1 Principles of Electric Power Conversion

In this introductory chapter, fundamentals of power electronics are outlined, including the scope, tools, and applications of this area of electrical engineering. The concept of generic power converter is introduced to illustrate the operating principles of power electronic converters and the types of power conversion performed. Components of voltage and current waveforms, and the related figures of merit, are defined. Two basic methods of magnitude control, that is, phase control and pulse width modulation (PWM), are presented. Calculation of current waveforms is explained. The single-phase diode rectifier is described as the simplest power electronic converter.

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1.1 WHAT IS POWER ELECTRONICS?

Modern society with its conveniences strongly relies on the ubiquitous availability of electric energy. The electricity performs most of the physical labor, provides the heating and lighting, activates electrochemical processes, and facilitates information collecting, processing, storage, and exchange.

Power electronics can be defined as a branch of electrical engineering devoted to conversion and control of electric power, using electronic converters based on semiconductor power switches. The power grid delivers an ac voltage of fixed frequency and magnitude. Typically, homes, offices, stores, and other small facilities are supplied from single-phase, low-voltage power lines, while three-phase supply systems with various voltage levels are available in industrial plants and other large commercial enterprises. The 60-Hz (50-Hz in most other parts of the world) fixed-voltage electric power can be thought of as raw power, which for many applications must be conditioned. The power conditioning involves conversion, from ac to dc or vice-versa, and control of the magnitude and/or frequency of voltages and currents. Using the electric lighting as a simple example, an incandescent bulb can directly be supplied with the raw power. However, a fluorescent lamp requires electronic ballast that starts and stabilizes the electric arc. The ballast is thus a power conditioner, necessary for proper operation of the lamp. If used in a movie theater, the incandescent bulb mentioned before is supplied from an ac voltage controller that allows dimming

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of the light just before the movie begins. Again, this controller constitutes an example of power conditioner, or power converter.

Raw dc power is usually supplied from batteries and, increasingly, from photovoltaic sources and fuel cells. Photovoltaic energy systems are usually connected to the grid, and the necessary power conditioning involves dc-to-ac voltage conversion and control of the ac voltage. If a dc source feeds an electric motor, as in a golf cart or an electric wheelchair, a power electronic converter between the battery and the motor performs voltage control and facilitates reverse power flow during braking or downhill ride.

The birth of power electronics can be traced back to the dawn of twentieth century when the first mercury arc rectifiers were invented. However, for conversion and control of electric power, *rotating electro-machine converters* were mostly used in the past. An electro-machine converter was an electric generator driven by an electric motor. If, for instance, adjustable dc voltage was to be obtained from fixed ac voltage, an ac motor operated a dc generator with controlled output voltage. Conversely, if ac voltage was required and the supply energy came from a battery pack, a speed-controlled dc motor and an ac synchronous generator were employed. Clearly, the convenience, efficiency, and reliability of such systems were inferior in comparison with today's *static power electronic converters* performing motionless energy conversion and control.

Today's power electronics has begun with the development of the *silicon controlled rectifier* (*SCR*), also called a *thyristor*, by the General Electric Company in 1958. The SCR is a unidirectional semiconductor power switch that can be turned on ("closed") by a low-power electric pulse applied to its controlling electrode, the gate. The available voltage and current ratings of SCRs are very high, but the SCR is inconvenient for use in dc-input power electronic converters. It is a *semi-controlled* switch, which when conducting current cannot be turned off ("opened") by a gate signal. Within the last few decades, several kinds of *fully controlled* semiconductor power switches that can be turned on and off have been introduced to the market.

Widespread introduction of power electronic converters to most areas of distribution and usage of electric energy is common for all developed countries. The converters condition the electric power for a variety of applications, such as electric motor drives, uninterruptable power supplies, heating and lighting, electrochemical and electro-thermal processes, electric arc welding, high-voltage dc transmission lines, active power filters and reactive power compensators in power systems, and high-quality supply sources for computers and other electronic equipment.

It is estimated that at least half of the electric power generated in the USA flows through power electronic converters, and an increase of this share to close to 100% in the next few decades is expected. In particular, a thorough revamping of the existing US national power grid is envisioned. Introduction of power electronic converters to all stages of the power generation, transmission, and distribution, coupled with extensive information exchange ("smart grid"), allows a dramatic increase of the grid's capabilities without investing in new power plants and transmission lines. The important role of power electronics in renewable energy systems and electric and hybrid vehicles is also worth stressing. It is safe to say that practically every electrical engineer encounters some power electronic converters in his/her professional career.

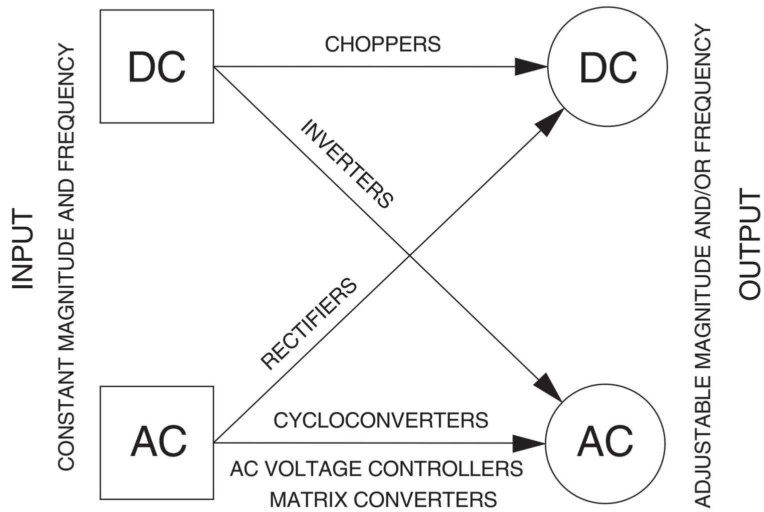

Figure 1.1 Types of electric power conversion and the corresponding power electronic converters.

Types of electric power conversion and the corresponding converters are presented in Figure 1.1. For instance, the ac-to-dc conversion is accomplished using rectifiers, which are supplied from an ac source and whose output voltage contains a fixed or adjustable dc component. Individual kinds of power electronic converters are described and analyzed in Chapters 4 through 8. Basic principles of power conversion and control are explained in the following sections of this chapter.

1.2 GENERIC POWER CONVERTER

Though not a practical apparatus, the hypothetical *generic power converter* shown in Figure 1.2 is a useful tool for illustration of the principles of electric power conversion and control. It is a two-port network of five switches. Switches S1 and

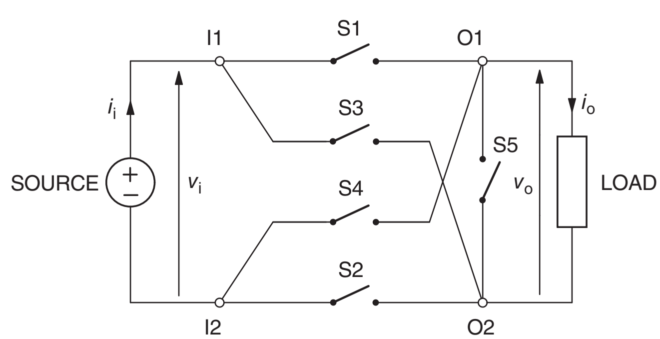

Figure 1.2 Generic power converter.

S2 provide *direct connection* between the input (supply) terminals, I1 and I2, and the output (load) terminals, O1 and O2, respectively, while switches S3 and S4 allow *cross connection* between these pairs of terminals. A voltage source, either dc or ac, supplies the electric power to a load through the converter. Practical loads often contain a significant inductive component, so a resistive–inductive (RL) load is assumed in the subsequent considerations. To ensure a closed path for the load current under any operating conditions, a fifth switch, S5, is connected between the output terminals of the converter and closed when switches S1 through S4 are open. It is assumed that the switches open or close instantaneously.

The supply source is an ideal voltage source and as such it may not be shorted. Also, the load current may not be interrupted. As the voltage across inductance is proportional to the rate of change of current, a rapid drop of that current would cause a high and potentially damaging overvoltage. Therefore, the generic converter can only assume the following three states:

State 0: Switches S1 through S4 are open and switch S5 is closed, shorting the output terminals and closing a path for the lingering load current, if any. The output voltage is zero. The input terminals are cut off from the output terminals so that the input current is also zero.

State 1: Switches S1 and S2 are closed, and the remaining ones are open. The output voltage equals the input voltage and the output current equals the input current.

State 2: Switches S3 and S4 are closed, and the remaining ones are open. Now, the output voltage and current are reversed with respect to their input counterparts.

Let us assume that the generic converter is to perform the ac-to-dc conversion. The sinusoidal input voltage, $v_{\rm i}$, whose waveform is shown in Figure 1.3, is given by

$$v_{\rm i} = V_{\rm i,p} \sin(\omega t), \tag{1.1}$$

where $V_{\rm i,p}$ denotes the peak value of that voltage and ω is the input radian frequency. The output voltage, $v_{\rm o}$, of the converter should contain a possibly large dc component.

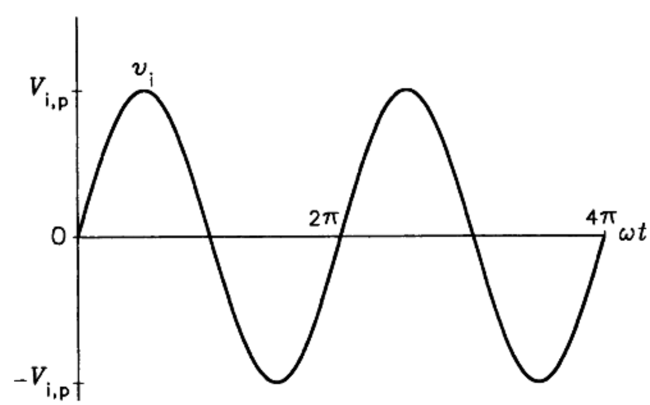

Figure 1.3 Input ac voltage waveform.

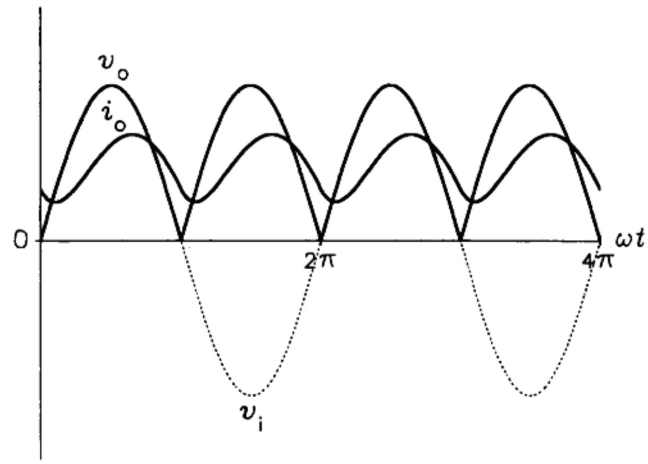

Figure 1.4 Output voltage and current waveforms in the generic rectifier.

Note that the output voltage is not expected to be of ideal dc quality, since such voltage and current waveforms are not feasible in the generic converter, as well as in practical power electronic converters. The same applies to the ideally sinusoidal output voltage and current in ac output converters. If within the first half-cycle of the input voltage, the converter is in state 1, and within the second half-cycle in state 2, the output voltage waveform will be such as depicted in Figure 1.4, that is,

$$v_o = |v_i| = V_{i,p}|\sin(\omega t)|. \tag{1.2}$$

The dc component is the average value of the voltage. Power electronic converters performing the ac-to-dc conversion are called *rectifiers*.

The output current waveform, i_o, can be obtained as a numerical solution of the load equation:

$$L\frac{di_o}{dt} + Ri_o = v_o. \tag{1.3}$$

Techniques for analytical and numerical computation of voltage and current waveforms in power electronic circuits are described at the end of this chapter. Here, only general features of the waveforms are outlined. The output current waveform of the considered generic rectifier is also shown in Figure 1.4. It can be seen that this waveform is closer to an ideal dc waveform than is the output voltage waveform because of the frequency-dependent load impedance. The kth harmonic, $v_{o,k}$, of the output voltage produces the corresponding harmonic, $i_{o,k}$, of the output current such that

$$I_{o,k} = \frac{V_{o,k}}{\sqrt{R^2 + (k\omega_o L)^2}}, \tag{1.4}$$

where $I_{o,k}$ and $V_{o,k}$ denote root mean square (rms) values of the current and voltage harmonics in question, respectively. In the considered rectifier, the fundamental radian frequency, ω_o, of the output voltage is twice as high as the input frequency, ω. The load impedance (represented by the denominator at the right-hand side of Eq. 1.4) for individual current harmonics increases with the harmonic number, k. Clearly, the dc component ($k = 0$) of the output current encounters the lowest impedance, equal to the load resistance only, while the load inductance attenuates only the ac component. In other words, the RL load acts as a low-pass filter.

Interestingly, if an ac output voltage is to be produced and the generic converter is supplied from a dc source, so that the input voltage is $v_i = V_i =$ const., the switches are operated in the same manner as in the previous case. Specifically, for every half period of the desired output frequency, states 1 and 2 are interchanged. In this way, the input terminals are alternately connected and cross-connected with the output terminals, and the output voltage acquires the ac (although not sinusoidal) waveform shown in Figure 1.5. The output current is composed of growth-function and decay-function segments, typical for transient conditions of an RL circuit subjected to dc excitation. Again, thanks to the attenuating effects of the load inductance, the current waveform is closer to the desired sinusoid than is the voltage waveform. In practice, the dc-to-ac power conversion is performed by power electronic *inverters*. In the case described, the generic inverter is said to operate in the *square-wave mode*.

If the input or output voltage is to be a three-phase ac voltage, the topology of the generic power converter portrayed here would have to be expanded, but it still would be a network of switches. Real power electronic converters are *networks of semiconductor power switches*, too. For various purposes, other elements, such as inductors, capacitors, fuses, and auxiliary circuits, are employed besides the switches in power circuits of practical power electronic converters. Yet, in most of these converters, the fundamental operating principle is the same as in the generic converter, that is, the

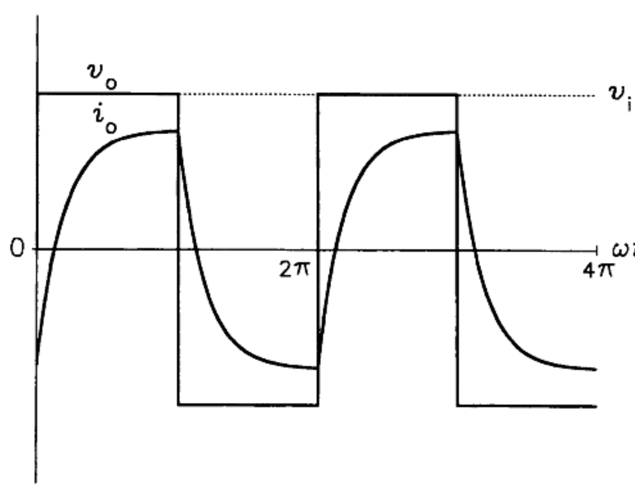

Figure 1.5 Output voltage and current waveforms in the generic inverter.

input and output terminals are being connected, cross-connected, and disconnected in a specific manner and sequence required for the given type of power conversion. Typically, as in the generic rectifier and inverter presented, the load inductance inhibits the switching-related undesirable high-frequency components of the output current.

Although a *voltage source* has been assumed for the generic power converter, some power electronic converters are supplied from *current sources*. In such converters, a large inductor is connected in series with the input terminals to prevent rapid changes of the input current. Analogously, voltage-source converters usually have a large capacitor connected across the input terminals to stabilize the input voltage. Inductors or capacitors are also used at the output of some converters to smooth the output current or voltage, respectively.

According to one of the tenets of circuit theory, two unequal ideal current sources may not be connected in series and two unequal ideal voltage sources may not be connected in parallel. Consequently, the load of a current-source converter may not appear as a current source while that of a voltage-source converter as a voltage source. As illustrated in Figure 1.6, it means that in a current-source power electronic converter a capacitor should be placed in parallel with the load. In addition to smoothing the output voltage, the capacitor prevents the potential hazards of connecting the input inductance conducting certain current with a load inductance conducting different current. In contrast, in voltage-source converters, no capacitor may be connected across the output terminals and it is the load inductance, or an extra inductor between the converter and the load, that is smoothing the output current.

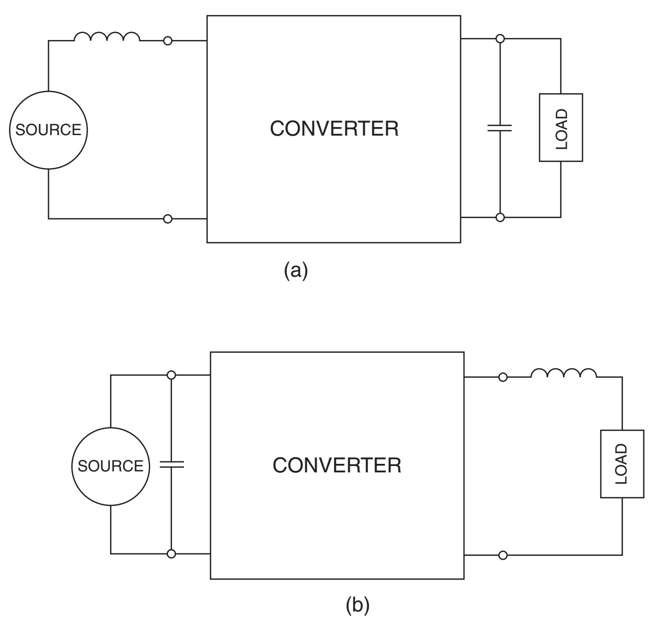

Figure 1.6 Basic configurations of power electronic converters: (a) current source, (b) voltage source.

