resistance*. For the structure, there are two basic concerns, both of which produce limits for the construction. The first concern is for structural collapse or significant structural loss. The second concern is for containment of the fire to control its spread. These concerns produce limits on the choice of materials (e.g., combustible or noncombustible) and some details of the construction (cover on reinforcement in concrete, fire insulation for steel beams, etc.).

The work in the design examples in Chapters 18–20 is based largely on criteria from ASCE 2013 (Ref. 1).

Live Loads

Live loads technically include all the nonpermanent loadings that can occur in addition to the dead loads. However, the term as commonly used usually refers only to the vertical gravity loadings on roof and floor surfaces. These loads occur in combination with the dead loads but are generally random in character and must be dealt with as potential contributors to various loading combinations, as discussed in Section 1.9.

Roof Loads

In addition to the dead loads they support, roofs are designed for a uniformly distributed live load. The minimum specified live load accounts for general loadings that occur during construction and maintenance of the roof. For special conditions, such as heavy snowfalls, additional loadings are specified.

The minimum roof live load in pounds per square foot (psf) is specified in ASCE 2013 (Ref. 1) in the form of an equation, as follows:

$$L_r = 20R_1R_2, \quad \text{in which } 12 \le L_r \le 20$$

In the equation R_1 is a reduction factor based on the tributary area supported by the structural member being designed (designated as A_t and quantified in square feet) and is determined as follows:

$$\begin{aligned} R_1 &= 1 \quad \text{for } A_t \le 200\ \text{ft}^2 \\ &= 1.2 - 0.001A_t \quad \text{for } 200\ \text{ft}^2 < A_t < 600\ \text{ft}^2 \\ &= 0.6 \quad \text{for } A_t \ge 600\ \text{ft}^2 \end{aligned}$$

Reduction factor R_2 accounts for the slope of a pitched roof and is determined as follows:

$$\begin{aligned} R_2 &= 1 \quad \text{for } F \le 4 \\ &= 1.2 - 0.05F \quad \text{for } 4 < F < 12 \\ &= 0.6 \quad \text{for } F \ge 12 \end{aligned}$$

The quantity F in the equations for R_2 is the number of inches of rise per foot for a pitched roof (e.g., $F = 12$ indicates a rise of 12 in. or an angle of 45°).

The design standard also provides data for roof surfaces that are arched or domed and for special loadings for snow or water accumulation. Roof surfaces must also be designed for wind pressures on the roof surface, both upward and downward. A special situation that must be considered is that of a roof with a low dead load and a significant wind load that exceeds the dead load.

Although the term *flat roof* is often used, there is generally no such thing; all roofs must be designed for some water drainage. The minimum required pitch is usually $1/4$ in./ft, or a slope of approximately 1 : 50. With roof surfaces that are close to flat, a potential problem is that of *ponding*, a phenomenon in which the weight of the water on the surface causes deflection of the supporting structure, which in turn allows for more water accumulation (in a pond), causing more deflection, and so on, resulting in an accelerated collapse condition.

Floor Live Loads

The live load on a floor represents the probable effects created by the occupancy. It includes the weights of human occupants, furniture, equipment, stored materials, and so on. All building codes provide minimum live loads to be used in the design of buildings for various occupancies. Since there is a lack of uniformity among different codes in specifying live loads, the local code should always be used. Table 1.3 contains a sample of values for floor live loads as given in ASCE 2013 (Ref. 1) and commonly specified by building codes.

Although expressed as uniform loads, code-required values are usually established large enough to account for ordinary concentrations that occur. For offices, parking garages, and some other occupancies, codes often require the consideration of a specified concentrated load as well as the distributed loading. This required concentrated load is listed in Table 1.3 for the appropriate occupancies.

Where buildings are to contain heavy machinery, stored materials, or other contents of unusual weight, these must be provided for individually in the design of the structure.

When structural framing members support large areas, most codes allow some reduction in the total live load to be used for design. These reductions, in the case of roof loads, are incorporated in the formulas for roof loads given previously. The following is the method given in ASCE 2013 (Ref. 1) for determining the reduction permitted for beams, trusses, or columns that support large floor areas.

