## Introduction

The most widely used option model is the Black and Scholes model. Although there are some shortcomings, the model is appreciated by many professional option traders and investors because of its simplicity, but also because, in many circumstances, it does generate a fair value for option prices in all kinds of markets.

The main shortcomings, most of which will be discussed later, are:
the model assumes a geometric Brownian motion where the market might deviate from that assumption (jumps); it assumes a normal distribution of daily (logarithmic) returns of an asset or Future while quite often there is a tendency towards a distribution with high peaks around the mean and fat tails; it assumes stable volatility while the market is characterised by changing (stochastic) volatility regimes; it also assumes all strikes of the options have the same volatility; it doesn't apply skew (adjustment of option prices) in the volatility smile/surface, and so on.

So in principle there may be a lot of caveats on the Black and Scholes model. However, because of its use by many market participants (with adjustments to make up for the shortcomings) in combination with its accuracy on many occasions, it may remain the basis option model for pricing options for quite some time.

This book aims to explore and explain the ins and outs of the Black and Scholes model (to be precise, the Black ' 76 model on Futures, minimising the impact of interest rates and leaving out dividends). It has been written for any person active in buying or selling options, involved in options from a business perspective or just interested in learning the background of options pricing, which is quite often seen as a black box. Although this is not an academic work, it could be worthwhile for academics to understand how options and their derivatives perform in practice, rather than in theory.

The book has a very practical approach and an emphasis on the distribution of the Greeks; these measure the sensitivity of the value of an option with regards to changes in parameters such as the strike, the underlying (Future), volatility (a measurement of the variation of the underlying), time to expiry or maturity, and the
interest rate. It further emphasises the implications of the Greeks and understanding them with regards to the impact they will/might have on the $\mathrm{P} \& \mathrm{~L}$ of an options portfolio. The aim is to give the reader a full understanding of the multidimensional aspects of trading options.

When measuring the sensitivity of the value of an option with regards to changes in the parameters one can discern many Greeks, but the most important ones are:

Delta: the price change of an option in relation to the change of the underlying;
Vega: the price change of an option in relation to volatility;
Theta (time decay): the price change of an option in relation to time;
Rho: the price change of an option in relation to interest rate.
These Greeks are called the first order Greeks. Next to that there are also second order Greeks, which are derivatives of the first order Greeks - gamma, vanna, vomma, etcetera - and third order Greeks, being derivatives of the second order Greeks - colour, speed, etcetera.

The most important of the higher order Greeks is gamma which measures the change of delta.

## TABLE 1.1

| Parameters | First order Greeks | Second order Greeks | Third order Greeks |
| :--- | :--- | :--- | :--- |
| Strike | Delta | Gamma | Colour |
| Underlying | Vega | Vanna | Speed |
| Volatility | Theta | Vomma | Ultima |
| Time to maturity | Rho | Charm | Zomma |
| Interest rate |  | Veta |  |
|  |  | Vera |  |

When Greeks are mentioned throughout the book, the term usually relates to delta, vega, theta and gamma, for they are the most important ones.

The book will also teach how to value at the money options, their surrounding strikes and their main Greeks, without applying the option model. Although much is based on rules of thumb and approximation, valuations without the model can be very accurate. Being able to value/approximate option prices and their Greeks off the top of the head is not the main objective; however, being able to do so must imply that one fully understands how pricing works and how the Greeks are distributed. This will enable the reader to consider and calculate how an option strategy might develop in a four dimensional way. The reader will learn about the consequences of options pricing with regards to changes in time, volatility, underlying and strike, all at the same time.

People on the verge of entering into an option strategy quite often prepare themselves by checking books or the internet. Too often they find explanations of a certain strategy which is only based on the payoff of an option at time of maturity -a


CHART 1.1 P\&L distribution of a short 40 put position at expiry
two-dimensional interpretation (underlying price versus profit loss). This can be quite misleading since there is so much to say about options during their lifetime, something some people might already have experienced when confronted with adverse market moves while running an option strategy with associated losses. A change in any one of the aforementioned parameters will result in a change in the value of an option. In a two-dimensional approach (i.e. looking at P\&L distribution at expiry) most of the Greeks are disregarded, while during the lifetime of the option they can make or break the strategy.

Throughout the book, any actor who is active in buying or selling options i.e. a private investor, trader, hedger, portfolio manager, etcetera - will be called a trader.

Many people understand losses deriving from bad investment decisions when buying options or the potentially unlimited losses of short options. However, quite often they fail to see the potentially devastating effects of misinterpretation of the Greeks.

For example, as shown in chart 1.1, a trader who sold a 40 put at $\$ 1.50$ when the Future was trading at 50 (volatility at $28 \%$, maturity 1 year), had the right view. During the lifetime of the option, the market never came below 40, the put expired worthless, and the trader consequently ended up with a profit of $\$ 1.50$.

The problem the trader may have experienced, however, is that shortly after inception of the trade, the market came off rapidly towards the 42 level. As a result of the sharp drop in the underlying, the volatility may have jumped from $28 \%$ to $40 \%$. The 40 put he sold at $\$ 1.50$ suddenly had a value of $\$ 5.50$, an unrealised loss of $\$ 4$. It would have at least made the trader nervous, but most probably he would have bought back the option because it hit his stop loss level or he was forced by


CHART 1.2 P\&L distribution of the combination trade at expiry
his broker, bank or clearing institution to deposit more margin; or even worse, the trade was stopped out by one of these institutions (at a bad price) when not adhering to the margin call.

So an adverse market move could have caused the trader to end up with a loss while being right in his strategy/view of the market. If he had anticipated the possibility of such a market move he might have sold less options or kept some cash for additional margin calls. Consequently, at expiry, he would have ended up with the $\$ 1.50$ profit. Anticipation obviously can only be applied when understanding the consequences of changing option parameters with regards to the price of an option.

