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Introduction to Harmonic Balance Finite Element Method (HBFEM)

1.1 Harmonic Problems in Power Systems

The harmonics problem in power systems is not a new problem. It has existed since the early 1900s – as long as AC power itself has been available. The earliest harmonic distortion issues were associated with third harmonic currents produced by saturated iron in machines and transformers, or so-called ferromagnetic loads. Later, arcing loads, like lighting and electric arc furnaces, were also shown to produce harmonic distortion. The final type, electronic loads, burst onto the power scene in the 1970s and 1980s, and has represented the fastest growing category ever since [1].

Since power system harmonic distortion is mainly caused by non-linear loads and power electronics used in the electrical power system [2, 3], the presence of non-linear loads and the increasing number of distributed generation power systems in electrical grids contributes to changing the characteristics of voltage and current waveforms in power systems (which differ from pure sinusoidal constant amplitude signals). The impact of non-linear loads and power electronics used in electrical power systems has been increasing during the last decade.

Such electrical loads, which introduce non-sinusoidal current consumption patterns (current harmonics), can be found in power electronics [4], such as: DC/AC inverters; switch mode power supplies; rectification front-ends in motor drives; electronic ballasts for discharge lamps; personal computers or electrical appliances; high-voltage DC (HVDC) power systems; impulse transformers; magnetic induction devices; and various

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Harmonic Balance Finite Element Method: Applications in Nonlinear Electromagnetics and Power Systems, First Edition. Junwei Lu, Xiaojun Zhao and Sotoshi Yamada.

Companion website: www.wiley.com/go/lu/HBFEM

electric machines. In addition, the harmonics can be generated in distributed renewable energy systems, geomagnetic disturbances (GMDs) and geomagnetic induced currents (GICs) [5, 6].

Harmonics in power systems means the existence of signals, superimposed on the fundamental signal, whose frequencies are integer numbers of the fundamental frequency. The presence of harmonics in the voltage or current waveform leads to a distorted signal for the voltage or current, and the signal becomes non-sinusoidal. Thus, the study of power system harmonics is an important subject for electrical engineers. Electricity supply authorities normally abrogate responsibility on harmonic matters by introducing standards or recommendations for the limitation of voltage harmonic levels at the points of common coupling between consumers.

1.1.1 Harmonic Phenomena in Power Systems

A better understanding of power system harmonic phenomena can be achieved by consideration of some fundamental concepts, especially the nature of non-linear loads, and the interaction of harmonic currents and voltages within the power system. By definition, harmonic (or non-linear) loads are those devices that naturally produce a non-sinusoidal current when energized by a sinusoidal voltage source. As shown in Figure 1.1, each "waveform" represents the variation in instantaneous current over time for two different loads each energized from a sinusoidal voltage source. This pattern is repeated continuously, as long as the device is energized, creating a set of largely-identical waveforms that adhere to a common time period. Both current waveforms were produced by turning on some type of load device. In the case of the current on the left, this device was probably an electric motor or resistance heater. The current on the right could have been produced by an electronic variable-speed drive, for example. The devices could be single- or three-phase, but only one phase current waveform is shown for illustration. The other phases would be similar.

A French mathematician, Jean Fourier, discovered a special characteristic of periodic waveforms in the early 19th century. The method describing the non-sinusoidal