The design live load on a member may be reduced in accordance with the formula

$$L = L_0 \left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}} \right)$$

where L = reduced live load supported, in psf
L_0 = unreduced live load, in psf
K_{LL} = live-load element factor (see Table 1.4)
A_T = tributary area supported, in ft^2

For members supporting one floor L shall not be less than $0.50L_0$, and L shall not be less than $0.40L_0$ for members supporting two or more floors.

In office buildings and certain other building types, partitions may not be permanently fixed in location but may be erected or moved from one position to another in accordance with the requirements of the occupants. In order to provide for this flexibility, it is customary to require an allowance of 15–20 psf, which is usually added to other dead loads.

Lateral Loads (Wind and Earthquake)

As used in building design, the term *lateral load* is usually applied to the effects of wind and earthquakes, as they induce horizontal forces on stationary structures. From experience and research, design criteria and methods in this area are continuously refined, with recommended practices being presented through the various model building codes.

Space limitations do not permit a complete discussion of the topic of lateral loads and design for their resistance. The following discussion summarizes some of the criteria for design in ASCE 2013 (Ref. 1). Examples of application of these criteria are given in the design examples of building structural design in Chapters 18–20. For a more extensive discussion the reader is referred to *Simplified Building Design for Wind and Earthquake Forces* (Ref. 2).

Wind

Where wind is a regional problem, local codes are often developed in response to local conditions. Complete design for wind effects on buildings includes a large number of both architectural and structural concerns. The following is a discussion of some of the requirements from ASCE 2013 (Ref. 1).

Basic Wind Speed. This is the maximum wind speed (or velocity) to be used for specific locations. It is based on recorded wind histories and adjusted for some statistical likelihood of occurrence. For the United States recommended minimum wind speeds are taken from maps provided in the ASCE standard. As a reference point, the speeds are those recorded at the standard measuring position of 10 m (approximately 33 ft) above the ground surface.

Wind Exposure. This refers to the conditions of the terrain surrounding the building site. The ASCE standard uses three categories, labeled B, C, and D. Qualifications for categories are based on the form and size of wind-shielding objects within specified distances around the building,

Simplified Design Wind Pressure (p_s). This is the basic reference equivalent static pressure based on the critical wind speed and is determined as follows:

$$p_s = \lambda I p_{S30}$$

where λ = adjustment factor for building height and exposure
I = importance factor
p_{S30} = simplified pressure, exposure B, height 30 ft, $I = 1$

The importance factor for ordinary circumstances of building occupancy is 1.0. For other buildings, factors are given for facilities that involve hazard to a large number of people, for facilities considered to be essential during emergencies (such as windstorms), and for buildings with hazardous contents.

The design wind pressure may be positive (inward) or negative (outward, suction) on any given surface. Both the sign and the value for the pressure are given in the design standard. Individual building surfaces, or parts thereof, must be designed for these pressures.

Design Methods. Two methods are described in the code for the application of wind pressures.

Method 1 (Simplified Procedure). This method is permitted to be used for relatively small, low-rise buildings of simple symmetrical shape. It is the method described here and used for the examples in Part V.

Method 2 (Analytical Procedure). This method is much more complex and is prescribed to be used for buildings that do not fit the limitations described for method 1.

Uplift. Uplift may occur as a general effect, involving the entire roof or even the whole building. It may also occur as a local phenomenon such as that generated by the overturning moment on a single shear wall.

Overturning Moment. Most codes require that the ratio of the dead-load resisting moment (called the restoring moment, stabilizing moment, etc.) to the overturning moment be 1.5 or greater. When this is not the case, uplift effects must be resisted by anchorage capable of developing the excess overturning moment. Overturning may be a critical problem for the whole building, as in the case of relatively tall and slender tower structures. For buildings braced by individual shear walls, trussed bents, and rigid-frame bents, overturning is investigated for the individual bracing units.

Drift. Drift refers to the horizontal deflection of the structure due to lateral loads. Code criteria for drift are usually limited to requirements for the drift of a single story (horizontal movement of one level with respect to the next level above or below). As in other situations involving structural deformations, effects on the building construction must be considered; thus, the detailing of curtain walls or interior partitions may affect design limits on drift.

Special Problems. The general design criteria given in most codes are applicable to ordinary buildings. More thorough investigation is recommended (and sometimes required) for special circumstances such as the following:

Tall Buildings. These are critical with regard to their height dimension as well as the overall size and number of occupants inferred. Local wind speeds and unusual wind phenomena at upper elevations must be considered. Tall buildings often require wind tunnel testing (either physical or with computer analysis) to determine appropriate wind loadings.

