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Introduction

1.1 Classification of Power Supplies

Power supply technology is an enabling technology that allows us to build and operate electronic circuits and systems [1–28]. All active electronic circuits, both digital and analog, require power supplies. Many electronic systems require several dc supply voltages. Power supplies are widely used in computers, telecommunications, instrumentation equipment, aerospace, medical, and defense electronics. A dc supply voltage is usually derived from a battery or an ac utility line using a transformer, rectifier, and a filter. The resultant raw dc voltage is not constant enough and contains a high ac ripple that is not appropriate for most applications. *Voltage regulators* are used to make the dc voltage more constant and to attenuate the ac ripple.

A power supply is a constant voltage source with a maximum current capability. There are two general classes of power supplies: *regulated* and *unregulated*. The output voltage of a regulated power supply is automatically maintained within a narrow range, e.g., 1 or 2% of the desired nominal value, in spite of line voltage, load current, and temperature variations. Regulated dc power supplies are called *dc voltage regulators*. There are also *dc current regulators*, such as battery chargers.

Figure 1.1 shows a classification of regulated power supply technologies. Two of the most popular categories of voltage regulators are *linear regulators* and *switching-mode power supplies* (SMPS). There are two basic linear regulator topologies: the series voltage regulator and the shunt voltage regulator. The switching-mode voltage regulators are divided into three categories: pulse-width modulated (PWM) dc–dc converters, resonant dc–dc converters, and switched-capacitor (also called charge-pump) voltage regulators. In linear voltage regulators, transistors are operated in the active region as dependent current sources with relatively high voltage drops at high currents, dissipating a large amount of power and resulting in low efficiency. Linear regulators are heavy and large, but they exhibit low noise level and are suitable for audio applications.

In switching-mode converters, transistors are operated as switches, which inherently dissipate much less power than transistors operated as dependent current sources. The voltage drop across the transistors is very low when they conduct high current and the transistors conduct a nearly zero current when the voltage drop across them is high. Therefore, the conduction losses are low and the efficiency of switching-mode converters is high, usually above 80% or 90%. However, switching losses reduce the efficiency at high frequencies. Switching losses increase proportionally to switching frequency. Linear and switched-capacitor regulator circuits (except for large capacitors) can be fully integrated and are used in low-power and low-voltage applications, usually below several watts and

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Pulse-Width Modulated DC–DC Power Converters, Second Edition. Marian K. Kazimierczuk.

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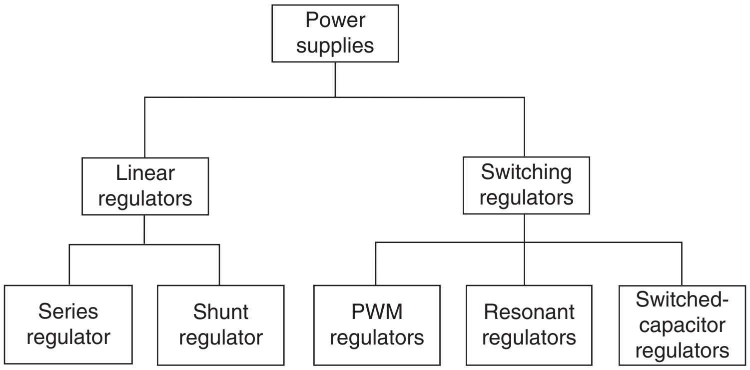

Figure 1.1 *Classification of power supply technologies.*

50 V. PWM and resonant regulators are used at high power and voltage levels. They are small in size, light in weight, and have high conversion efficiency.

Figure 1.2 shows block diagrams of two typical ac–dc power supplies that convert the widely available ac power to dc power. The power supply of Figure 1.2(a) contains a dc linear voltage regulator, whereas the power supply of Figure 1.2(b) contains a switching-mode voltage regulator. The power supply shown in Figure 1.2(a) consists of a low-frequency step-down power line transformer, a front-end rectifier, a low-pass filter, a linear voltage regulator, and a load. The nominal voltage of the ac utility power line is 110 V_{rms} in the United States and 230 V_{rms} in Europe. However, the actual line voltage varies within a range of about ±20% of the nominal voltage. The frequency of the ac line voltage is very low (50 Hz in Europe, 60 Hz in United States, 400 Hz in aircraft applications, and 20 kHz in space applications). The line transformer provides dc isolation from the ac power line and reduces a relatively high line voltage to a lower voltage (ranging usually from 5 to 28 V_{rms}). Since the frequency of the ac line voltage is very low, the line transformer is heavy and bulky. The output voltage of the front-end rectifier/filter is unregulated and it varies because the peak voltage of the ac line varies. Therefore, a voltage regulator is required between the rectifier/filter and the load. There still exists a need for *universal power supplies* that can accept any utility line voltage in the world, ranging from 85 to 264 V_{rms}.

The power supply shown in Figure 1.2(b) consists of a front-end rectifier, a low-pass filter, an isolated dc–dc switching-mode voltage regulator, and a load. It is run directly from the ac line. The ac voltage is rectified directly

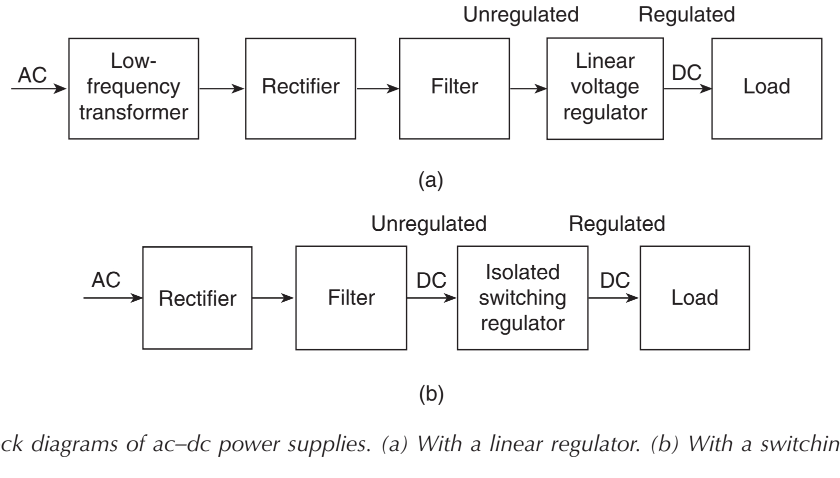

Figure 1.2 *Block diagrams of ac–dc power supplies. (a) With a linear regulator. (b) With a switching-mode voltage regulator.*

from the ac power line, which does not require a bulky low-frequency line transformer. Hence, such a circuit is called an *off-line power supply* (plug into the wall). The switching-mode voltage regulator contains a high-frequency transformer to obtain dc isolation for the entire power supply. Since the switching frequency is much higher than that of the ac line frequency, the size and weight of a high-frequency transformer as well as inductors and capacitors is reduced. The switching frequency usually ranges from 25 to 500 kHz. To avoid audio noise, the switching frequency should be above 20 kHz. A PWM switching-mode voltage regulator generates a high-frequency rectangular voltage wave, which is rectified and filtered. The duty cycle (or the pulse width) of the rectangular wave is varied to control the dc output voltage. Therefore, these voltage regulators are called PWM dc–dc converters.

