1 Linear Equations and Functions

Fundamentals of Calculus, First Edition. Carla C. Morris and Robert M. Stark.

Companion Website: http://www.wiley.com/go/morris/calculus

1.1 SOLVING LINEAR EQUATIONS

Mathematical descriptions, often as **algebraic expressions**, usually consist of alphanumeric characters and special symbols.

↑ The name "algebra" has fascinating origins in early Arabic language (Historical Notes).

For example, physicists describe the distance, s, that an object falls under gravity in a time, t, by $s = (1/2)gt^2$. Here, the letters s and t represent **variables** since their values may change while, g, the acceleration of gravity, is considered as constant. While any letters can represent variables, typically, later letters of the alphabet are customary. Use of x and y is generic. Sometimes, it is convenient to use a letter that is descriptive of a variable, as t for time.

Earlier letters of the alphabet are customary for fixed values or **constants**. However, exceptions are common. The equal sign, a special symbol, is used to form an **equation**. An equation equates algebraic expressions. Numerical values for variables that preserve equality are called **solutions** to the equations.

For example, $5x + 1 = 11$ is an equation in a single variable, x. It is a **conditional** equation since it is only true when $x = 2$. Equations that hold for all values of the variable are called **identities**. For example, $(x + 1)^2 = x^2 + 2x + 1$ is an identity. By solving an equation, values of the variables that satisfy the equation are determined.

An equation in which only the first powers of variables appear is a **linear equation**. Every linear equation in a single variable can be solved using some or all of these properties:

Substitution – Substituting one expression for an equivalent one does not alter the original equation. For example, $2(x - 3) + 3(x - 1) = 21$ is equivalent to $2x - 6 + 3x - 3 = 21$ or $5x - 9 = 21$.

Addition – Adding (or subtracting) a quantity to each side of an equation leaves it unchanged. For example, $5x - 9 = 21$ is equivalent to $5x - 9 + 9 = 21 + 9$ or $5x = 30$.

Multiplication – Multiplying (or dividing) each side of an equation by a non-zero quantity leaves it unchanged. For example, $5x = 30$ is equivalent to $(5x)(1/5) = (30)(1/5)$ or $x = 6$.

↓ Here are examples of linear equations: $5x - 3 = 11$, $y = 3x + 5$, $3x + 5y + 6z = 4$. They are linear in one, two, or three variables, respectively. It is the unit exponent on the variables that identifies them as linear.

↓ By "solving an equation" we generally intend the numerical values of its variables.

To Solve Single Variable Linear Equations

1. **Resolve fractions.**
2. **Remove grouping symbols.**
3. **Use addition (and/or subtraction) to have variable terms on one side of the equation.**
4. **Divide the equation by the variable's coefficient.**
5. **As a check, verify the solution in the original equation.**

Example 1.1.1 Solving a Linear Equation

Solve $(3x/2) - 8 = (2/3)(x - 2)$.

Solution:
To remove fractions, multiply both sides of the equation by 6, the least common denominator of 2 and 3. (Step 1 above)
The revised equation becomes

$$9x - 48 = 4(x - 2).$$

Next, remove grouping symbols (Step 2). That leaves

$$9x - 48 = 4x - 8.$$

Now, subtract 4x and add 48 to both sides (Step 3). Now,

$$9x - 4x - 48 + 48 = 4x - 4x - 8 + 48 \text{ or } 5x = 40.$$

Finally, divide both sides by the coefficient 5 (Step 4). One obtains $x = 8$.
The result, $x = 8$, *is checked by substitution in the original equation (Step 5):*

$$3(8)/2 - 8 = (2/3)(8 - 2)$$

$$4 = 4 \text{ checks!}$$

The solution $x = 8$ *is correct!*

Equations often have more than one variable. To solve linear equations in several variables simply bring a variable of interest to one side. Proceed as for a single variable regarding the other variables as constants for the moment.

↓ If y is the variable of interest in $3x + 5y + 6z = 2$, it can be written as $y = (2 - 3x - 6z)/5$ regarding x and z as constants for now.

Example 1.1.2 Solving for y

Solve for y: $5x + 4y = 20$.

Solution:
Move terms with y to one side of the equation and any remaining terms to the opposite side. Here, $4y = 20 - 5x$. Next, divide both sides by 4 to yield $y = 5 - (5/4)x$.

Example 1.1.3 Simple Interest

"Interest equals Principal times Rate times Time" expresses the well-known Simple Interest Formula, $I = PRT$. Solve for the time, T.

Solution:
Grouping, $I = (PR)T$ so PR becomes a coefficient of T. Dividing by PR gives $T = I/PR$.

Mathematics is often called "the language of science" or "the universal language". To study phenomena or situations of interest, mathematical expressions and equations are used to create **mathematical models**. Extracting information from the mathematical model provides solutions and insights. Mathematical modeling ideas appear throughout the text. These suggestions may aid your modeling skills.

To Solve Word Problems

1. **Read problems carefully.**
2. **Identify the quantity of interest (and possibly useful formulas).**
3. **A diagram may be helpful.**
4. **Assign symbols to variables and other unknown quantities.**
5. **Use symbols as variables and unknowns to translate words into an equation(s).**
6. **Solve for the quantity of interest.**
7. **Check your solution and whether you have answered the proper question.**

Example 1.1.4 Investment

Ms. Brown invests \$5000 at 6% annual interest. Model her resulting capital for one year.

Solution:
Here the principal (original investment) is \$5000. The interest rate is 0.06 (expressed as a decimal) and the time is 1 year.
Using the simple interest formula, $I = PRT$, Ms. Brown's interest is

$$I = (\$5000)(0.06)(1) = \$300.$$

After one year a model for her capital is $P + PRT = \$5000 + \$300 = \$5300$.

Example 1.1.5 Gasoline Prices

Recently East Coast regular grade gasoline was priced about \$3.50 per gallon. West Coast prices were about \$0.50/gallon higher.

a) *What was the average regular grade gasoline price on the East Coast for 10 gallons?*
b) *What was the average regular grade gasoline price on the West Coast for 15 gallons?*

Solution:

a) *On average, a model for the East Coast cost of ten gallons was $(10)(3.50) = \$35.00$.*
b) *On average, a model for the West Coast of fifteen gallons was $(15)(\$4.00) = \$60.00$.*

♦ Consumption as a function of disposable income can be expressed by the linear relation $C = mx + b$, where C is consumption (in \$); x, disposable income (in \$); m, marginal propensity to consume and b, a scaling constant. This consumption model arose in Keynesian economic studies popular during The Great Depression of the 1930s.

EXERCISES 1.1

In Exercises 1–6 identify equations as an identity, a conditional equation, or a contradiction.

1. $3x + 1 = 4x - 5$
2. $2(x + 1) = x + x + 2$
3. $5(x + 1) + 2(x - 1) = 7x + 6$
4. $4x + 3(x + 2) = x + 6$
5. $4(x + 3) = 2(2x + 5)$
6. $3x + 7 = 2x + 4$

In Exercises 7–27 solve the equations.

7. $5x - 3 = 17$
8. $3x + 2 = 2x + 7$
9. $2x = 4x - 10$
10. $x/3 = 10$
11. $4x - 5 = 6x - 7$
12. $5x + (1/3) = 7$
13. $0.6x = 30$
14. $(3x/5) - 1 = 2 - (1/5)(x - 5)$
15. $2/3 = (4/5)x - (1/3)$
16. $4(x - 3) = 2(x - 1)$
17. $5(x - 4) = 2x + 3(x - 7)$
18. $3x + 5(x - 2) = 2(x + 7)$
19. $3s - 4 = 2s + 6$
20. $5(z - 3) + 3(z + 1) = 12$
21. $7t + 2 = 4t + 11$
22. $(1/3)x + (1/2)x = 5$
23. $4(x + 1) + 2(x - 3) = 7(x - 1)$
24. $1/3 = (3/5)x - (1/2)$
25. $\dfrac{x + 8}{2x - 5} = 2$
26. $\dfrac{3x - 1}{7} = x - 3$
27. $8 - \{4[x - (3x - 4) - x] + 4\} = 3(x + 2)$

In Exercises 28–35 solve for the indicated variable.

28. Solve: $5x - 2y + 18 = 0$ for y.
29. Solve: $6x - 3y = 9$ for x.
30. Solve: $y = mx + b$ for x.
31. Solve: $3x + 5y = 15$ for y.
32. Solve: $A = P + PRT$ for P.
33. Solve: $V = LWH$ for W.
34. Solve: $C = 2\pi r$ for r.
35. Solve: $Z = \dfrac{x - \mu}{\sigma}$ for x.

Exercises 36–45 feature mathematical models.

36. The sum of three consecutive positive integers is 81. Determine the largest integer.
37. Sally purchased a used car for \$1300 and paid \$300 down. If she plans to pay the balance in five equal monthly installments, what is the monthly payment?
38. A suit, marked down 20%, sold for \$120. What was the original price?
39. If the marginal propensity to consume is $m = 0.75$ and consumption, C, is \$11 when disposable income is \$2, develop the consumption function.
40. A new addition to a fire station costs \$100,000. The annual maintenance cost increases by \$2500 with each fire engine housed. If \$115,000 has been allocated for the addition and maintenance next year, how many additional fire engines can be housed?

41. Lightning is seen before thunder is heard as the speed of light is much greater than the speed of sound. The flash's distance from an observer can be calculated from the time between the flash and the sound of thunder.
The distance, d (in miles), from the storm can be modeled as $d = 4.5t$ where time, t, is in seconds.
   a) If thunder is heard two seconds after lightning is seen, how far is the storm?
   b) If a storm is 18 miles distant, how long before thunder is heard?
42. A worker has forty hours to produce two types of items, A and B. Each unit of A takes three hours to produce and each item of B takes two hours. The worker made eight items of B and with the remaining time produced items of A. How many of item A were produced?
43. An employee's Social Security Payroll Tax was 6.2% for the first $87,000 of earnings and was matched by the employer. Develop a linear model for an employee's portion of the Social Security Tax.
44. An employee works 37.5 hours at a $10 hourly wage. If Federal tax deductions are 6.2% for Social Security, 1.45% for Medicare Part A, and 15% for Federal taxes, what is the take-home pay?
45. The body surface area (BSA) and weight (Wt) in infants and children weighing between 3 kg and 30 kg has been modeled by the linear relationship
BSA = 1321 + 0.3433Wt (where BSA is in square centimeters and weight in grams)
   a) Determine the BSA for a child weighing 20 kg.
   b) A child's BSA is 10,320 cm^2. Estimate its weight in kilograms.