Flexible Structures. These may be affected in a variety of ways, including vibration or flutter as well as simple magnitude of movements.

Unusual Shapes. Open structures, structures with large overhangs or other projections, and any building with a complex shape should be carefully studied for the special wind effects that may occur. Wind tunnel testing may be advised or even required by some codes.

Earthquakes

During an earthquake, a building is shaken up and down and back and forth. The back-and-forth (horizontal) movements are typically more violent and tend to produce major destabilizing effects on buildings; thus, structural design for earthquakes is mostly done in terms of considerations for horizontal (called lateral) forces. The lateral forces are actually generated by the weight of the building—or, more specifically, by the mass of the building that represents both an inertial resistance to movement and the source for kinetic energy once the building is actually in motion. In the simplified procedures of the equivalent static force method, the building structure is considered to be loaded by a set of horizontal forces consisting of some fraction of the building weight. An analogy would be to visualize the building as being rotated vertically 90° to form a cantilever beam, with the ground as the fixed end and with a load consisting of the building weight.

In general, design for the horizontal force effects of earthquakes is quite similar to design for the horizontal force effects of wind. The same basic types of lateral bracing (shear walls, trussed bents, rigid frames, etc.) are used to resist both force effects. There are indeed some significant differences, but in the main a system of bracing that is developed for wind bracing will most likely serve reasonably well for earthquake resistance as well.

Because of its considerably more complex criteria and procedures, we have chosen not to illustrate the design for earthquake effects in the examples in this book. Nevertheless, the development of elements and systems for the lateral bracing of the building in the design examples here is quite applicable in general to situations where earthquakes are a predominant concern. For structural investigation, the principal difference is in the determination of the loads and their distribution in the building. Another major difference is in the true dynamic effects, critical wind force being usually represented by a single, major, one-direction punch from a gust, while earthquakes represent rapid back-and-forth actions. However, once the dynamic effects are translated into equivalent static forces, design concerns for the bracing systems are very similar, involving considerations for shear, overturning, horizontal sliding, and so on.

For a detailed explanation of earthquake effects and illustrations of the investigation by the equivalent static force method, the reader is referred to *Simplified Building Design for Wind and Earthquake Forces* (Ref. 2).

1.9 LOAD COMBINATIONS

The various types of load sources, as described in the preceding section, must be individually considered for quantification. However, for design work the possible combination of loads must also be considered. Using the appropriate combinations, the design load for individual structural elements must be determined. The first step in finding the design load is to establish the critical combinations of load for the individual element. Using ASCE 2013 (Ref. 1) as a reference, the following combinations are to be considered. As this process is different for the two basic methods of design, they are presented separately.

Allowable Stress Method

For this method the individual loads are used directly for the following possible combinations:

Dead load only
Dead load + live load
Dead load + roof load
Dead load + 0.75(live load) + 0.75(roof load)
Dead load + wind load or 0.7(earthquake load)
Dead load + 0.75(live load) + 0.75(roof load) + 0.75(wind load) or 0.7(earthquake load)
0.6(dead load) + wind load
0.6(dead load) + 0.7(earthquake load)

The combination that produces the critical design situation for individual structural elements depends on the load magnitudes and the loading condition for the elements. Demonstrations of examples of the use of these combinations are given in the building design cases in Part V and isolated problems in Part II.

Strength Design Method

Some adjustment of the percentage of loads (called factoring) is utilized in load combinations within the allowable stress method. However, factoring is done with all the loads for the strength method. The need here is to produce a load higher than the true anticipated load (called the *service load*)—the difference representing a margin of safety. The structural elements will be designed at their failure limits with the design load, and they really should not fail with the actual expected loads. For the strength method the following combinations are considered:

1.4(dead load)
1.2(dead load) + 1.6(live load) + 0.5(roof load)
1.2(dead load) + 1.6(roof load) + live load or 0.8(wind load)
1.2(dead load) + 1.6(wind load) + (live load) + 0.5(roof load)
1.2(dead load) + 1.0(earthquake load) + live load + 0.2(snow load)
0.9(dead load) + 1.0(earthquake load) or 1.6(wind load)

Use of these load combinations is demonstrated in the building design cases in Part V and in isolated problems in Parts III and IV.