Power converters are required to convert one form of electric energy to another. A dc–dc converter is a power supply that converts a dc input voltage into a desired regulated dc output voltage. The dc input may be an unregulated or regulated voltage. Often, the input of a dc–dc converter is a battery or a rectified ac line voltage. A voltage regulator should provide a constant voltage to the load, even if line voltage, load current, and temperature vary. Unlike in linear voltage regulators, the output voltage in PWM dc–dc converters may be either lower or higher than the input voltage and are called either *step-down* or *step-up* converters. In a step-down converter, the output voltage is lower than the input voltage. In a step-up converter, the output voltage is higher than the input voltage. Some converters may act as both step-down and step-up converters. The output voltage source may be of the same polarity (noninverting) or opposite polarity (inverting) to that of the polarity of the input voltage. The dc–dc converters may have *common negative* or *common positive* input and output terminals. Converters may have a *single output* or *multiple outputs*. In addition, there are *fixed* or *adjustable* output voltage power supplies. Fixed output voltage supplies (e.g., 1.8 V) are used for power electronic circuits that require a specific supply voltage. Power supplies with adjustable output voltage (e.g., from 0 to 30 V) are convenient for laboratory tests. In some applications, *programmable* power supplies with digitally selected output voltages are required. Power supplies may be *nonisolated* or *isolated*. Transformers can be used to obtain dc isolation between the input and output and between the different outputs. Common requirements of most power supplies are: high efficiency, high power density, high reliability, and low cost.

1.2 Basic Functions of Voltage Regulators

The simplest voltage regulator is a Zener diode regulator, shown in Figure 1.3. It is a shunt regulator. However, the performance of the Zener diode regulator is not satisfactory for most applications. Therefore, negative feedback techniques are usually used in voltage regulators to improve the performance. A block diagram of a voltage regulator with negative feedback is shown in Figure 1.4. It consists of a power stage (a dc–dc converter), a feedback network, a reference voltage V_{ref}, and a control circuit (also called an error amplifier). The feedback network monitors the output voltage and reduces the error signal. The control circuit compares the feedback voltage with the reference voltage, generates an error voltage, amplifies it, and adjusts the transistor base current to keep the output voltage V_O constant.

The load current I_O may vary over a very wide range: $I_{Omin} \le I_O \le I_{Omax}$. Consequently, the load resistance $R_L = V_O/I_O$ also varies over a wide range: $R_{Lmin} \le R_L \le R_{Lmax}$, where $R_{Lmin} = V_O/I_{Omax}$ and $R_{Lmax} = V_O/I_{Omin}$. Most regulated power supplies have a short-circuit or current-overload protection circuit, which limits the output

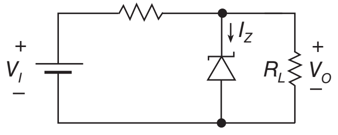

Figure 1.3 *Zener diode voltage regulator.*

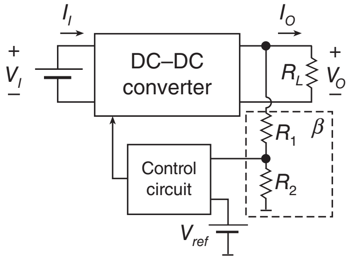

Figure 1.4 *Block diagram of a voltage regulator with negative feedback.*

current to a safe level to protect the power supply and/or the load. The input voltage of a voltage regulator is usually unregulated and can vary over a wide range: $V_{Imin} \le V_I \le V_{Imax}$. For example, the dc input voltage in telecommunication power supplies is $36 \le V_I \le 72$ V with a nominal input voltage $V_{Inom} = 48$ V. The input voltage source may be a battery, a rectified single-phase or three-phase ac line voltage. The output voltage of a battery decreases when the battery is discharged. The peak voltage of a utility line varies as much as 10% or 20%, causing the rectified dc voltage to vary. The operating temperature of semiconductor and passive devices may also change from T_{min} to T_{max}, affecting the performance of power supplies.

The basic functions of a dc–dc converter are as follows:

(1) to provide conversion of a dc input voltage V_I to the desired dc output voltage within a tolerance range, for example, $V_O = 1.2\ \text{V} \pm 1\%$;
(2) to regulate the output voltage V_O against variations in the input voltage V_I, the load current I_O (or the load resistance R_L), and the temperature;
(3) to reduce the output ripple voltage below the specified level;
(4) to ensure fast response to rapid changes in the input voltage and load current (or load resistance);
(5) to provide dc isolation;
(6) to provide multiple outputs;
(7) to minimize the electromagnetic interference (EMI) below levels specified by EMI standards.

1.3 Power Relationships in DC–DC Converters

The input current i_I of many switching-mode dc–dc converters is pulsating. The dc component of the converter input current is given by

$$I_I = \frac{1}{T}\int_0^T i_I dt. \tag{1.1}$$

Hence, the dc input power of a dc–dc converter is

$$P_I = \frac{1}{T}\int_0^T V_I i_I dt = V_I \frac{1}{T}\int_0^T i_I dt = V_I I_I. \tag{1.2}$$

The ac components of the output voltage and current are assumed to be very small and can be neglected. Therefore, dc output power of a dc–dc converter is

$$P_O = V_O I_O \tag{1.3}$$

and the power loss in the converter is

$$P_{LS} = P_I - P_O. \tag{1.4}$$

The efficiency of the dc–dc converter is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{LS}} = \frac{1}{1 + \frac{P_{LS}}{P_O}} \tag{1.5}$$

from which

$$\frac{P_{LS}}{P_O} = \left(\frac{1}{\eta} - 1\right). \tag{1.6}$$

The normalized power loss P_{LS}/P_O decreases as the converter efficiency increases. For example, for $\eta = 25\%$, $P_{LS}/P_O = 300\%$, but for $\eta = 95\%$, $P_{LS}/P_O = 5.26\%$.

1.4 DC Transfer Functions of DC–DC Converters

The dc voltage transfer function (also called the dc voltage conversion ratio or the dc voltage gain) of a dc–dc converter is

$$M_{VDC} = \frac{V_O}{V_I} \tag{1.7}$$

and the dc current transfer function of a dc–dc converter is

$$M_{IDC} = \frac{I_O}{I_I}. \tag{1.8}$$

Hence, the efficiency of a dc–dc converter is

$$\eta = \frac{P_O}{P_I} = \frac{I_O V_O}{I_I V_I} = M_{IDC} M_{VDC}. \tag{1.9}$$

From (1.7), (1.8), and (1.9),

$$V_O = M_{VDC} V_I \tag{1.10}$$

and

$$I_I = \frac{I_O}{M_{IDC}} = \frac{M_{VDC}}{\eta} I_O. \tag{1.11}$$

These equations can be represented by the dc circuit model of a dc–dc converter shown in Figure 1.5.

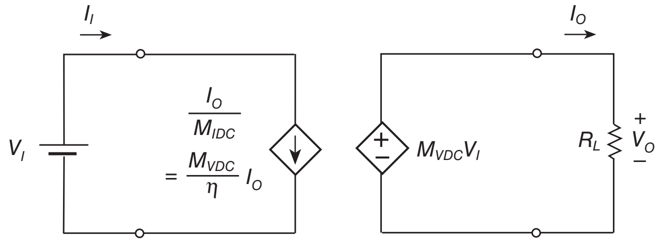

Figure 1.5 A dc model of a dc–dc converter.

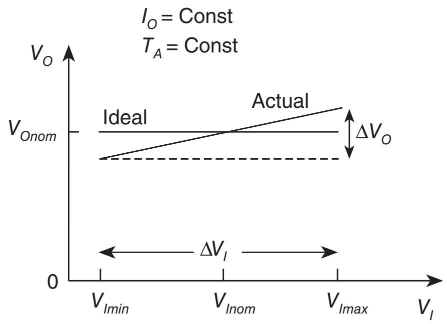

Figure 1.6 *Output voltage V_o versus input voltage V_I for voltage regulators illustrating line regulation.*

1.5 Static Characteristics of DC Voltage Regulators

The quality of a power supply can be described by three parameters: line regulation, load regulation, and thermal regulation. The output voltage V_O of most voltage regulators increases as the input voltage V_I increases, as shown in Figure 1.6. Therefore, one figure-of-merit of voltage regulators for steady-state operation is *line regulation*, which is a measure of the regulator's ability to maintain the predescribed nominal output voltage V_{Onom} under slowly varying input voltage conditions.