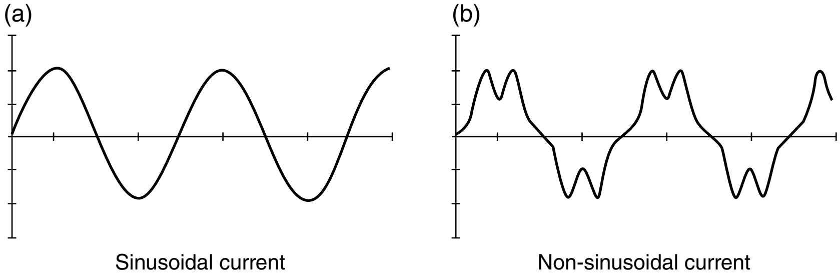

Figure 1.1 (a) Sine wave. (b) Distorted waveform or non-sinusoidal

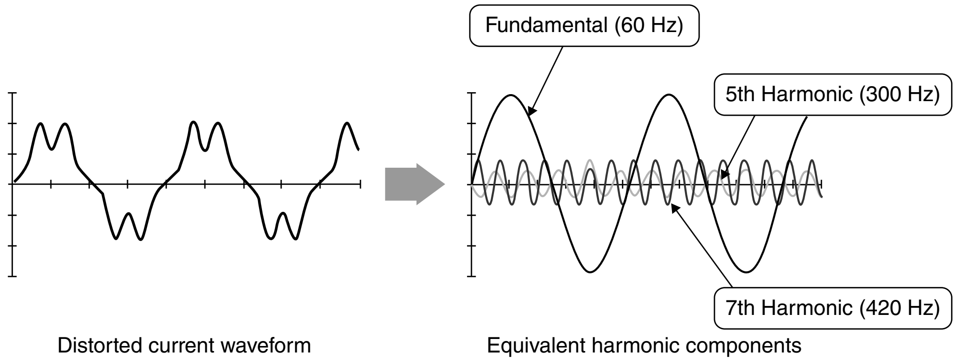

Figure 1.2 Distorted waveform and number of harmonics by Fourier series

waveform is called its Fourier Series. The Fourier theorem breaks down a periodic wave into its component frequencies. Periodic waveforms are those waveforms comprised of identical values that repeat in the same time interval, as shown in Figure 1.2. Fourier discovered that periodic waveforms can be represented by a series of sinusoids summed together. The frequency of these sinusoids is an integer multiple of the frequency represented by the fundamental periodic waveform.

The distorted (non-linear) waveform, however, deserves further scrutiny. This waveform meets the continuous, periodic requirement established by Fourier. It can be described, therefore, by a series of sinusoids. This example waveform is represented by only three harmonic components, but some real-world waveforms (square wave, for example) require hundreds of sinusoidal components to describe them fully. The magnitude of these sinusoids decreases with increasing frequency, often allowing the power engineer to ignore the effect of components above the 50th harmonic.

1.1.2 Sources and Problems of Harmonics in Power Systems

Harmonic sources generated in power systems can be divided into two categories: established and known; and new and future. Table 1.1 presents sources and problems of harmonics. Harmonic problems in power systems can be traced to a number of factors [3], such as: (a) the substantial increase of non-linear loads resulting from new technologies such as silicon-controlled rectifiers (SCRs), power transistors, and microprocessor controls, which create load-generated harmonics throughout the system; and (b) a change in equipment design philosophy.

In the past, equipment designs tended to be under-rated or over-designed. Nowadays, in order to be competitive, power devices and equipment are more critically designed and, in the case of iron-core devices, their operating points are more focused on non-linear regions. Operation in these regions results in a sharp rise in harmonics.

Table 1.1 Sources and problems of harmonics

Established and known	New and future
Tooth ripple or ripples in the voltage waveform of rotating machines.	Energy conservation measures, such as those for improved motor efficiency and load-matching, which employ power semiconductor devices and switching for their operation. These devices often produce irregular voltage and current waveforms that are rich in harmonics.
Variations in air-gap reluctance over synchronous machine pole pitch.	Motor control devices, such as speed controls for traction.
Flux distortion in the synchronous machine from sudden load changes.	High-voltage direct current power conversion and transmission.
Non-sinusoidal distribution of the flux in the air gap of synchronous machines.	Interconnection of wind and solar power converters with distribution systems.
Transformer magnetizing currents.	Static var compensators which have largely replaced synchronous condensers as continuously variable-var sources.
Network non-linearities from loads such as rectifiers, inverters, welders, arc furnaces, voltage controllers, frequency converters, etc.	The development and potential use of electric vehicles that require a significant amount of power rectification for battery charging.
N/A	The potential use of direct energy conversion devices, such as magneto-hydrodynamics, storage batteries, and fuel cells that require DC/AC power converters.
N/A	Cyclo-converters used for low-speed high-torque machines.
N/A	Pulse-burst-modulated heating elements for large furnaces.

1.1.3 Total Harmonic Distortion (THD)

The reduced impedance at the peak voltage results in a large, sudden rise in current flow until the impedance is suddenly increased, resulting in a sudden drop in current. Because the voltage and current waveforms are no longer related, they are said to be “non-linear”. These non-sinusoidal current pulses introduce unanticipated reflective currents back into the power distribution system, and the currents operate at frequencies other than the fundamental 50/60 Hz. Ideally, voltage and current waveforms are perfect sinusoids. However, because of the increased non-linear load and power electronic devices based on switch mode power supplies and motor drives, these waveforms quite often become distorted. This deviation from a perfect sine wave can be represented by harmonics – sinusoidal components having a frequency that is an integral multiple of the fundamental frequency, as shown in Figure 1.3. Thus, a non-sinusoidal wave has distortion and harmonics. To quantify the distortion, the term total harmonic distortion (THD) is used, and this expresses the distortion as a percentage of the fundamental voltage and current waveforms.