1.3 WAVEFORM COMPONENTS AND FIGURES OF MERIT

Terms such as the "dc component," "ac component," and "harmonics" mentioned in the preceding section deserve closer examination. Knowledge of the basic components of voltage and current waveforms allows evaluation of performance of a converter. Certain relations of these components are commonly used as performance indicators, or *figures of merit*.

A time function, $\psi(t)$, here a waveform of voltage or current, is said to be *periodic* with a period T if

$$\psi(t) = \psi(t + T), \tag{1.5}$$

that is, if the pattern (shape) of the waveform is repeated every T seconds. In the realm of power electronics, it is often convenient to analyze voltages and currents in the *angle domain* instead of the *time domain*. The so-called *fundamental frequency*, f_1, in Hz, is defined as

$$f_1 = \frac{1}{T}, \tag{1.6}$$

and the corresponding *fundamental radian frequency*, ω, in rad/s, as

$$\omega = 2\pi f_1 = \frac{2\pi}{T}. \tag{1.7}$$

Now, a periodic function $\psi(\omega t)$ can be defined as such that

$$\psi(\omega t) = \psi(\omega t + 2\pi). \tag{1.8}$$

The *rms value*, Ψ, of waveform $\psi(\omega t)$ is defined as

$$\Psi \equiv \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \psi^2(\omega t) d\omega t}, \tag{1.9}$$

and the *average value*, or *dc component*, Ψ_{dc}, of the waveform as

$$\Psi_{dc} \equiv \frac{1}{2\pi} \int_0^{2\pi} \psi(\omega t) d\omega t. \tag{1.10}$$

When the dc component is subtracted from the waveform, the remaining waveform, $\psi_{ac}(\omega t)$, is called the *ac component*, or *ripple*, that is,

$$\psi_{ac}(\omega t) = \psi(\omega t) - \Psi_{dc}. \tag{1.11}$$

The ac component has an average value of zero and the fundamental frequency of f_1.

The rms value, Ψ_{ac}, of $\psi_{ac}(\omega t)$ is defined as

$$\Psi_{ac} \equiv \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \psi_{ac}^2(\omega t)d\omega t}, \tag{1.12}$$

and it can be shown that

$$\Psi^2 = \Psi_{dc}^2 + \Psi_{ac}^2. \tag{1.13}$$

For waveforms of the desirable ideal dc quality, such as the load current of a rectifier, a figure of merit called a *ripple factor*, *RF*, is defined as

$$\mathrm{RF} = \frac{\Psi_{ac}}{\Psi_{dc}}. \tag{1.14}$$

A low value of the ripple factor indicates high quality of a waveform.

Before proceeding to other waveform components and figures of merit, the terms and formulas introduced so far will be illustrated using the waveform of output voltage, v_o, of the generic rectifier, shown in Figure 1.4. The waveform pattern is repeating itself every π radians and, within the 0 to π interval, $v_o = v_i$. Therefore, the average value, $V_{o,dc}$, of the output voltage can most conveniently be determined by calculating the area under the waveform from $\omega t = 0$ to $\omega t = \pi$ and dividing it by the length, π, of the considered interval. Thus,

$$V_{o,dc} = \frac{1}{\pi}\int_0^{\pi} V_{i,p}\sin(\omega t)d\omega t = \frac{2}{\pi}V_{i,p} = 0.64V_{i,p}. \tag{1.15}$$

Note that the formula above differs from Eq. (1.10). Since $\omega_1 = \omega_o = 2\omega$, the integration is performed in the 0 to π interval of ωt instead of the 0 to 2π interval of $\omega_1 t$.

Similarly, the rms value, V_o, of the output voltage can be calculated as

$$V_o = \sqrt{\frac{1}{\pi}\int_0^{\pi}[V_{i,p}\sin(\omega t)]^2 d\omega t} = \frac{V_{i,p}}{\sqrt{2}} = 0.71V_{i,p}. \tag{1.16}$$

The result in Eq. (1.16) agrees with the well-known relation for a sine wave as ${v_o}^2 = {v_i}^2$.

Based on Eqs. (1.13) and (1.14), the rms value, $V_{\mathrm{o,ac}}$, of ac component of the voltage in question can be calculated as

$$V_{\mathrm{o,ac}} = \sqrt{V_{\mathrm{o}}^2 - V_{\mathrm{o,dc}}^2} = \sqrt{\left(\frac{V_{\mathrm{i,p}}}{\sqrt{2}}\right)^2 - \left(\frac{2}{\pi}V_{\mathrm{i,p}}\right)^2} = 0.31V_{\mathrm{i,p}}, \tag{1.17}$$

and the ripple factor, $\mathrm{RF_V}$, of the voltage as

$$\mathrm{RF_V} = \frac{V_{\mathrm{o,ac}}}{V_{\mathrm{o,dc}}} = \frac{0.31V_{\mathrm{i,p}}}{0.64V_{\mathrm{i,p}}} = 0.48. \tag{1.18}$$

Decomposition of the analyzed waveform into the dc and ac components is shown in Figure 1.7.

To analytically determine the ripple factor, $\mathrm{RF_I}$, of the output current, the output current waveform, $i_{\mathrm{o}}(\omega t)$, would have to be expressed in a closed form. Instead, numerical computations were performed on the waveform in Figure 1.4, and $\mathrm{RF_I}$ was found to be 0.31. This value is 36% lower than that of the output voltage. This is an example only, but output currents in power electronic converters routinely have higher quality than the output voltages. It is worth mentioning that the obtained value of $\mathrm{RF_I}$ is poor. Practical high-quality dc current waveforms have the ripple factor in the order of few percentage points, and below the 5% level the current is considered as ideal. The current ripple factor depends on the type of converter, and it decreases with an increase in the inductive component of the load. Components of the current waveform evaluated are shown in Figure 1.8.

The ripple factor is of no use for quality evaluation of ac waveforms, such as the output current of an inverter, which ideally should be pure sinusoids. However, as

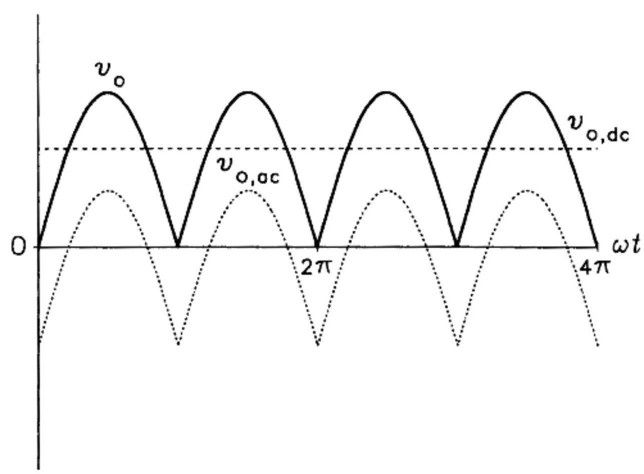

Figure 1.7 Decomposition of the output voltage waveform in the generic rectifier.

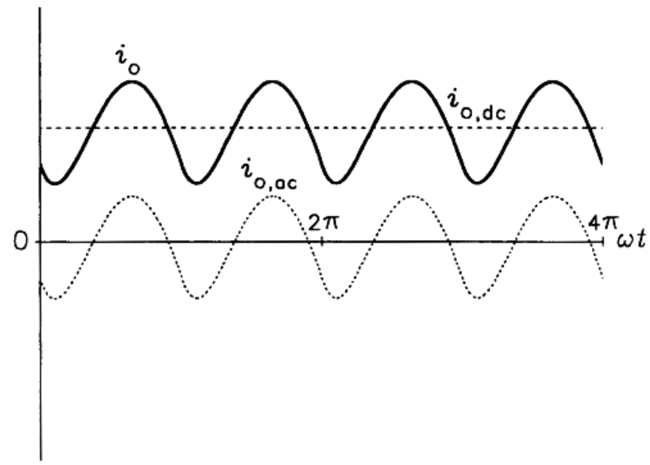

Figure 1.8 Decomposition of the output current waveform in the generic rectifier.

already mentioned and exemplified by the waveforms in Figure 1.5, purely sinusoidal voltages and currents cannot be produced by switching power converters. Therefore, an appropriate figure of merit must be defined as a measure of deviation of a practical ac waveform from its ideal counterpart.

Following the theory of Fourier series (see Appendix B), the ac component, $\psi_{ac}(t)$, of a periodic function, $\psi(t)$, can be expressed as an infinite sum of harmonics, that is, sine waves whose frequencies are multiples of the fundamental frequency, f_1, of $\psi(t)$. In the angle domain,

$$\psi_{ac}(\omega t) = \sum_{k=1}^{\infty} \psi_k(k\omega t) = \sum_{k=1}^{\infty} \Psi_{k,p} \cos(k\omega t + \varphi_k), \tag{1.19}$$

where k is the *harmonic number*, and $\Psi_{k,p}$ and φ_k denote the peak value and phase angle of the kth harmonic, respectively. The first harmonic, $\psi_1(\omega t)$, is called a *fundamental*. Terms "fundamental voltage" and "fundamental current" are used throughout the book to denote the fundamental of a given voltage or current.

The peak value, $\Psi_{1,p}$, of fundamental of a periodic function, $\psi(\omega t)$, is calculated as

$$\Psi_{1,p} = \sqrt{\Psi_{1,c}^2 + \Psi_{1,s}^2}, \tag{1.20}$$

where

$$\Psi_{1,c} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega t) \cos(\omega t) d\omega t, \tag{1.21}$$

$$\Psi_{1,s} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega t) \sin(\omega t) d\omega t, \tag{1.22}$$

and the rms value, Ψ_1, of the fundamental is

$$\Psi_1 = \frac{\Psi_{1,p}}{\sqrt{2}}. \tag{1.23}$$

Since the fundamental of a function does not depend on the dc component of the function, the ac component, $\psi_{ac}(\omega t)$, can be used in Eqs. (1.21) and (1.22) in place of $\psi(\omega t)$.

When the fundamental is subtracted from the ac component, the so-called *harmonic component*, $\psi_h(\omega t)$, is obtained as

$$\psi_h(\omega t) = \psi_{ac}(\omega t) - \psi_1(\omega t). \tag{1.24}$$

The rms value, Ψ_h, of $\psi_h(\omega t)$, called a *harmonic content* of function $\psi(\omega t)$, can be calculated as

$$\Psi_h = \sqrt{\Psi_{ac}^2 - \Psi_1^2} = \sqrt{\Psi^2 - \Psi_{dc}^2 - \Psi_1^2} \tag{1.25}$$

and used for calculation of the so-called *total harmonic distortion*, *THD*, defined as

$$\text{THD} \equiv \frac{\Psi_h}{\Psi_1}. \tag{1.26}$$

The concept of total harmonic distortion is also widely employed outside power electronics as, for instance, in the characterization of quality of audio equipment. Conceptually, the total harmonic distortion constitutes an ac counterpart of the ripple factor.

In the generic inverter, whose output waveforms have been shown in Figure 1.6, the rms value, V_o, of output voltage equals the dc input voltage, V_i. Since v_o is either V_i or $-V_i$, $v_o{}^2 = V_i{}^2$. The peak value, $V_{o,1,p}$, of fundamental output voltage is

$$V_{o,1,p} = V_{o,1,s}, \tag{1.27}$$

because the waveform in question has the odd symmetry (see Appendix B). Consequently,

$$V_{o,1,p} = \frac{2}{\pi}\int_0^{\pi} V_i \sin(\omega t) d\omega t = \frac{4}{\pi} V_i = 1.27 V_i. \tag{1.28}$$

Now, the fundamental output voltage, $v_{o,1}(\omega t)$, can be expressed as

$$v_{o,1}(\omega t) = V_{o,1,p} \sin(\omega t) = \frac{4}{\pi} V_i \sin(\omega t). \tag{1.29}$$

The rms value, $V_{o,1}$, of fundamental output voltage is

$$V_{o,1} = \frac{V_{o,1,p}}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi} V_i = 0.9V_i, \tag{1.30}$$

and the harmonic content, $V_{o,h,}$ is

$$V_{o,h} = \sqrt{V_o^2 - V_{o,1}^2} = \sqrt{V_i^2 - \left(\frac{2\sqrt{2}}{\pi} V_i\right)^2} = 0.44V_i. \tag{1.31}$$

Thus, the total harmonic distortion of the output voltage, THD_V, is

$$THD_V = \frac{V_{o,h}}{V_{o,1}} = \frac{0.44V_i}{0.9V_i} = 0.49. \tag{1.32}$$

The high value of THD_V is not surprising since the output voltage waveform of the generic inverter operating in the square-wave mode significantly differs from a sine wave. Decomposition of the waveform analyzed is illustrated in Figure 1.9.

The numerically determined total harmonic distortion, THD_I, of the output current, i_o, in this example is 0.216, that is, less than that of the output voltage by as much as 55%. Indeed, as seen in Figure 1.10 that shows decomposition of the current waveform, the harmonic component is quite small in comparison with the fundamental. As in the generic rectifier, it shows the attenuating influence of the load inductance on the output current. In practical inverters, the output current is considered to be of high quality if THD_I does not exceed 0.05 (5%).

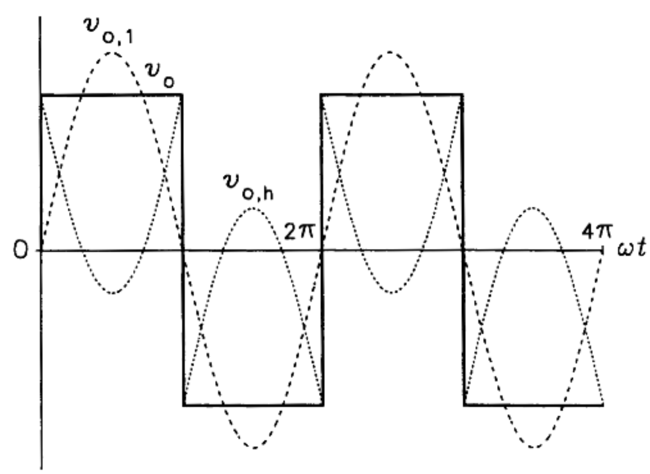

Figure 1.9 Decomposition of the output voltage waveform in the generic inverter.

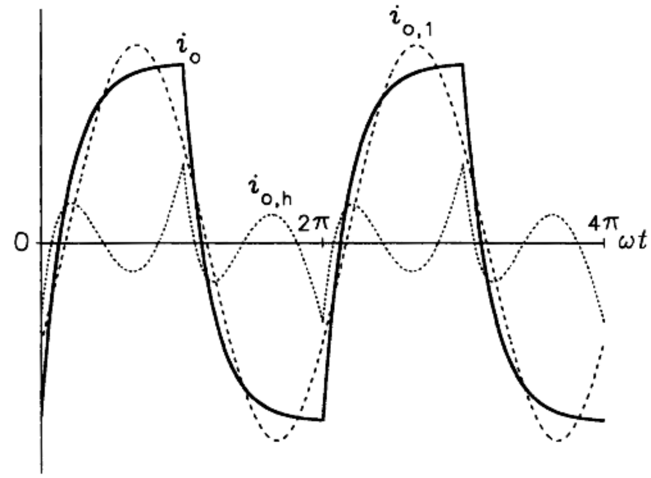

Figure 1.10 Decomposition of the output current waveform in the generic inverter.

Other figures of merit often employed for performance evaluation of power electronic converters are

(1) *Power efficiency*, η, of the converter is defined as

$$\eta \equiv \frac{P_o}{P_i}, \tag{1.33}$$

where P_o and P_i denote the output and input powers of the converter, respectively.

(2) *Conversion efficiency*, η_c, of the converter is defined as

$$\eta_c \equiv \frac{P_{o,dc}}{P_i} \tag{1.34}$$

for dc output converters, and

$$\eta_c \equiv \frac{P_{o,1}}{P_i} \tag{1.35}$$

for ac output converters. Symbol $P_{o,dc}$ denotes the dc output power, that is, the product of the dc components of the output voltage and current, while $P_{o,1}$ is the ac output power carried by the fundamental components of the output voltage and current.

(3) *Input power factor*, PF, of the converter is defined as

$$\mathrm{PF} = \frac{P_\mathrm{i}}{S_\mathrm{i}}, \tag{1.36}$$

where S_i is the apparent input power. The power factor can also be expressed as

$$\mathrm{PF} = K_\mathrm{d} K_\Theta. \tag{1.37}$$

Here, K_d denotes the so-called *distortion factor* (not to be confused with the total harmonic distortion, THD), defined as the ratio of the rms fundamental input current, $I_{\mathrm{i},1}$, to the rms input current, I_i, and K_Θ is the *displacement factor*, that is, cosine of the phase shift, Θ, between the fundamentals of input voltage and current.

