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INTRODUCTION

Optics, as a field of science, is in its third millennium of life; yet in spite of its age, it remains remarkably vigorous and youthful. During the middle of the twentieth century, various events and discoveries gave new life, energy, and richness to the field. Especially important in this regard were (i) the introduction of the concepts and tools of Fourier analysis and communications theory into optics, primarily in the late 1940s and throughout the 1950s; (ii) the invention of the laser in late 1950s and its commercialization starting in the early 1960s; (iii) the origin of the field of nonlinear optics in the 1960s; (iv) the invention of the low-loss optical fiber in the early 1970s and the revolution in optical communications that followed; and (v) the rise of the young fields of nanophotonics and biophotonics. It is the thesis of this book that in parallel with these many advances, another important change has also taken place gradually but with an accelerating pace, namely, the infusion of statistical concepts and methods of analysis into the field of optics. It is to the role of such concepts in optics that this book is devoted.

The field we shall call "statistical optics" has a considerable history of its own. Many fundamental statistical problems were solved in the late nineteenth century and applied to acoustics and optics by Lord Rayleigh. The need for statistical methods in optics increased dramatically with the discovery of the quantized nature of light and, particularly, with the statistical interpretation of quantum mechanics introduced by Max Born. The introduction by E. Wolf in 1954 of an elegant and broad framework for considering the coherence properties of waves laid a foundation upon which many of the important statistical problems in optics could be treated in a unified

Statistical Optics, Second Edition. Joseph W. Goodman.

way. Also worth mentioning is the semiclassical theory of light detection, pioneered by L. Mandel, which tied together, in a comparatively simple way, knowledge of the statistical fluctuations of classical wave quantities (fields, intensities) and fluctuations associated with the interaction of light and matter. This history is far from complete but is dealt with in more detail in the individual chapters that follow.

1.1 DETERMINISTIC VERSUS STATISTICAL PHENOMENA AND MODELS

In the normal course of events, a student of physics or engineering first encounters optics in an entirely deterministic framework. Physical quantities are represented by mathematical functions that are either completely specified in advance or are assumed precisely measurable. These physical quantities are subjected to well-defined transformations that modify their form in perfectly predictable ways. For example, if a monochromatic light wave with a known complex field distribution is incident on a transparent aperture in an otherwise opaque screen, the resulting complex field distribution some distance behind the screen can be calculated using the well-established diffraction formulas of wave optics. In this approach, inaccuracies in the results arise only due to inaccuracies of the deterministic models used to describe the diffraction process.

The students emerging from such an introductory course may feel confident that they have grasped the basic physical concepts and laws and are ready to find a precise answer to almost any problem that comes their way. To be sure, they have probably been warned that there are certain problems, arising particularly in the detection of weak light waves, for which a statistical approach is required. But a statistical approach to problem solving appears at first glance to be a “second class” approach, for statistics is generally used when we lack sufficient information to carry out the aesthetically more pleasing “exact” solution. The problem may be inherently too complex to be solved analytically or numerically, or the boundary conditions may be poorly defined. Surely, the preferred way to solve a problem must be the deterministic way, with statistics entering only as a sign of our weakness or limitations. Partially as a consequence of this viewpoint, the subject of statistical optics is usually left for the more advanced students, particularly those with a mathematical flair.

Although the origins of the above viewpoint are quite clear and understandable, the conclusions reached regarding the relative merits of deterministic and statistical analysis are very greatly in error, for several important reasons. First, it is difficult, if not impossible, to conceive of a real engineering problem in optics that does not contain some element of uncertainty requiring statistical analysis. Even the lens designer, who traces rays through application of precise physical laws accepted for centuries, must ultimately worry about quality control! Thus statistics is certainly not a subject to be left primarily to those more interested in mathematics than in physics and engineering.

Furthermore, the view that the use of statistics is an admission of one’s limitations and thus should be avoided is based on too narrow a view of the nature of statistical phenomena. Experimental evidence indicates, and indeed the great majority of physicists believe, that the interaction of light and matter is *fundamentally* a statistical phenomenon, which in principle cannot be predicted with perfect precision in advance. Thus statistical phenomena play a role of the greatest importance in the world around us, independent of our particular mental capabilities or limitations.

Finally, in defense of statistical analysis, we must say that, whereas both deterministic and statistical approaches to problem solving require the construction of mathematical models of physical phenomena, the models constructed for statistical analysis are inherently more general and flexible. Indeed, they invariably contain the deterministic model as a special case! For a statistical model to be accurate and useful, it should fully incorporate the current state of our knowledge regarding the physical parameters of concern. Our solutions to statistical problems will be no more accurate than the models we use to describe both the physical laws involved and the state of knowledge or ignorance.

The statistical approach is indeed somewhat more complex than the deterministic approach, for it requires knowledge of the elements of probability theory. In the long run, however, statistical models are far more powerful and useful than deterministic models in solving physical problems of genuine practical interest. Hopefully, the reader will agree with this viewpoint by the time this book has been digested.

1.2 STATISTICAL PHENOMENA IN OPTICS

Statistical phenomena are so plentiful in optics that there is no difficulty in compiling a long list of examples. Because of the wide variety of these problems, it is difficult to find a general scheme for classifying them. Here we attempt to identify several broad aspects of optics that require statistical treatment. These aspects are conveniently discussed in the context of an optical imaging problem.

Most optical imaging problems are of the following type. Nature assumes some particular state (e.g., a certain collection of atoms and/or molecules in a distant region of space, a certain distribution of reflectance over terrain of unknown characteristics, or a certain distribution of transmittance in a sample of interest). By operating on optical waves that arise as a consequence of this state of Nature, we wish to deduce exactly what that state is.