1.10 DETERMINATION OF DESIGN LOADS

Figure 1.23 shows the plan layout for the framed structure of a multistory building. The vertical structure consists of columns and the horizontal floor structure of a deck and beam system. The repeating plan unit of 24×32 ft is called a column bay. Assuming lateral bracing of the building to be achieved by other structural elements, the columns and beams shown here will be designed for dead load and live load only.

The load to be carried by each element of the structure is defined by the unit loads for dead load and live load and the *load periphery* for the individual elements. The load periphery for an element is established by the layout and dimensions of the framing system. Referring to the labeled elements in Figure 1.23, the load peripheries are as follows:

Beam A: $8 \times 24 = 192$ ft^2
Beam B: $4 \times 24 = 96$ ft^2

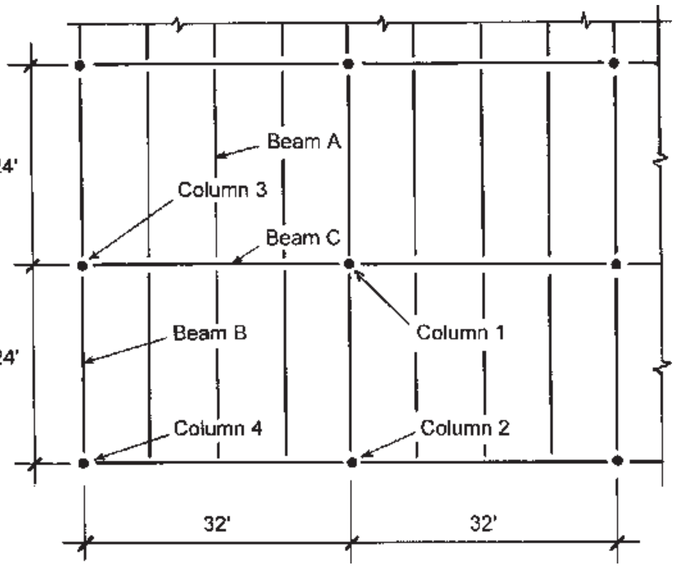

Figure 1.23 Reference for determination of distributed loads.

Beam C: $24 \times 24 = 576$ ft^2 (Note that beam C carries only three of the four beams per bay of the system, the fourth being carried directly by the columns.)

Column 1: $24 \times 32 = 768$ ft^2

Column 2: $12 \times 32 = 384$ ft^2

Column 3: $16 \times 24 = 384$ ft^2

Column 4: $12 \times 16 = 192$ ft^2

For each of these elements the unit dead load and unit live load from the floor are multiplied by the floor areas computed for the individual elements. Any possible live-load reduction (as described in Section 1.8) is made for the individual elements based on their load periphery area.

Additional dead load for the elements consists of the dead weight of the elements themselves. For the columns and beams at the building edge, another additional dead load consists of the portion of the exterior wall construction supported by the elements. Thus, column 2 carries an area of the exterior wall defined by the multiple of the story height times 32 ft. Column 3 carries 24 ft of wall and column 4 carries 28 ft of wall (12 + 16).

The column loads are determined by the indicated supported floor, to which is added the weight of the columns. For an individual story column this would be added to loads supported above this level—from the roof and any upper levels of floor.

The loads as described are used in the defined combinations described in Section 1.9. If any of these elements are involved in the development of the lateral bracing structure, the appropriate wind or earthquake loads are also added.

Floor live loads may be reduced by the method described in Section 1.8. Reductions are based on the tributary area supported and the number of levels supported by members.

Computations of design loads using the process described here are given for the building design cases in Part V.

1.11 DESIGN METHODS

Use of allowable stress as a design condition relates to the classic method of structural design known as the *working stress method* and now called the *allowable stress design* (ASD) method. The loads used for this method are generally those described as *service loads*; that is, they are related to the service (use) of the structure. Deformation limits are also related to service loads.

Even from the earliest times of use of stress methods, it was known that for most materials and structures the true ultimate capacity was not predictable by use of elastic stress methods. Compensating for this with the working stress method was mostly accomplished by considerations for the establishment of the limiting design stresses. For more accurate predictions of true failure limits, however, it was necessary to abandon elastic methods and to use true ultimate strength behaviors. This led eventually to the so-called *strength method* for design, presently described as the LRFD method, or *load and resistance factor design* method.