The power efficiency, η, of a converter simply indicates what portion of the power supplied to the converter reaches the load. In contrast, the conversion efficiency, η_c, expresses the relative amount of *useful* output power and, therefore, constitutes a more valuable figure of merit than the power efficiency. Since the input voltage to a converter is usually constant, the power factor serves mainly as a measure of utilization of the input current, drawn from the source that supplies the converter. With a constant power consumed by the converter, a high power factor implies a low current and, consequently, low power losses in the source. Most likely, the reader is familiar with the term "power factor" as the cosine of the phase shift between the voltage and current waveforms, as used in the theory of ac circuits. However, it must be stressed that it is true for purely sinusoidal waveforms only, and the general definition of the power factor is given by Eq. (1.36).

In an ideal power converter, all the three figures of merit defined above would be in equal unity. To illustrate the relevant calculations, the generic rectifier will again be employed. Since ideal switches have been assumed, no losses are incurred in the generic converter, so that the input power, P_i, and output power, P_o, are equal and the power efficiency, η, of the converter is always unity.

For the RL-load assumed for the generic rectifier, it can be shown that the conversion efficiency, η_c, is a function of the current ripple factor, $\mathrm{RF_I}$. Specifically,

$$\begin{aligned}\eta_\mathrm{c} &= \frac{P_\mathrm{o,dc}}{P_\mathrm{i}} = \frac{P_\mathrm{o,dc}}{\frac{P_\mathrm{o}}{\eta}} = \eta\frac{RI^2_\mathrm{o,dc}}{RI^2_\mathrm{o}} = \eta\frac{I^2_\mathrm{o,dc}}{I^2_\mathrm{o,dc} + I^2_\mathrm{o,ac}}\\ &= \frac{\eta}{1 + \left(\frac{I_\mathrm{o,ac}}{I_\mathrm{o,dc}}\right)^2} = \frac{\eta}{1 + (\mathrm{RF_I})^2},\end{aligned} \tag{1.38}$$

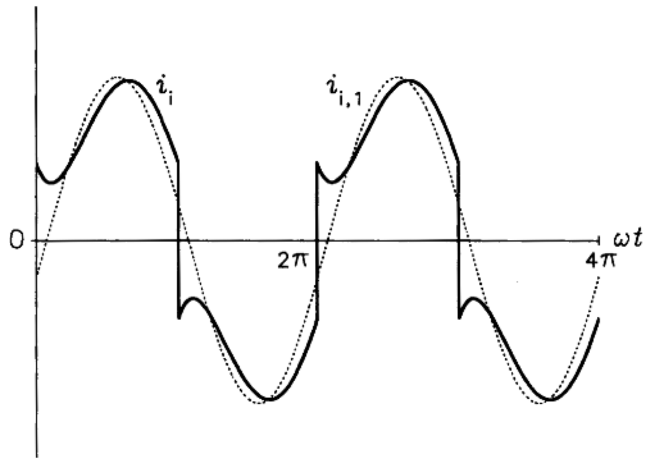

Figure 1.11 Decomposition of the input current waveform in the generic rectifier.

where R is the load resistance. The ripple factor for the example current in Figure 1.4 has already been found to be 0.307. Hence, with $\eta = 1$, the conversion efficiency is $1/(1 + 0.307^2) = 0.914$.

The input current waveform, $i_i(\omega t)$, of the rectifier is shown in Figure 1.11 with the numerically found fundamental, $i_{i,1}(\omega t)$. The converter either directly passes the input voltage and current to the output or inverts them. Therefore, the rms values of input voltage, V_i, and current, I_i, are equal to those of the output voltage, V_o, and current, I_o. The apparent input power, S_i, is a product of the rms values of input voltage and current. Hence,

$$PF = \frac{P_i}{S_i} = \frac{\frac{P_o}{\eta}}{V_i I_i} = \frac{RI_o^2}{\eta V_o I_o} = \frac{RI_o}{\eta V_o}, \tag{1.39}$$

and specific values of R, I_o, and V_o are needed to determine the power factor. If, for example, the peak input voltage, $V_{i,p}$, in the example rectifier described is 100 V and the load resistance, R, is 1.3 Ω, then, according to Eq. (1.16), the dc output voltage, V_o, is 70.7 V. The numerically computed rms output current, I_o, is 51.3 A. Consequently, PF = (1.3 × 51.3)/(1 × 70.7) = 0.943 (lag), the “lag” indicating that the fundamental input current lags the fundamental input voltage (compare Figures 1.1 and 1.11).

1.4 PHASE CONTROL AND SQUARE-WAVE MODE

Based on the idea of generic converter, whose switches connect, cross-connect, or disconnect the input and output terminals, and short the output terminals in the last case, the principles of the ac-to-dc and dc-to-ac power conversion were explained in

Section 1.2. The question how to control the magnitude of the output voltage and, consequently, that of the output current has not yet been answered though.

The reader is likely familiar with electric transformers and autotransformers that allow magnitude regulation of ac voltage and current. These are heavy and bulky apparatus designed for a fixed frequency and impractical for wide-range magnitude control. Moreover, their principle of operation inherently excludes transformation of dc quantities. In the early days of electrical engineering, adjustable resistors were predominantly employed for voltage and current control. Today, the *resistive control* can still be encountered in relay-based starters for electric motors and obsolete adjustable-speed drive systems. On the other hand, small rheostats and potentiometers are still widely used in low-power electric and electronic circuits, in which the power efficiency is not of major importance.

Resistive control does not have to involve real resistors. Actually, any of the existing transistor-type power switches could serve this purpose. Between the state of saturation, in which a transistor offers minimum resistance in the collector–emitter path, and the blocking state resulting in practically zero collector and emitter currents, a wide range of intermediate states is available. Therefore, such a switch can be viewed as a controlled resistor and one may wonder if, for instance, the transistor switches used in power electronic converters could be operated in the same way as are the transistors in low-power analog electronic circuits.

To show why the resistive control *should not* be used in high-power applications, two basic schemes, depicted in Figure 1.12, will be considered. For simplicity, it is assumed that the circuits shown are to provide control of a dc voltage supplied to a resistive load. The dc input voltage, V_i, is constant, while the output voltage, V_o, is to be adjustable within the zero to V_i range. Generally, for power converters with a controlled output quantity (voltage or current), the so-called *magnitude control ratio*, M, can be defined as

$$M \equiv \frac{\Psi_{o,adj}}{\Psi_{o,adj(max)}}, \tag{1.40}$$

where $\Psi_{o,adj}$ denotes the value of the adjustable component of the output quantity, for example, the dc component of the output voltage in a controlled rectifier, while $\Psi_{o,adj(max)}$ is the maximum available value of this component. Usually, it is required

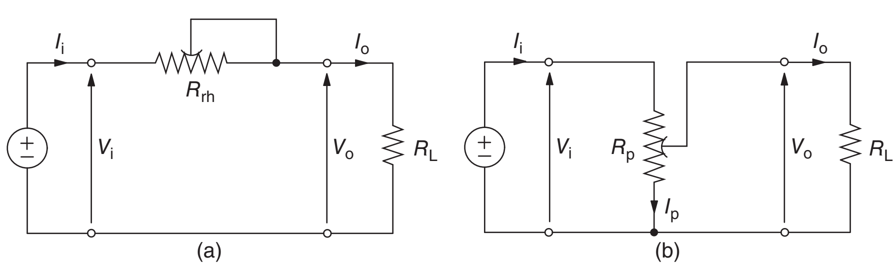

Figure 1.12 Resistive control schemes: (a) rheostatic control, (b) potentiometric control.

that M be adjustable from certain minimum value to unity. In certain converters, the minimum value can be negative, down to –1, implying polarity reversal of the controlled variable.

The magnitude control ratio should not be confused with the so-called *voltage gain*, K_V, which, generally, represents the ratio of the output voltage to the input voltage. Specifically, the average value is used for dc voltages and the peak value of the fundamental for ac voltages. Hence, for instance, the voltage gain of a rectifier is defined as the ratio of the dc output voltage, $V_{o,dc}$, to the peak input voltage, $V_{i,p}$. In the resistive control schemes considered, $K_V = V_o/V_i$ and, since the maximum available value of the output voltage equals V_i, the voltage gain equals the magnitude control ratio, M.

Figure 1.12a illustrates the *rheostatic control*. The active part, R_{rh}, of the controlling rheostat forms a voltage divider with the load resistance, R_L. Here,

$$M = \frac{V_o}{V_i} = \frac{R_L}{R_{rh} + R_L}, \tag{1.41}$$

and since the input current, I_i, equals the output current, I_o, the efficiency, η, of the power transfer from the source to the load is

$$\eta = \frac{R_L I_o^2}{(R_{rh} + R_L)I_i^2} = \frac{R_L I_o^2}{(R_{rh} + R_L)I_o^2} = \frac{R_L}{R_{rh} + R_L} = M. \tag{1.42}$$

The identity relation between η and M is a serious drawback of the rheostatic control as decreasing the output voltage causes an equal reduction of the efficiency.

The *potentiometric control*, shown in Figure 1.12b and based on the principle of current division, fares even worse. Note that the input current, I_i, is greater than the output current, I_o, by the amount of the potentiometer current, I_p. The power efficiency, η, is

$$\eta = \frac{V_o I_o}{V_i I_i} = M\frac{I_o}{I_i}, \tag{1.43}$$

that is, less than M.

It can be seen that the main trouble with the resistive control is that the load current flows through the controlling resistance. As a result, power is lost in that resistance, and the power efficiency is reduced to a value equal to, or less than, the magnitude control ratio. In practical power electronic systems, this is unacceptable. Imagine, for example, a 120-kVA converter (not excessively large against today's standards) that at $M = 0.5$ loses so much power in the form of heat as do 40 typical, 1.5-kW domestic heaters! Efficiencies of power electronic converters are seldom lower than 90% in low-power converters and exceed 95% in high-power ones. Apart from the economic considerations, large power losses in a converter would require an extensive cooling system. Even in the contemporary high-efficiency power conversion schemes, the cooling is often quite a problem since the semiconductor power switches are of relatively small size and, consequently, of limited thermal capacity. Therefore, they tend to overheat quickly if cooling is inadequate.

The resistive control allows adjustment of the *instantaneous values* of voltage and current, which is important in many applications, for example, those requiring amplification of analog signals, such as radio, TV, and tape recorder. There, transistors and operational amplifiers operate on the principle of resistive control, and because of the low levels of power involved, the low efficiency is of a minor concern. In power electronic converters, as illustrated later, it is sufficient to control the *average value* of dc waveforms and *rms value* of ac waveforms. This can be accomplished by periodic application of state 0 of the converter (see Section 1.2), in which the connection between the input and output is broken and the output terminals are shorted. In this way, the output voltage is made zero within specified intervals of time and, depending on the length of zero intervals, its average or rms value is more or less reduced compared to that of the full waveform.

Clearly, the mode of operation described can be implemented by appropriate use of switches of the converter. Note that there are no power losses in *ideal* switches, because when a switch is on (closed), there is no voltage across it, while when it is off (open), there is no current through it. For this reason, both the power conversion and the control in power electronic converters are accomplished by means of switching. Analogously to the switches in the generic power converter, the semiconductor devices used in practical converters are allowed to assume two states only. The device is either fully conducting, with a minimum voltage drop between its main electrodes (on-state), or fully blocking, with a minimum current passing between these electrodes (off-state). That is why the term “semiconductor power switches” is used for the devices employed in power electronic converters.

The only major difference between the ideal switches in the hypothetical generic converter in Figure 1.2 and practical semiconductor switches is in the unidirectionality of the latter devices. In the on-state, a current in the switch can only flow in one direction, for instance, from the anode to the cathode in an SCR. Therefore, an idealized semiconductor power switch can be thought of as a series connection of an ideal switch and an ideal diode.

Historically, for a major part of twentieth century, only semicontrolled power switches, such as mercury arc rectifiers, gas tubes (thyratrons), and SCRs, had been available for power-conditioning purposes. As mentioned in Section 1.1, a semicontrolled switch once turned on (“fired”) cannot be turned off (“extinguished”) as long as the conducted current drops below certain minimum level for a sufficient amount of time. This condition is required for extinguishing the arc in the thyratrons and mercury arc rectifiers, or for the SCRs to recover the blocking capability. If a switch operates in an ac circuit, the turn-off occurs naturally when the current changes polarity from positive to negative. After a turn-off, a semicontrolled switch must be re-fired in every cycle when the anode–cathode voltage becomes positive, that is, when the switch becomes *forward biased*.

As the forward bias of a switch in an ac input power electronic converter lasts a half-cycle of the input voltage, the firing can be delayed by up to a half-cycle from the instant when the bias changes from reverse to forward. This creates an opportunity for

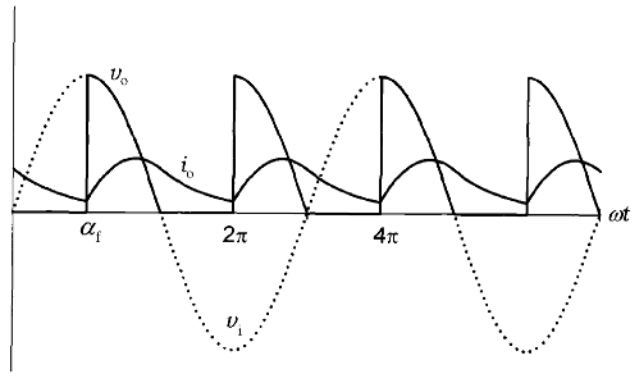

Figure 1.13 Output voltage and current waveforms in the generic rectifier with the firing angle of 90°.

controlling the average or rms value of output voltage of the converter. To demonstrate this method, the generic power converter will again be employed. For simplicity, the firing delay of the converter switches, previously assumed zero, is now set to a quarter of the period of the ac input voltage, that is, 90° in the angle domain.

Controlled ac-to-dc power conversion is illustrated in Figure 1.13. As explained in Section 1.2, when switches S1 through S4 of the generic converter are open, switch S5 must close to provide a path for the load current which, because of the inductance of the load, may not be interrupted. Thus, states 1 and 2 of the converter are separated by state 0. It can be seen that the sinusoidal half-waves of the output voltage in Figure 1.4 have been replaced with quarter-waves. As a result, the dc component of the output voltage has been reduced by 50%. Clearly, a longer delay in closing switches S1–S2 and S3–S4 would further reduce this component until, with the delay of 180°, it would drop to zero. The generic power converter operates now as the *controlled rectifier*. Practical controlled rectifiers based on SCRs do not need to employ an equivalent of switch S5. As already explained, an SCR cannot be turned off when conducting a current. Therefore, state 1 can only be terminated by switching to state 2, and the other way round, so that one pair of switches takes over the current from another pair. Both these states provide for the closed path for the output current. State 0, if any, occurs only when the output current has died out.

In the angle domain, the firing delay is referred to as a *delay angle*, or *firing angle*, and the method of output voltage control described is called the *phase control*, since the firing occurs at a specified phase of the input voltage waveform. In practice, the phase control is limited to power electronic converters based on SCRs. Fully controlled semiconductor switches allow more effective control by means of the so-called *pulse width modulation* (*PWM*), described in the next section.

The voltage control characteristic, $V_{o,dc}(\alpha_f)$, of the generic controlled rectifier can be determined as

$$V_{o,dc}(\alpha_f) = \frac{1}{\pi}\int_0^{\pi} v_o(\omega t)d\omega t = \frac{1}{\pi}\int_{\alpha_f}^{\pi} V_{i,p}\sin(\omega t)d\omega t = \frac{V_{i,p}}{\pi}[1 + \cos(\alpha_f)]. \tag{1.44}$$

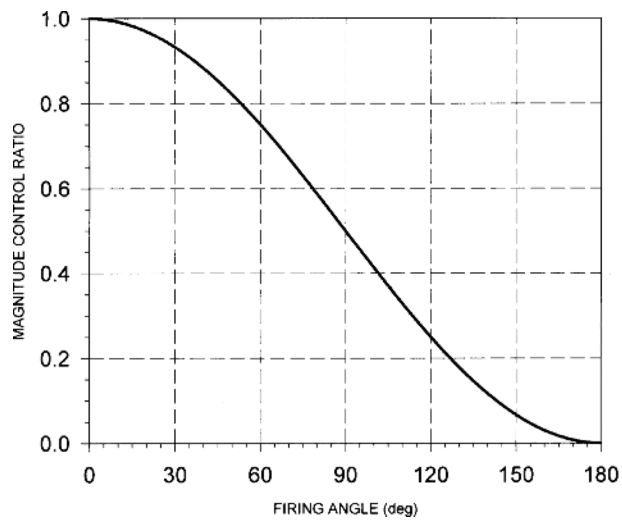

Figure 1.14 Control characteristic of the generic phase-controlled rectifier.

If $\alpha_f = 0$, Eq. (1.44) becomes identical with Eq. (1.15). The control characteristic, which is nonlinear, is shown in Figure 1.14.