Statistics is involved in this task in a wide variety of ways, as can be discovered by reference to Fig. 1.1.

First, and most fundamentally, the state of Nature is known to us a priori only in a statistical sense. If it were known exactly, there would be no need for any measurement in the first place. Thus the state of Nature is random, and in order to properly assess the performance of the system, we must have a statistical model, ideally representing the set of possible states, together with associated probabilities of the occurrence of those states. Usually, a less complete description of the statistical properties of the object will suffice.

Our measurement system operates not on the state of Nature per se, but rather on, an optical representation of that state (e.g., radiated light, transmitted light, or

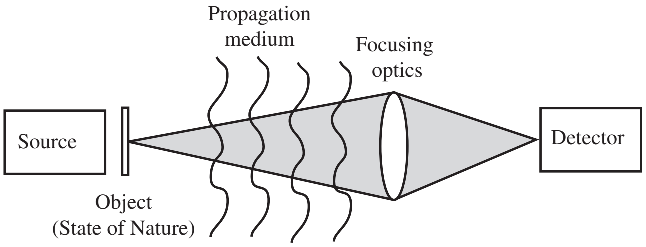

Figure 1.1 An optical imaging system.

reflected light). The representation of the state of Nature by an optical wave has statistical attributes itself, primarily as a result of the statistical or random properties of all light waves. Because of fundamentally statistical attributes of the mechanisms that generate light, all optical sources produce radiation that is to some degree random in its properties. At one extreme, we have the chaotic and unordered emission of light by a thermal source, such as an incandescent lamp; at the other extreme, we have the comparatively ordered emission of light by a continuous-wave (CW) gas laser. In the latter case, the light comes close to containing only a single frequency and traveling in a single direction. Nonetheless, any real laser emits light with statistical properties, including random fluctuations of both amplitude and phase of the radiation. Statistical fluctuations of light are of great importance in many optical experiments and indeed play a central role in the character of the image produced by the system depicted in Fig. 1.1.

After interacting with the state of Nature, the radiation travels through an intervening medium until it reaches the focusing optics. The parameters of that medium may or may not be well known. If the medium is a perfect vacuum, it introduces no additional statistical aspects to the problem. On the other hand, if the medium is the Earth's atmosphere and the optical path length is more than a few meters, random fluctuations of the atmospheric index of refraction can have dramatic effects on the wave and can seriously degrade the image obtained by the system. Statistical methods are required to quantify this degradation.

The light eventually reaches the focusing optics. How well are the exact parameters of this system known? Any lack of knowledge of the parameters of the system must be taken into account in our statistical model for the measurement system. For example, there may be unknown errors in the wavefront deformation introduced by passage through the focusing optics. Such errors can be modeled statistically over an ensemble of possible lenses and should be taken into account in assessment of the performance of the system.

The radiation finally reaches an optical detector, where there is an interaction of light and matter. Random fluctuations of the detected energy are readily observed, particularly at low light levels, and can be attributed to a variety of sources, including the discrete nature of the interaction between light and matter as well as the presence of internal additive electronic noise (e.g., thermal noise associated with resistors in the detector circuitry). The result of the measurement is thus related to the image falling on the detector only in a statistical way.

We conclude that at all stages of the optical problem, including illumination, transmission, image formation, and detection, statistical treatment may be needed to assess the performance of the system. Our goal in this book is to lay the necessary foundation and to illustrate the application of statistics to the many diverse areas of optics where it is needed.

1.3 AN OUTLINE OF THE BOOK

Eight chapters follow this chapter. Since many scientists and engineers working in the field of optics may feel the need to strengthen their basic knowledge of statistical tools, Chapter 2 presents a review of probability theory and Chapter 3 contains a review of the theory of random processes. *The reader already familiar with these subjects may wish to proceed directly to Chapter 4, using the earlier material only for reference when needed.*

Discussion of optical problems begins in Chapter 4, which deals with the "first-order" statistics (i.e., the statistics at a single point in space and time) of several kinds of light waves, including light generated by thermal sources and light generated by lasers. Also included is a formalism that allows characterization of the polarization properties of an optical wave.

Chapter 5 introduces the concepts of time coherence and space coherence (which are "second-order" properties of light waves) and deals at length with the propagation of coherence under various conditions. Chapter 6 extends this theory to coherence of order higher than two and illustrates the need for fourth-order coherence functions in a variety of different optical problems, including a classical analysis of the intensity interferometer.

Chapter 7 is devoted to the theory of image formation in partially coherent light. Several analytical approaches to the problem are introduced, including the approach widely used in microlithography. The concept of interferometric imaging, as widely practiced in radio astronomy, is also introduced in this chapter and is used to lend insight into the incoherent image formation process.

Chapter 8 is concerned with the effects of transparent random media, such as the Earth's atmosphere, on the quality of images formed by optical systems. The origin of the random refractive-index fluctuations in the atmosphere is reviewed, and statistical models for such fluctuations are introduced. The effects of these fluctuations on optical waves are dealt with, and image degradations introduced by the atmosphere are treated from a statistical viewpoint. Stellar speckle interferometry, a method for partially overcoming the effects of atmospheric turbulence, is discussed in some detail, as are several related methods for achieving more complete image recovery.

Finally, Chapter 9 treats the semiclassical theory of light detection and illustrates the theory with analyses of the sensitivity limitations of amplitude interferometry, intensity interferometry, and speckle interferometry.

Appendices A–E present supplemental background material and analysis.