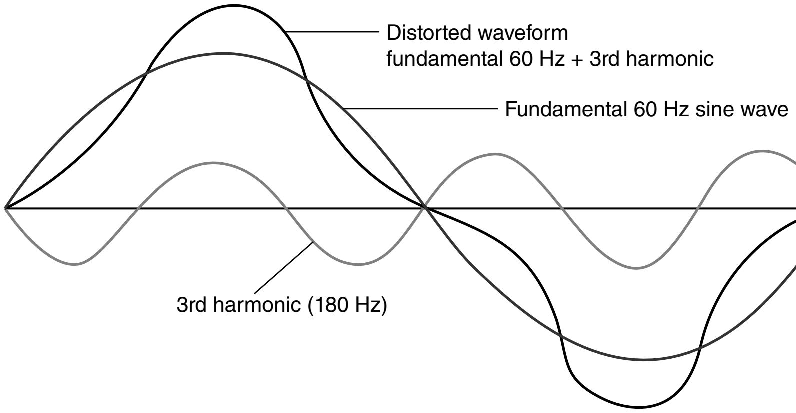

Figure 1.3 Harmonic distortion of the electrical current waveform, where the distorted waveform is composed of fundamental and 3rd harmonics

Harmonics have frequencies that are integer multiples of the waveform's fundamental frequency. For example, given a 60 Hz fundamental waveform, the 2nd, 3rd, 4th and 5th harmonic components will be at 120 Hz, 180 Hz, 240 Hz and 300 Hz, respectively. Thus, harmonic distortion is the degree to which a waveform deviates from its pure sinusoidal values as a result of the summation of all these harmonic elements. The ideal sine wave has zero harmonic components. In that case, there is nothing to distort this perfect wave. Total harmonic distortion, or THD, is the summation of all harmonic components of the voltage or current waveform, compared against the fundamental component of the voltage or current wave:

$$THD = \frac{\sqrt{\left(V_2^2 + V_3^2 + V_4^2 + \cdots V_n^2\right)}}{V_1} \tag{1-1}$$

The formula above (Equation 1-1) shows the calculation for THD on a voltage signal. The end result is a percentage comparing the harmonic components to the fundamental component of a signal. The higher the percentage, the more distortion that is present on the mains signal. The concept that a distorted waveform (including a square wave) can be represented by a series of sinusoids is difficult for many engineers, but it is absolutely essential for understanding the harmonic analysis and mitigation to follow. It is important for the power engineer to keep the following facts in mind:

- The equivalent harmonic components are just a representation – the instantaneous current as described by the distorted waveform is what is actually flowing on the wire.

- This representation is necessary, because it facilitates analysis of the power system. The effect of sinusoids on typical power system components (transformers, conductors, capacitors) is much easier to analyze than distorted signals.
- Power engineers comfortable with the concept of harmonics often refer to individual harmonic components as if each really exists as a separate entity. For example, a load might be described as producing "30 A of 5th harmonic." What is intended is not that the load under consideration produced 30 A of current at 300 Hz, but rather that the load produced a distorted (but largely 60 Hz) current, one sinusoidal component of which has a frequency of 300 Hz with an rms magnitude of 30 A.
- The equivalent harmonic components, while imaginary, fully and accurately represent the distorted current. As one test, try summing the instantaneous current of the harmonic components at any point in time. Compare this value to the value of the distorted waveform at the same time (see Figure 1.3). These values are equal.

The current drawn by non-linear loads passes through all of the impedance between the system source and load. This current produces harmonic voltages for each harmonic as it flows through the system impedance. The sum of these harmonic voltages produces a distorted voltage when combined with the fundamental. The voltage distortion magnitude is dependent on the source impedance and the harmonic voltages produced. Figure 1.4 illustrates how the distorted voltage is created. As illustrated, non-linear loads are typically modeled as a source of harmonic current.

With low source impedance, the voltage distortion will be low for a given level of harmonic current. If the harmonic current increases, however, system impedance changes due to the harmonic resonance (discussed below) can significantly increase voltage distortion.

IEEE Std. 519-1992, which is titled *IEEE Recommended Practices and Requirements for Harmonic Control in Electric Power Systems*, is the main document for harmonics in North America. This standard serves as an excellent tutorial on harmonics. The most

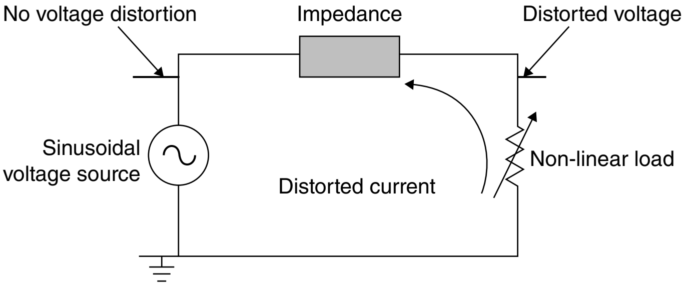

Figure 1.4 Creation of distorted current

important part of this document to the industrial user is Chapter 10 (“Recommended Practices for Individual Consumers” [7]).