The ac-to-ac conversion performed by the generic converter for adjusting the rms value of an ac output voltage is illustrated in Figure 1.15. Power electronic converters used for this type of power conditioning are called *ac voltage controllers*. Practical ac voltage controllers are mostly based on the so-called *triacs*, whose internal structure is equivalent to two SCRs connected in antiparallel. Phase-controlled ac voltage controllers do not require a counterpart of switch S5.

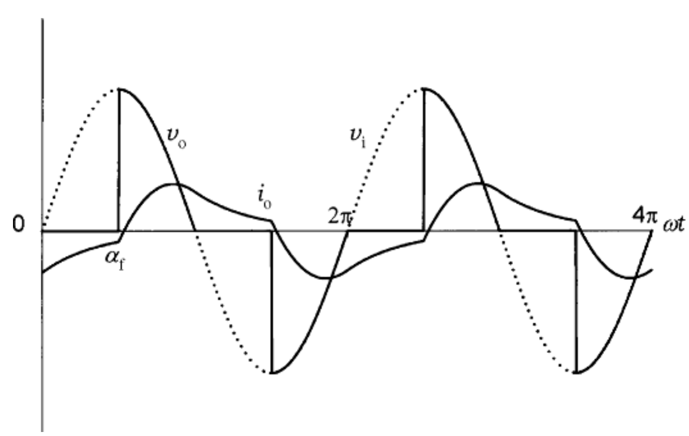

Figure 1.15 Output voltage and current waveforms in the generic ac voltage controller with the firing angle of 90°.

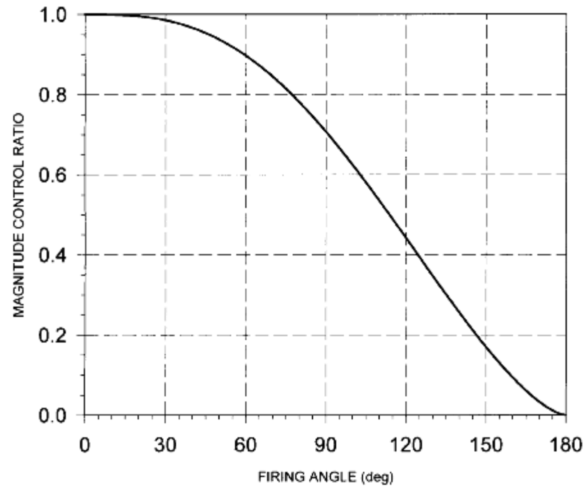

Figure 1.16 Control characteristic of the phase-controlled generic ac voltage controller.

The voltage control characteristic, $V_o(\alpha_f)$, of a generic ac voltage controller, given by

$$V_o(\alpha_f) = \sqrt{\frac{1}{\pi}\int_0^{\pi} v_o^2(\omega t)d\omega t} = \sqrt{\frac{1}{\pi}\int_{\alpha_f}^{\pi}[V_{i,p}\sin(\omega t)]^2 d\omega t}$$
$$= V_{i,p}\sqrt{\frac{1}{2\pi}\left[\pi - \alpha_f + \frac{\sin 2\alpha_f}{2}\right]} \qquad (1.45)$$

is shown in Figure 1.16. Again, the characteristic is nonlinear. The fundamental output voltage, $V_{o,1}$, can be shown to depend nonlinearly on the firing angle, too.

The concept of using the zero-output state 0 to control the magnitude of output voltage can be extended to the generic inverter. Analogously to the firing angle α_f, the active states 1 and 2 can be delayed by the *delay angle* α_d (the same name is also used as an alternative to "firing angle" in ac input converters). The resultant *square-wave mode* is illustrated in Figure 1.17 for the delay angle of 30°. Using the Fourier series, the rms value, $V_{o,1}$, of the fundamental output voltage is found to be

$$V_{o,1} = \frac{2\sqrt{2}}{\pi} V_i \cos(\alpha_d) \approx 0.9 V_i \cos(\alpha_d). \qquad (1.46)$$

1.5 PULSE WIDTH MODULATION

As explained in the preceding section, control of the output voltage of a power electronic converter by means of an adjustable firing delay has been primarily dictated

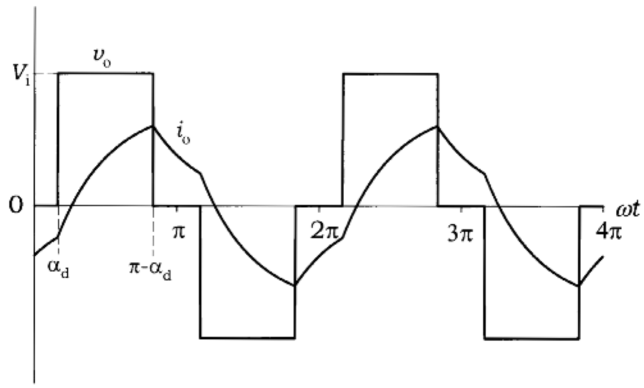

Figure 1.17 Output voltage and current waveforms in the generic inverter with the delay angle of 30°.

by the operating properties of semicontrolled power switches. The phase control, although conceptually simple, results in serious distortion of the output current of a converter. The distortion increases with the length of delay. Clearly, the distorted current is a consequence of the distorted voltage. However, the shape of the output voltage waveforms in power electronic converters is of much less practical concern than that of the output current waveforms. Precisely, *it is the current that does the job*, whether it is the emission of a light bulb, torque production in an electric motor, or electrolytic process in an electrochemical plant.

As mentioned before, most practical loads contain inductance. As such a load constitutes a low-pass filter, high-order harmonics of the output voltage of a converter have weaker impact on the output current waveform than do the low-order ones. Therefore, the quality of the current strongly depends on the amplitudes of low-order harmonics in the frequency spectrum of output voltage. Such spectra for the phase-controlled generic rectifier and ac voltage controller are illustrated in Figure 1.18. Amplitudes of the harmonics are expressed in the per-unit format, with

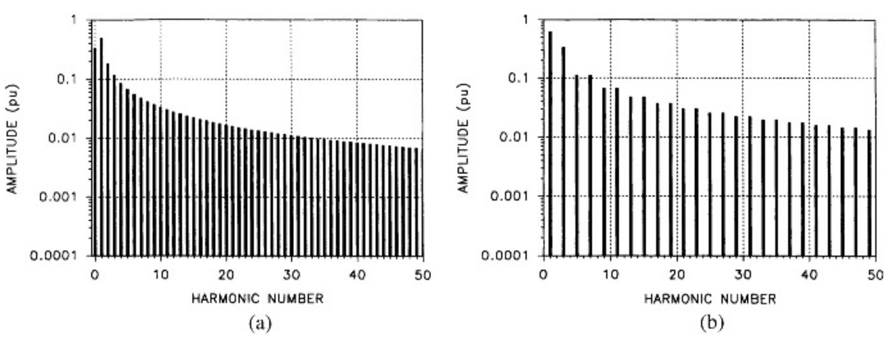

Figure 1.18 Harmonic spectra of output voltage with the firing angle of 90° in (a) phase-controlled generic rectifier, (b) phase-controlled generic ac voltage controller.

the peak value, $V_{i,p}$, of input voltage taken as the base voltage. Both spectra display several high-amplitude low-order harmonics.

In ac input converters, the distortion of the *input* current is of equal importance. Distorted currents drawn from the power system cause the so-called *harmonic pollution* of the system resulting in faulty operation of system protection relays and electromagnetic interference (EMI) with communication systems. To alleviate these problems, utility companies require that input filters be installed, which raises the total cost of power conversion. The lower is the frequency of harmonics to be attenuated, the larger and more expensive filters are needed. The already mentioned alternate method of voltage and current control by *pulse width modulation* (*PWM*) results in better spectral characteristics of converters and smaller filters. Therefore, PWM schemes are increasingly adopted in modern power electronic converters.

The principle of PWM can best be explained considering dc-to-dc power conversion performed by the generic converter supplied with a fixed dc voltage. The converter controls the dc component of the output voltage. As shown in Figure 1.19, this is accomplished by using the converter switches in such a way that the output voltage consists of a train of pulses (state 1 of the generic converter) interspersed with notches (state 0). Fittingly, the respective practical power electronic converters are called *choppers*. Low-power converters used in power supplies for electronic equipment are usually referred to as *dc voltage regulators* or, simply, *dc-to-dc converters*.

In the case illustrated, the pulses and notches are of equal duration, that is, switches S1 and S2 operate with the so-called *duty ratio* of 0.5. A duty ratio, d, of a switch is defined as

$$d \equiv \frac{t_{ON}}{t_{ON} + t_{OFF}}, \tag{1.47}$$

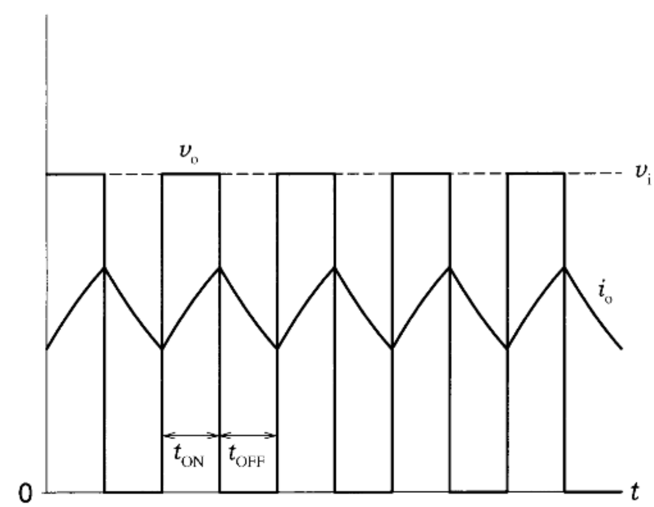

Figure 1.19 Output voltage and current waveforms in the generic chopper.

where t_{ON} denotes the on-time, that is, the interval within which the switch is closed, and t_{OFF} is the off-time, that is, the interval within which the switch is open. Here, switch S5 also operates with the duty ratio of 0.5. However, if the duty ratio of switches S1 and S2 were, for example, 0.6, the duty ratio of switch S5 would have to be 0.4. Switches S3 and S4 of the generic converter are not utilized (unless a reversal of polarity of the output voltage is required), so their duty ratio is zero.

It is easy to see that the average value (dc component), $V_{o,dc}$, of the output voltage is proportional to the fixed value, V_i, of the input voltage and to the duty ratio, d_{12}, of switches S1 and S2, that is,

$$V_{o,dc} = d_{12}V_i. \tag{1.48}$$

Since the possible range of a duty ratio is zero (switch open all the time) to unity (switch closed all the time), adjusting the duty ratio of appropriate switches allows setting $V_{o,dc}$ at any level between zero and V_i. As follows from Eq. (1.48), the voltage control characteristic, $V_{o,dc} = f(d_{12})$, of the generic chopper is linear.

The switching frequency, f_{sw}, defined as

$$f_{sw} \equiv \frac{1}{t_{ON} + t_{OFF}} \tag{1.49}$$

does not affect the dc component of the output voltage. However, the quality of output current strongly depends on f_{sw}. As illustrated in Figure 1.20, if the number of pulses per second is doubled compared to that in Figure 1.19, the increased switching frequency results in the reduction of the current ripple by about 50%. The time

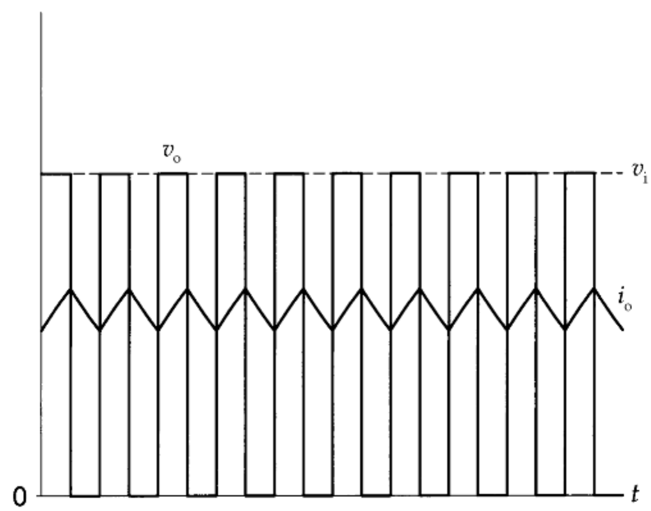

Figure 1.20 Output voltage and current waveforms in the generic chopper with the switching frequency twice as high as that in Figure 1.19.

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between consecutive state changes of the converter is simply short enough to prevent significant current changes between consecutive "jumps" of the output voltage.

The reduction of current ripple can also be explained by the harmonic analysis of the output voltage. Note that the fundamental output frequency equals the switching frequency. As a result, harmonics of the ac component of the voltage appear around frequencies that are integer multiples of f_{sw}. The corresponding inductive reactances of the load are proportional to these frequencies. Therefore, if the switching frequency is sufficiently high, the ac component of the output current is so strongly attenuated that the current is practically of ideal dc quality.

Voltage control using PWM can be employed in all the other types of electric power conversion described in this chapter. Instead of taking out solid chunks of the output voltage waveforms by the firing delay as shown in Figures 1.13 and 1.15, a number of narrow segments can be removed in the PWM operating mode illustrated in Figure 1.21. Here, the ratio, N, of the switching to input frequency is 12. This is also the number of pulses of output voltage per cycle. The output current waveforms have significantly higher quality than those in Figures 1.13 and 1.15, exemplifying the operational superiority of pulse width modulated converters over phase-controlled ones.

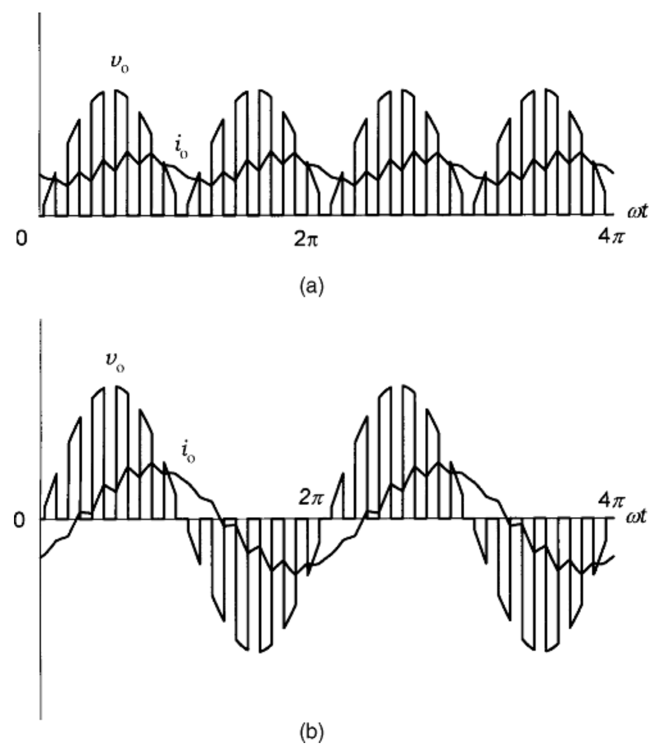

Figure 1.21 Output voltage and current waveforms in (a) generic PWM rectifier, (b) generic PWM ac voltage controller ($N = 12$).

It should be mentioned that, for clarity, in most examples of PWM converters throughout the book, the switching frequencies employed in preparing the figures are lower than the practical ones. Typically, depending on the type of power switches, switching frequencies are of the order of few kHz, seldom less than 1 kHz and, in the so-called *supersonic converters*, they are higher than 20 kHz. Therefore, if, for example, a switching frequency in a 60-Hz PWM ac voltage controller is 3.6 kHz, the output voltage is “sliced” into 60 segments per cycle, instead of the 12 segments shown in Figure 1.21b. Generally, a switching frequency should be several times higher than the reciprocal of the dominant (the longest) time constant of the load.

It can be shown that the linear relation (1.47) pertaining to the chopper can be extended on other types of PWM converters operating with fixed duty ratios of switches, such as rectifiers and ac voltage controllers. In all these converters,

$$V_{\mathrm{o,adj}} = dV_{\mathrm{o,adj(max)}}, \tag{1.50}$$

that is, the magnitude control ratio, M, equals the duty ratio, d, of switches connecting the input and output terminals. Depending on the type of converter, symbol $V_{\mathrm{o,adj}}$ represents the adjustable dc component or fundamental ac component of the output voltage, while $V_{\mathrm{o,adj(max)}}$ is the maximum available value of this component. However, concerning the *rms value*, V_{o}, of output voltage, the dependence on the duty ratio is radical, that is,

$$V_{\mathrm{o}} = \sqrt{d}V_{\mathrm{o,max}}, \tag{1.51}$$

where $V_{\mathrm{o(max)}}$ is the maximum available rms value of output voltage. Specific values of $V_{\mathrm{o,adj(max)}}$ and $V_{\mathrm{o(max)}}$ depend on the type of converter and the magnitude of the input voltage. Control characteristics of the generic PWM rectifier (Eq. (1.48)) and an ac voltage controller (Eq. (1.51)) are shown in Figure 1.22 for comparison with those of phase-controlled converters depicted in Figures 1.14 and 1.16.