The *line regulation* is the ratio of the output voltage change ΔV_O to a corresponding change in the input voltage

$$LNR = \left(\frac{\Delta V_O}{\Delta V_I}\right)\Bigg|_{I_O=Const \text{ and } T_A=Const} \left(\frac{\text{mV}}{\text{V}}\right) \tag{1.12}$$

where T_A is the ambient temperature. For example, for a linear voltage regulator LM140, $\Delta V_O = 10$ mV at $I_O = 0.5$ A, $T_A = 25°\text{C}$, and $7.5 \text{ V} \leq V_I \leq 20$ V. Hence, $LNR = 10/(20 - 7.5) = 0.8$ mV/V.

The *percentage line regulation (PLNR)* is defined as the ratio of the percentage change in the output voltage to a corresponding change in the input voltage

$$PLNR = \frac{\frac{\Delta V_O}{V_{Onom}} \times 100\%}{\Delta V_I}\Bigg|_{I_O=Const \text{ and } T_A=Const} \left(\frac{\%}{\text{V}}\right) \tag{1.13}$$

where T_A is the ambient temperature. Ideally, the line regulation should be zero, in which case the output voltage is independent of the input voltage. In practice, the line regulation (*LNR*) should be less than 0.1%. For example, for a linear voltage regulator LM317, the typical value of the line regulation is $PLNR = 0.01\%/\text{V}$ at $I_O = 20$ mA, $T_A = 25°\text{C}$, and $3 \text{ V} \leq (V_I - V_O) \leq 40$ V.

The output voltage V_O of voltage regulators decreases as the load current I_O increases due to a varying load resistance, as shown in Figure 1.7. Hence, the second figure-of-merit of voltage regulators for steady-state operation is *load regulation*, which is a measure of the regulator's ability to maintain a constant output voltage V_{Onom} under slowly varying load conditions over a certain range of load current, usually from zero load current to a maximum load current I_{Omax}.

The *load regulation* is given by

$$\begin{aligned} LOR &= \frac{\Delta V_O}{\Delta I_O}\Bigg|_{V_I=Const \text{ and } T_A=Const} \left(\frac{\text{mV}}{\text{A}}\right) \\ &= \frac{V_{Omax} - V_{Omin}}{I_{Omax} - I_{Omin}}\Bigg|_{V_I=Const \text{ and } T_A=Const} \left(\frac{\text{mV}}{\text{A}}\right). \end{aligned} \tag{1.14}$$

The load regulation *LOR* should be less than 1%.

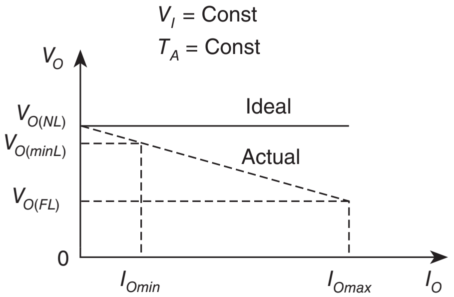

Figure 1.7 *Output voltage V_O versus output current I_O for voltage regulators illustrating load regulation.*

The *percentage load regulation* for voltage regulators that have no minimum load requirement is defined as

$$PLOR = \frac{V_{O(NL)} - V_{O(FL)}}{V_{O(FL)}} \times 100\%\Bigg|_{V_I=Const \text{ and } T_A=Const} \quad (\%) \tag{1.15}$$

where $V_{O(NL)}$ is the no-load (open-circuit) output voltage and $V_{O(FL)}$ is the full-load output voltage, which corresponds to a maximum load current I_{Omax}. In some voltage regulators, such as PWM converters operated in the continuous conduction mode, the minimum load current I_{Omin} is not zero. The output voltage at the minimum load current is $V_{O(minL)}$. In this case, the load regulation is defined as

$$PLOR = \frac{V_{O(minL)} - V_{O(FL)}}{V_{O(FL)}} \times 100\%\Bigg|_{V_I=Const \text{ and } T_A=Const} \quad (\%). \tag{1.16}$$

For an ideal voltage regulator, the load regulation is zero. For example, for a linear voltage regulator LM117, $PLOR_2 = 0.3\%$ for 5 mA$\leq I_O \leq$ 100 mA and $T_A = 25°\text{C}$.

The line regulation and the load regulation can be combined into a *line/load regulation*

$$LLR = \frac{\frac{\Delta V_O}{V_{Onom}} \times 100\%}{\Delta I_O}\Bigg|_{V_I=Const \text{ and } T_A=Const} \quad \left(\frac{\%}{\text{A}}\right). \tag{1.17}$$

Sometimes power supply manufacturers specify the equivalent dc output resistance R_o. A dc model of a real voltage source consists of an ideal voltage source V and an output resistance R_o, as shown in Figure 1.8. The output voltage is given by

$$V_O = V - R_o I_O \tag{1.18}$$

from which

$$\Delta V_O = -R_o \Delta I_O. \tag{1.19}$$

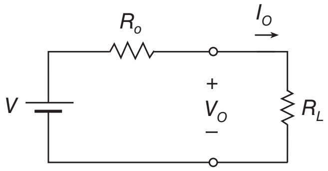

Figure 1.8 *DC model of voltage source with an output resistance.*

Hence, the *incremental* or *dynamic output resistance* is defined as the ratio of change in the output voltage to the corresponding change in the load current

$$R_o = -\frac{\Delta V_O}{\Delta I_O} = -\frac{V_{Omin} - V_{Omax}}{I_{Omax} - I_{Omin}} = -LOR(\Omega). \tag{1.20}$$

When $I_{Omin} = 0$, the dc output resistance is given by

$$R_o = \frac{V_{O(NL)} - V_{O(FL)}}{I_{Omax}}. \tag{1.21}$$

The output resistance of a voltage regulator should be as low as possible so that a change in the output current ΔI_O will result only in a small change in the output voltage $\Delta V_O = -R_o \Delta I_O$. Ideally, R_o should be zero, resulting in the output voltage that is independent of the load current. At high frequencies (or for fast changes in the load current), the output resistance has a complex output impedance. From Figure 1.8, the output voltage at the full load resistance $R_{FL} = R_{Lmin}$ is

$$V_{O(FL)} = V_{O(NL)} \frac{R_{FL}}{R_o + R_{FL}}. \tag{1.22}$$

Hence, the percentage load regulation when the voltage regulator operates from full load to no-load can be expressed as

$$PLOR = \frac{V_{O(NL)} - V_{O(FL)}}{V_{O(FL)}} \times 100\% = \frac{V_{O(FL)} \left(\frac{R_o + R_{FL}}{R_{FL}}\right) - V_{O(FL)}}{V_{O(FL)}} \times 100\% = \frac{R_o}{R_{FL}} \times 100\%. \tag{1.23}$$

A very low output resistance can be obtained by using negative feedback with shunt connection of the power stage and the feedback network at the output. The relationship between the open-loop output resistance R_o and the closed-loop output resistance R_{of} is

$$R_{of} = \frac{R_o}{1 + \beta A} \tag{1.24}$$

where A is the dc (or low-frequency) voltage gain of the forward path and β is the transfer function of the feedback network.

A third figure-of-merit of voltage regulators is the *thermal regulation* defined as

$$THR = \left.\frac{\frac{\Delta V_O}{V_{Onom}} \times 100\%}{\Delta P_D}\right|_{I_O = Const \text{ and } V_I = Const} \left(\frac{\%}{\text{W}}\right) \tag{1.25}$$

where ΔP_D is the change in power dissipation. For example, for a linear voltage regulator LM317, $THR = 0.04\%/\text{W}$.