A far more complex strategy, with a striking difference in P\&L distribution at expiry compared to the $\mathrm{P} \& \mathrm{~L}$ distribution during its lifetime, is a combination where the trader is short the 50 call once ( 10,000 lots) and long the 60 call twice $(20,000$ lots) and at the same time short the 50 put once ( 10,000 lots) and long the 40 put twice ( 20,000 lots). He received around $\$ 45,000$ when entering into the strategy. The P\&L distribution of the combination trade at expiry is shown above in Chart 1.2.

The combination trade will perform best when the market is at 50 (around $\$ 45,000$ profit) and will have its worst performance when the market is either at 40 or at 60 at expiry (around $\$ 55,000$ loss).

The strategy has been set up with 1 year to maturity; a lot can happen in the time between inception of the trade and its expiry. In an environment where the Future will rapidly change and where as a result of the fast move in the market the volatility might increase, the P\&L distribution of the strategy could look, in a three dimensional way, as follows ( $\mathrm{P} \& \mathrm{~L}$ versus time to maturity versus underlying level):

Chart 1.3 shows the $\mathrm{P} \& \mathrm{~L}$ distribution of the combination trade in relation to time. When looking at expiry, at the axis "Days to expiry" at 0 , the $\mathrm{P} \& \mathrm{~L}$ distribution


CHART 1.3 Combination trade, long 20,000 40 puts and 60 calls, short 10,00050 puts and 50 calls
is the same as the distribution depicted in the two-dimensional chart 1.2. The best performance is at 50 in the Future, resulting in a $\mathrm{P} \& \mathrm{~L}$ of around $\$ 45,000$ and the worst case scenario is when the Future is at 40 or at 60 , when the loss will mount up to around $\$ 55,000$. However, when the maturity is 365 "days to expiry" and the market starts moving and consequently the volatility will, for instance, increase, the performance will overall be positive, there will be some profit at the 50 level in the Future. This is the smallest amount, but still a few thousand up: a profit of around $\$ 35,000$ when the Future is at 60 and around $\$ 10,000$ when the Future is at 40 . These P\&L numbers keep changing during the lifetime of the strategy. For instance, the $\$ 35,000$ profit at 60 in the Future (at 365 days to expiry) will turn into a $\$ 55,000$ loss at expiry when the market stays at that level - losses in time, called the time decay or theta. Also, when the trade has been set up, the P\&L of the portfolio increases with higher levels in the Future, so there must be some sort of delta active (change of value of the portfolio in relation to the change of the Future). Next to that, the P\&L distribution displays a convex line between 50 and 65 at 365 days to expiry, which means that the delta will change as well; changes in the delta are called the gamma.

So the P\&L distribution of this structure is heavily influenced by its Greeks: the delta, gamma, vega and theta - a very dynamic distribution. Thus, without an understanding of the Greeks this structure would not be understandable when looking at the $\mathrm{P} \& \mathrm{~L}$ distribution from a more dimensional perspective.

It is of utmost importance that one realises that changing market conditions can make an option portfolio with a profitable outlook change into a position with an almost certainly negative $\mathrm{P} \& \mathrm{~L}$; or the other way around, as shown in the example above. Therefore it is a prerequisite that when trading/investing in options one understands the Greeks.

This book will, in the first chapters, discuss probability distribution, volatility and put call parity, then the main Greeks: delta, gamma, vega and theta. The first order Greeks, together with gamma (a second order Greek), are the most important and will thus be discussed at length. As the other Greeks are derived from these, they will be discussed only briefly, if at all. Once the regular Greeks are understood one can easily ponder the second and third order Greeks and understand how they work.

In the introduction of a chapter on a Greek, the formula of this Greek will be shown as well. The intention is not to write about mathematics, its purpose is to show how parameters like underlying, volatility and time will influence that specific Greek. So a mathematical equation like the one for gamma, $\gamma=\frac{\varphi(d 1)}{F \sigma \sqrt{T}}$, should not bring despair. In the chapter itself it will be fully explained.

In the last chapter, trading strategies will be discussed, from simple strategies towards complex structures. The main importance, though, is that the trader must have a view about the market; without this it is hard to determine which strategy is appropriate to become a potential winner. An option strategy should be the result of careful consideration of the market circumstances. How well the option strategy performs is fully related to the trader making the right assessment on the market's direction or market circumstances. A potential winning option strategy could end up disastrously with an unanticipated adverse market move. Each strategy could be a winner, but at the same time a loser as well.

The terms "in", "at" or "out" of the money will be mentioned throughout the book. "At the money" refers to an option which strike is situated at precisely the level of the underlying. When not meant to be precisely at the money, the term "around the at the money" will be applied. "Out of the money" options are calls with higher strikes and puts with lower strikes compared to the at the money strike; "in the money" options are calls with lower strikes and puts with higher strikes compared to the at the money strike.

The options in the book are treated as European options, hence there will be no early exercise possible (exercising the right entailed by the option before maturity date), as opposed to American options. Obviously, American option prices might differ from European (in relation to dividends and the interest rate level), however discussing this falls beyond the scope of the book. When applying European style there will be no effect on option pricing with regards to dividend.

For the asset/underlying, a Future has been chosen; it already has a future dividend pay out and the interest rate component incorporated in its value.

For reasons of simplicity and also for making a better representation of the effect of the Greeks, 10,000 units has been applied as the basis volume for at the money options where each option represents the right to buy (i.e. a call) or sell (i.e. a put) one Future. The 10,000 basis volume could represent a fairly large private investor or a fairly small trader in the real world. For out of the money options, larger quantities will be applied, depending on the face value of the portfolio/position.