The electric consumption is a significant part of the total energy consumption and, consequently, the complete chain of generation, transportation and usage of electricity should be optimized. The usage of electrical energy is often optimized by controlling the output of electrical equipment towards the desired value. Advances in power electronic (PE) energy conversion have led to an optimization of electrical equipment. Practical examples of PE-controlled energy conversion are dimmable halogen lighting, low- and high-pressurized discharge lights, AC drives for induction machines (IM), and so on. In addition to the advantages of PE in terms of energy optimization, a lot of PE is also used for DC power supply, such as IT equipment, DC arcing or electrolysis [8, 9].

Harmonics are a distortion of the normal electrical current waveform, generally transmitted by non-linear loads. Switch-mode power supplies (SMPS), variable speed motors and drives, photocopiers, personal computers, laser printers, fax machines, battery chargers and UPSs are examples of non-linear loads. Single-phase non-linear loads are prevalent in modern office buildings, while three-phase, non-linear loads are widespread in factories and industrial plants, and in DC-biased power transformers in HVDC power systems [10].

The study of these harmonics problems is normally focused on the electrical circuit level. A large number of articles and reports have been published in this area. However, the harmonics problem in the component level (or electromagnetic fields) has not been fully investigated, due to a lack of understanding of the characteristics of non-linear electromagnetic fields and a lack of theory and methodology dealing with harmonics generated from non-linear electromagnetic fields. Only a very limited number of papers and reports related to HBFEM used in solving the harmonic problems in electromagnetic field [11, 12]. Detailed HBFEM theory development and various application problem-solving examples are presented in later chapters.

1.2 Definitions of Computational Electromagnetics and IEEE Standards 1597.1 and 1597.2

1.2.1 “The Building Block” of the Computational Electromagnetics Model [13, 14]

The objective of computational electromagnetics (CEM) is to create a representation of real-life problems that can be examined and analyzed by computer resources, as an alternative to building a system, exciting it, and measuring the generated fields. Once the problem has been defined, the important physical characteristics must be identified. All CEM models can be broken into three parts: the source of EM energy, the geometry of the model components, and the remaining problem space. The following elements of a physical CEM model should be taken into account during the simulation:

1.2.1.1 The Sources of EM Energy

- ***Source*** – Sources include both intended and unintended sources that electromagnetically couple to and drive conductors (such that energy is conducted into areas that can energize and drive the electric machine to make a correct operation, or can cause problems with the correct operation of the victim devices).
- ***Physical Source Modeling*** – Sources may be characterized by their electrical size, the distance from materials with which they interact, their geometry, and the excitation applied to them.
- ***Source Excitation*** – Like fully specified circuit model sources, field sources must also be defined by their amplitude and impedance.

1.2.2 The Geometry of the Model and the Problem Space

The major concern of every CEM model is the geometry of the problem to be solved. A less complex representation must be created, which includes all the important details while avoiding unnecessary details. In addition to the fixed portions of the geometry, it is often necessary to include variables such as the range of positions in which a nearby wire – or any other conductor – could be placed. Together with the geometry of a problem, the properties of all materials used must also be included in the model. If the computational domain were of infinite extent, the simulation of free space would be involved. This can be achieved by using mesh truncation techniques or absorbing boundary conditions. These techniques require that extra free space is added around the model components.

1.2.3 Numerical Computation Methods

Substantial advancements have been made in enhancing the important numerical techniques – for example: the method of moments (MoM); the finite-difference time-domain (FDTD) method; the finite-element method (FEM); the proposed harmonic balance method (HBFEM) in this book; and the transmission line matrix (TLM) method. Many numerical methods were invented decades ago but, in all cases, additional novel ideas were required to make them applicable to today's real-world electromagnetic problems.

- **The quasi-static field** can be expressed by several different partial differential equations (PDEs). Although existing computational electromagnetic solvers provide preliminary insight, a multi-physics simulation system is needed to model coupled problems in their entity. **Multi-physics problems** are often related to more than two fields, such as thermal and **E** fields, or the **H** field, thermal dynamic field, and so forth. In the quasi-static field, the following methods are often used in FEM based EM computation:

- **Time-domain techniques** use a band-limited impulse to excite the simulation across a wide frequency range. The result obtained from a time-domain code is the model's response to this impulse. Where frequency-domain information is required, a Fourier transform is applied to the time-domain data.
- **Frequency-domain codes** solve for one frequency at a time. This is usually adequate for antenna work or electric machine simulation, and for examining specific issues. Frequency-domain codes are, in general, faster than their time-domain cousins. Therefore, several frequency-domain simulations can usually be run in the time it would take for a single time-domain simulation. However, in nonlinear EM field problems, there is a coalition between each frequency domain, particularly for solving harmonic problems in nonlinear time periodic problems. This can be called the **multi-frequency-domain or HBFEM**.