The *static* or *dc input resistance* of a dc voltage regulator at a given operating point Q is

$$R_{in(DC)} = \frac{V_I}{I_I}. \tag{1.26}$$

Since

$$P_O = \frac{V_O^2}{R_L} \tag{1.27}$$

and

$$P_I = \frac{V_I^2}{R_{in(DC)}} \tag{1.28}$$

the converter efficiency can be expressed as

$$\eta = \frac{P_O}{P_I} = \frac{\frac{V_O^2}{R_L}}{\frac{V_I^2}{R_{in(DC)}}} = \left(\frac{V_O}{V_I}\right)^2 \frac{R_{in(DC)}}{R_L} = M_{VDC}^2 \frac{R_{in(DC)}}{R_L}. \tag{1.29}$$

Hence, one obtains the dc input resistance of dc voltage regulators as a function of load resistance R_L and the dc–dc voltage transfer function

$$R_{in(DC)} = \frac{\eta R_L}{M_{VDC}^2}. \tag{1.30}$$

1.6 Dynamic Characteristics of DC Voltage Regulators

Voltage regulators should minimize the amount of ripple voltage at the output. The parameter that describes this feature is called the *ripple rejection ratio* defined as

$$RRR = \frac{V_{ri}}{V_r} \tag{1.31}$$

where V_r is the output ripple resulting from an input ripple V_{ri}. For example, for a linear voltage regulator LM317, $RRR = 80$ dB $= 10^4$ at $f = 120$ Hz. If the input ripple $V_{ri} = 1$ V, then the output ripple is $V_r = V_{ri}/RRR = 1/10^4 = 0.1$ mV.

Dynamic transient performance of voltage regulators is described by *line transient response* and *load transient response*. In general, transient response is the shape of a signal as it moves between two steady-state points. Figure 1.9 shows a circuit for testing line transient response of voltage regulators. A test is made at a fixed load current I_O, usually 50% of its rated full-load current I_{Omax}. The input voltage v_I contains step changes of magnitude Δv_I superimposed on its dc component V_I, as shown in Figure 1.10(a). As a result, the output voltage v_O contains transients just after the step changes in the input voltage, as shown in Figure 1.10(b). When the input voltage v_I abruptly increases, the output voltage v_O also increases initially and then returns to a steady-state value. On the other hand, when the input voltage v_I abruptly decreases, the output voltage also decreases initially and then returns to a steady-state value. The abrupt change in the input voltage may cause an oscillatory (or underdamped) response characterized by overshoot and undershoot through the limits of a static regulation band. The response may be overdamped or critically damped. A closed-loop step response should be nonoscillatory. An oscillatory step response of a closed-loop circuit indicates that the margins of stability are too low or the circuit is unstable. The settling time t_s and the transient component V_{pk} should be below the specified levels.

Figure 1.11 shows a circuit for testing a transient response to a sudden electrical load changes. The input voltage V_I is held constant, usually at the nominal value V_{Inom}. Step changes in the load current are obtained using an active load that acts like a current sink. Its waveform is a square wave with a dc offset, as shown in Figure 1.12(a). The step changes in the load current cause a transient response in the converter output voltage. When the load

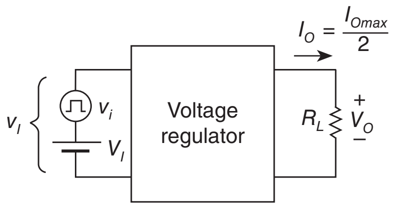

Figure 1.9 *Circuit for testing the line transient response of voltage regulators.*

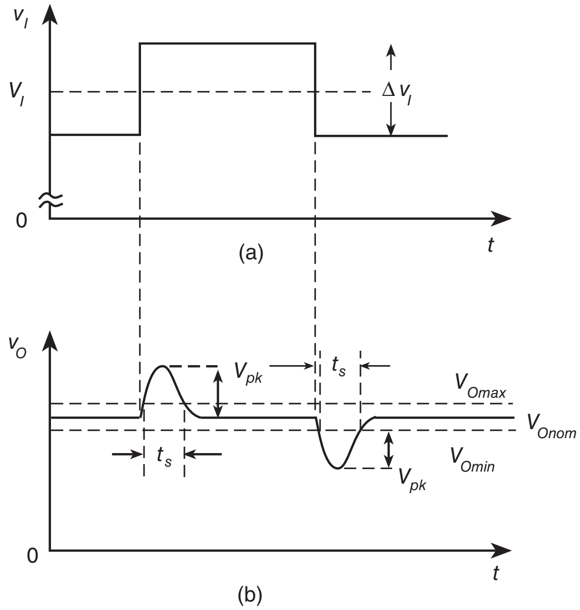

Figure 1.10 *Waveforms illustrating line transient response of voltage regulators. (a) Waveform of the input voltage v_I. (b) Waveform of the output voltage v_O.*

current is abruptly decreased, the output voltage initially increases and then returns to its steady-state value. The two parameters of output voltage are the peak transient voltage V_{pk} and the settling time t_s. The settling time t_s should be less than 200–500 ms and V_{pk} should be below a specified value. Usually, nonoscillatory response is expected in closed-loop power supplies to ensure sufficient stability margins.

Another circuit for testing the load transient response is shown in Figure 1.13. The input voltage V_I is held constant, usually at the nominal value V_{Inom}. A step change in the load current may be obtained by switching the load resistance R_L. A resistor R_1 is connected in parallel with a series combination of a resistor R_2 and a fast switch, for example, a power metal-oxide-semiconductor-field-effect transistor (MOSFET). If the switch is OFF, the load resistance is high, equal to $R_{L1} = R_1$, and the steady-state load current is low, equal to $I_{O1} = V_O/R_{L1}$. If the switch is ON, the load resistance is low, equal to $R_{L2} = R_1R_2/(R_1 + R_2)$, and the steady-state load current is high, equal to $I_{O2} = V_O/R_{L2}$. Therefore, when the load resistance is switched from R_{L1} to R_{L2} and vice versa, the load current i_O experiences step changes in magnitude ΔI_O superimposed on the dc load current I_O, for example, from $0.1I_{Omax}$ to $0.9I_{Omax}$. This causes the output voltage to change just after the step change in the load current, as shown in Figure 1.12(b). When the load current i_O abruptly increases, the output voltage v_O initially decreases and then returns to a steady-state value and vice versa. In general, the response may be underdamped (or oscillatory), critically damped, or overdamped, but a nonoscillatory response is normally required.

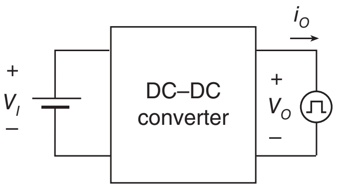

Figure 1.11 *Circuit for testing the transient response to a sudden electrical load change using an active current sink.*

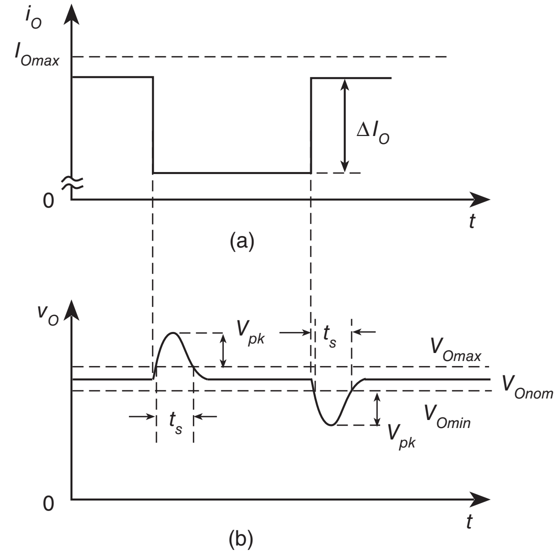

Figure 1.12 *Waveforms illustrating load transient response of voltage regulators. (a) Waveform of the load current* i_O*. (b) Waveform of the output voltage* v_O*.*