   Current, J.D.,"A Linear Equation for Estimating the Body Surface Area in Infants and Children.", The Internet Journal of Anesthesiology 1998:Vol2N2.

1.2 LINEAR EQUATIONS AND THEIR GRAPHS

Mathematical models express features of interest. In the managerial, social, and natural sciences and engineering, linear equations often relate quantities of interest. Therefore, a thorough understanding of linear equations is important.

The standard form of a linear equation is $ax + by = c$ where a, b, and c are real valued constants. It is characterized by the first power of the exponents.

Standard Form of a Linear Equation

$$ax + by = c$$

a, b, c are real numbered constants; a and b, not both zero

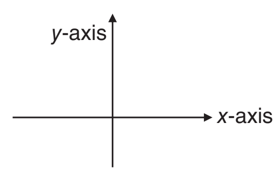
Example 1.2.1 Ordered Pair Solutions

Do the points (3, 5) and (1, 7) satisfy the linear equation $2x + y = 9$?

Solution:
A point satisfies an equation if equality is preserved. The point (3, 5) yields: $2(3) + 5 \neq 9$. Therefore, the ordered pair (3, 5) is not a solution to the equation $2x + y = 9$.
For (1, 7), the substitution yields $2(1) + 7 = 9$. Therefore, (1, 7) is a point on the line $2x + y = 9$.

↓ An ordered (coordinate) pair, (x, y) describes a (graphical) point in the x, y plane. By convention, the x value always appears first.

A **graph** is a pictorial representation of a function. It consists of points that satisfy the function. **Cartesian Coordinates** are used to represent the relative positions of points in a plane or in space. In a plane, a point P is specified by the coordinates or ordered pair (x, y) representing its distance from two perpendicular intersecting straight lines, called the x-axis and the y-axis, respectively (figure).



Cartesian coordinates are so named to honor the mathematician **René Descartes** (Historical Notes).

The **graph of a linear equation is a line**. It is uniquely determined by two distinct points. Any additional points can be checked as the points must be **collinear** (i.e., lie on the same line). The coordinate axes may be differently scaled. To determine the ***x*-intercept** of a line (its intersection with the x-axis), set $y = 0$ and solve for x. Likewise, for the ***y*-intercept** set $x = 0$ and solve for y.

↓ For the linear equation $2x + y = 9$, set $y = 0$ for the x-intercept ($x = 4.5$) and $x = 0$ for the y-intercept ($y = 9$) . As noted, intercepts are intersections of the line with the respective axes.

Example 1.2.2 Intercepts and Graph of a Line

Locate the x and y-intercepts of the line $2x + 3y = 6$ and graph its equation.

Solution:
When $x = 0$, $3y = 6$ so the y-intercept is $y = 2$. When $y = 0$, $2x = 6$ so the x-intercept is $x = 3$. The two intercepts, (3, 0) and (0, 2), as two points, uniquely determine the line. As

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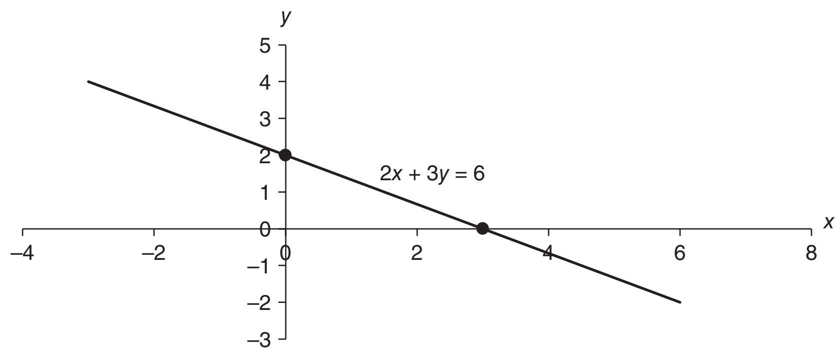
a check, arbitrarily choose a value for x, say $x = -1$. Then, $2(-1) + 3y = 6$ or $3y = 8$ so $y = 8/3$. Therefore $(-1,\ 8/3)$ is another point on the line. Check that these three points lie on the same line.



↑ Besides the algebraic representation of linear equations used here, many applications use elegant matrix representations. So $2x + 3y = 6$ (algebraic) can also be expressed as $(2\quad 3)\begin{pmatrix} x \\ y \end{pmatrix} = (6)$ in matrix format.

Price and quantity often arise in economic models. For instance, the demand $D(p)$ for an item is related to its unit price, p, by the equation $D(p) = 240 - 3p$. In graphs of economic models price appears on the (vertical) y-axis and quantity on the (horizontal) x-axis.

Example 1.2.3 Intercepts of a Demand Function

Given $D(p) = 240 - 3p$.

- *a) Determine demand when price is 10.*
- *b) What is demand when the goods are free?*
- *c) At what price will consumers no longer purchase the goods?*
- *d) For what values of price is D(p) meaningful?*

Solution:

- *a) Substituting $p = 10$ yields a demand of 210 units.*
- *b) When the goods are free $p = 0$ and $D(p) = 240$. Note that this is an intercept.*

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c) Here, $D(p) = 0$ and $p = 80$ is the price that is too high and results in no demand for the goods. Note, this is an intercept.

d) Since price is at least zero, and the same for demand, therefore, $0 \leq p \leq 80$ and $0 \leq D(p) \leq 240$.

When either a or b in $ax + by = c$ is zero, the standard equation reduces to a single value for the remaining variable. If $y = 0$, $ax = c$ so $x = c/a$; a vertical line. If $x = 0$, $by = c$ so $y = c/b$; a horizontal line.

↓ Remember: horizontal lines have zero slopes while vertical lines have infinite slopes.

Vertical and Horizontal Lines

For

$$ax + by = c$$

When $b = 0$ the graph of $ax = c$ (or $x =$ constant) is a vertical line.

When $a = 0$ the graph of $by = c$ (or $y =$ constant) is a horizontal line.

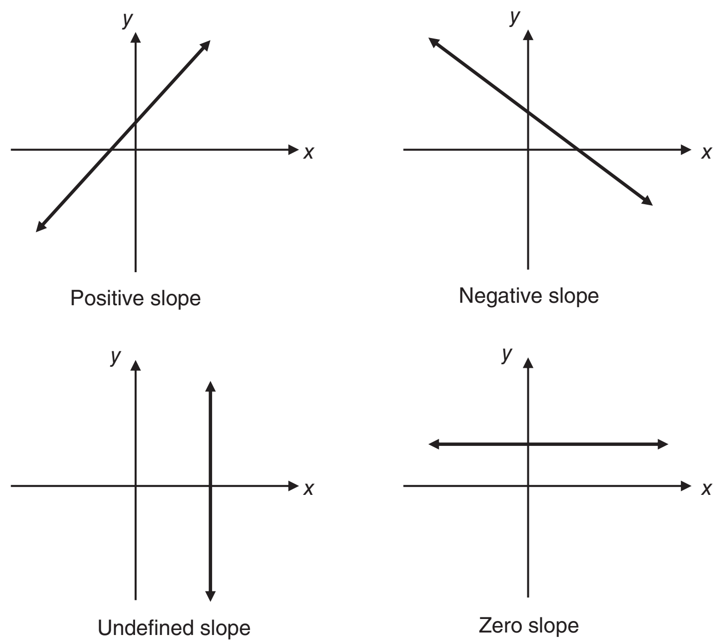
It is often useful to express equations of lines in different (and equivalent) algebraic formats. The **slope**, m, of a line can be described in several ways; "the rise divided by the run", or "the change in y, denoted by Δy, divided by the change in x, Δx". From left to right, positive sloped lines "rise" (/) while negative sloped lines "fall" (\).

⧫ In usage here, Δ denotes "a small change" or "differential." Later, the same symbol is used for a "finite difference." Unfortunately, the dual usage, being nearly universal, compels its usage. However, usage is usually clear from the context.

Slope

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The equation of a line can be expressed in different, but equivalent, ways. The **slope-intercept form** of a line is $y = mx + b$, where m is the slope and b its y-intercept. A horizontal line has zero slope. A vertical line has an infinite (undefined) slope, as there is no change in x for any value of y.



Slope-Intercept Form

$$y = mx + b$$

where m is the slope and b the y-intercept

A linear equation in standard form, $ax + by = c$, is written in slope-intercept form by solving for y.

Example 1.2.4 Slope-Intercept Form

Write $2x + 3y = 6$ in slope-intercept form and identify the slope and y-intercept.

Solution:
Solving, $y = (-2/3)x + 2$. By inspection, the slope is $-2/3$ ("line falls") and the y-intercept is (0, 2); in agreement with the previous Example.

Linear equations are also written in **point-slope form:** $y - y_1 = m(x - x_1)$. Here, (x_1, y_1) is a given point on the line and m is the slope.

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) a point on the line.

Example 1.2.5 Point-Slope Form

Determine the equation of a line in point-slope form passing through (2, 4) and (5, 13).

Solution:
First, the slope $m = \frac{13-4}{5-2} = \frac{9}{3} = 3$. *Now, using (2, 4) in the point-slope form we have* $y - 4 = 3(x - 2)$. *[Using the point (5, 13) yields the equivalent* $y - 13 = 3(x - 5)$*]. For the slope-intercept form, solving for y yields* $y = 3x - 2$. *The standard form is* $3x - y = 2$.

↓ Various representations of linear equations are equivalent but can seem confusing. Usage depends on the manner in which information is provided. If the slope and y-intercept are known, use the slope-intercept form. If coordinates of a point through which the line passes is known, use the point-slope form. Using a bit of algebraic manipulation, you can simply remember to use $y = mx + b$.

⧫ Incidentally, as noted earlier, while the generic symbols x and y are most common for variables, other letters are also used to denote variables. The equation $I = PRT$ was introduced earlier to express accrued interest. Economists use q and p for quantity and price, respectively, and scientists often use F and C for Fahrenheit and Celsius temperatures, respectively, and so on.