The procedures of the stress method are still applicable in many cases—especially for design for deformation limitations. However, the LRFD methods are now very closely related to more accurate use of test data and risk analysis and purport to be more realistic with regard to true structural safety.

The Allowable Stress Design (ASD) Method

The ASD method generally consists of the following:

1. The service (working) load conditions are visualized and quantified as intelligently as possible. Adjustments may be made here by the determination of various statistically likely load combinations (dead load plus live load plus wind load, etc.), by consideration of load duration, and so on.
2. Stress, stability, and deformation limits are set by standards for the various responses of the structure to the loads: in tension, bending, shear, buckling, deflection, uplift, overturning, and so on.
3. The structure is then evaluated (investigated) for its adequacy or is proposed (designed) for an adequate response.

An advantage obtained in working with the stress method is that the real usage condition (or at least an intelligent guess about it) is kept continuously in mind. The principal disadvantage comes from its detached nature regarding real failure conditions, since most structures develop much different forms of stress and strain as they approach their failure limits.

The Strength Design Method (LRFD)

In essence, the ASD method consists of designing a structure to *work* at some established appropriate percentage of its total capacity. The strength method consists of designing a structure to *fail*, but at a load condition well beyond what it should have to experience in use. A major reason for favoring the strength methods is that the failure of a structure is relatively easily demonstrated by physical testing. What is truly appropriate as a working condition, however, is pretty much a theoretical speculation. The strength method is now largely preferred in professional design work. It was first developed mostly for design of concrete structures but has now generally taken over all areas of structural design.

Nevertheless, it is considered necessary to study the classic theories of elastic behavior as a basis for visualization of the general ways that structures work. Ultimate responses are usually some form of variant from the classic responses (because of inelastic materials, secondary effects, multimode responses, etc.). In other words, the usual study procedure is to first consider a classic, elastic response and then to observe (or speculate about) what happens as failure limits are approached.

For the strength method, the process is as follows:

1. The service loads are quantified as in step 1 for the stress method and then are multiplied by an adjustment factor (essentially a safety factor) to produce the *factored load*.
2. The form of response of the structure is visualized and its ultimate (maximum, failure) resistance is quantified in appropriate terms (resistance to compression, to buckling, to bending, etc.). This quantified resistance is also subject to an adjustment factor called the *resistance factor*. Use of resistance factors is discussed in Part II for wood structures, in Part III for steel structures, and in Part IV for concrete structures.
3. The usable resistance of the structure is then compared to the ultimate resistance required (an investigation procedure), or a structure with an appropriate resistance is proposed (a design procedure).

Choice of Design Method

Applications of design procedures in the stress method tend to be simpler and more direct appearing than in the strength methods. For example, the design of a beam may amount to the simple inversion of a few stress or strain equations to derive some required properties (section modulus for bending, area for shear, moment of inertia for deflection, etc.). Applications of strength methods tend to be more obscure, simply because the mathematical formulations for describing failure conditions are more complex than the refined forms of the classic elastic methods.

As strength methods are increasingly used, however, the same kinds of shortcuts, approximations, and round-number rules of thumb will emerge to ease the work of designers. And, of course, use of the computer combined with design experience will permit designers to utilize highly complex formulas and massive databases with ease—all the while hopefully keeping some sense of the reality of it all.

Arguments for use of the stress method or the strength method are essentially academic. An advantage of the stress method may be a closer association with the in-use working conditions of the structure. On the other hand, strength design has a tighter grip on true safety through its focus on failure modes and mechanisms. The successful structural designer, however, needs both forms of consciousness and will develop them through the work of design, whatever methods are employed for the task.

Deformations of structures (such as deflection of beams) that are of concern for design will occur at the working stress level. Visualization and computation of these deformations require the use of basic techniques developed for the ASD method, regardless of whether ASD or LRFD is used for the design work in general.

Work in this book demonstrates the use of both the ASD and LRFD methods. Either method can be used for wood, steel, or concrete structures. However, professional structural design for steel and concrete is now done almost exclusively with the LRFD method. For wood design, work is still done with both methods, although the trend is steadily toward the use of the LRFD method.