Many voltage regulators are operated with a constant load resistance R_L (or a constant load current I_O) for relatively long time intervals. In addition, these regulators have a negative feedback controller, which maintains a constant output voltage V_O. Therefore, the dc output power $P_O = V_O^2/R_L$ is also constant. Such operating conditions are called constant power load. If the output power P_O and the efficiency η are constant, the input power $P_I = P_O/\eta = V_I I_I$ is also constant. The dc input voltage of a dc voltage regulator can be expressed by

$$V_I = \frac{P_I}{I_I} = \frac{P_O}{\eta I_I}. \tag{1.32}$$

Figure 1.14 shows a plot of the input voltage V_I as a function of the dc input current I_I at a constant output power P_O. If the input voltage V_I is increased, the input current $I_I = P_I/V_I$ decreases under constant power load conditions. Therefore, the slope of the I_I–V_I characteristic is negative at any operating point Q. The *dynamic input resistance* (also called the *ac* or *incremental input resistance*) of the voltage regulator with a constant input power P_I for slow changes of the input voltage and current at a given operating point Q (i.e., for low frequencies) is given by

$$R_i = \frac{d(V_I)}{dI_I} = \frac{d}{dI_I}\left(\frac{P_I}{I_I}\right) = -\frac{P_I}{I_I^2} = -\frac{V_I}{I_I} = -R_{in(DC)}. \tag{1.33}$$

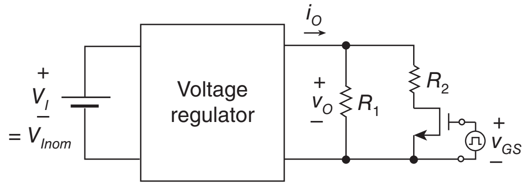

Figure 1.13 *Circuit for testing the load transient response with a switched load resistance from* R_1 *to* $R_1||R_2$ *using a MOSFET.*

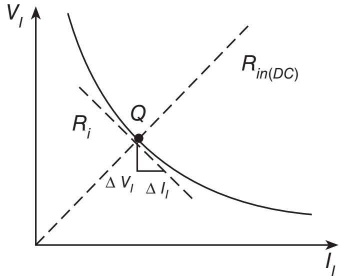

Figure 1.14 Voltage–current characteristic of a constant power source.

Note that for a constant input power, the dynamic input resistance is just the negative of the static input resistance. The dynamic input resistance of a voltage regulator with a constant output power P_O and a constant efficiency η is

$$R_i = \frac{d(V_I)}{dI_I} = \frac{d}{dI_I}\left(\frac{P_O}{\eta I_I}\right) = \frac{P_O}{\eta}\frac{d}{dI_I}\left(\frac{1}{I_I}\right) = -\frac{P_O}{\eta I_I^2} = -\frac{R_L I_O^2}{\eta I_I^2}. \tag{1.34}$$

From (1.9), one obtains

$$\frac{I_O}{I_I} = \eta\frac{V_I}{V_O} = \frac{\eta}{\frac{V_O}{V_I}} = \frac{\eta}{M_{VDC}}. \tag{1.35}$$

Substitution of (1.35) into (1.34) produces

$$R_i = -\frac{\eta R_L}{M_{VDC}^2}. \tag{1.36}$$

It can be seen that the dynamic input resistance of a dc voltage regulator with a constant power load is negative and directly proportional to the load resistance R_L. The dynamic input resistance is a negative reflected load resistance.

1.7 Linear Voltage Regulators

There are two basic topologies of linear voltage regulators: the series voltage regulator and the shunt voltage regulator. These topologies are shown in Figure 1.15. A band gap reference voltage source V_{ref} is applied to the noninverting input of the op-amp. The input voltage of the op-amp is the difference between the noninverting input voltage V^+ and the inverting input voltage V^- given by $V_{i(op-amp)} = V^+ - V^-$. Since the input voltage of an op-amp with negative feedback is almost zero, the voltage across the resistor R_2 is controlled by the reference voltage source V_{ref}. Thus,

$$V_{R2} = V_O\frac{R_2}{R_1 + R_2} \approx V_{ref}. \tag{1.37}$$

Rearrangement of this equation gives the output voltage for both linear voltage regulators

$$V_O \approx V_{ref}\left(\frac{R_1}{R_2} + 1\right). \tag{1.38}$$

The range of the output current of linear voltage regulators is from 0 to a maximum value I_{Omax}, usually determined by a current limiting circuit.

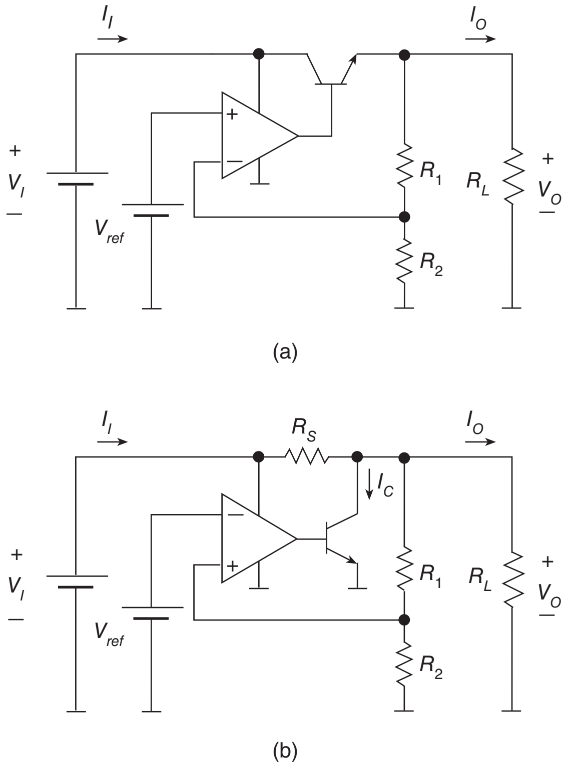

Figure 1.15 *Basic circuits of linear voltage regulators. (a) Series voltage regulator. (b) Shunt voltage regulator.*

1.7.1 Series Voltage Regulator

The series voltage regulator is shown in Figure 1.15(a). It employs a *pass transistor* whose collector-to-emitter voltage V_{CE} is controlled to compensate for varying the input voltage. Referring to Figure 1.15(a),

$$V_I = V_{CE} + V_O. \tag{1.39}$$

Since V_O is constant,

$$\Delta V_I = \Delta V_{CE}. \tag{1.40}$$

Thus, a change in the input voltage will result in the same change in the voltage drop across the pass transistor. The pass transistor behaves like a variable resistor R_v. The series voltage regulator can be represented as a voltage divider composed of the variable resistor R_v and the load resistor R_L. When the output voltage V_O decreases, the variable resistance also decreases, causing the output voltage across the load resistance to increase, and vice versa. The voltage drop across R_v can be expressed as

$$V_{CE} = V_I - V_O = R_v I_O. \tag{1.41}$$

At a fixed load current I_O, a change in the input voltage is given by

$$\Delta V_{CE} = \Delta V_I = \Delta R_v I_O. \tag{1.42}$$

When the input voltage is changed by ΔV_I, the resistance is changed by $\Delta R_v = \Delta V_I / I_O$.