Example 1.2.6 Temperature Conversion

Water boils at 212°F(100°C) and freezes at 32°F (0°C). What linear equation relates Celsius and Fahrenheit temperatures?

Solution:
Denote Celsius temperatures by C and Fahrenheit temperatures by F. The ordered pairs are (100, 212) and (0, 32). Using the slope-intercept form of a line, $m = \frac{32-212}{0-100} = \frac{-180}{-100} = \frac{9}{5}$. *The second ordered pair, (0, 32) is its y-intercept. Therefore,* $F = 9/5\,C + 32$ *is the widely used relation to enable conversion of Celsius to Fahrenheit temperatures. An Exercise seeks the Fahrenheit to Celsius relation.*

♦ A common mathematical model for depreciation of equipment or buildings is to relate current value, y (dollars) to age, x (years). Straight Line Depreciation (SLD) is a common choice. In an SLD model, annual depreciation, d, is the same each year of useful life. Any remaining value is the "salvage value," s. Therefore, $y = dx + s$ is the desired model.

Example 1.2.7 Salvage Value

Equipment value at time t is $V(t) = -10{,}000t + 80{,}000$ and its useful life expectancy is 6 years. Develop a model for the original value, salvage value, and annual depreciation.

Solution:
The original value, at $t = 0$, is \$80,000. The salvage value, at $t = 6$, the end of useful life, is \$20,000 = $(-10{,}000(6) + 80{,}000)$. The slope, which is the annual depreciation, is \$10,000.

Lines having the same slope are **parallel**. Two lines are **perpendicular** if their slopes are negative reciprocals.

Parallel and Perpendicular Lines

$$y = m_1x + b_1 \qquad\qquad y = m_2x + b_2$$

Two lines are parallel if their slopes are equal

$$m_1 = m_2$$

Two lines are perpendicular if their slopes are negative reciprocals

$$m_1 = -\frac{1}{m_2}$$

(regardless of their y-intercepts b_1 and b_2)

Example 1.2.8 Parallel or Perpendicular Lines

Are these pairs of lines parallel, perpendicular, or neither?

a) $3x - y = 1$ and $y = (1/3)x - 4$
b) $y = 2x + 3$ and $y = (-1/2)x + 5$
c) $y = 7x + 1$ and $y = 7x + 3$

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Solution:

a) *The two slopes are required. By inspection, the slope of the second line is 1/3. The line $3x - y = 1$ in slope-intercept form is $y = 3x - 1$ so its slope is 3. Since the slopes are neither equal nor negative reciprocals the lines intersect.*
b) *The slopes are 2 and $(-1/2)$. Since these are negative reciprocals the two lines are perpendicular.*
c) *The lines have the same slope (and different intercepts) so they are parallel.*

♦ "What Makes an Equation Beautiful", once the title of a New York Times article isn't likely to excite widespread interest; especially in linear equations.

However, linear equations are main building blocks for more advanced – and more interesting equations.

Some physicists were asked, "Which equations are the greatest?" According to the article some were nominated for the breadth of knowledge they capture, for their historical importance, and some for reshaping our perception of the universe.

EXERCISES 1.2

1. Determine the x and y-intercepts for the following:
   a) $5x - 3y = 15$
   b) $y = 4x - 5$
   c) $2x + 3y = 24$
   d) $9x - y = 18$
   e) $x = 4$
   f) $y = -2$
2. Determine slopes and y-intercepts for the following:
   a) $y = (2/3)x + 8$
   b) $3x + 4y = 12$
   c) $2x - 3y - 6 = 0$
   d) $6y = 4x + 3$
   e) $5x = 2y + 10$
   f) $y = 7$
3. Determine the slopes of lines defined by the points:
   a) (3, 6) and (−1, 4)
   b) (1, 6) and (2, 11)
   c) (6, 3) and (12, 7)
   d) (2, 3) and (2, 7)
   e) (2, 6) and (5, 6)
   f) (5/3, 2/3) and (10/3, 1)
4. Determine the equation for the line
   a) with slope 4 that passes through (1, 7).
   b) passing through (2, 7) and (5, 13).
   c) with undefined slope passing through (2, 5/2).
   d) with x-intercept 6 and y-intercept −2.
   e) with slope 5 and passing through (0, −7).
   f) passing through (4, 9) and (7, 18).

5. Plot graphs of:
   a) $y = 2x - 5$
   b) $x = 4$
   c) $3x + 5y = 15$
   d) $2x + 7y = 14$
6. Plot graphs of:
   a) $2x - 3y = 6$
   b) $y = -3$
   c) $y = (-2/3)x + 2$
   d) $y = 4x - 7$
7. Are the pairs of lines parallel, perpendicular, or neither?
   a) $y = (5/3)x + 2$ and $5x - 3y = 10$
   b) $6x + 2y = 4$ and $y = (1/3)x + 1$
   c) $2x - 3y = 6$ and $4x - 6y = 15$
   d) $y = 5x - 4$ and $3x - y = 4$
   e) $y = 5$ and $x = 3$
8. Determine equation for the line
   a) through (2, 3) and parallel to $y = 5x - 1$.
   b) through (1, 4) and perpendicular to $2x + 3y = 6$.
   c) through (5, 7) and perpendicular to $x = 6$.
   d) through (4, 1) and parallel to $x = 1$.
   e) through (2, 3) and parallel to $2y = 5x + 4$.
9. When does a linear equation lack an x-intercept? Have more than one x-intercept? Lack a y-intercept? Have more than one y-intercept?
10. Model the conversion of Fahrenheit to Celsius temperatures. Hint: Use the freezing and boiling temperatures for water.
11. A new machine cost \$75,000 and has a salvage value of \$21,000 after nine years. Model its straight line depreciation.
12. A new car cost \$28,000 and has a trade-in value of \$3000 after 5 years. Use straight line depreciation.
13. The distance a car travels depends on the quantity of gasoline available. A car requires seven gallons to travel 245 miles and 12 gallons to travel 420 miles. What linear relationship expresses distance (miles) as a function of gasoline usage (gallons)?
14. Office equipment is purchased for \$50,000 and after ten years has a salvage value of \$5000. Model its depreciation with a linear equation.
15. Monthly rent on a building is \$1100 (a fixed cost). Each unit of the firm's product costs \$5 (a variable cost). Form a linear model for the total monthly cost to produce x items.
16. A skateboard sells for \$24. Determine the revenue function for selling x skateboards.

17. A car rental company charges \$50 per day for a medium sized car and 28 cents per mile driven.
    a) Model the cost for renting a medium sized car for a single day.
    b) How many miles can be driven that day for \$92?
18. At the ocean's surface water and air pressures are equal (15 lb / in^2). Below the surface water pressure increases by 4.43 lb / in^2 for every 10 feet of depth.
    a) Express water pressure as a function of ocean depth.
    b) At what depth is the water pressure 80 lb / in^2?
19. A man's suit sells for \$84. The cost to the store is \$70. A woman's dress sells for \$48 and costs the store \$40. If the store's markup policy is linear, and is reflected in the price of these items, model the relationship between retail price, R, and store cost, C.
20. Demand for a product is linearly related to its price. If the product is priced at \$1.50 each, 40 items can be sold. Priced at \$6, only 22 items are sold. Let x be the price and y the number of items sold. Model a linear relationship between price and items sold.

1.3 FACTORING AND THE QUADRATIC FORMULA

Factoring is a most useful mathematical skill. A first step in factoring expressions is to seek the **greatest common factor** (GCF) among terms of an algebraic expression. For example, for $5x + 10$, the GCF is 5. So, to factor this expression one writes $5(x + 2)$.

Often one is unsure whether an expression is factorable. One aid is to note that a linear (first power) expression is only factorable for a common constant factor. The expression $5(x + 2)$ is completely factored because the term $(x + 2)$ is linear and does not have any constant common factor. When higher power (order) terms are involved, it is more difficult to know whether additional factoring is possible. We develop a few guidelines in this Section and with practice you should have more success with this essential skill.

The GCF may not be a single expression. When factoring $10x^3y^2 + 30x^2y^3$, 10 is a common factor. However, the variables x and y must be considered. The GCF is $10x^2y^2$ and leads to $10x^2y^2(x + 3y)$. The expression $(x + 3y)$ cannot be factored.

Example 1.3.1 Finding the GCF

Find the greatest common factor for these expressions.

a) $5x^4(a + b)^3 + 10x^2y(a + b)^2$
b) $2x^3 + 10x^2 - 48x$
c) $(x + y)^{10}(x^2 + 4x + 7)^3 + (x + y)^8(x^2 + 4x + 7)^4$

Solution:

a) *Here, 5, x^2, and $(a + b)^2$ are common factors to each term. The GCF is $5x^2(a + b)^2$ and results in $5x^2(a + b)^2[x^2(a + b) + 2y]$.*

→

b) Since 2 and x are factors, the GCF is 2x and $2x(x^2 + 5x - 24)$ *is the first stage of factoring.*

c) Here, $(x + y)^8$ *and* $(x^2 + 4x + 7)^3$ *are common factors. The GCF is* $(x + y)^8\,(x^2 + 4x + 7)^3$ *and results in* $(x + y)^8(x^2 + 4x + 7)^3[(x + y)^2 + (x^2 + 4x + 7)]$.

The number of terms in an expression is a hint as some factoring rules depend upon the number of terms involved.

For two terms, one seeks a sum or difference of squares or of cubes. With three terms, factoring is usually by trial and error as there are fewer dependable rules and mastery comes from experience (and luck!). With four terms, it may be possible to group terms in "compatible" pairs for continued factoring. In other instances, grouping three terms may be useful.

Sum and Differences of Squares and Cubes

1. $x^2 - a^2 = (x + a)(x - a)$
2. $x^2 + a^2$ **is not factorable**
3. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$
4. $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

↓ Here are a couple of illustrations:

$$x^3 - 16a^2x = x(x^2 - 16a^2) = x(x - 4a)(x + 4a)$$

$$x^4y^4 + xy = xy(x^3y^3 + 1) = xy(xy + 1)(x^2y^2 - xy + 1)$$

Example 1.3.2 Sum and Difference of Squares

Factor these expressions of sums or differences of squares.

a) $x^2 - 100$ *b)* $2x^4y - 50y$ *c)* $(x + y)^2(a + b) - 9(a + b)$.