1.2.4 High-Performance Computation and Visualization (HPCV) in CEM

With the rapid growth of microelectronics and computer technologies, cluster-based high-performance parallel computers are becoming more and more powerful and cost-effective. This provides a new opportunity to apply computational electromagnetics technologies to challenging problems in EM computer modeling and simulation. Since the computational technique extends from numeric analysis to visualization analysis, the demand for innovative visualization techniques becomes higher and higher. Visualization is closely related to high-performance computation using visualization techniques to deal with the complex dynamic electromagnetic system problems.

Visualization techniques for computational electromagnetics in 2D and 3D promise to radically change the way data is analyzed. To minimize eddy current loss and other problems in nonlinear EM fields, the optimization algorithms have been considered in current computational electromagnetic (CEM) modeling approaches. In fact, the action of EM computer modeling and simulation involves several physical effects. Detailed knowledge of all these effects is a prerequisite for effective and efficient design. The first step in reducing the design time and allowing for aggressive design strategies is to use EM computer modeling techniques that will let designers try "what if" experiments in hours instead of months.

1.2.5 IEEE Standards 1597.1 and 1597.2 for Validation of CEM Computer Modeling and Simulations

IEEE P1597.1 and P1597.2 Standards, developed by the EMC community, were released in 2008 and 2010 respectively. IEEE Standard 1597.1-2008 is related to the

IEEE Standard for validation of computational electromagnetics computer modeling and simulations. IEEE Standard 1597.2-2010 was released for IEEE recommended practice for validation of computational electromagnetics computer modeling and simulations. The following highlighted descriptions are based on IEEE Standards 1597.1 and 1597.2 [15, 16].

The development of IEEE standards, and recommended practices for computational electromagnetics (CEM) computer modeling and simulation and code validation, has been a topic of much interest within the EMC community particularly since the mid-1980s [17]. This has been due to advances in computer hardware and software technologies,+ as well as the arrival of new CEM codes and applications. The areas of concern include, but are not limited to, high-frequency areas such as analyzing printed circuit boards (PCBs), radiated and conducted emissions/immunity, system-level electromagnetic compatibility (EMC), radar cross-section (RCS) of complex structures, and the simulation of various EM environment effects problems. In particular, there are concerns regarding the lack of well-defined methodologies to achieve code-to-code or even simulation-to-measurement validations with a consistent level of accuracy.

Since the mid-1960s, a number of CEM techniques have been developed, and numerical codes have been generated, to analyze various related electromagnetic problems, including electromagnetic compatibility (EMC). While each is based on classical electromagnetic theory and implements Maxwell's equations in one form or another, these techniques, and the manner in which they are used to analyze a given problem, can produce quite different results. A well-defined, mature, and robust methodology for validating computational electromagnetic techniques, with a consistent level of accuracy, is lacking. Indeed, this has eluded the EMC community for many years, and methods have been sought to address this deficiency. The EMC community has persisted regarding the validity, accuracy and applicability of existing numerical techniques to the general class of EMC problems of interest.

The IEEE P1597.1 Standard defines a method to validate computational electromagnetics (CEM) computer modeling and simulation (M&S) techniques, codes and models. It is applicable to a wide variety of electromagnetic (EM) applications, including (but not limited to) the fields of electromagnetic compatibility (EMC), radar cross-section (RCS), signal integrity (SI), and antennas. Validation of a particular solution data set can be achieved by comparing the data set obtained by measurements, alternate codes, canonical, or analytic methods.

IEEE P1597.2™, recommended practice for validation of computational electromagnetics computer modeling and simulation, has been developed to provide examples and problem sets for use in the validation of CEM computer modeling and simulation techniques, codes and models. It is applicable to a wide variety of electromagnetic applications. The recommended practice, in conjunction with this standard, shows how to validate a particular solution data set by comparing it to the data set obtained by measurements, alternate codes, canonical, or analytic methods. The key areas addressed include model accuracy, convergence, and techniques or code validity for a given set of canonical, benchmark, and standard validation models.