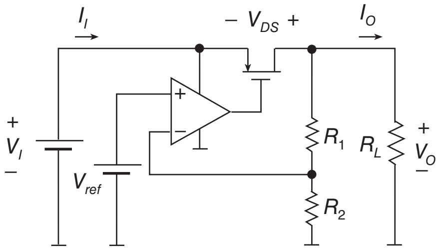

Figure 1.16 *Typical low drop-out (LDO) voltage regulator topology.*

The efficiency of a series voltage regulator can be derived by observing that $I_O \approx I_I$

$$\eta = \frac{P_O}{P_I} = \frac{I_O V_O}{I_I V_I} \approx \frac{V_O}{V_I} = M_{VDC}. \tag{1.43}$$

It can be seen that the efficiency of a series voltage regulator is equal to the dc voltage transfer function M_{VDC}. If the input voltage V_I is much higher than the output voltage V_O, the efficiency is very low. For example, if $V_I = 20$ V and $V_O = 5$ V, then $\eta = 5/20 = 25\%$. This is a very low efficiency. However, if $V_I = 8$ V and $V_O = 5$ V, then $\eta = 5/8 = 62.5\%$. The power loss in the pass transistor is expressed by

$$P_{LS} \approx I_O(V_I - V_O). \tag{1.44}$$

Thus, the power loss increases with increasing load current I_O and the voltage drop across the pass transistor $\Delta V = V_I - V_O$.

The series voltage regulator will work properly as long as V_I does not drop too low, which causes the op-amp to saturate. The op-amp must be in the linear region to function properly as a control circuit. The minimum voltage difference between the unregulated input voltage and the regulated output voltage $V_{DO} = V_{Imin} - V_O$ at which the circuit ceases to regulate against further reduction in the input voltage is called the *drop-out voltage*. For most series voltage regulators, this voltage is about 2 V, but in some voltage regulators V_{DO} can be as low as 0.1 V. Voltage regulators with a low drop-out (LDO) voltage are called LDO regulators. In these regulators, a *pnp* or an NMOS transistor is used as a pass component, as shown in Figure 1.16. The series voltage regulator is quiet because its transistor always operates in the pinch-off region as a dependent current source and does not generate a lot of noise like that in switching-mode power supplies. In addition, a series voltage regulator is simple to design and build.

1.7.2 Shunt Voltage Regulator

The shunt voltage regulator is shown in Figure 1.15(b). It employs a shunt transistor, in which the current is controlled to compensate for the change in the input voltage or the load current. The output voltage is held constant by varying the collector current I_C of the shunt transistor. The shunt transistor acts like a variable resistor. When the output voltage V_O decreases, the op-amp output voltage also decreases, the shunt transistor conducts less heavily, and the variable resistance increases. Thus, less current is diverted from the load, causing an increase in the load current and the output voltage. Using Kirchhoff's current law (KCL),

$$I_I = I_C + I_O. \tag{1.45}$$

When the load current I_O is changed at a fixed input voltage V_I, the input current $I_I = (V_I - V_O)/R_s$ is constant and therefore

$$\Delta I_C = -\Delta I_O. \tag{1.46}$$

Equation (1.45) can be rewritten as

$$\frac{V_I - V_O}{R_s} = I_C + I_O. \tag{1.47}$$

When the input voltage changes at a fixed load current I_O,

$$\frac{\Delta V_I}{R_s} = \Delta I_C, \tag{1.48}$$

from which

$$\Delta V_I = R_s \Delta I_C = \Delta V_{Rs}. \tag{1.49}$$

Thus, the output voltage V_O is held constant by varying the voltage drop across the series resistor R_s, which in turn is controlled by varying the collector current I_C of the shunt transistor.

The shunt regulator is inherently short-circuit proof. The output current under short-circuit conditions is given by

$$I_{O(sc)} = \frac{V_I}{R_s}. \tag{1.50}$$

The power loss in resistor R_s is

$$P_{Rs} = (V_I - V_O)I_I = (V_I - V_O)(I_O + I_C). \tag{1.51}$$

The power loss in the shunt transistor is

$$P_Q = V_O I_C = V_O(I_I - I_O) \tag{1.52}$$

and the efficiency is defined as

$$\eta = \frac{P_O}{P_I} = \frac{V_O I_O}{V_I I_I} = \frac{V_O}{V_I}\frac{I_O}{I_O + I_C}. \tag{1.53}$$

Thus, the shunt voltage regulator is less efficient than the series voltage regulator due to the power loss in both series resistor R_s and shunt transistor. However, the line transient response of the shunt regulator is better than that of the series regulator. The shunt voltage regulator must be protected against input overvoltage conditions.

The major characteristics of IC linear voltage regulators are as follows:

(1) Simple circuit
(2) Very small size and low weight
(3) Cost effective
(4) Low noise level
(5) Wide bandwidth and fast step response to load and line changes
(6) Low input and output voltages, usually below 40 V
(7) Low output current, usually below 3 A
(8) Low output power, usually below 25 W
(9) Low efficiency (especially for $V_I \gg V_O$), usually between 20% and 60%
(10) Only step-down linear voltage regulators are possible
(11) Only noninverting linear voltage regulators are possible
(12) Large low-frequency (50 or 60 Hz) transformers are required in AC–DC power supplies with linear voltage regulators.

1.8 Topologies of PWM DC–DC Converters

Switched-mode technology employs a wide variety of topologies. Figure 1.17 shows a family of single-ended PWM dc–dc converters, such as buck, boost, buck–boost, flyback, forward, Ćuk (boost–buck), SEPIC (single-ended primary input converter), and dual-SEPIC [7] (also called zeta or inverse-SEPIC) converters. The SEPIC converter is noninverting step-down/step-up converter. Its voltage ratio is $M_{VDC} = V_O/V_I = D/(1 - D)$.

The flyback converter is a transformer version of the buck–boost converter, and the forward converter is a transformer version of the buck converter. The flyback and dual-SEPIC converters are identical on the primary side of the transformer. Also, the Ćuk and SEPIC converters are identical on the primary side of the transformer. The flyback and SEPIC converters are identical on the secondary side of the transformer. Also, the Ćuk and dual-SEPIC converters are identical on the secondary side of the transformer.

Figure 1.18 depicts the multiple-switch PWM dc–dc converters: half-bridge, full-bridge, and push–pull converters. Switched-mode converters use duty-cycle control of a switching element to block the flow of energy from