Solution:

a) This is a difference of squares. Therefore, the expression factors as $(x + 10)(x - 10)$.

b) First, the common factor 2y yields $(2y)[x^4 - 25]$. *Within the brackets is a difference of two squares,* $(x^2)^2$ *and* $(5)^2$. *Continuing,* $(2y)(x^2-5)(x^2 + 5)$ *is the factorization.*

⟶

c) *A common factor* $(a+b)$, *yields* $(a+b)[(x+y)^2-9]$. *Within the brackets is a difference of two squares* $(x+y)^2$ *and* $(3)^2$. *Continuing, yields a completely factored expression* $(a+b)[(x+y)+3][x+y)-3]$.

Example 1.3.3 Sum and Difference of Cubes

Factor these expressions of sums or differences of cubes.

a) x^3+1000 b) $5x^3-625$ c) $2x^3y^2(a+b)^3+2x^3y^2$

Solution:

a) *Factoring of a sum of two cubes* $(x)^3$ *and*$(10)^3$ *yields* $(x+10)(x^2-10x+100)$.
b) *First, the GCF of 5 yields* $5(x^3-125)$. *Next, for a difference of cubes, the factoring yields* $5(x-5)(x^2+5x+25)$. *(Note that the trinomial factor,* $x^2+5x+25$, *cannot be simplified).*
c) *Here, the GCF is* $2x^3y^2$ *so* $2x^3y^2[(a+b)^3+1]$ *is the first stage result. Next, noting the difference of cubes yields the completely factored* $2x^3y^2[(a+b)+1][(a+b)^2-(a+b)+1]$.

Factoring trinomials is more complicated; however, some ideas may be useful. One approach is to use trial and error that is somewhat the reverse of the FOIL technique used to multiply two binomial factors to yield a trinomial. For instance, to factor $x^2-10x+21$ the lead term, x^2, suggests two linear factors that multiply x by itself. Next, focus on the constant and its factors. To multiply two numbers to yield +21 they must have the same sign. The coefficient of the linear term being negative, each of the two factors must be negative. We seek two negative factors of 21. Possible choices are −1 and −21, or −3 and −7. The latter is the correct option since −3 − 7 sums to the linear coefficient −10. The factored trinomial is $(x-3)(x-7)$.

Quadratic trinomials x^2+bx+c, with unit coefficient of x^2 are fairly simple to factor. Always be alert for a GCF before attempting to factor. Expressions that cannot be factored are known as being **prime** or **irreducible**.

↓ Here are a couple of illustrations:
For $x^2+11x+28$, note that 4 and 7 are factors of 28 that sum to 11. Therefore, $(x+7)(x+4)$ is the factorization. For $2x^2+7x-4$ the coefficient of x^2 immediately suggests a start with $(2x+?)(x+?)$.
The −4 suggests trial and error with numbers +4 and −1 or −4 and +1. The result is $(2x-1)(x+4)$.

↑ You may know the quadratic formula and wonder why it has not been used so far. It can always be used to factor second order trinomials and appears later in this Section.

Example 1.3.4 Factoring Trinomials

Factor these trinomials:

a) $x^2 + 20x + 36$

b) $2x^3 - 24x^2 + 54x$

c) $x^2 - 3x - 10$

d) $x^2 + 9x + 12$

Solution:

a) *As 36 is positive, the two factors have like positive sign since 20x is positive. Therefore, all positive pairs of factors of 36 are prospects. Here, 2 and 18 are the factors which sum to 20. The trinomial is factored as* $(x + 2)(x + 18)$.

b) *First the GCF 2x is factored to yield* $2x(x^2 - 12x + 27)$. *The constant term and the −12 indicate the need of two negative factors of 27 which sum to 12. The two factors are −9 and −3 so the expression factors as* $(2x)(x - 3)(x - 9)$.

c) *Here, the constant term indicates that the factors are of opposite sign. We seek two factors of −10, one positive and one negative that sum to −3. These are −5 and +2 so the trinomial factors as* $(x - 5)(x + 2)$.

d) *Here we seek two positive factors of 12 that add to 9. This is not possible. Therefore, this trinomial is not factorable.*

When the lead term coefficient is not unity, trial and error factoring seems more difficult. For instance, to factor $2x^2 + 5x - 12$ the lead term can only result from multiplying $2x$ by x. The constant is negative so one positive factor and one negative factor are needed here as

$$(2x \pm \quad)(x \pm \quad).$$

The possibilities are $(-1,\ 12)$, $(1,\ -12)$, $(-2,\ 6)$, $(2,\ -6)$, $(-3,\ 4)$, $(3,\ -4)$, $(-4,\ 3)$, $(4,\ -3)$, $(-6,\ 2)$, $(6,\ -2)$, $(-12,\ 1)$, $(12,\ -1)$. Some of these twelve options are quickly eliminated. The first term with $2x$, cannot have a factor that is an even number as it would contain a multiple of 2 to factor at the start. This leaves the four possibilities $(-1,\ 12)$, $(1,\ -12)$, $(-3,\ 4)$, and $(3,\ -4)$. Investigating these possibilities yields $(-3,\ 4)$ as the correct pair. The trinomial factors as $(2x - 3)(x + 4)$.

Example 1.3.5 Factoring Trinomials (revisited)

Factor these trinomials:

a) $3x^2 + 13x + 10$ b) $6x^2 - 11x - 10$ c) $60x^3 + 74x^2 - 168x$.

Solution:

a) *First, use* $(3x + \quad)(x + \quad)$ *for the first term. Next, since 10 and 13 are both positive only two positive factors of ten need to be investigated. There are four possibilities (1, 10), (2, 5), (5, 2), and (10, 1). Investigating the (2, 5) and (5, 2) possibilities first, determine that neither fits. Finally, looking at the remaining two pairs the trinomial is factored as* $(3x + 10)(x + 1)$.

b) *Here, there are two possibilities for* $6x^2$. *They are* $(2x + \quad)(3x + \quad)$ *or* $(6x + \quad)(x + \quad)$. *Sometimes it is helpful to use factors closer together unless the middle term coefficient is large. Here,* $(2x + \quad)(3x + \quad)$ *is first choice. The* -10 *indicates positive and negative factors to try to obtain the* -11 *coefficient of the middle term. Therefore* $(-1, 10)$, $(1, -10)$, $(-5, 2)$ *and* $(5, -2)$ *are the possibilities to evaluate since none of them have a multiple of 2 for a factor. Checking further, the trinomial factors* $(2x - 5)(3x + 2)$ *emerge.*

c) *Here, the GCF is 2x so* $(2x)[30x^2 + 37x - 84]$. *The* $30x^2$ *can come from multiplying* $(x + \quad)(30x + \quad)$, $(2x + \quad)(15x + \quad)$, $(3x + \quad)(10x + \quad)$, *or* $(5x + \quad)(6x + \quad)$. *Quite a list of options! Again, as a rule of thumb, first try the factors closer together. If that fails, proceed to the next closest pair and so on. We try* $(2x)(5x + \quad)(6x + \quad)$ *first. The* -84 *can be found in many ways. However, a multiple of 2 or 3 cannot be the second factor since the 6x term would have a common factor then. This narrows the options to* $(-12, 7)$, $(12, -7)$, $(-84, 1)$, *and* $(84, -1)$. *If none of these are correct, investigate the 3x and 10x as factors. The 5x and 6x option is correct to yield* $(2x)(5x + 12)(6x - 7)$ *as the completely factored trinomial.*

Factoring trinomials when the lead coefficient is not unity becomes easier with practice. One learns to eliminate possibilities by the value of the second coefficient or by noticing that certain factors cannot be a common factor in the terms. To factor expressions with four terms, they are often grouped for common factors.

For instance, suppose one wants to factor $ax + 5x + ay + 5y$. The four terms of this expression lack a common factor (GCF). However, pairs of factors do share a common factor. Group the front pair and back pair as $(ax + 5x) + (ay + 5y)$ to begin the factoring. Now, factoring the pairs yields $(x)(a + 5) + (y)(a + 5)$. These terms have $(a + 5)$ as a GCF to yield $(a + 5)(x + y)$ to complete the factoring. An equivalent alternative groups the terms as $(ax + ay) + (5x + 5y)$. Factoring the GCF of the pairs yields
$(a)(x + y) + (5)(x + y) = (x + y)(a + 5)$.

Example 1.3.6 Factoring by Grouping

Completely factor these expressions by grouping.

a) $ax - 7x + ay - 7y$

b) $x^2 + 9x - a^2 - 9a$

c) $2x^3 - 10x + 3x^2 - 15$

d) $x^2 + 10x + 25 - y^2$

Solution:

a) *Grouping as* $(ax - 7x) + (ay - 7y)$ *yields* $x(a - 7) + y(a - 7)$. *The common factor is* $(a - 7)$. *Therefore, the expression factors as* $(a - 7)(x + y)$.

b) *Grouping as* $(x^2 + 9x) - (a^2 + 9a)$ *yields* $x(x + 9) - a(a + 9)$; *not helpful. Try regrouping by powers, as* $(x^2 - a^2) + (9x - 9a)$ *which yields* $(x + a)(x - a) + 9(x - a)$. *This has a common factor* $(x - a)$. *The expression factors as* $(x - a)[(x + a) + 9]$.

c) *Grouping as* $(2x^3 - 10\,x) + (3x^2 - 15)$ *yields* $2x(x^2 - 5) + 3(x^2 - 5)$. *It has a common factor* $(x^2 - 5)$. *Therefore, the expression factors as* $(x^2 - 5)(2x + 3)$.

d) *Grouping in pairs makes no sense here since no two pairs have a common factor. Grouping as* $(x^2 + 10x + 25) - y^2$, *the first three terms of the expression are the perfect square trinomial* $(x + 5)^2$. *Now,* $(x + 5)^2 - y^2$ *is a difference of squares and factors as* $[(x + 5) + y][(x + 5) - y]$.

The Quadratic Formula

The **quadratic equation**, whose largest exponent is two, is an important trinomial. It has the form: $y = ax^2 + bx + c$, where a, b, c are real constants. The graph of this quadratic is a **parabola**.