In fact, computer predictions have been compared to measurements to provide a first-order validation, but there is also much interest in how the techniques, when applied to a given problem or a class of problems, compare to each other and the fundamental theory upon which they are based. Hence, additional efforts are needed to establish a standardized method for validating these techniques, and to instill confidence in them. Therefore, the purpose of this first-of-its-kind standard is to define the specific process and steps that will be used to validate CEM techniques and to significantly reduce uncertainty (as it pertains to their implementation and application to practical EMC problem-solving tasks). The standardized process, based on the Feature Selective Validation (FSV) method, is used to validate various techniques against each other, as well as against measurement baselines (in order to determine the degree of agreement or convergence, and to identify the potential error sources that would lead to divergent trends).

In general, CEM techniques and codes, and the manner in which they are used to analyze a given problem, can produce quite different results. These results are affected by the way in which the underlying physic formalisms have been implemented within the codes, including the mathematical basis functions, numerical solution methods, numerical precision, and the use of building blocks (primitives) to generate computational models. Despite all CEM codes having their basis in Maxwell’s equations in one form or another, their accuracy and convergence rate depends on how the physics equations are cast (e.g., integral or differential form, frequency or time domain), what numerical solver approach is used (full or partial wave, banded or partitioned matrix, non-matrix), inherent modeling limitations, approximations, and so forth. The physics formalism, available modeling primitives (canonical surface or volumetric objects, wires, patches, facets), analysis frequency, and time or mesh discretization further combine to affect accuracy, solution convergence and overall validity of the computer model.

The critical areas that must be addressed include model accuracy, convergence, and techniques or code validity for a given set of canonical, benchmark and standard validation models. For instance, uncertainties may arise when the predicted results using one type of CEM technique do not agree favorably or consistently with the results of other techniques or codes of comparable type, or even against measured data on benchmark models. Furthermore, it can be difficult to compare the results between certain techniques or codes, despite their common basis in Maxwell’s equations. Exceptions can be cited, in particular, when comparing the results of “similar” codes grouped according to their physics, solution methods, and modeling element domains. Nevertheless, disparities even among codes in a certain “class” have been observed. Many examples can be cited where fairly significant deviations have been observed between analytical or computational techniques and empirical-based methods. Differences are not unexpected, but the degree of disparity in certain cases cannot be readily explained nor easily discounted. This has led to the often asked question: “Which result is accurate?”

1.3 HBFEM Used in Nonlinear EM Field Problems and Power Systems

Nonlinear phenomena in EM fields are caused by nonlinear materials used in electric machines. The nonlinear materials are normally field strength-dependent, which can cause harmonics. Therefore, when the time-periodic quasi-static EM field is applied to the nonlinear material, the electromagnetic properties of the material will be functions of the EM field, which is also time-dependent. On the other hand, harmonics can also be generated by power electronic devices and drives, which are largely used in power systems, renewable energy systems and microgrids. These power electronic devices and drives are used for power rectification, power conversion (e.g. DC/DC converter) and inversion (e.g. DC/AC inverter). In fact, HBFEM can be used to effectively solve these harmonic problems in nonlinear EM fields and power systems.

Since the harmonic balance FEM technique was introduced to analyze low-frequency electromagnetic (EM) field problems in the late 1980s [11, 12], various harmonic problems in nonlinear EM fields and power systems have been investigated and solved by using HBFEM [18–24]. Harmonic balance techniques were combined with the finite element method (FEM) to accurately solve the problems arising from time-periodic, steady-state nonlinear magnetic fields. The method can be used for weak and strong nonlinear time-periodic EM fields, as well as harmonic problems in renewable energy systems and microgrids with distributed energy resources.

The harmonic balance FEM (HBFEM) method uses a linear combination of sinusoids to build the solution, and represents waveforms using the sinusoid – coefficients combined with the finite element method. It can directly solve the steady-state response of the EM field in the multi-frequency domain. Thus, the method is often considerably more efficient and accurate in capturing coupled nonlinear effects than the traditional FEM time-domain approach when the field exhibits widely separated harmonics in the frequency spectrum domain (e.g. pulse width modulation (PWM)-based power electronic devices and drives). The HBFEM consists of approximating the time-periodic solution (magnetic potentials, currents, voltages, etc.) with a truncated Fourier series. Besides the frequency components of the excitation (e.g. applied voltages or current), the solution contains harmonics due to nonlinearity (magnetic saturation and nonlinear lumped electrical components), movement (e.g. rotation in electric machines), and power electronic devices and drives.

In order to solve time-periodic nonlinear magnetic field problems, a novel numerical computation method called HBFEM was developed, which is the combination of FEM and the harmonic balance method. The principle of a new approach of HBFEM is to drive a basic formulation of the harmonic balance finite element method (HBFEM). For simplicity of fundamental formulation, a time-periodic nonlinear magnetic field is assumed as two-dimensional in the (x, y) plane, and is quasi-stationary. Therefore, the vector potential A = (0, 0, A) satisfies in the region of interest surrounded with some boundary conditions. To calculate such a quasi-static magnetic field, the following equation (1-2) can be used:

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J_s+\sigma\frac{\partial A}{\partial t} \tag{1-2}$$

where ν and σ are magnetic reluctivity and conductivity.