Harmonic spectra of the output voltage of the generic PWM rectifier and ac voltage controller are shown in Figure 1.23, again in the per-unit format. Here, the switching frequency is 24 times higher than the input frequency, that is, the output voltage has 24 pulses per cycle. The duty ratio of converter switches is 0.5. Comparing these spectra with those in Figure 1.18, essential differences can be observed. Amplitudes of the low-order harmonics have been suppressed, especially in the spectrum for the ac voltage controller. Higher harmonics appear in clusters centered about integer multiples of 12 for the rectifier and of 24 for the ac voltage controller. The reduced amplitudes of low-order harmonics of the output voltage are translated into enhanced quality of the output current waveforms.

It should be pointed out that the simple PWM described, with a constant duty ratio of switches, has only been used for illustration of the basic principles of PWM converters. In practice, constant duty ratios are typical for choppers and common for ac voltage controllers. PWM rectifiers and inverters employ more sophisticated PWM

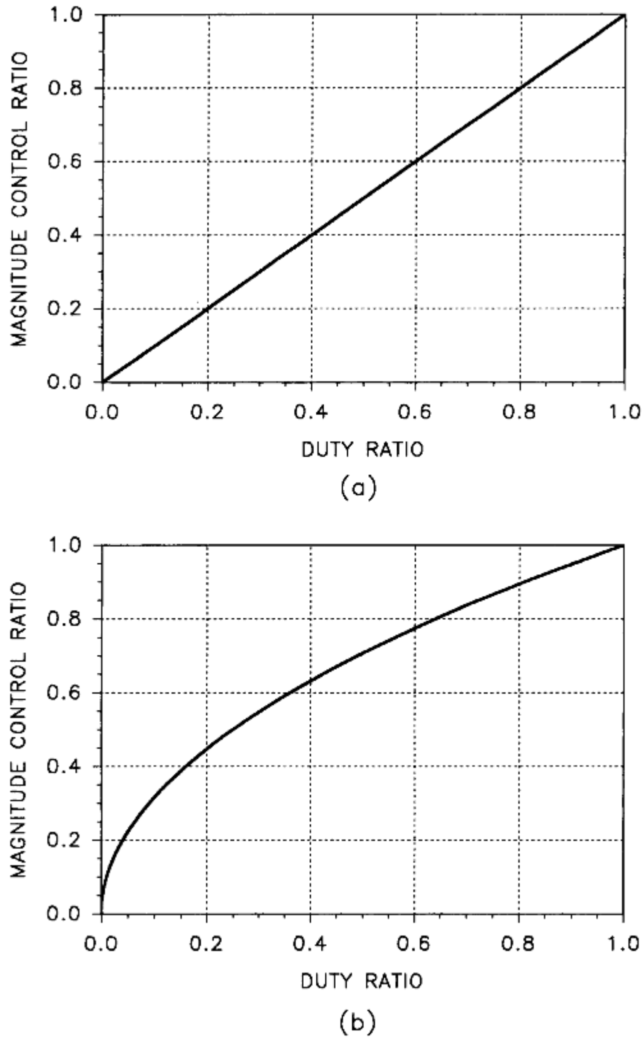

Figure 1.22 Control characteristics of (a) generic PWM rectifier, (b) generic PWM ac voltage controller.

techniques, in which the duty ratios of switches change throughout the cycle of the output voltage. Such techniques can also be applied in PWM ac voltage controllers.

PWM techniques characterized by variable duty ratios of converter switches will be explained in detail in Chapters 4 and 7. Example waveforms of the output voltage and current of a generic inverter with $N = 10$ voltage pulses per cycle and with variable duty ratios are shown in Figure 1.24. The magnitude control ratio, M, pertaining to the amplitude of the fundamental output voltage, is 1 in Figure 1.24a and 0.5 in Figure 1.24b. The varying widths of the voltage pulses and proportionality of these widths to the magnitude control ratio can easily be observed. The output current

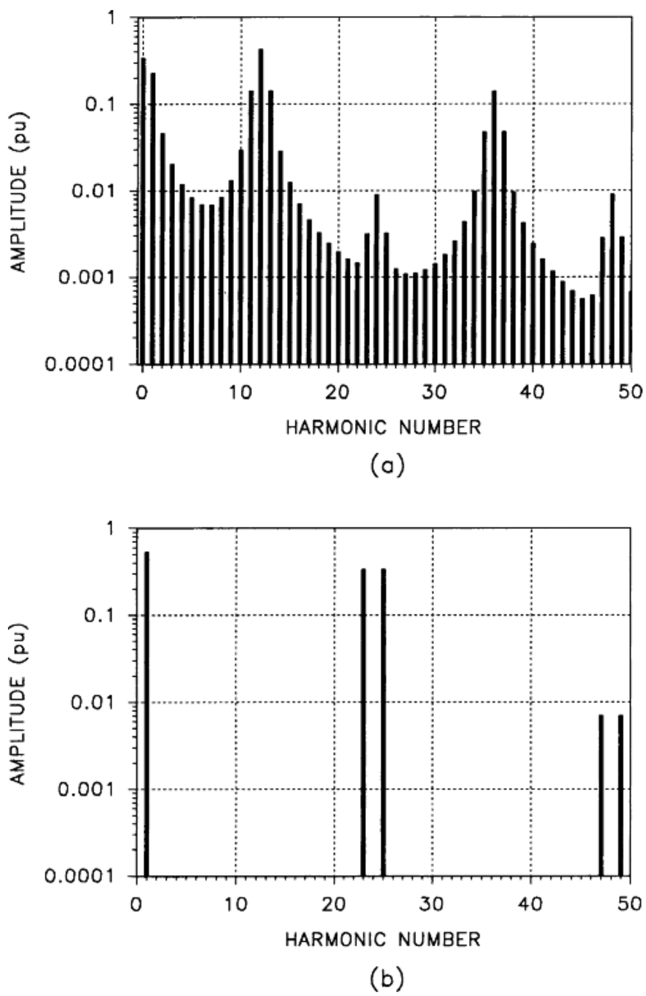

Figure 1.23 Harmonic spectra of output voltage in (a) generic PWM rectifier, (b) generic PWM ac voltage controller ($N = 24$).

waveforms, although rippled, are much closer to ideal sine waves than those in the square-wave operating mode of the inverter (see Figure 1.5). The harmonic content of the current would decrease further with an increase in the switching frequency.

All the examples of PWM converters presented in this section show that *the higher the switching frequency, the better quality of the output current is obtained*. However, the allowable switching frequency in practical power electronic converters is limited by several factors. First, any power switch requires certain amount of time for the transitions from the on-state to off-state and vice-versa. Therefore, the operating frequency of a switch is restricted, the maximum value depending on the type and ratings of the switch. Second, the control system of a converter has a limited operating speed. Finally, as explained in Chapter 2, the so-called *switching losses* in practical switches

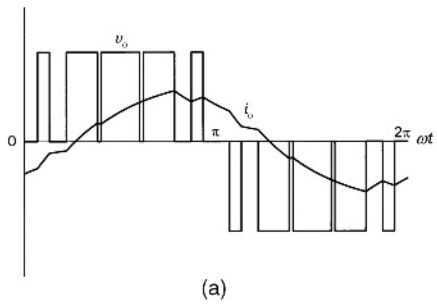

Figure 1.24 Output voltage and current waveforms in the generic PWM inverter: (a) $M = 1$, (b) $M = 0.5$ ($N = 10$).

increase with the switching frequency, reducing the efficiency of power conversion. Therefore, the switching frequency employed in a PWM converter should represent a sensible tradeoff between the quality and efficiency of operation of the converter.

1.6 COMPUTATION OF CURRENT WAVEFORMS

Sources of raw electric power, such as generators or batteries, are predominantly voltage sources. Since, as already demonstrated, a static power converter is a network of switches, waveforms of output voltages are easy to determine from the converter's principle of operation. It is not so with output currents, which depend on both the voltages and load.

As exemplified by the generic converter, power electronic converters operate most of the time in the so-called *quasi-steady state*, which is a sequence of transient states. The converters are variable-topology circuits, and each change of topology initiates a new transient state. Final values of currents and voltages in one topology become initial values in the next topology. Since linear electric circuits can be described by ordinary linear differential equations, the task of finding a current waveform becomes a classic initial value problem. As shown later, in place of differential equations, difference equations can conveniently be applied to converters operating in the PWM mode.

If the load of a converter is linear and the output voltage can be expressed in a closed form, the output current waveform corresponding to individual states of the converter can also be expressed in a closed form. However, when a computer is used for converter analysis, numerical algorithms can be employed to compute consecutive points of the current waveform. Both the analytical and numerical approaches will be illustrated in the subsequent sections.

1.6.1 Analytical Solution

To show the typical way of finding the closed-form expressions for the output current in a power electronic converter, the generic rectifier, generic inverter, and

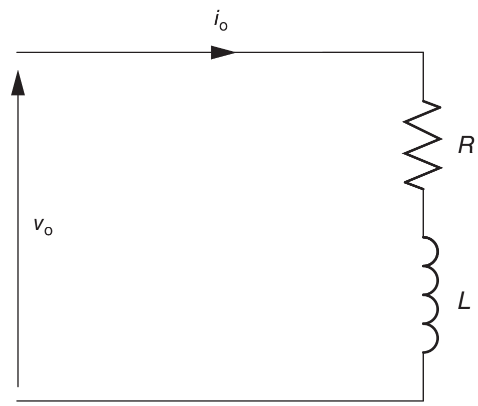

Figure 1.25 Resistive–inductive (RL) load circuit.

generic PWM ac voltage controller will be considered. In all three cases, an RL-load is assumed.

The RL-load circuit of the generic converter is shown in Figure 1.25. If the converter operates as a rectifier, the input and output terminals are cross-connected when the ac input voltage, given by Eq. (1.1), is negative. Consequently, the output voltage, $v_o(t)$, and current, $i_o(t)$, are periodic with the period of $T/2$, where T is the period of the input voltage, equal to $2\pi/\omega$. Therefore, all the subsequent considerations of the generic rectifier are limited to the interval 0 to $T/2$.

The Kirchhoff Voltage Law for the circuit in Figure 1.25 can be written as

$$Ri_o(t) + L\frac{di_o(t)}{dt} = V_{i,p}\sin(\omega t). \tag{1.52}$$

Equation (1.52) can be solved for $i_o(t)$ using the Laplace transformation or, less tediously, taking advantage of the known property of linear differential equations allowing $i_o(t)$ to be expressed as

$$i_o(t) = i_{o,F}(t) + i_{o,N}(t), \tag{1.53}$$

where $i_{o,F}(t)$ and $i_{o,N}(t)$ denote the so-called *forced* and *natural* components of $i_o(t)$, respectively. The convenience of this approach lies in the fact that the forced component constitutes the steady-state solution of Eq. (1.52), that is, the steady-state current excited in the circuit in Figure 1.26 by the sinusoidal voltage $v_i(t)$. As well known to every electrical engineer,

$$i_{o,F}(t) = \frac{V_{i,p}}{Z}\sin(\omega t - \varphi), \tag{1.54}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2} \tag{1.55}$$

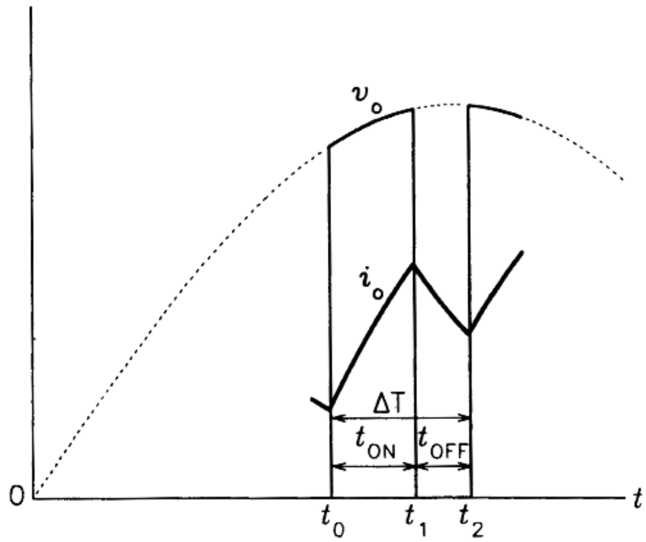

Figure 1.26 Fragments of the output voltage and current waveforms in the generic PWM ac voltage controller.

and

$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) \tag{1.56}$$

are the impedance and phase angle of the RL-load, respectively. The shortcut from Eq. (1.52) to the forced solution (1.54) saves a lot of work.

The natural component, $i_{o,N}(t)$, represents a solution of the homogenous equation

$$Ri_{o,N}(t) + L\frac{di_{o,N}(t)}{dt} = 0 \tag{1.57}$$

obtained from Eq. (1.52) by equating the right-hand side (the excitation) to zero. It can be seen that $i_{o,N}(t)$ must be such that the linear combination of it and its derivative is zero. Clearly, an exponential function

$$i_{o,N}(t) = Ae^{BT} \tag{1.58}$$

is the best candidate for the solution. Substituting $i_{o,N}(t)$ in Eq. (1.57) yields

$$RAe^{BT} + LABe^{BT} = 0 \tag{1.59}$$

from which, $B = -R/L$, and

$$i_{o,N}(t) = Ae^{-\frac{R}{L}t}. \tag{1.60}$$

Thus, based on Eqs. (1.53) and (1.54),

$$i_o(t) = \frac{V_{i,p}}{Z}\sin(\omega t - \varphi) + Ae^{-\frac{R}{L}t}. \tag{1.61}$$

To determine the constant A, note that in the quasi-steady state of the rectifier, $i_o(0) = i_o(T/2) = i_o(\pi/\omega)$ (see Figure 1.4). Hence,

$$\frac{V_{i,p}}{Z}\sin(-\varphi) + A = \frac{V_{i,p}}{Z}\sin(\pi - \varphi) + Ae^{-\frac{R}{L}\frac{\pi}{\omega}}, \tag{1.62}$$

where

$$-\frac{R\pi}{L\omega} = -\frac{\pi}{\tan(\varphi)}. \tag{1.63}$$

Equation (1.62) can now be solved for A, yielding

$$A = \frac{2V_{i,p}\sin(\varphi)}{Z\left[1 - e^{-\frac{\pi}{\tan(\varphi)}}\right]} \tag{1.64}$$

and

$$i_o(t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) + \frac{2\sin(\varphi)}{1 - e^{-\frac{\pi}{\tan(\varphi)}}}e^{-\frac{R}{L}t}\right]. \tag{1.65}$$

The shown approach to derivation of the rather complex relation (1.65) takes much less time and effort than those required when the Laplace transformation is employed. The reader is encouraged to confirm this observation by his/her own computations.

When the generic power converter works as an inverter, the output voltage equals V_i when the converter is in state 1 and $-V_i$ in state 2. If T now denotes the period of the output voltage, it can be assumed that the state 1 lasts from 0 to $T/2$ and state 2 from $T/2$ to T. As it is only the polarity of output voltage that changes every half-cycle, the output current, $i_{o(2)}(t)$, in the second half-cycle is equal to $-i_{o(1)}(t-T/2)$, where $i_{o(1)}(t)$ is the output current in the first half-cycle. Consequently, it is sufficient to determine $i_{o(1)}(t)$ only.

The forced component, $i_{o,F(1)}(t)$, of current $i_{o(1)}(t)$, excited in the RL-load by the dc voltage $v_o = V_i$ is given by the Ohm's Law as

$$i_{o,F(1)} = \frac{V_i}{R}. \tag{1.66}$$

The natural component, which depends solely on the load, is the same as that of the output current of the generic rectifier. Thus,

$$i_{o(1)}(t) = \frac{V_i}{R} + Ae^{-\frac{R}{L}}. \tag{1.67}$$

Constant A can be found from the condition $i_{o(1)}(0) = -i_{o(1)}(T/2)$ (see Figure 1.5), that is,

$$\frac{V_i}{R} + A = -\left[\frac{V_i}{R} + Ae^{-\frac{\pi}{\tan(\varphi)}}\right], \tag{1.68}$$

which yields

$$A = \frac{2}{1 + e^{-\frac{\pi}{\tan(\varphi)}}}\frac{V_i}{R}. \tag{1.69}$$

This gives

$$i_o(t) = \left\{\begin{matrix} \frac{V_i}{R}\left[1 - \frac{2}{1+e^{-\frac{\pi}{\tan(\varphi)}}}e^{-\frac{R}{L}t}\right] & \text{for} \quad 0 < t \le \frac{T}{2} \\ -\frac{V_i}{R}\left[1 - \frac{2}{1+e^{-\frac{\pi}{\tan(\varphi)}}}e^{-\frac{R}{L}\left(t-\frac{T}{2}\right)}\right] & \text{for} \quad \frac{T}{2} < t \le T \end{matrix}\right\}. \tag{1.70}$$

In the final example, a PWM converter, specifically the PWM ac voltage controller, is dealt with. Within a single cycle of output voltage, the converter undergoes a large number of state changes, and within each corresponding interval of time, the waveform of output current is described by a different equation. Therefore, to express the waveform in a useful format, iterative formulas are employed. The general form of such a formula is $i_o(t + \Delta t) = f[i_o(t), t]$, where Δt is a small increment of time, meaning that consecutive values of the output current are calculated from the previous values.