To determine any x-intercepts (called "roots" or zeros of the quadratic) set $y = 0$ and solve $0 = ax^2 + bx + c$. Sometimes this can be accomplished by factoring the trinomial and then setting its factors to zero. However, often it may not possible to factor a trinomial. Fortunately, it may factor using the **quadratic formula**.

↓ That is, for $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *(Quadratic Formula).*

For instance, to find the factors of $x^2 - 5x + 4$ using the quadratic formula, substitute $a = 1$, $b = -5$, and $c = 4$ as $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)} = \dfrac{5 \pm 3}{2}$. So $x = 4$ and $x = 1$ and the factors are $(x - 4)$ and $(x - 1)$. Given the equation $x^2 - 5x + 4 = 0$, $x = 4$ and $x = 1$ are its roots.

Example 1.3.7 The Quadratic Formula

Derive the Quadratic Formula.
Hint: Complete the square of $ax^2 + bx + c = 0$ *to solve for its roots.*

Solution:
First, rewrite the equation as $ax^2 + bx = -c$.
To complete the square, set the coefficient of x^2 *to unity ("1") by dividing by "a" as*

$$\frac{ax^2 + bx}{a} = \frac{-c}{a} = x^2 + \frac{b}{a}x = \frac{-c}{a}$$

Next, square 1/2 the coefficient of the linear term, x. Next, add the result to both sides. That is, square
$\frac{1}{2}\left(\frac{b}{a}\right)$ *to become* $\left[\left(\frac{1}{2}\right)\left(\frac{b}{a}\right)\right]^2 = \left[\frac{b}{2a}\right]^2 = \frac{b^2}{4a^2}$ *and add to each side as*

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac + b^2}{4a^2}$$

Now, the left side is the perfect square,

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Next, take the square root of each side

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the familiar quadratic formula!

The Quadratic Formula

The roots (solutions) of

$$\mathbf{ax^2 + bx + c = 0}$$

are $\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a \neq 0)}$

Example 1.3.8 Zeros of Quadratics

Use the quadratic formula to determine the roots of:

a) $x^2 + 20x + 50 = 0$

b) $x^2 + 12 = 13x$

c) $x^2 - 3x = -5$

d) $6x^2 - 13x - 5 = 0.$

Solution:

a) *Note the corresponding values of a, b, and c are 1, 20 and 50, respectively. The quadratic formula yields*

$$x = \frac{-(20) \pm \sqrt{(20)^2 - 4(1)(50)}}{2(1)} = \frac{-20 \pm \sqrt{200}}{2} = \frac{20 \pm 10\sqrt{2}}{2}$$
$$= 10 \pm 5\sqrt{2}$$

The two irrational roots $10 + 5\sqrt{2}$*and* $10 - 5\sqrt{2}$ *indicate the quadratic was not factorable.*

b) *First, rewrite the equation as* $x^2 - 13x + 12 = 0$. *The values of a, b, and c are 1, −13, and 12, respectively. The quadratic formula yields*

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(12)}}{2(1)} = \frac{13 \pm \sqrt{121}}{2} = \frac{13 \pm 11}{2} \quad or$$
$$x = \frac{13 + 11}{2} = 12, \quad \frac{13 - 11}{2} = 1.$$

Note in this example that the radicand is a perfect square. The two rational solutions indicate that the equation is factorable $[(x - 12)(x - 1)]$.

c) *First, rewrite the equation as* $x^2 - 3x + 5 = 0$. *The values of a, b, and c are 1, −3, and 5, respectively. The quadratic formula yields* $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)} = \frac{3 \pm \sqrt{-11}}{2}$. *The negative radicand indicates that there are no real valued solutions(the solutions are complex numbers).*

d) *The equation is already set to zero so the values of a, b, and c are 6, −13, and −5, respectively. The quadratic formula yields* $x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(-5)}}{2(6)} = \frac{13 \pm \sqrt{289}}{12} = \frac{13 \pm 17}{12}$. $x = \frac{13 + 17}{12} = \frac{5}{2}$, $\frac{13 - 17}{12} = \frac{-1}{3}$. *The two rational roots here indicate the original equation was factorable as*

$$\left[\left(x - \frac{5}{2}\right)\left(x + \frac{1}{3}\right)\right] \text{ or as } [(2x - 5)(3x + 1)].$$

Example 1.3.9 A Quadratic Supply Function

Use the quadratic formula to determine the price below which producers would not manufacture a product whose ***supply function*** *is $S(p) = p^2 + 10p - 200$. The supply function, $S(p)$, relates manufactured supply and unit selling price, p.*

Solution:
First, note the values of a, b, and c are 1, 10 and −200, respectively. Using the quadratic formula,

$$p = \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-200)}}{2(1)} = \frac{-10 \pm \sqrt{900}}{2} = \frac{-10 \pm 30}{2}.$$

The two rational roots are −20 and 10. However, price cannot be negative. For prices above $p = 10$ the supply function is positive. Therefore, producers will not manufacture the product when the price is below 10.

EXERCISES 1.3

In Exercises 1–8 factor out the greatest common factor.

1. $8x - 24$
2. $20c^2 - 10c$
3. $5x^3 - 10x^2 + 15x$
4. $36y^4 - 12y^3$
5. $5a^3b^2c^4 + 10a^3bc^3$
6. $x(a + b) + 2y(a + b)$
7. $20x^3y^5z^6 + 15x^4y^3z^7 + 20x^2y^4z^5$
8. $4(x + y)^3(a + b)^5 + 8(x + y)^2(a + b)^5$

In Exercises 9–16 factor completely using the rules for sums or differences of squares or cubes.

9. $x^2 - 25$
10. $2x^3 - 8x$
11. $3x^2 + 27$
12. $x^4 - 81$
13. $2x^3 - 16$
14. $x^3 + 125$
15. $7x^2(a + b) - 28(a + b)$
16. $3x^3(x + y)^4 + 24(x + y)^4$

In Exercises 17–32 factor each trinomial completely or indicate primes.

17. $x^2 + 5x + 4$
18. $x^2 - 37x + 36$
19. $x^2 + 3x + 1$
20. $x^2 - 10x + 9$
21. $x^2 - 6x - 16$
22. $x^2 - 7x + 6$
23. $2x^2 + 12x + 16$
24. $x^3 - x^2 - 12x$

25. $a^2b^2 + 9ab + 20$

26. $x^2(a + b) - 15x(a + b) + 36(a + b)$

27. $2x^2y^2 + 28xy + 90$

28. $x^2 - 20x + 36$

29. $x^2 + 7x + 5$

30. $x^2 - 19x + 48$

31. $x^4 - 5x^2 + 4$

32. $x^6 - 7x^3 - 8$

In Exercises 33–36 factor by grouping.

33. $x^2 - a^2 + 5x - 5a$

34. $x^2 - y^2 + 3x - 3y$

35. $4ab - 8ax + 6by - 12xy$

36. $x^2 + 6x + 9 - y^2$

In Exercises 37–46 solve using the quadratic formula.

37. $x^2 + 9x + 8 = 0$

38. $x^2 + 5x + 3 = 0$

39. $x^2 + 17x + 72 = 0$

40. $x^2 + 3x + 5 = 0$

41. $x^2 + 4x + 7 = 0$

42. $x^2 + 11x = -24$

43. $x^2 + 18 = 9x$

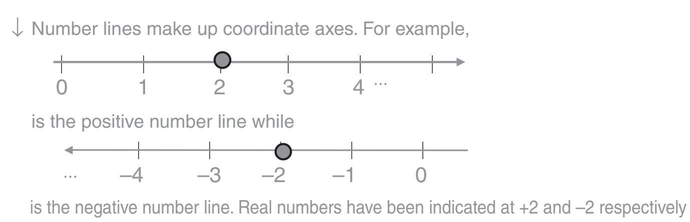
44. $x^2 - 4x = 45$

45. $2x^2 - 3x + 1 = 0$

46. $5x^2 - 16x + 3 = 0$

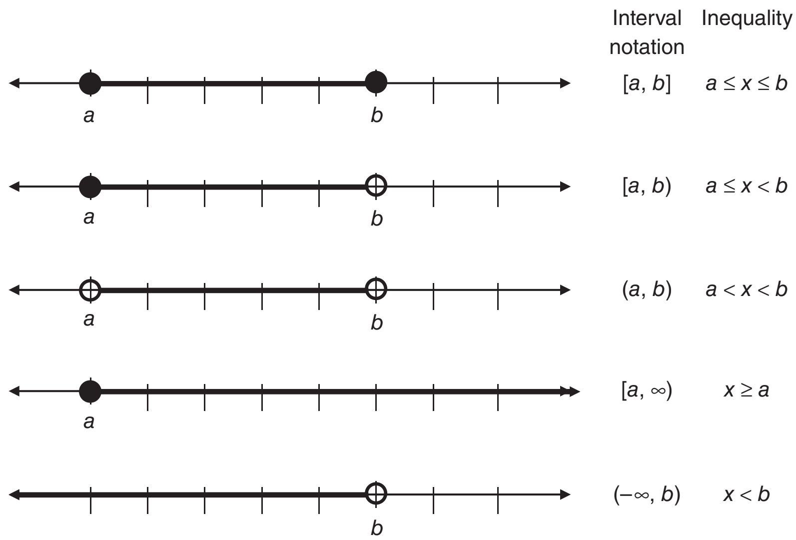
1.4 FUNCTIONS AND THEIR GRAPHS

Recall that any real number can be displayed on a number line. Often, solutions are displayed as intervals, especially inequalities. We assume real numbers here.



An **open interval** does not include endpoints. Thus, $x > 5$ is an example of an open interval since $x = 5$ is excluded. In a **closed interval** endpoints are included. Thus, $-4 \leq x \leq -2$ is a closed interval as the endpoints are included. An example of a **half-open** interval is $3 < x \leq 7$.

The table displays intervals on a number line and in **interval notation.** In interval notation, **parentheses**, ( ), represent open endpoints and **brackets**, [ ], closed endpoints. Parentheses are used for $\pm\infty$. An open endpoint represents an absent point while a closed one is an actual point.