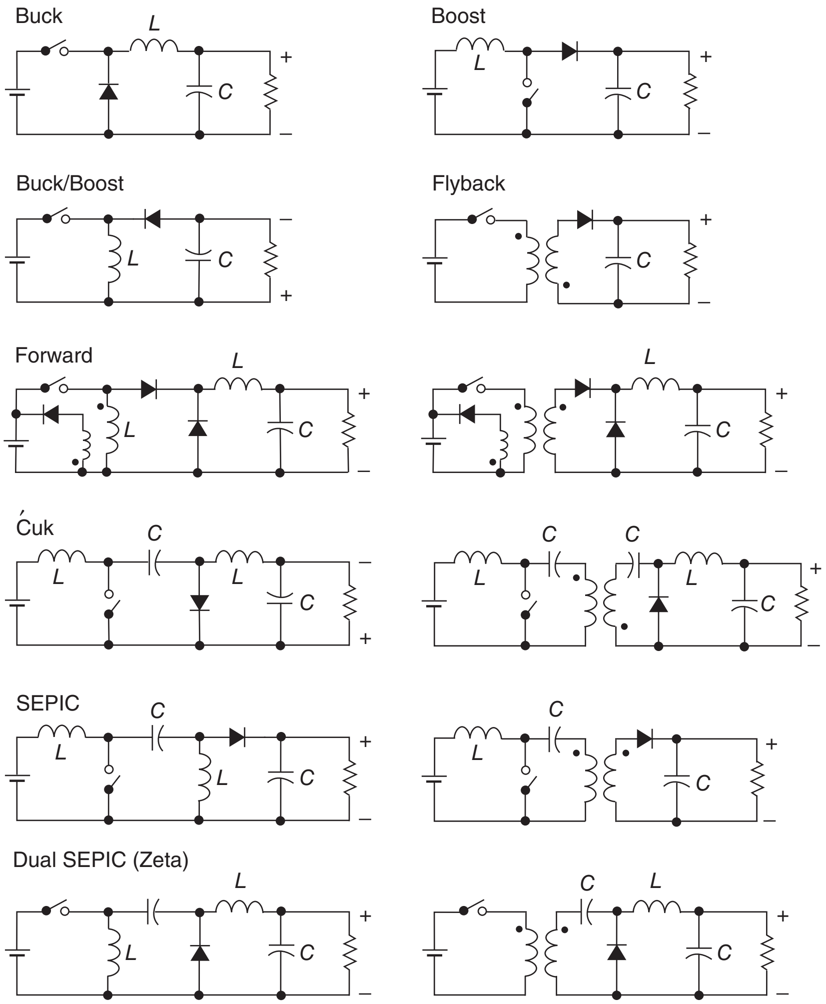

Figure 1.17 *Single-ended PWM dc–;dc nonisolated and isolated converters.*

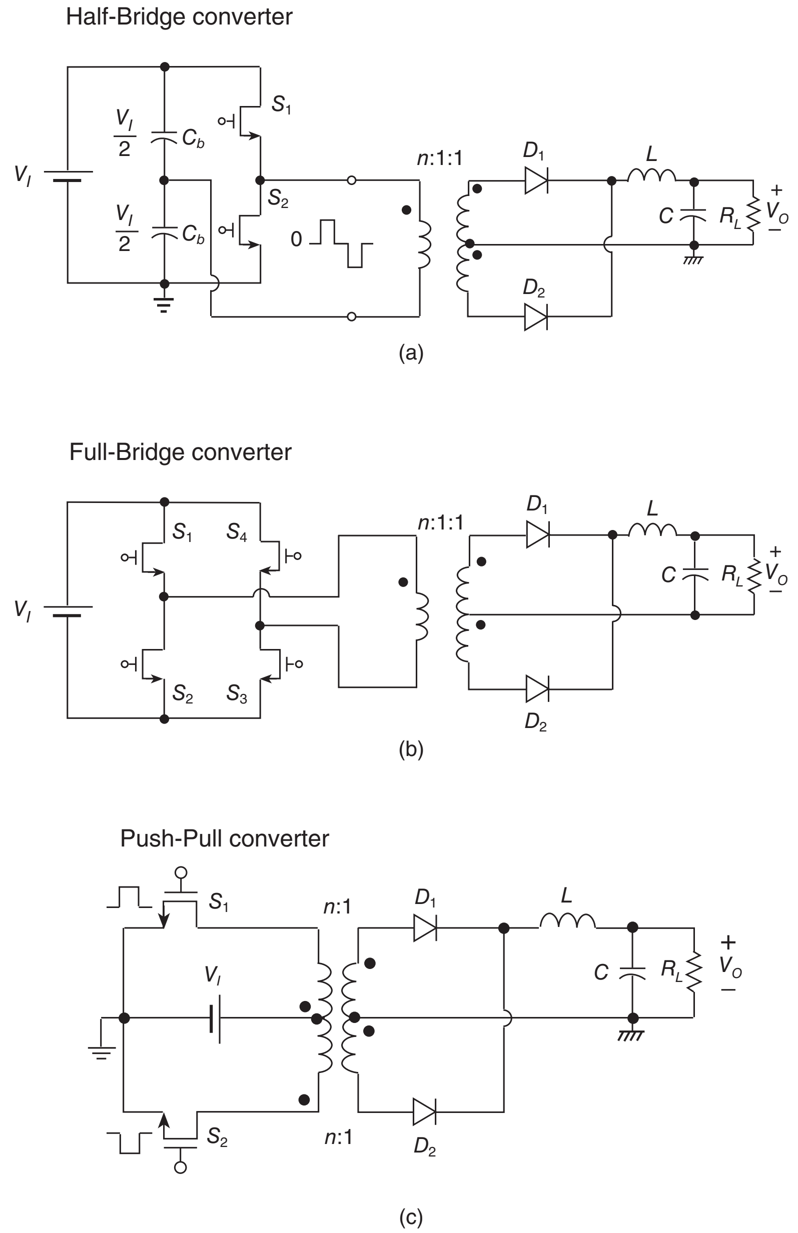

Figure 1.18 *Multiple-switch isolated PWM dc–dc converters.*

the input to the output and thus achieve voltage regulation. The advantages of these converters include significant reduction of a transformer and energy storage components. Since switched-mode converters can operate at high frequencies, a small transformer with a ferrite core can be used. The reduced size is very important in many applications, such as aerospace, computers, and wireless technologies. However, there is a penalty paid due to the increased noise, which is present at both input and output of the supply due to the switching action of semiconductor devices. In addition, the control circuit is much more complicated than that used in linear regulators.

Power MOSFETs are often used as controllable switches. In 1979, International Rectifier patented the first commercially viable power MOSFET, called the HEXFET. Fast recovery diodes, ultrafast recovery [28], and hyperfast recovery *pn* junction diodes, or Schottky diodes are used in switching dc–dc power converters. In 1976, Silicon General introduced the industry's first PWM controller IC, the SG1524.

1.9 Relationships Among Current, Voltage, Energy, and Power

The average value of current $i(t)$ is given by

$$I_{AV} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{2\pi}\int_0^{2\pi} i(\omega t)d(\omega t) \tag{1.54}$$

and the rms value of the current is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)d(\omega t)}. \tag{1.55}$$

Likewise, the average value of voltage $v(t)$ is expressed by

$$V_{AV} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t)d(\omega t) \tag{1.56}$$

and the rms value of the voltage is given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\omega t)d(\omega t)}. \tag{1.57}$$

The instantaneous power is

$$p(t) = i(t)v(t). \tag{1.58}$$

The energy dissipated in a component or delivered by a source over a time interval t_1 is

$$W = \int_0^{t_1} p(t)dt = \int_0^{t_1} i(t)v(t)dt. \tag{1.59}$$

For periodic waveforms in steady state, the average real power absorbed by a component or delivered by a source is the time average value of the instantaneous power over a period T of the operating frequency

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T i(t)v(t)dt = \frac{W}{T} = fW. \tag{1.60}$$

For periodic waveforms in steady state, the average charge stored in a capacitor over one period is zero

$$Q = \int_0^T i_C(t)dt = 0. \tag{1.61}$$

This is called the principle of *capacitor charge balance* or *capacitor ampere-second balance*. Thus, the average current through a capacitor for steady-state operation is zero

$$I_{C(AV)} = \frac{Q}{T} = \frac{1}{T}\int_0^T i_C(t)dt = 0. \tag{1.62}$$

For periodic waveforms in steady state, the average magnetic flux linkage of an inductor over one period is zero

$$\lambda = \int_0^T v_L(t)dt = 0. \tag{1.63}$$

This is called the principle of *inductor flux linkage balance* or *inductor volt-second balance*. Hence, the average voltage across an inductor in steady state is zero