Based on the harmonics balance theory, the governing equations of the quasi-static field containing harmonics can also be solved by using a FEM-based numerical approach. Assuming $\nabla\varphi = 0$ in the two-dimensional case, and using Galerkin's method to discretize, the governing equation for two dimensional problems can be written in an integral form that is given as:

$$G=\iint_S\left\{\frac{\partial N_i}{\partial x}\nu\frac{\partial A}{\partial x}+\frac{\partial N_i}{\partial y}\nu\frac{\partial A}{\partial y}\right\}dxdy-\iint_S\left\{J_s-\sigma\frac{\partial A}{\partial t}\right\}N_i dxdy=0 \tag{1-3}$$

where $N_i(x, y)$ is the interpolating function.

When the applied voltage waveform is a sinusoidal signal, the current can be considered as a non-sinusoidal waveform. The waveform may be distorted due to nonlinear load or power electronic devices. Therefore, the current excitation source will include harmonic components, and the resultant magnetic field will contain all harmonic components. The vector potential ***A*** and current density ***J*** are approximated as a summation of all harmonic solutions. According to the harmonic balance method, all variables (i.e. vector potentials, flux densities and applied current) are approximated as a summation of all harmonic solutions. Therefore, the time-periodic solution (harmonic problem) can be found when an alternating magnetizing current is applied. In fact, HBFEM has been successfully used to solve various nonlinear magnetic field problems, and the computation results have been verified by experimental results listed below (detailed results will be discussed in Chapter 3 and 4).

1.3.1 HBFEM for a Nonlinear Magnetic Field With Current Driven

The HBFEM differs from traditional finite element time-domain methods, transient analysis and other time harmonic methods. The harmonic balance method uses a linear combination of sinusoids to generate a solution, and represents waveforms using the coefficients of the sinusoids. It is combined with the finite element method to solve time-periodic, steady-state nonlinear electromagnetic field problems. The HBFEM directly solves the steady-state response of the electromagnetic field in the frequency domain, and so is often considerably more efficient than traditional time-domain methods when fields exhibit widely separated time constants and mildly nonlinear behavior. The electromagnetic field with harmonics satisfies Maxwell's equations. The magnetic core of the transformer and inductor, with nonlinear characteristics and hysteresis, is excited by a current source of current density *Js*.

When a nonlinear magnetic system is excited by a sinusoidal waveform, a number of harmonics will be generated in this nonlinear magnetic system. For the non-DC biased case, only odd harmonics can be generated in the magnetic field where the B-H curve, with and without hysteresis characteristics, is used.

1.3.2 HBFEM for Magnetic Field and Electric Circuit Coupled Problems

In most cases, pulse width modulation and zero-current switched resonant converters (including LLC converters, DC biased HVDC transformers and HV transformers) can be considered as a voltage source to the magnetic system, which is always coupled to the external circuits. In that case, the current in the input circuits will be unknown, and the saturation of the current waveform occurs because of the nonlinear characteristic of the magnetic core and power electronic device.

1.3.3 HBFEM for a Nonlinear Magnetic Field with Voltage Driven

When a high-frequency transformer of switching mode power supply is excited by a voltage source, such as pulse-width modulation (PWM) converters and zero-current switched (ZCS) resonant converters, the numerical analysis of the magnetic field should be carried out by taking account of the voltage source and the external circuits. If the excitation waveform is a square wave or triangular wave, it can be considered as a linear combination of harmonics. For a DC-biased transformer problem in the HVDC power system, the voltage source and the external circuits will be considered in the HBFEM simulation, which is a magnetic field- and electric circuit-coupled problem.

1.3.4 HBFEM for a Three-Phase Magnetic Tripler Transformer

A magnetic frequency tripler is a nonlinear magnetic system which is used for the production of triple-frequency output from a three-phase fundamental frequency source based on the nonlinear magnetic saturation characteristics. Although magnetic frequency triplers have been used extensively for certain applications, the design of these devices has, until the earlier 1990s, been largely empirical. The earlier, and some recent, papers [25–27] have discussed the analyses of magnetic frequency-tripling devices, based on an equivalent-circuit approach under various load conditions and the Preisach model.