In developing the formulas, two simplifying assumptions were made: first, that the current waveform is piecewise linear; second, that the initial value, $i_o(0)$, of the output current is equal to that of a hypothetical sinusoidal current that would be generated in the same load by the fundamental of output voltage of the converter. Thus, if the input voltage of the generic ac voltage controller is given by Eq. (1.1), the initial value of the output current is

$$i_o(0) = M\frac{V_{i,p}}{Z}\sin(\omega t - \varphi)|_{t=0} = -M\frac{V_{i,p}}{Z}\sin(\varphi). \tag{1.71}$$

The output voltage waveform of the controller, shown in Figure 1.21b, is a chopped sinusoid. A single switching cycle of that waveform is depicted in Figure 1.26, which also shows the coinciding fragment of the output current waveform. The time interval,

t_0 to t_2, corresponding to the switching cycle is called a *switching interval*. From t_0 to t_1, the controller is in state 1 and the output voltage equals the input voltage, while from t_1 to t_2, the output voltage is zero, the controller assuming state 0. Denoting the length of the switching interval, $t_2 - t_0$, by ΔT, the on-time, t_{ON}, is $M\Delta T$, and the off-time, t_{OFF}, is $(1 - M)\Delta T$.

The differential equation of the load circuit is

$$Ri_o(t) + L\frac{di_o(t)}{dt} = v_o(t), \tag{1.72}$$

which, for the instant $t = t_0$, can be rewritten as

$$Ri_o(t_0) + L\frac{\Delta i_o}{\Delta t} = V_{i,p}\sin(\omega t_o), \tag{1.73}$$

where $\Delta i_o = i_o(t_1) - i_o(t_0)$ and $\Delta t = t_1 - t_0$, that is, as a *difference equation*. Taking into account that $\Delta t = M\Delta T$, Eq. (1.73) can be solved for $i_o(t_0)$ to yield

$$i_o(t_1) = i_o(t_0) + \frac{M}{L}[V_{i,p}\sin(\omega t_o) - Ri_o(t_0)]\Delta T. \tag{1.74}$$

Similarly, at $t = t_1$, the load circuit equation

$$Ri_o(t_1) + L\frac{\Delta i_o}{\Delta t} = 0, \tag{1.75}$$

where $\Delta i_o = i_o(t_2) - i_o(t_1)$ and $\Delta t = t_2 - t_1$, can be rearranged to

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{R}{L}(1 - M)\Delta T\right]. \tag{1.76}$$

Formulas (1.71), (1.74), and (1.76) allow computation of consecutive segments of the piecewise linear waveform of the output current. If, as typical in practical PWM converters, the switching frequency, $f_{sw} = 1/\Delta T$, is at least one order of magnitude higher than the input or output frequency, the accuracy of those approximate relations is sufficient for all practical purposes.

1.6.2 Numerical Solution

When a computer program is used to simulate a converter, the output voltage is calculated as a series of values, $v_{o,0}$, $v_{o,1}$, $v_{o,2}$, …, for the consecutive instants, t_0, t_1, t_2, … These instants should be close apart so that $t_{n+1} - t_n \ll \tau$, where τ denotes the shortest time constant of the simulated system. Then, the respective values of the output current, $i_{o,1}$, $i_{o,2}$, $i_{o,3}$, …, can be computed as responses of the load circuit to step excitations $v_{o,0}u(t - t_0)$, $v_{o,1}u(t - t_1)$, $v_{o,2}u(t - t_2)$, …, where $u(t - t_n)$ denotes a unit step function at $t = t_n$.

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For instance, for the RL-load considered in the previous section, the differential equation of the load circuit for $t \geq t_n$ can be written as

$$Ri_{\mathrm{o}}(t) + L\frac{di_{\mathrm{o}}(t)}{dt} = v_{\mathrm{o},n}u(t - t_n), \tag{1.77}$$

where $u(t - t_n) = 1$. The forced component, $i_{\mathrm{o,F}}(t)$, of the solution, $i_{\mathrm{o}}(t)$, of Eq. (1.77) is given by

$$i_{\mathrm{o,F}}(t) = \frac{v_{\mathrm{o},n}}{R}, \tag{1.78}$$

and the natural component, $i_{\mathrm{o,N}}(t)$, by

$$i_{\mathrm{o,N}}(t) = Ae^{-\frac{R}{L}t}. \tag{1.79}$$

Hence,

$$i_{\mathrm{o}}(t) = \frac{v_{\mathrm{o},n}}{R} + Ae^{-\frac{R}{L}t}, \tag{1.80}$$

where constant A can be determined from equation

$$i_{\mathrm{o}}(t_n) = i_{\mathrm{o},n} = \frac{v_{\mathrm{o},n}}{R} + Ae^{-\frac{R}{L}t_n} \tag{1.81}$$

as

$$A = \left(i_{\mathrm{o},n} - \frac{v_{\mathrm{o},n}}{R}\right)e^{-\frac{R}{L}t_n}. \tag{1.82}$$

Substituting $t = t_{n+1}$ and Eq. (1.82) in Eq. (1.80), and denoting $i_{\mathrm{o}}(t_n + 1)$ by $i_{\mathrm{o},n+1}$ yields

$$i_{\mathrm{o},n+1} = \frac{v_{\mathrm{o},n}}{R} + \left(i_{\mathrm{o},n} - \frac{v_{\mathrm{o},n}}{R}\right)e^{-\frac{R}{L}(t_{n+1} - t_n)}, \tag{1.83}$$

which is an iterative formula, $i_{\mathrm{o},n+1} = f(i_{\mathrm{o},n}, t_{n+1} - t_n)$, that allows easy computation of consecutive points of the output current waveform.

The other common loads and corresponding numerical formulas for the output current are

Resistive load (R-load):

$$i_{\mathrm{o},n} = \frac{v_{\mathrm{o},n}}{R}. \tag{1.84}$$

Inductive load (L-load):

$$i_{o,n+1} = i_{o,n} + \frac{v_{o,n}}{L}(t_{n+1} - t_n). \tag{1.85}$$

Resistive-EMF load (RE-load):

$$i_{o,n} = \frac{v_{o,n} - E_n}{R}, \tag{1.86}$$

where E_n denotes the value of load EMF at $t = t_n$. The EMF, in series with resistance R, is assumed to oppose the output current.

Inductive-EMF load (LE-load):

$$i_{o,n+1} = i_{o,n} + \frac{v_{o,n} - E_n}{L}(t_{n+1} - t_n), \tag{1.87}$$

where the load EMF is connected in series with inductance L.

Resistive-inductive-EMF load (RLE-load):

$$i_{o,n+1} = \frac{v_{o,n} - E_n}{R} + \left(i_{o,n} - \frac{v_{o,n} - E_n}{R}\right) e^{-\frac{R}{L}(t_{n+1} - t_n)}, \tag{1.88}$$

where the load is represented by a series connection of resistance R, inductance L, and EMF E.

The presented considerations can be adapted for analysis of the less common current-source converters. There, those are currents that are easily determinable from the operating principles of a converter, while voltage waveforms require analytical or numerical solution of appropriate differential equations.

In the engineering practice, specialized computer programs are used for modeling and analysis of power electronic converters. The free-download software package LTspice for simulation of electronic circuits is called for in most of the computer assignments in this book (see Appendix A). Other commercial versions of the original UC Berkeley's SPICE are also available on the software market. Many advanced simulation programs, of which Saber is the best known example, have been developed in several countries specifically for modeling of switching power converters. Program EMTP, used primarily by utilities for the power system analysis, allows simulation of power electronic converters, which is particularly useful for studies of impact of the converters on the power system operation. General-purpose dynamic simulators, such as Simulink, Simplorer, or ACSL, can successfully be used for analysis of not only the converters themselves, but also whole systems that include converters, such as electric motor drives.

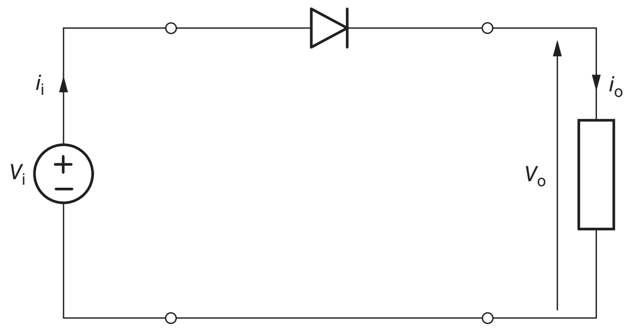

Figure 1.27 Single-pulse diode rectifier.

1.6.3 Practical Example: Single-Phase Diode Rectifiers

To illustrate the considerations presented in the preceding two sections and to start introduction of practical power electronic converters, single-phase diode rectifiers will be described. They are the simplest static converters of electrical power. The power diodes employed in the rectifiers can be thought of as uncontrolled semiconductor power switches. A diode in a closed circuit begins conducting (turns on) when its anode voltage becomes higher than the cathode voltage, that is, when the diode is forward biased. The diode ceases to conduct (turns off) when its current changes polarity.

The *single-pulse* (single-phase half-wave) diode rectifier is shown in Figure 1.27. Functionally, it is analogous to a reduced generic power converter having only switches S1 and S2, which stay closed as long as they conduct the load current. If the load is purely resistive (R-load), and the sinusoidal input voltage is given by Eq. (1.1), the output current waveform resembles that of the output voltage, as shown in Figure 1.28.

Comparing Figures 1.28 and 1.4 it can easily be seen that the average output voltage, $V_{o,dc}$, in the single-pulse rectifier is only half as large as that in the generic rectifier described in Section 1.2, that is, $V_{o,dc} = V_{i,p}/\pi \approx 0.32\ V_{i,p}$ (see Eq. 1.15). The single “pulse” of output current per cycle of the input voltage explains the name of the rectifier.

The voltage gain is even lower when the load contains an inductance (RL-load). Equation (1.60) can be used to describe the output current, but only until it reaches zero at $\omega t = \alpha_e$, where α_e is called an *extinction angle*. Thus,

$$i_o(0) = \frac{V_{i,p}}{Z}\sin(-\varphi) + A = 0, \tag{1.89}$$

from which

$$A = \frac{V_{i,p}}{Z}\sin(\varphi) \tag{1.90}$$

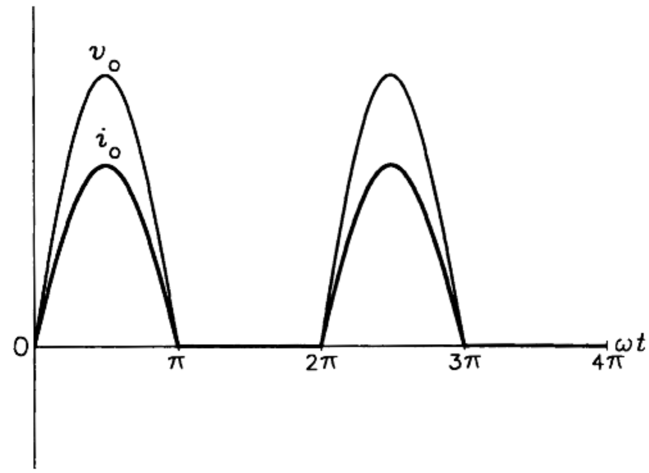

Figure 1.28 Output voltage and current waveforms in the single-pulse diode rectifier with an R-load.

and

$$i_o(t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) + e^{-\frac{R}{L}t}\sin(\varphi)\right], \tag{1.91}$$

when $i_o > 0$, that is, when $0 < \omega t \le \alpha_e$. Substituting α_e/ω for t and zero for i_o, the extinction angle can be found from equation

$$\frac{V_{i,p}}{Z}\left[\sin(\alpha_e - \varphi) + e^{-\frac{\alpha_e}{\tan(\varphi)}}\sin(\varphi)\right] = 0. \tag{1.92}$$

Clearly, no closed-form expression for α_e exists, and the extinction angle, which is a function of the load angle φ, can only be found numerically. This problem will be analyzed in greater detail in Chapter 4.

The output voltage and current waveforms of a single-pulse rectifier with an RL-load are shown in Figure 1.29. As the extinction angle is greater than 180°, the output voltage is negative in the $\omega t = \pi$ to $\omega t = \alpha_e$ interval, and its overall average value is lower than that with the resistive load. This can be remedied by connecting the so-called *freewheeling diode*, DF, across the load, as shown in Figure 1.30.

The freewheeling diode, which corresponds to switch S5 in the generic power converter, shorts the output terminals when the output voltage reaches zero and provides a path for the output current in the π to α_e interval. Until the voltage reaches zero, the waveform of output voltage is the same as that in Figure 1.29 and that of output current is described by Eq. (1.91). Later, the current dies out while freewheeled by diode DF. As now $v_o = 0$, the equation of the current is simply

$$i_o(t) = Ae^{-\frac{R}{L}t}, \tag{1.93}$$

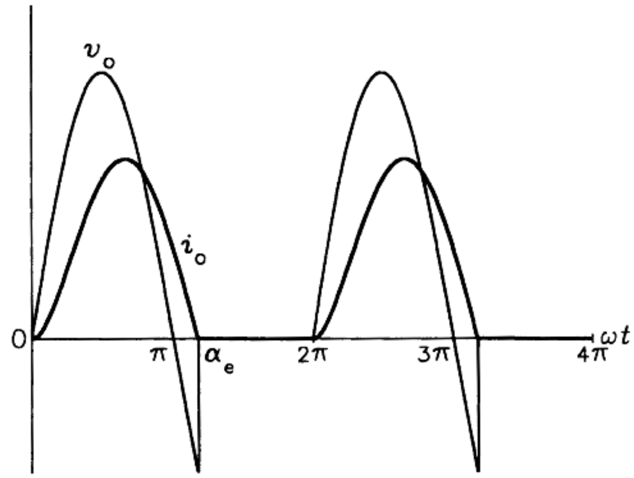

Figure 1.29 Output voltage and current waveforms in the single-pulse diode rectifier with an RL-load.

where constant A can be found by equaling currents given by Eqs. (1.91) and (1.93) at $\omega t = \pi$, that is,

$$\frac{V_{i,p}}{Z}\left[1 - e^{-\frac{\pi}{\tan(\varphi)}}\right]\sin(\varphi) = Ae^{-\frac{\pi}{\tan(\varphi)}}. \tag{1.94}$$

From Eq. (1.94),

$$A = \frac{V_{i,p}}{Z}\left[e^{\frac{\pi}{\tan(\varphi)}} - 1\right]\sin(\varphi) \tag{1.95}$$

and

$$i_o(t) = \frac{V_{i,p}}{Z}\left[e^{\frac{\pi}{\tan(\varphi)}} - 1\right]e^{-\frac{R}{L}t}\sin(\varphi). \tag{1.96}$$

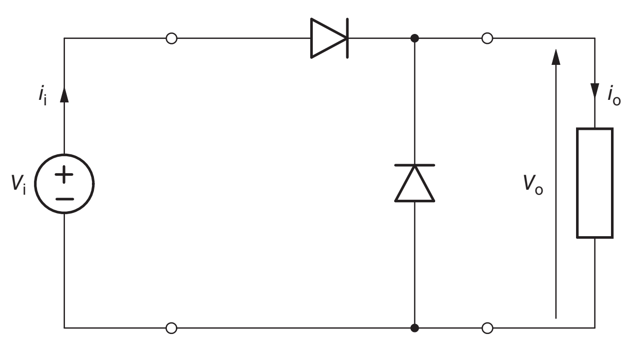

Figure 1.30 Single-pulse diode rectifier with a freewheeling diode.

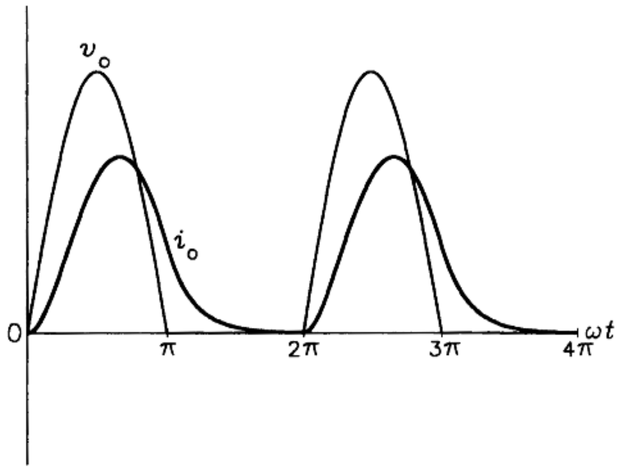

Figure 1.31 Output voltage and current waveforms in the single-pulse diode rectifier with a freewheeling diode and an RL-load.

Waveforms of the output voltage and current of a single-pulse rectifier with the freewheeling diode are shown in Figure 1.31.

More radical enhancement of the single-pulse rectifier, shown in Figure 1.32, consists in connecting a capacitor across the output terminals. The capacitor charges up when the input voltage is high and it discharges through the load when the input voltage drops below a specific level that depends on the capacitor and load. Typical waveforms of the output voltage, v_o, and capacitor current, i_C, with a resistive load are shown in Figure 1.33. Advanced readers are encouraged to try and obtain analytical expressions for these waveforms using the approach sketched in this chapter.