$$V_{L(AV)} = \frac{\lambda}{T} = \frac{1}{T}\int_0^T v_L(t)dt = 0. \tag{1.64}$$

The instantaneous energy stored in a capacitor is

$$w_C(t) = \frac{1}{2}Cv_C^2(t) \tag{1.65}$$

and in an inductor is

$$w_L(t) = \frac{1}{2}Li_L^2(t). \tag{1.66}$$

1.10 Summary

- The main function of voltage regulators is the regulation of the dc output voltage against changes in the load current, the input voltage, and the temperature.
- Additional functions of voltage regulators are the dc isolation, ripple voltage reduction, and fast transient response to rapid changes in the load current and the input voltage.
- Voltage regulators can be categorized into linear voltage regulators, switching-mode dc–dc converters, and switched-capacitor voltage regulators.
- Linear voltage regulators have simple circuit, low power levels, low noise (EMI), low output ripple voltage, excellent load and line regulation, wide bandwidth and fast transient response to load and line changes, but have low efficiency and are only step-down regulators.
- There are series and shunt linear voltage regulators.
- In linear voltage regulators, transistors are operated as dependent current sources.
- In PWM dc–dc converters, transistors are operated as switches. Therefore, the voltage is low when the current is high, and the current is zero when the voltage is high, yielding low conduction loss and high efficiency.
- PWM converters are sources of EMI because of the hard switching action of transistors and diodes.
- Switching voltage regulators have high efficiency, high power density, and high power levels. They can be step-down or step-up converters and can have multiple-output voltages, but they have slow response to load and line changes, produce high level of EMI, and have high output ripple voltage.
- Flyback converters are used at power levels in the range 0–50 W. Forward converters are used in the range 50–500 W. Half-bridge converters are used in the range 100–1000 W. Full-bridge converters are used for power level above 500 W.
- Power converters are sources of EMI/RFI noise.
- EMI noise can be conducted and radiated. The conducted noise is in the range from 9 kHz to 30 MHz. The radiated noise is in the range from 30 MHz to 1 GHz.
- EMI noise can be differential-mode noise and common-mode noise.

References

[1] R. D. Middlebrook and S. Ćuk, *Advances in Switched-Mode Power Conversion*, vols. I, II, and III. Pasadena, CA: TESLAco, 1981.
[2] E. R. Hnatek, *Design of Solid-State Power Supplies*, 2nd Ed. New York, NY: Van Nostrand, 1981.
[3] K. K. Sum, *Switching Power Conversion*. New York: Marcel Dekker, 1984.
[4] G. Chryssis, *High-Frequency Power Supplies: Theory and Design*. New York: McGraw-Hill, 1984.
[5] R. P. Severns and G. Bloom, *Modern DC-to-DC Switchmode Power Converter Circuits*. New York: Van Nostrand, 1985.

[6] K. Billings, *Switchmode Power Supply Handbook*. New York: McGraw-Hill, 1989.
[7] J. Jóźwik and M. K. Kazimierczuk, "Dual SEPIC PWM switching-mode dc/dc converter," *IEEE Transactions on Industrial Electronics*, vol. IE-36, pp. 64–70, February 1989.
[8] A. I. Pressman, *Switching Power Supply Design*. New York: McGraw-Hill, 1991.
[9] J. G. Kassakian, M. F. Schlecht, and G. C. Verghese, *Principles of Power Electronics*. Reading, MA: Addison-Wesley, 1991.
[10] M. K. Kazimierczuk and D. Czarkowski, *Resonant Power Converters*, 2nd Ed. New York: John Wiley & Sons, 2011.
[11] D. W. Hart, *Introduction to Power Electronics*. Upper Saddle River, NJ: Prentice Hall, 1997.
[12] A. M. Trzynadlowski, *Introduction to Modern Power Electronics*. New York: John Wiley & Sons, 1998.
[13] P. T. Krein, *Elements to Power Electronics.* New York: Oxford University Press, 1998.
[14] R. W. Erickson and D. Maksimović, *Fundamentals of Power Electronics*, 2nd Ed. Norwall, MA: Kluwer Academic Publisher, 2001.
[15] N. Mohan, T. M. Undeland, and W. P. Robbins, *Power Electronics: Converters, Applications and Design*, 3rd Ed. New York: John Wiley & Sons, 2004.
[16] M. H. Rashid, *Power Electronics, Circuits, Devices, and Applications*, 3rd Ed. Upper Saddle River, NJ: Prentice Hall, 2004.
[17] W. Shepherd and L. Zhang, *Power Converter Circuits*. New York: Marcel Dekker, 2004.
[18] S. Ang and A. Oliva, *Power-Switching Converters*, 2nd Ed. Boca Raton, FL: CRC/Taylor & Francis, 2004.
[19] A. Aminian and M. K. Kazimierczuk, *Electronic Devices: A Design Approach*. Upper Saddle River, NJ: Prentice Hall, 2004.
[20] I. Batarseh, *Power Electronics Circuits*. New York: John Wiley & Sons, 2004.
[21] M. H. Rashid, Editor, *Power Electronics Handbook*. New York: Academic Press, 2006.
[22] M. H. Rashid and H. M. Rashid, *SPICE for Power Electronics and Electric Power*, 2nd Ed. Boca Raton, FL: CRC/Taylor & Francis, 2006.
[23] C. P. Basso, *Switch-Mode Power Supply*. New York: McGraw-Hill, 2008.
[24] M. K. Kazimierczuk, *RF Power Amplifiers*. Chichester, UK: John Wiley & Sons, 2014.
[25] M. K. Kazimierczuk, *High-Frequency Magnetic Components*, 2nd Ed. Chichester, UK: John Wiley & Sons, 2014.
[26] S. Buso and P. Mattavelli, *Digital Control in Power Electronics*. Morgan & Claypool Publishers, 2006.
[27] S. Maniktala, *Switching Power Supply Design and Optimization*. New York: McGraw-Hill, 2005.
[28] M. K. Kazimierczuk, "Reverse recovery power *pn* junction diodes," *International Journal of Circuits, Systems, and Computers*, vol. 5, no. 4, pp. 747–755, December 1995.

Review Questions

1.1 List the main functions of dc–dc converters.

1.2 Give a classification of power supplies.

1.3 Define line regulation of voltage regulators.

1.4 Define load regulation of voltage regulators.

1.5 Define thermal regulation of voltage regulators.

1.6 Define the dc input resistance of voltage regulators.

1.7 Define the dynamic input resistance of voltage regulators.

1.8 Define the ripple rejection ratio of voltage regulators.

1.9 What is the line transient response of voltage regulators?

1.10 What is the load transient response of voltage regulators?

1.11 How are transistors operated in linear voltage regulators?

1.12 What are the basic topologies of linear voltage regulators?

1.13 Give the expression for the efficiency of the series voltage regulator.

1.14 What is the range of efficiency for linear voltage regulators?

1.15 What is the range of output power for linear voltage regulators?

1.16 Can you build a step-up linear voltage regulator?

1.17 What is the size and weight of a transformer in power supplies with linear voltage regulators?

1.18 What is the noise level in linear voltage regulators?

1.19 What are LDO voltage regulators?

1.20 How are transistors and diodes operated in switching-mode dc–dc power converters?

1.21 How is the dc isolation achieved in switching-mode power supplies?

1.22 Compare the efficiency of linear and PWM switching-mode voltage regulators.

Problems

1.1 A voltage regulator experiences a 100 mV change in the output voltage, when its input voltage changes by 10 V at $I_O = 0.2$ A and $T_A = 25°\text{C}$. The nominal output voltage is $V_{Onom} = 3.3$ V. Determine the line regulation and the percentage line regulation.

1.2 A voltage regulator is rated for an output current $I_O = 0$–50 mA. Under the no-load condition, the output voltage is 5 V. Under the full-load condition, the output voltage 4.99 V. Find load regulation, the percentage load regulation, the dc output resistance, and load/line regulation.

1.3 A series linear voltage regulator is operated under the following conditions: $V_I = 6$–15 V, $V_O = 3.3$V, and $I_O = 0$–0.4 A. Find the minimum and maximum efficiency of the voltage regulator at full load.

1.4 A voltage regulator has $R_L = 10\ \Omega$, $V_I = 10$ V, $V_O = 5$ V, and $\eta = 90\%$. Find the dc input resistance.

1.5 A boost PWM converter operating in CCM is rated for an output current of 0.5–1 A. The output voltage at minimum load current is 20 V and at full load current is 19.95 V. Find the load regulation in (mV/A), the percentage load regulation in (%), and the dc output resistance (Ω).