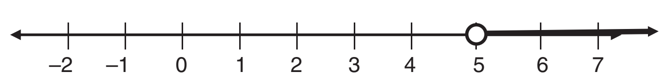
Example 1.4.1 Interval Notation

Display these inequalities on a number line and express them in interval notation.

a) $x > 5$ *b)* $-2 \leq x \leq 3$

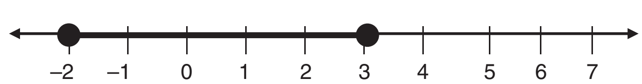
Solution:

a)



Notice the open endpoint at $x = 5$*. The interval notation for the inequality is* $(5, \infty)$.

b)



Notice the closed endpoints are used here because of the indicated equality. The interval notation for the inequality is $[-2, 3]$.

Interval notation requires some care. For example, a bracket with $\pm\infty$ implies closure at infinity which is not sensible!

Functions

You may have encountered several ways to define **functions**, (usually denoted as $y = f(x)$). A function is a rule that assigns to each value of x a unique value of y. Stated differently, each element of the **domain** is assigned one element of the **range**. Another, drawing on the language of computers or economics, is that to each "input" (x) there is a corresponding "output" (y).

The domain can be any (sensible) input. For example, the domain for polynomials is any real number. Remember, a denominator cannot be zero. If there are radicals, the *radicand* (expression under the radical sign) and the root determine the domain. The *index* of a radical is its "root." That is, the index is 2 for a square root, 3 for a cube root, and 4 for a fourth root, and so on. There is no sign restriction on domains of radicals with an odd index. If the index is even, as in square roots or fourth roots, the radicand cannot be negative.

Example 1.4.2 Finding Domains

Find the domain for each of the following functions.

a) $x^3 + 5x^2 + 7x + 3$

b) $\sqrt[3]{4x - 7}$

c) $\sqrt[4]{4x - 7}$

d) $\dfrac{5x + 1}{(x - 1)(x + 2)}$.

Solution:

a) *Recall, a polynomial is an expression whose exponents are whole numbers* $\{0, 1, 2, \ldots\}$. *Therefore,* $x^3 + 5x^2 + 7x + 3$ *is a polynomial and, as such, its domain is all real numbers; or in interval notation* $(-\infty, \infty)$.

b) *The domain for* $\sqrt[3]{4x - 7}$ *is the real numbers since the index* $(n = 3)$ *is odd.*

c) *Not all real numbers can be the domain for* $\sqrt[4]{4x - 7}$ *since its index* $(n = 4)$ *is even. The radicand cannot be negative so* $4x - 7 \geq 0$ *yields* $x \geq 7/4$. *In interval notation, the domain is* $[7/4, \infty)$.

d) *The real numbers are the domain for* $\dfrac{5x + 1}{(x - 1)(x + 2)}$ *except for* $x = 1$ *or* $x = -2$. *In interval notation the domain is* $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

Example 1.4.3 Function Values

If $f(x) = x^2 - 3$ *determine the following:*

a) $f(0)$ b) $f(2)$ c) $f(a)$ d) $f(a + 2)$.

Solution:

a) *Replace x by 0 to yield* $0^2 - 3 = -3$. *So,* $f(0) = -3$.

b) *Replace x by 2 to yield* $2^2 - 3 = 1$. *So,* $f(2) = 1$.

c) *Replace x by a to yield* $a^2 - 3$. *So,* $f(a) = a^2 - 3$.

⟶

d) Replace x by a + 2 to yield $(a + 2)^2 - 3 = (a^2 + 4a + 4) - 3$. *So,* $f(a + 2) = a^2 + 4a + 1$.

Example 1.4.4 Function Notation and Piecewise Intervals

Suppose f(x) is the piecewise function

$$f(x) = \begin{cases} x^2 + 1 & x > 4 \\ 5x + 3 & -1 \le x \le 4 \\ 7 & x < -1 \end{cases}$$

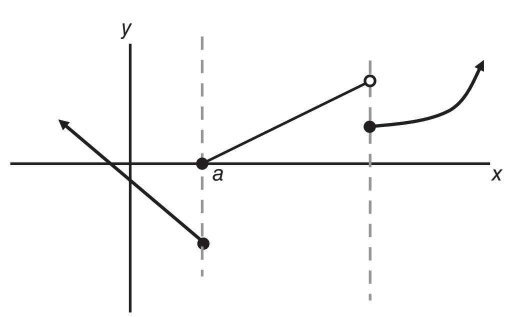
determine the following: a) $f(-3)$ b) $f(0)$ c) $f(2)$ d) $f(4)$ e) $f(7)$

Solution:

a) *The value* $x = -3$ *belongs in the interval* $x < -1$. *Substituting* $x = -3$ *into the third portion of the function above doesn't alter the constant value of 7.*
b) *The value* $x = 0$ *lies in the interval* $-1 \le x \le 4$. *Substituting* $x = 0$ *into* $5x + 3$ *yields* $f(0) = 3$.
c) *The value 2 also lies in the interval* $-1 \le x \le 4$. *Substituting* $x = 2$ *into* $5x + 3$ *yields* $f(2) = 13$.
d) *The value* $x = 4$ *also lies in the interval* $-1 \le x \le 4$ *(note the equality for* $x = 4$*). Substituting* $x = 4$ *into* $5x + 3$ *yields* $f(4) = 23$.
e) *The value 7 lies in the interval* $x > 4$. *Substituting* $x = 7$ *into* $x^2 + 1$ *yields* $f(7) = 50$.

To determine whether or not a graph actually represents a function, it must pass the **vertical line test**: *if an intersection with any vertical line is not unique, the graph cannot be that of a function.*

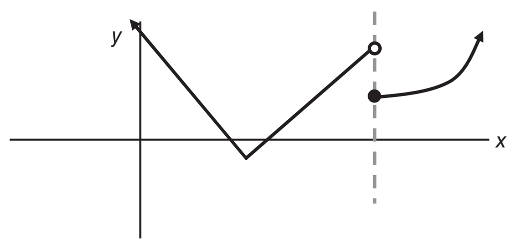
The graph below cannot be that of a function as it fails the vertical line test at $x = a$.



↑ The closed endpoint indicates that a point is included in the graph while an open endpoint is not an actual point of the graph. Therefore, one open endpoint and one closed circle at a value of x does not violate the conditions for the vertical line test.

Example 1.4.5 Determining Functions

Determine whether or not the graph shown below is that of a function.



Solution:
It is a function that passes the vertical line test (vertical line passes through one open and one closed circle). There is a unique value of y for every value of x.

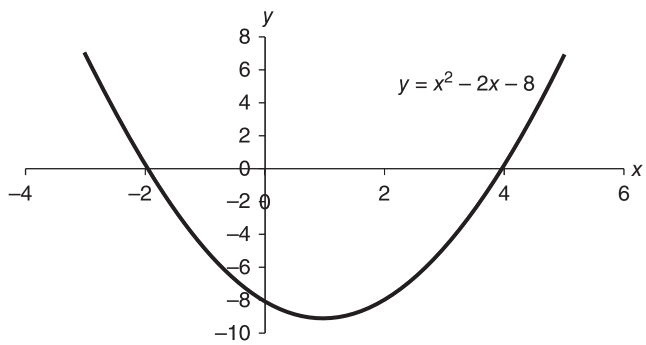
Graphs of Functions

In Section 1.2 you learned to graph linear functions using intercepts and, as a check, to plot at least three points. In general, the graph of quadratics (2nd degree) or higher degree polynomials should have at least five plotted points to be reasonable representations. To graph such functions, the y-intercept, x-intercepts, and key points (such as the vertex for a parabola) are useful.

Example 1.4.6 Graph of a Parabola

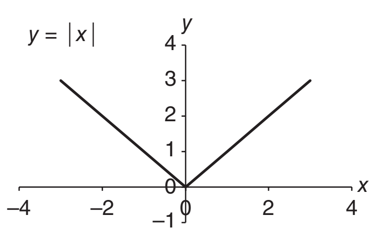
Sketch $y = x^2 - 2x - 8$.

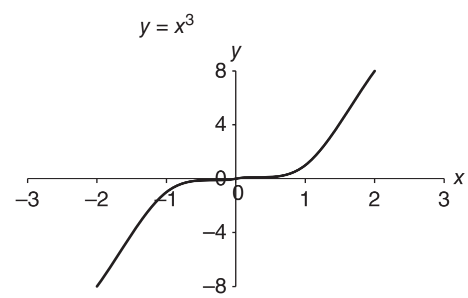
Solution:
The axis of symmetry is $x = -b/2a$. *Here,* $a = 1$ *and* $b = -2$ *so the axis of symmetry is at* $x = 1$. *Recall, that when* $a > 0$ *the parabola opens upward (concave upward). The y-intercept is the constant term,* -8*, here. The vertex, or turning point, of the parabola occurs at the axis of symmetry. Substituting* $x = 1$ *yields* $y = -9$ *so the vertex occurs at the ordered pair* $(1, -9)$. *To determine the x-intercepts, y is set to zero. Here,* $0 = x^2 - 2x - 8$. *Solving yields* $0 = (x - 4)(x + 2)$ *so the x -intercepts are at* $x = 4$ *and* $x = -2$. *Now, four points of the parabola are identified. A fifth point, using symmetry, is* $(2, -8)$ *and substituting* $x = 2$ *into the original equation correctly yields* -8.

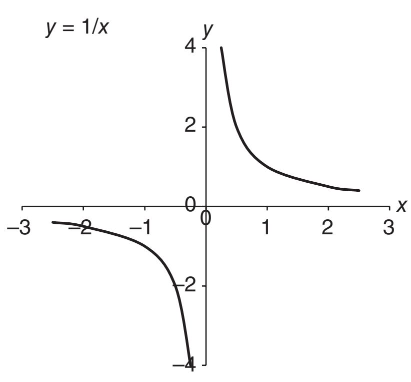


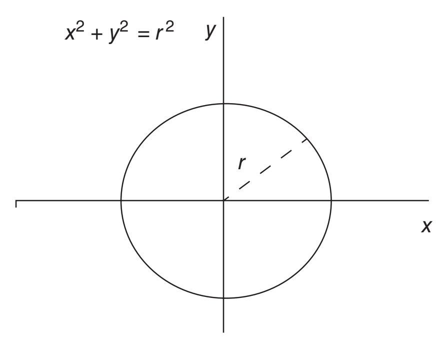
↑ When there are two x-intercepts a useful check is to verify that their average is the x value for the axis of symmetry. In the previous Example $(-2+4)/2 = 1$ and $x = 1$ was the axis of symmetry.