However, the above methods are based on the equivalent circuit theory, magnetic nonlinear characteristics, hysteresis losses, eddy current losses, and magnetic flux distribution for each harmonic component cannot be calculated and presented. Therefore, the EM full wave solution can be obtained from an HBFEM-based numerical computation. HBFEM can provide magnetic flux distribution and eddy current losses at each harmonic [21].

1.3.5 HBFEM for a Three-Phase High-Speed Motor

The HBFEM taking account of external circuits and motion can be effectively used to solve the high-speed and hybrid induction motor problem [21]. A comparison is made between experimental and numerical results for the static model, and this has shown good consistency.

The high-speed hybrid induction motor consists of three-phase input windings and two-phase magnetic frequency tripler as an output, and an induction motor. The induction motor has two pairs of magnetic frequency triplers and four magnetic poles, with the air gap in the middle leg of the cores. The three-phase magnetizing windings are connected as a Scott connection, and four additional coils, connected with the capacitors for increasing output power, are put in the poles. When a 60 Hz commercial source is applied to the hybrid induction motor, the rotation speed (10 800 rpm) will be gained between the poles. The principle of the three-phase input windings and two-phase magnetic frequency tripler as an output is that two single-phase triplers (composed of three-legged cores) are connected in a Scott connection. Therefore, the input voltages shifted at 90 degrees are applied to two single-phase triplers.

Since the high-speed hybrid motor has a very complex configuration, an optimal design is requested in the design of electric machines. Optimal design ensures that the flux of the fundamental harmonic components does not pass through the poles and rotor, and only the three-times frequency flux passes through the poles and moves the rotor. In the HBFEM numerical analysis, the half model is used as an analysis area. The magnetizing windings are applied with a 60 Hz commercial three-phase voltage source. The harmonic components will be generated in the core when the core becomes saturated.

1.3.6 HBFEM for a DC-Biased 3D Asymmetrical Magnetic Structure Simulation

In high-frequency switching power supplies, the leakage inductance, skin and proximity effects, winding self-capacitance and inter-winding capacitance can cause some serious problems in high-frequency transformers. Detailed information about the distribution of eddy currents, flux density and harmonics distribution in the magnetic core and windings has to be known when designing a high-frequency transformer. Furthermore, the nonlinear nature and hysteresis of the core material can cause waveform distortion. These distortions cause further harmonics, which will increase power losses in both the winding and magnetic core, resulting in a loss of efficiency, as well as the possibility of parasitic resonance in the system.

A typical port core transformer has an axi-symmetrical structure and a B-H hysteresis curve. The transformer is excited by quasi-sinusoidal waveforms, which includes AC fundamentals, harmonics and DC components. When the magnetic core becomes saturated, the waveforms will be distorted, and harmonics will be generated in the magnetic field and circuits. The hysteresis loss in winding is also increased, due to the effect of high-frequency harmonics. This kind of problem can be easily solved by using HBFEM [22].

1.3.7 HBFEM for a DC-Biased Problem in HV Power Transformers

A typical DC transmission system consists of a DC transmission line connecting two AC systems. A converter at one end of the line converts AC power into DC power, while a similar converter at the other end reconverts the DC power into AC power. One converter acts as a rectifier, the other as an inverter. The basic purpose of the converter transformer on the rectifier side is to transform the AC network voltage to yield the DC voltage required by the converter. Three-phase transformers, connected in either wye-wye or wye-delta, are used.

The model has a voltage-driven source connected to the magnetic system, which is always coupled to the external circuits. The current in the input circuits will be unknown, but saturation of the current waveform occurs because of the nonlinear characteristic of the magnetic core. Considering a three-phase transformer, connected in wye-wye, a computer simulation model with a neutral *NN* (and external circuits for both primary and secondary windings) is obtained using the HBFEM technique. According to the Galerkin procedure, system matrix equations of HBFEM for the HVDC transformer can be obtained through Faraday's and Kirchhoff's laws for the transformer with external circuits [23, 24].

During geomagnetic disturbances, variations in the geomagnetic field induce quasi-DC voltages in the network, which drive geomagnetically induced currents (GIC) along transmission lines and through transformer windings to ground wherever there is a path for them to flow. The flow of these quasi-DC currents in transformer windings causes half-cycle saturation of transformer cores, which leads to increased transformer hotspot heating, harmonic generation, and reactive power absorption – each of which can affect system reliability. As part of the assessment of geomagnetic disturbances (GMDs) impacts on the Bulk-Power System, it is necessary to model the GIC produced by different levels of geomagnetic activity [28–30]. The HBFEM can be used for solving geomagnetically-induced currents (GIC) and harmonic problem directly, while commercially available GIC modeling software packages cannot solve the harmonic problem. The detailed theory and numerical model for GIC modeling will be explained later in Chapters 3 and 6.

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