Practical single-pulse rectifiers do not belong in the realm of true power electronics, as the output capacitors would have to be excessively large. The average output voltage, $V_{o,dc}$, can be increased to $2V_{i,p}/\pi \approx 0.64V_{i,p}$ in a two-pulse (single-phase full-wave) diode rectifier in Figure 1.34. Diodes D1 through D4 correspond to the respective switches in the generic converter, directly connecting and cross-connecting the input terminals with the output terminals in dependence on polarity of the input

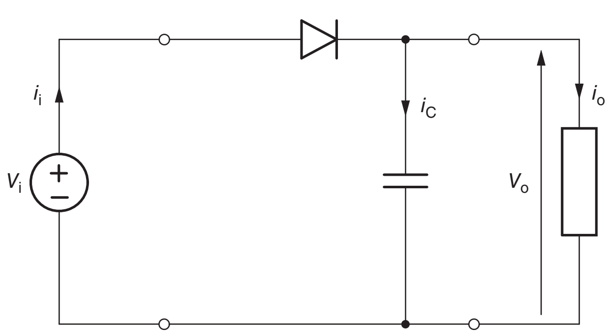

Figure 1.32 Single-pulse diode rectifier with an output capacitor.

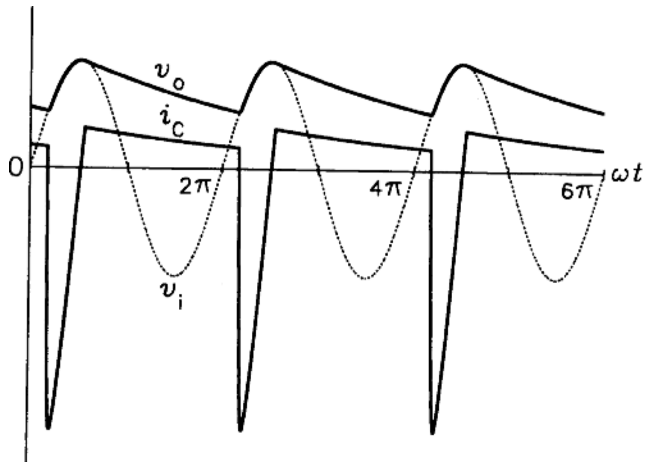

Figure 1.33 Output voltage and current waveforms in the single-pulse diode rectifier with output capacitor and RL-load.

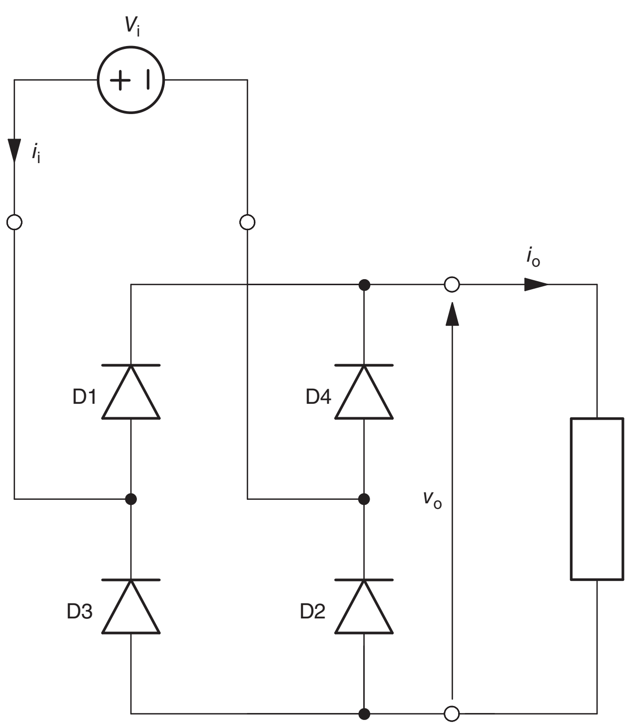

Figure 1.34 Two-pulse diode rectifier.

voltage. As a result, the output voltage and current waveforms are the same as in the generic converter (see Figure 1.4). In low-power rectifiers, a capacitor can be placed across the load, similarly as in the single-pulse rectifier in Figure 1.32, to smooth out the output voltage and increase its dc component. However, the operating characteristics of single-phase rectifiers are distinctly inferior to those of three-phase rectifiers, to be presented in Chapter 4.

SUMMARY

Power conversion and control are performed in power electronic converters, which are networks of semiconductor power switches. Three basic states of a voltage-source converter can be distinguished (although in some converters only two states suffice): the input and output terminals are directly connected, cross-connected, or separated. In the last case, the output terminals must be shorted to maintain a closed path for the output current. An appropriate sequence of states results in conversion of the given input (supply) voltage to the desired output (load) voltage.

Current-source converters are also feasible although less common than the voltage-source ones. A load of a current-source converter must appear as a voltage source, and that of a voltage-source converter as a current source. In practice, the current-source and voltage-source requirements imply a series-connected inductor and a parallel-connected capacitor, respectively.

Control of the output voltage in voltage-source power electronic converters is realized by periodic use of state 0 of the converter. This results in the removal of certain portions of the output voltage waveform. Two approaches are employed: phase control and PWM. The latter technique, promoted by the availability of fast fully controlled semiconductor power switches, is increasingly employed in modern power electronics. The PWM results in higher quality of power conversion and control than does the phase control.

The output voltage waveforms in voltage-source converters are usually easy to determine. Current waveforms, however, require analytical or numerical solution of differential or, especially for PWM converters, difference equations describing the quasi-steady state operation of the converters. A dual approach is applicable to current-source converters. Simulation software packages are extensively used in the engineering practice.

Single-phase diode rectifiers are the simplest ac-to-dc power converters. However, their voltage gain, especially that of the single-pulse rectifier, is low, and ripple factors of the output voltage and current are high. An improvement can be accomplished by installing an output capacitor, but it is feasible in low-power rectifiers only.

EXAMPLES

Example 1.1 The generic power converter is supplied from a 120-V, 50-Hz ac voltage source and its output voltage waveform is shown in Figure 1.35. It can be seen that the converter performs ac-to-ac conversion resulting in the output fundamental

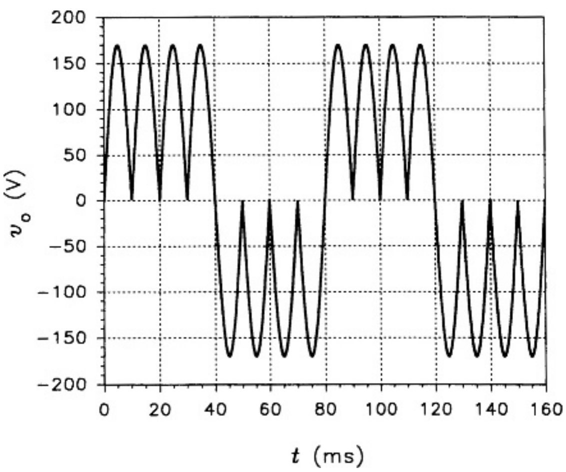

Figure 1.35 Output voltage waveform in the generic cycloconverter in Example 1.1.

frequency reduced with respect to the input frequency. This kind of power conversion is characteristic for *cycloconverters*. The generic cycloconverter in this example operates in a simple *trapezoidal mode*, in which the ratio of the input frequency to output fundamental frequency is an integer. What is the fundamental frequency of the output voltage? Sketch the timing diagram of switches of the converter.

Solution: The output fundamental frequency in question is four times lower than the input frequency of 50 Hz, that is, 12.5 Hz. The timing diagram of switches of the cycloconverter is shown in Figure 1.36. Switch S5 is open all the time as either switches S1 and S2 or S3 and S4 are closed.

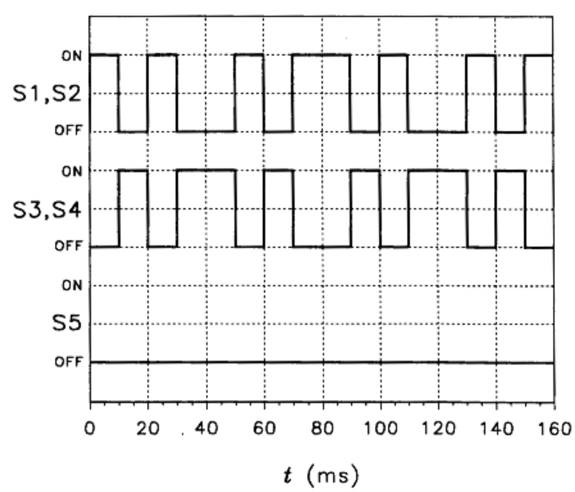

Figure 1.36 Timing diagram of switches in the generic cycloconverter in Example 1.1.

Example 1.2 For the output voltage waveform in Figure 1.35, find:

(a) rms value, V_o
(b) peak value, $V_{o,1,p}$, and rms value, $V_{o,1}$, of fundamental
(c) harmonic content, $V_{o,h}$
(d) total harmonic distortion, THD_V

Solution: The peak value of input voltage is $\sqrt{2} \times 120 = 170$ V. Denoting the fundamental output radian frequency by ω, the output voltage waveform can be expressed as

$$v_o(\omega t) = \begin{cases} |170| \sin(4\omega t) & \text{for} \quad 2n\frac{\pi}{4} \le \omega t < (2n+1)\frac{\pi}{4} \\ -|170| \sin(4\omega t) & \text{for} \quad (2n+1)\frac{\pi}{4} \le \omega t < (2n+2)\frac{\pi}{4} \end{cases},$$

where $n = 0, 1, 2, \ldots$, and the rms value, V_o, of the voltage is the same as that of the input voltage, that is, 120 V. The waveform has the odd and half-wave symmetries, so the peak value, $V_{o,1,p}$, of the fundamental is

$$V_{o,1,p} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} v_o(\omega t) \sin(\omega t) d\omega t = \frac{4}{\pi} \int_0^{\frac{\pi}{4}} 170 \sin(4\omega t) d\omega t - \frac{4}{\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 170 \sin(4\omega t) d\omega t$$

$$= \frac{4 \times 170}{\pi} \left\{ \left[\frac{\sin(3\omega t)}{6} - \frac{\sin(5\omega t)}{10} \right]_0^{\frac{\pi}{4}} - \left[\frac{\sin(3\omega t)}{6} - \frac{\sin(5\omega t)}{10} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right\} = 139 \text{ V}.$$

The rms value of the fundamental is

$$V_{o,1} = \frac{V_{o,1,p}}{\sqrt{2}} = \frac{139}{\sqrt{2}} = 98 \text{ V}$$

and, since there is no dc component, the harmonic content is

$$V_{o,k} = \sqrt{V_o^2 - V_{o,1}^2} = \sqrt{120^2 - 98^2} = 69 \text{ V}.$$

Thus, the total harmonic distortion is

$$THD_V = \frac{V_{o,h}}{V_{o,1}} = \frac{69}{98} = 0.7.$$

Example 1.3 For the generic phase-controlled rectifier, find the relation between the ripple factor of the output voltage, RF_V, and the firing angle, α_f.

Solution: Eq. (1.44) gives the relation between the dc component, $V_{o,dc}$, of the output voltage in question and the firing angle, while Eq. (1.45), concerning the rms value, V_o, of output voltage of a generic ac voltage controller, can directly be applied to the generic rectifier (why?). Consequently,

$$\mathrm{RF_V}(\alpha_f) = \frac{V_{o,ac}(\alpha_f)}{V_{o,dc}(\alpha_f)} = \frac{\sqrt{V_o^2(\alpha_f) - V_{o,dc}^2(\alpha_f)}}{V_{o,dc}^2(\alpha_f)} = \sqrt{\frac{V_o^2(\alpha_f)}{V_{o,dc}^2(\alpha_f)} - 1}$$

$$= \sqrt{\frac{\frac{1}{2\pi}\left[\pi - \alpha_f + \frac{\sin(2\alpha_f)}{2}\right]}{\frac{1}{\pi^2}[1+\cos(\alpha_f)]^2} - 1} = \sqrt{\frac{\pi}{2}\frac{\pi - \alpha_f + \frac{\sin(2\alpha_f)}{2}}{[1+\cos(\alpha_f)]^2} - 1}.$$

Graphical representation of the derived relation is shown in Figure 1.37. It can be seen that when the firing angle exceeds 150°, the voltage ripple rapidly increases because the dc component approaches zero.

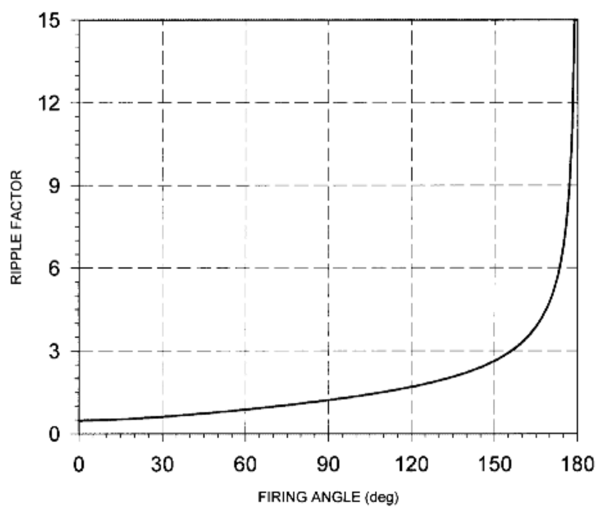

Figure 1.37 Voltage ripple factor versus firing angle in the generic rectifier in Example 1.3.

Example 1.4 The generic converter is supplied from a 100-V dc source and operates as a chopper with the switching frequency, f_{sw}, of 2 kHz. The average output voltage, $V_{o,dc}$, is –60 V. Determine duty ratios of all switches of the converter and the corresponding on- and off-times, t_{ON} and t_{OFF}.

Solution: The negative polarity of the output voltage implies that switches S3, S4, and S5 perform the modulation, while switches S1 and S2 are permanently open (turned off). Therefore, the duty ratio, d_{12}, of the latter switches is zero. Adapting Eq. (1.48) to switches S3 and S4, the duty ratio, d_{34}, of these switches is

$$d_{34} = \frac{V_{\text{o,dc}}}{V_1} = \frac{60}{100} = 0.6.$$

The negative sign at 60 V is omitted since the very use of switches S3 and S4 causes reversal of the output voltage (obviously, a duty ratio can only assume values between zero and unity). Switch S5 is turned on when the other switches are off. Hence, its duty ratio, d_5, is

$$d_5 = 1 - d_{34} = 1 - 0.6 = 0.4.$$

From Eq. (1.48),

$$t_{\text{ON}} + t_{\text{OFF}} = \frac{1}{f_{\text{sw}}} = \frac{1}{2 \times 10^3} = 0.0005 \text{ s} = 0.5 \text{ ms}$$

and from Eq. (1.45),

$$t_{\text{ON},34} = d_{34}(t_{\text{ON}} + t_{\text{OFF}}) = 0.6 \times 0.5 = 0.3 \text{ ms},$$

while

$$t_{\text{ON},5} = d_5(t_{\text{ON}} + t_{\text{OFF}}) = 0.4 \times 0.5 = 0.2 \text{ ms}.$$

Consequently,

$$t_{\text{OFF},34} = 0.5 - 0.3 = 0.2 \text{ ms}$$

and

$$t_{\text{OFF},5} = 0.5 - 0.2 = 0.3 \text{ ms}.$$

Example 1.5 The generic converter is supplied from a 120-V, 60-Hz ac voltage source and operates as a PWM rectifier with an RL-load, where $R = 2\ \Omega$ and $L =$ 5 mH. The switching frequency is 720 Hz and the magnitude control ratio is 0.6. Develop iterative formulas for the output current and calculate values of that current at the switching instants for one cycle of the output voltage.

Solution: Inspecting similar formulas, (1.74) and (1.76), for the generic PWM ac voltage controller, it can be seen that they are easy to adapt to the PWM rectifier by replacing the $\sin(\omega t_0)$ term in Eq. (1.74) with $|\sin(\omega t_0)|$. Then, the output current of the rectifier can be computed from equations

$$i_o(t_1) = i_o(t_0) + \frac{M}{L}[V_{i,p}|\sin(\omega t_0)| - Ri_o(t_0)]\Delta T$$

and

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{R}{L}(1 - M)\Delta T\right].$$

The input frequency, ω, is $2\pi \times 60 = 377$ rad/s, and the length, ΔT, of a switching interval is 1/720 s. Thus, the general iterative formulas above give

$$\begin{aligned} i_o(t_1) &= i_o(t_0) + \frac{0.6}{5 \times 10^{-3}}\left[\sqrt{2} \times 120|\sin(377t_0)| - 2i_o(t_0)\right] \times \frac{1}{720} \\ &= 0.667 i_o(t_0) + 28.3|\sin(377t_0)| \end{aligned}$$

and

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{2}{5 \times 10^{-3}}(1 - 0.6)\frac{1}{720}\right] = 0.778 i_o(t_1).$$

As seen in Figure 1.20a, the period of the output voltage of the rectifier equals to the half of that of the input voltage, that is, 1/120 s = 8.333 ms. Comparing it with the switching frequency, the number of switching intervals per cycle of the output voltage turns out to be six. With $M = 0.6$, the on-time, t_{ON}, equals $t_1 - t_0$, is 0.6/720 s = 0.833 ms, and the off-time, t_{OFF}, equals $t_2 - t_1$, is 0.4/720 s = 0.556 ms.