Other graphs useful to recognize are $y = |x|$, $y = x^3$, $y = 1/x$, and $x^2 + y^2 = r^2$ (shown below).









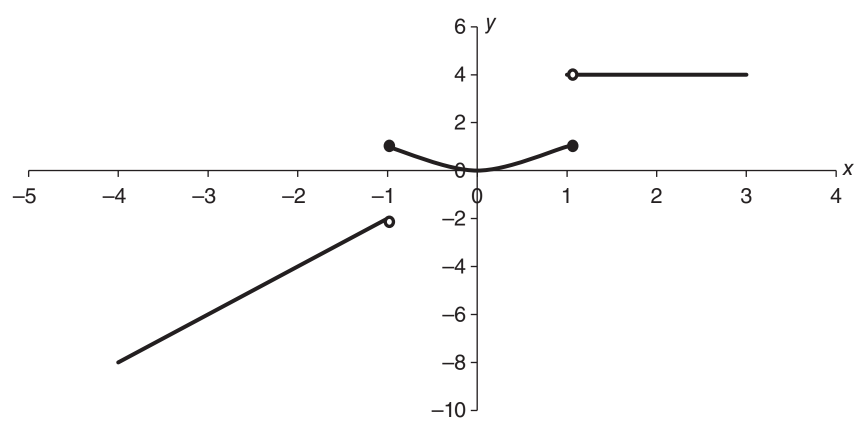
Sometimes one uses the graph of a **piecewise function**. You can use portions of the familiar graphs with the domains restricted for the employed pieces.

Example 1.4.7 A Piecewise (segmented) Graph

$$\text{Graph the function} f(x) = \begin{cases} 2x & x < -1 \\ x^2 & -1 \leq x \leq 1 \\ 4 & x > 1 \end{cases}$$

Solution:

The graph is the line $y = 2x$ as long as $x < -1$. The restriction on x means that an open circle is used at $(-1, -2)$. The second segment of the graph is parabolic with closed circles at $(-1, 1)$ and $(1, 1)$. The third and final segment of the graph is a portion of the horizontal line $y = 4$ with an open circle at $(1, 4)$.



The Algebra of Functions

Many functions are composed of other functions. For example, for sales volume x, the profit function, $P(x)$, is the revenue $R(x)$ minus the cost $C(x)$ functions, $P(x) = R(x) - C(x)$.

Functions may be added, subtracted, multiplied or divided. For instance, $f(x) + g(x)$ is the addition of $g(x)$ to $f(x)$ and can be written $(f + g)(x)$. Likewise, other operations are denoted by

$$f(x) - g(x) = (f - g)(x),\ \ f(x)g(x) = (f \cdot g)(x), \text{ and } \frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x).$$

Algebra of Functions

The notation for addition, subtraction, multiplication, and division of two functions f(x) and g(x) are, respectively,

1. $(f + g)(x) = f(x) + g(x)$
2. $(f - g)(x) = f(x) - g(x)$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example 1.4.8 Algebra of Functions

Suppose $f(x) = x^3 - 7x^2 + 6x + 3$ *and* $g(x) = 2x^2 - 3x$. *Determine*

a) $(f+g)(x)$

b) $(f-g)(x)$

c) $(f \cdot g)(x)$

d) $\left(\frac{f}{g}\right)(x)$

Solution:

a) $(f+g)(x) = (x^3 - 7x^2 + 6x + 3) + (2x^2 - 3x) = x^3 - 5x^2 + 3x + 3.$

b) $(f-g)(x) = (x^3 - 7x^2 + 6x + 3) - (2x^2 - 3x) = x^3 - 9x^2 + 9x + 3.$

c) $(f \cdot g)(x) = (x^3 - 7x^2 + 6x + 3)(2x^2 - 3x) = 2x^5 - 17x^4 + 33x^3 - 12x^2 - 9x.$

d) $\left(\frac{f}{g}\right)(x) = \dfrac{x^3 - 7x^2 + 6x + 3}{2x^2 - 3x}.$

Composite functions result from one function being the variable for another. A composite function is denoted by $(f \circ g)(x) = f(g(x))$. In other words, the notation means that $g(x)$ is substituted for x in the function $f(x)$. Likewise, $(g \circ f)(x) = g(f(x))$. In other words, substitute $f(x)$ for x in the function $g(x)$. Take care not to confuse the composite operation with the multiplication operation.

Example 1.4.9 Composite Functions

Suppose $f(x) = x^5 - 3x^3 + 4x + 1$ *and* $g(x) = 5x^3 - 7x + 8$. *Determine* $(f \circ g)(x)$ *and* $(g \circ f)(x)$

Solution:

$$\begin{aligned}(f \circ g)(x) &= f(5x^3 - 7x + 8)\\ &= (5x^3 - 7x + 8)^5 - 3(5x^3 - 7x + 8)^3 + 4(5x^3 - 7x + 8) + 1.\\ (g \circ f)(x) &= g(x^5 - 3x^3 + 4x + 1)\\ &= 5\,(x^5 - 3x^3 + 4x + 1)^3 - 7(x^5 - 3x^3 + 4x + 1) + 8.\end{aligned}$$

EXERCISES 1.4

In Exercises 1–6 identify intervals on a number line.

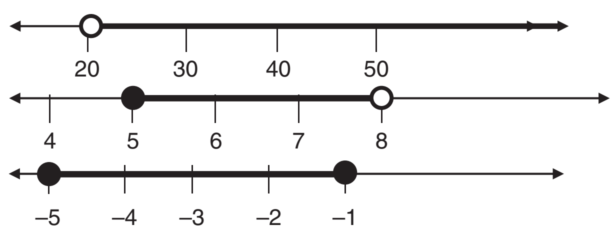
1. $(3, 7)$ 2. $[-3, 6]$ 3. $[5, \infty)$ 4. $(-\infty, 10]$ 5. $(-2, 1]$ 6. $[-5, 9)$

In Exercises 7–11 use interval notation to describe the inequalities.

7. $x > 4$ 8. $x \leq -3$ 9. $-3 < x < 7$ 10. $x \geq -5$ 11. $1 \leq x < 8$

In Exercise 12–14 use interval notation to describe the inequalities depicted on the number line.

12.
13.
14.



In Exercises 15–20 determine domains for the given functions.

15. $f(x) = 2x^4 - 3x^2 + 7$

16. $f(x) = \sqrt[5]{x - 3}$

17. $f(x) = \sqrt{2x - 5}$

18. $f(x) = 5x^3 - 2x + 1$

19. $f(x) = \dfrac{3x + 5}{(x - 1)(x + 3)}$

20. $f(x) = \dfrac{5}{\sqrt[3]{x - 4}}$

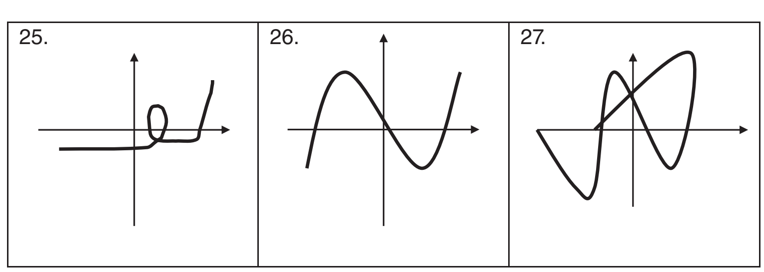
21. If $f(x) = 7x^3 + 5x + 3$ determine $f(0)$, $f(1)$, and $f(x + 3)$.

22. If $f(x) = 4x^2 + 3x + 2$ determine $f(-1)$, $f(2)$, and f(a).

23. If $f(x) = x^5 + 11$ determine $f(-1)$, $f(a^2)$, and $f(x + h)$.

24. If $f(x) = x^2 + 6x + 8$ determine $f(a)$, $f(x + h)$, and $f(x) + h$.

In Exercises 25–27 determine whether or not the graph is that of a function.



In Exercises 28–33 graph the indicated functions

28. $f(x) = 3x - 9$

29. $f(x) = x^2 - 4$

30. $f(x) = x^2 - 3x + 2$

31. $f(x) = x^3 - 8$

32. $f(x) = \frac{2}{x}$

33. $f(x) = \begin{cases} 2x & x < -1 \\ |x| & -1 \leq x < 3 \\ 5 & x \geq 3 \end{cases}$

34. Let $f(x) = 5x^4 - 3x^2 + 2$ and $g(x) = 3x^3 + 7x^2 + 2x$. Determine the following:

a) $(f + g)(x)$ b) $(f - g)(x)$ c) $\left(\frac{f}{g}\right)(x)$ d) $(f \circ g)(x)$

35. Let $f(x) = 4x^5 - 2x^3 + 2x$ and $g(x) = 3x^5 + 7x^3 + 8$. Determine the following:

a) $(g - f)(x)$ b) $(f - g)(x)$ c) $(f \cdot g)(x)$ d) $(g \circ f)(x)$

36. Let $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{1}{x-3}$. Determine the following:

a) $(f + g)(x)$ b) $(f \cdot g)(x)$ c) $(f \circ g)(x)$ d) $(g \circ f)(x)$

37. Let $f(x) = 2x^5$ and $g(x) = x^2 + 4$. Determine the following:

a) $f(x) + h$ b) $g(x + h)$ c) $f(a)g(a)$ d) $f(x + 1)g(x + 2)$

1.5 LAWS OF EXPONENTS

For any b ($\neq 0$), and any positive integer, x, we define

$$b^x = b \cdot b \cdots b$$

x times $x = 1, 2, \ldots$

That is, b^x is b multiplied by itself x times. For example, $2^6 = (2)(2)(2)(2)(2)(2) = 64$. Some laws of exponents are summarized below and most are most likely to be familiar.

Laws of Exponents

1. $b^0 = 1$

2. $b^{-x} = \frac{1}{b^x}$

3. $b^x b^y = b^{x+y}$

4. $\frac{b^x}{b^y} = b^{x-y}$

5. $(b^x)^y = b^{xy}$

6. $(ab)^x = a^x b^x$

7. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

8. $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x = \frac{b^x}{a^x}$

In general, positive exponents are preferred to simplify expressions.