To start the computations, the initial value, $i_o(0)$, is required as $i_o(t_0)$ in the first switching interval. It can be estimated as the dc component, $I_{o,dc}$, of the output current, given by

$$I_{o,dc} = \frac{M\frac{2}{\pi}V_{i,p}}{R} = \frac{0.6 \times \frac{2}{\pi} \times \sqrt{2} \times 120}{2} = 32.4 \text{ A}.$$

Now, the computations of $i_o(t)$ can be performed for the consecutive switching intervals; the t_2 instant for a given interval being the t_0 instant for the next interval.

First interval ($t_0 = 0$, $t_1 = 0.833$ ms, $t_2 = 1.389$ ms):

$$i_o(0.833 \text{ ms}) = 0.667 \times 32.4 + 28.3|\sin(377 \times 0)| = 21.6 \text{ A}$$

and

$$i_o(1.389 \text{ ms}) = 0.778 \times 21.6 = 16.8 \text{ A}.$$

Second interval ($t_0 = 1.389$ ms, $t_1 = 2.222$ ms, $t_2 = 2.778$ ms):

$$i_o(2.222\text{ ms}) = 0.667 \times 16.8 + 28.3|\sin(377 \times 1.389 \times 10^{-3})| = 25.4\text{ A}$$

and

$$i_o(2.778\text{ ms}) = 0.778 \times 25.4 = 19.8\text{ A}.$$

Third interval ($t_0 = 2.778$ ms, $t_1 = 3.611$ ms, $t_2 = 4.167$ ms):

$$i_o(3.611\text{ ms}) = 0.667 \times 19.8 + 28.3|\sin(377 \times 2.778 \times 10^{-3})| = 37.7\text{ A}$$

and

$$i_o(4.167\text{ ms}) = 0.778 \times 37.7 = 29.3\text{ A}.$$

Fourth interval ($t_0 = 4.167$ ms, $t_1 = 5$ ms, $t_2 = 5.556$ ms):

$$i_o(4.167\text{ ms}) = 0.667 \times 29.3 + 28.3|\sin(377 \times 4.167 \times 10^{-3})| = 47.8\text{ A}$$

and

$$i_o(5.556\text{ ms}) = 0.778 \times 47.8 = 37.2\text{ A}.$$

Fifth interval ($t_0 = 2.778$ ms, $t_1 = 3.611$ ms, $t_2 = 4.167$ ms):

$$i_o(6.389\text{ ms}) = 0.667 \times 37.2 + 28.3|\sin(377 \times 5.556 \times 10^{-3})| = 49.3\text{ A}$$

and

$$i_o(6.944\text{ ms}) = 0.778 \times 49.3 = 38.4\text{ A}.$$

Sixth interval ($t_0 = 6.944$ ms, $t_1 = 7.778$ ms, $t_2 = 8.333$ ms):

$$i_o(7.778\text{ ms}) = 0.667 \times 38.4 + 28.3|\sin(377 \times 6.944 \times 10^{-3})| = 39.8\text{ A}$$

and

$$i_o(8.333\text{ ms}) = 0.778 \times 39.8 = 31.0\text{ A}.$$

As the rectifier is assumed to operate in the quasi-steady state, the last, final value, $i_o(8.333\text{ ms})$, should be equal to the initial value, $i_o(0)$. It is not exactly so, which implies an incorrect assumption of the initial value of 32.4 A. Note that the impact of the initial value on the final value is minimal, since at each step of the computations the previous value of the output current is multiplied by either 0.667 or 0.778.

Thus, after the 12 steps resulting from the 6 switching intervals, the initial value error, $\Delta i_o(0)$, is translated into the respective final value error, $\Delta i_o(8.333\text{ ms})$, of $(0.667 \times 0.778)^6\ \Delta i_o(0) \approx 0.02\ \Delta i_o(0)$ only. Therefore, it is a safe guess that the obtained final value of 31 A is only slightly greater than the actual initial value. Assuming now that $i_o(0)$ is 30.9 A, and repeating the iterative calculations, yields the following points of the output current waveform:

0	30.9 A
0.833 ms	20.6 A
1.389 ms	16.0 A
…	…
6.944 ms	38.3 A
7.778 ms	39.7 A
8.333 ms	30.9 A.

Waveforms of the output voltage and current are shown in Figure 1.38. Clearly, the calculations presented can easily be computerized, greatly reducing the amount of work involved.

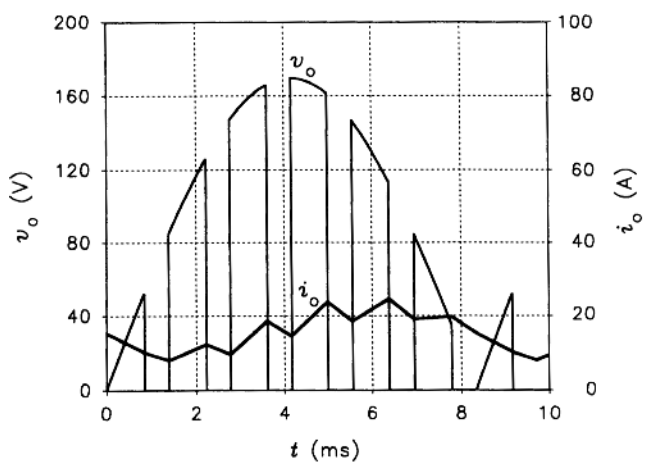

Figure 1.38 Output voltage and current waveforms in the generic PWM rectifier in Example 1.5.

PROBLEMS

P1.1 Refer to Figure 1.13 and sketch the waveforms of the output voltage of the generic rectifier with the firing angle of:

(a) 36°

(b) 72°

(c) 108°

(d) 144°

P1.2 Refer to Figure 1.15 and sketch the waveforms of the output voltage of the generic ac voltage controller with the firing angle of:

(a) 36°

(b) 72°

(c) 108°

(d) 144°

P1.3 From the harmonic spectrum in Figure 1.18a, find:

(a) per-unit value of the dc component of output voltage of the generic rectifier with the firing angle of 90° (use Eq. 1.44 to verify the result)

(b) peak and rms per-unit values of the fundamental output voltage with the same firing angle as in (a)

P1.4 From the harmonic spectrum in Figure 1.18b, read the peak per-unit value of fundamental output voltage of the generic ac voltage controller with the firing angle of 90°, and find:

(a) rms per-unit value of the output voltage (use Eq. 1.44)

(b) per-unit harmonic content of the output voltage

(c) total harmonic distortion of the output voltage

P1.5 For the generic inverter in the square-wave operating mode, derive an equation for the peak per-unit value of kth harmonic of the output voltage (take the dc input voltage, V_{i}, as the base voltage). Calculate peak per-unit values of the first 10 harmonics.

P1.6 A generic rectifier is supplied from a 230-V, 60-Hz ac source and operates with the firing angle of 30°. For the output voltage of the rectifier, find:

(a) dc component

(b) rms value of the ac component

(c) ripple factor

P1.7 For the generic rectifier in P1.6, find the fundamental frequency of the output voltage.

P1.8 For the generic ac voltage controller, derive an equation for the peak per-unit value of kth harmonic of the output voltage as a function of the firing angle (take the peak value, $V_{\mathrm{i,p}}$, of input voltage as the base voltage). Use the spectrum in Figure 1.18b to verify the equation for the firing angle of 90° and the five lowest-order harmonics.

P1.9 Review the output voltage waveforms in Figures 1.13 and 1.15 and the corresponding harmonic spectra in Figure 1.18. Which harmonics present in the spectrum for the generic phase-controlled rectifier are absent in the spectrum for the generic ac voltage controller? Why?

P1.10 The generic converter operates as a chopper and is supplied from a 200-V dc source. What is the magnitude control ratio of the chopper and what are the duty ratios of individual switches if the average output voltage is 80 V?

P1.11 Repeat Problem 1.10 for the output voltage of –95 V.

P1.12 The generic converter operates as a chopper with the magnitude control ratio of 0.4. Duration of a pulse of the output voltage is 125 µs. Find the switching frequency.

P1.13 The generic converter operates as a chopper with 2500 pulses of output voltage per second and with the magnitude control ratio of 0.7. Find the durations of a pulse and a notch of the output voltage.

P1.14 The generic PWM rectifier operates with the average output voltage reduced by 30% with respect to the maximum available value of this voltage. Find the magnitude control ratio and the ratio of the width of pulse of the output voltage to the notch width.

P1.15 The generic PWM rectifier operates as in Problem 1.14 with 100 pulses of the output voltage per period of the 60-Hz input voltage. Find the width of a pulse and the switching frequency.

P1.16 The generic PWM ac voltage controller is supplied from a 60-Hz ac voltage source. The fundamental output voltage is reduced by two-thirds with respect to the supply voltage, and the switching frequency is 5 kHz. Find:

(a) number of pulses of the output voltage per cycle of this voltage
(b) duty ratios of switches S1, S2, and S5
(c) pulse width

P1.17 Refer to Figure 1.23a and determine the states (on or off) of all the switches of the generic PWM inverter at:

(a) $\omega t = \pi/2$ rad
(b) $\omega t = \pi$ rad
(c) $\omega t = 3\pi/2$ rad

P1.18 Waveforms in Figure 1.24 for the generic PWM inverter, with 10 pulses of the output voltage per cycle, were obtained in the following way:

(1) The 360° period of the voltage was divided into 10 equal switching intervals, 36° each
(2) Denoting by α_n the central angle of nth switching interval ($\alpha_1 = 18°$, $\alpha_2 = 54°$, etc.), the duty ratio, d_n, of operating switches S1–S2 or S3–S4 for this interval was calculated as

$$d_n = M|\sin(\alpha_n)|$$

where M denotes the magnitude control ratio.

For eight pulses per cycle and $M = 0.6$, find widths (in degrees) of individual pulses of the output voltage and sketch to scale the resultant voltage waveform, $v_o(\omega t)$. The voltage pulses should be located in the middle of switching intervals.

P1.19 Refer to Figure 1.35 and sketch the output voltage waveform of a generic *phase-controlled* cycloconverter with the input/output frequency ratio of 3 and firing angle of 45°. Mark the states of the converter.

P1.20 Refer to Figure 1.35 and sketch the output voltage waveform of a generic PWM cycloconverter with the input/output frequency ratio of 2 and the magnitude control ratio of 0.5 (for convenience, assume a low number of pulses of the output voltage).

P1.21 The generic PWM rectifier is supplied from a 460-V, 60-Hz ac line and operates with the magnitude control ratio of 0.6 and switching frequency of 840 Hz. The rectifier feeds a dc motor, which, under the given operating conditions, can be represented as an RLE-load with $R = 0.5\ \Omega$, $L = 10$ mH, and $E = 200$ V. Determine the piecewise linear waveform of the output current for one cycle of output voltage.

P1.22 The generic PWM ac voltage controller with an RL-load is supplied from a 220-V, 50-Hz source and operates with 10 switching intervals per cycle and the magnitude control ratio of 0.75. The resistance and inductance of the load are 22 Ω and 55 mH, respectively. Determine the piecewise linear waveform of the output current for one cycle of the output voltage.

COMPUTER ASSIGNMENTS

The generic power converter, which represents an idealized theoretical concept, cannot be modeled precisely using Spice, which is designed for the simulation of practical circuits. In contrast to the ideal, infinitely fast switches assumed for the generic converter, Spice switch models have finite time of transition from one state to another. To avoid interruptions of the output current, the output terminals of the converter must, therefore, be shorted by switch S5 just before the output is separated from the input by opening switches S1–S2 or S3–S4. As a result, the supply source is temporarily shorted too, albeit very briefly, producing large impulses ("spikes") of the input current. Such short-circuit currents are not present in practical, correctly designed and controlled power electronic converters. However, the output voltage and current in the generic converter can be simulated quite accurately.

To calculate the figures of merit, make use of the avg(x) (average value of x) and rms(x) (rms value of x) functions. Also, employ the *Fourier* option for the X-axis to obtain harmonic spectra of voltages and currents. Refer to Appendix A and Reference 3 for instructions on Spice simulations.

Assignments with the asterisk "*" denote circuit files available at the Publisher's website listed in the Preface and Appendix A.

CA1.1* Run Spice file *Gen_Ph-Contr_Rect.cir* for the generic phase-controlled rectifier. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

(a) dc component
(b) rms value
(c) rms value of the ac component
(d) ripple factor

Observe oscillograms of the input and output voltages and currents. Explain what causes spikes in the oscillograms of the input quantities, and what limits the amplitude of the current spikes.

CA1.2 Develop a Spice circuit file for the generic inverter operating in the square-wave mode with an adjustable fundamental output frequency. Perform a simulation for the output frequency of 50 Hz and find for the output voltage of the inverter:

(a) rms value
(b) rms value of the fundamental (from harmonic spectrum)
(c) harmonic content
(d) total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

CA1.3 Develop a Spice circuit file for the generic ac voltage controller. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

(a) rms value
(b) rms value of the fundamental (from harmonic spectrum)
(c) harmonic content
(d) total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

CA1.4 Refer to Example 1.1 and Problem P1.19 and develop a Spice circuit file for a generic phase-controlled cycloconverter with the input frequency of 50 Hz and fundamental output frequency of 25 Hz. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

(a) rms value
(b) rms value of the fundamental (from harmonic spectrum)
(c) harmonic content
(d) total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

CA1.5 Develop a Spice circuit file for the generic chopper. Perform simulations for the switching frequency of 1 kHz and magnitude control ratios of 0.6 and –0.3 and find for the output voltage of the converter:

(a) dc component
(b) rms value
(c) rms value of the ac component
(d) ripple factor

Observe oscillograms of the input and output voltages and currents.

CA1.6* Run Spice program *Gen_PWM_Rect.cir* for the generic PWM rectifier. Perform simulations for 12 pulses of the output voltage per cycle of the input voltage and magnitude control ratio of 0.5, and find for the output voltage of the converter:

(a) dc component
(b) rms value
(c) rms value of the ac component
(d) ripple factor

Observe oscillograms of the input and output voltages and currents. Explain what causes the spikes in the oscillograms of the input quantities, and what limits the amplitude of the current spikes.

CA1.7 Develop a Spice circuit file for the generic PWM ac voltage controller. Perform a simulation for 12 pulses of output voltage per cycle and the magnitude control ratio of 0.5, and find for the output voltage of the converter:

(a) rms value
(b) rms value of the fundamental (from harmonic spectrum)
(c) harmonic content
(d) total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

CA1.8 Refer to Example 1.1 and Problem P1.20 and develop a Spice circuit file for a generic PWM cycloconverter. Perform a simulation for the input frequency of 50 Hz, fundamental output frequency of 25 Hz, magnitude control ratio of 0.5, and 10 pulses of output voltage per cycle of the input voltage. Find for the output voltage of the converter:

(a) rms value
(b) rms value of the fundamental (from harmonic spectrum)
(c) harmonic content
(d) total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

CA1.9 Develop a computer program for calculation of harmonics of a given periodic waveform, $\psi(\omega t)$. Data points that represent one cycle of the waveform are stored in an ASCII file in the ωt, $\psi(\omega t)$ format. Generate and store the voltage waveform in Figure 1.13 (generic phase-controlled rectifier) and use your program to obtain the harmonic spectrum of the waveform. Compare the results with those in Figure 1.18a.

CA1.10 Develop a computer program for calculation of the following parameters of a given periodic waveform, $\psi(\omega t)$:

(a) rms value

(b) dc component

(c) rms value of the ac component

(d) rms value of the fundamental

(e) harmonic content

(f) total harmonic distortion

Data points that represent one cycle of the waveform are stored in an ASCII file in the ωt, $\psi(\omega t)$ format. Generate and store the voltage waveform in Figure 1.15 (generic phase-controlled ac voltage controller) and apply the program to compute parameters (a) through (f).

CA1.11 Develop a computer program for determination of a current waveform generated in a given load by a given voltage. The load can be of the R, RL, RE, LE, or RLE type, and the voltage waveform, $v(t)$, is given as either a closed-form function of time, $v = f(t)$, or an ASCII file of (t, v) pairs.

CA1.12* Run Spice program *Diode_Rect_1P.cir* for a single-pulse diode rectifier. Repeat the simulation for the rectifier with a freewheeling diode and with an output capacitor ("comment out" the unused components). In each case, determine the average output voltage and ripple factor of that voltage.

FURTHER READING

[1] Rashid, M. H., *Power Electronics Handbook*, 2nd ed., Academic Press, Boston, MA, 2010, Chapter 1.

[2] Rashid, M. H., *Power Electronics: Circuits, Devices, and Applications*, 4th ed., Prentice Hall, Upper Saddle River, NJ, 2013, Chapter 1.

[3] Rashid, M. H., *SPICE for Power Electronics and Electric Power*, 3rd ed., CRC Press, Boca Raton, FL, 2012.