Example 1.5.1 Using Exponent Laws

Use laws of exponents to calculate

a) $3^2 3^3$

b) 7^0

c) $\frac{5^4}{5^3}$

d) $\frac{1}{2^{-5}}$.

Solution:

a) *Here, exponents are added to yield* $3^5 = 243$.
b) *Any nonzero number with an exponent of zero is 1.*
c) *Here, the exponents are subtracted to yield* $5^{4-3} = 5^1 = 5$.
d) *The expression is rewritten as* $2^5 = 32$ *to use a positive exponent.*

Example 1.5.2 Using Exponent Laws (revisited)

Use laws of exponents to calculate

a) $x^3 x^7$

b) x^{-2}

c) $(x^3)^4$

d) $\frac{x^5 x^2}{x^{-4}}$

e) $\left(\frac{x^3}{y^5}\right)^{-3}$.

Solution:

a) *Here, add exponents to yield* x^{10}.
b) *Using positive exponents this expression is rewritten as* $1/x^2$.
c) *Here, multiply exponents to yield* x^{12}.
d) *Here, too laws are used.* $\frac{x^5 x^2}{x^{-4}} = \frac{x^7}{x^{-4}} = x^7 x^4 = x^{11}$.
e) *Here it is easier to invert and separate the fraction as*

$$\left(\frac{y^5}{x^3}\right)^3 = \frac{(y^5)^3}{(x^3)^3} = \frac{y^{15}}{x^9}.$$

Fractional exponents are sometimes written in radical format. For instance, $4^{1/2} = \sqrt{4} = 2$ and $125^{1/3} = \sqrt[3]{125} = 5$. The previous laws of exponents apply to any real numbered exponent; not only integer valued ones.

Additional Laws of Exponents

$$\sqrt[x]{b} = b^{1/x} \qquad \sqrt[x]{b^y} = b^{y/x}$$

In numerical expressions it is often more useful to write $\sqrt[x]{b^y}$ *as* $(b^{1/x})^y$. In other words, first the x^{th} root reduces the number b, then raises it to the y^{th} power. For instance, to evaluate $16^{3/4}$, the fourth root of 16 is 2. Next, cube 2 to get 8. This is easier (without a calculator) than cubing 16 and seeking its fourth root.

In evaluating $-16^{3/2}$ first take the square root of 16, which is 4, and its cube is 64. Finally, use the negative sign for the result, −64. This contrasts with evaluating $(-16)^{3/2}$ where the square root of −16 is a complex number and beyond our scope.

The next Example continues rules of exponents with fractional exponents and numerical coefficients.

Example 1.5.3 Using Fractional Exponents

Simplify the following expressions:

a) $\left(\dfrac{27x^6}{8y^3}\right)^{2/3}$ *b)* $\sqrt[3]{x^2}(5x^{1/3} + x^{7/3})$ *c)* $\left(\dfrac{25x^{5/2}y^{9/2}}{x^{-7/2}y^{1/2}}\right)^{3/2}$

Solution:

a) First, evaluate the numerical coefficients and then the individual variables. Here,

$$\left(\frac{27}{8}\right)^{2/3} = \left(\frac{3}{2}\right)^2 = \frac{9}{4},\ (x^6)^{2/3} = x^4,\ \text{and}\ (y^3)^{2/3} = y^2.$$

Therefore, the expression simplifies to $\dfrac{9x^4}{4y^2}$.

b) First, rewrite $\sqrt[3]{x^2}$ *as* $x^{2/3}$ *to yield* $x^{2/3}(5x^{1/3} + x^{7/3})$. *Simplify using the distributive law as* $5x + x^3$.

c) First, simplify the expression within the parenthesis. Here, the expression becomes $(25x^6y^4)^{3/2} = 125x^9y^6$.

EXERCISES 1.5

In Exercises 1–12 compute numerical values.

1. $1^{3/7}$
2. $(-1)^{4/5}$
3. $25^{3/2}$
4. $-36^{1/2}$
5. $64^{5/6}$
6. $(-27)^{4/3}$
7. $(2/3)^{-2}$
8. $(1.75)^0$
9. $(0.008)^{1/3}$
10. $7^{2/3}7^{4/3}$
11. $\dfrac{15^3}{5^3}$
12. $(25^{3/4})^{2/3}$

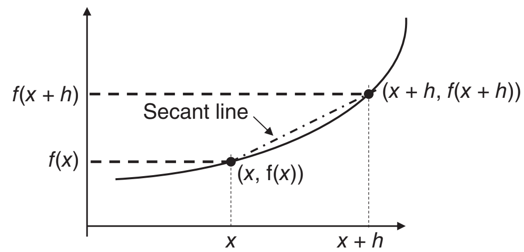
In Exercises 13–30 use the laws of exponents to simplify the expressions.

13. x^3x^5
14. $x^{2/5}x^{13/5}$
15. $(2xy)^3$
16. $(-4xy)^2$
17. $\dfrac{x^3x^5}{x^{-4}}$
18. $\dfrac{x^8x^{-5}}{x^4}$
19. $\dfrac{x^4y^5}{x^2y^{-2}}$
20. $\left(\dfrac{x^2x^5}{y^{-3}}\right)^2$
21. $\left(\dfrac{2x^3}{y^2}\right)^2$
22. $\left(\dfrac{3x^4}{y^3}\right)^4$
23. $\sqrt[3]{x^5}\sqrt[3]{x^4}$
24. $\sqrt[4]{x^6y^3}\sqrt[4]{x^2y^9}$
25. $(81x^4y^8)^{1/4}$
26. $(-27x^{-3}y^{-6})^{-1/3}$
27. $\dfrac{(16x^4y^5)^{3/2}}{\sqrt{y}}$
28. $\dfrac{(-8x^5y^6)^{2/3}}{\sqrt[3]{x}}$
29. $\dfrac{(8x^5y^7)^{2/3}}{\sqrt[3]{xy^2}}$
30. $\dfrac{(64x^2y^3)^{-2/3}}{x^{5/3}y^{-6}}$

1.6 SLOPES AND RELATIVE CHANGE

A **secant line** joins two points on the curve $y = f(x)$. Its slope, “the rise over the run” is

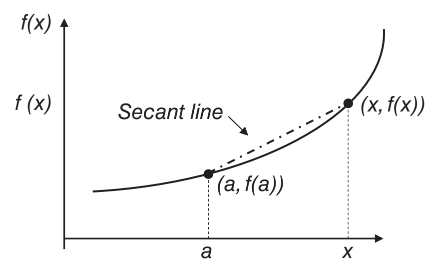
$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}, \; h \neq 0$$



This relative change in f(x) with a change, h, in x is a **difference quotient**. It has a basic role in differential calculus.

Example 1.6.1 Another Difference Quotient

Express the slope of the secant line as a difference quotient.



Solution:

The slope of this secant line is:

$$\frac{f(x) - f(a)}{x - a}.$$

Example 1.6.2 Difference Quotients

When $f(x) = x^2 + 2x + 6$ *determine*

a) $\frac{f(x+h) - f(x)}{h}$ *b)* $\frac{f(x) - f(a)}{x - a}.$

Solution:

a) First, write $f(x + h)$ *as* $(x + h)^2 + 2(x + h) + 6 = x^2 + 2xh + h^2 + 2x + 2h + 6$*. Note that* $(x + h)$ *has been substituted for* x*.*

→

Next, the numerator is obtained by subtracting f(x), as

$$f(x+h) - f(x) = (x^2 + 2xh + h^2 + 2x + 2h + 6) - (x^2 + 2x + 6)$$
$$= 2xh + h^2 + 2h$$

Forming the difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{h(2x + h + 2)}{h} = 2x + h + 2.$$

(The denominator, h, is a factor of the numerator).

b) *Again, in sequence,*

$$f(a) = a^2 + 2a + 6 \text{ so } f(x) - f(a) = (x^2 + 2x + 6) - (a^2 + 2a + 6)$$
$$= x^2 + 2x - a^2 - 2a.$$

Since the numerator has $(x - a)$ *as a factor, regroup by exponent as*

$$\frac{f(x) - f(a)}{x - a} = \frac{(x^2 - a^2) + (2x - 2a)}{x - a} = \frac{(x - a)(x + a) + 2(x - a)}{x - a}$$
$$= \frac{(x - a)[(x + a) + 2]}{x - a}$$
$$= x + a + 2.$$

↓ Do not confuse $f(x + h)$ and $f(x) + h$. The latter adds h to $f(x)$ while the former replaces x by $x + h$.

↑ The denominator of a difference quotient is a factor of the numerator so the difference quotient can be simplified.

EXERCISES 1.6

1. If $f(x) = 6x + 11$ determine

   a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

2. If $f(x) = 9x + 3$ determine

   a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

3. If $f(x) = 7x - 4$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

4. If $f(x) = 10x + 1$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

5. If $\mathrm{f(x)} = \mathrm{x}^2 - 7\mathrm{x} + 4$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

6. If $f(x) = x^2 + 3x + 9$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

7. If $f(x) = x^2 + 6x - 8$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

8. If $f(x) = 3x^2 - 5x + 2$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

9. If $f(x) = 5x^2 - 2x - 3$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

10. If $f(x) = x^3 - 1$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

11. If $f(x) = x^3 - 4x + 5$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

12. If $f(x) = x^3 + 3x + 9$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

13. If $f(x) = 2x^3 - 7x + 3$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

14. If $f(x) = 1/x^2$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

15. If $f(x) = \dfrac{3}{x^3}$ determine

a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$.

HISTORICAL NOTES

According to the Oxford Dictionary of the English Language (OED), the word **Algebra** became an accepted form among many similar ones in the seventeenth century. It is believed to have origins in early Arabic language from root words as "reunion of broken parts" and "to calculate." While the word "algebra" has contexts other than this chapter as, for example, in the surgical treatment of fractures and bone setting, and as the branch of mathematics which investigates the relations and properties of numbers by means of general symbols; and, in a more abstract sense, a calculus of symbols combined according to certain defined laws. Algebra is a principal branch of mathematics.

René Descartes (1596–1650) – Descartes was born in France in 1596, the son of an aristocrat. He traveled in Europe studying a variety of subjects including mathematics, science, law, medicine, religion, and philosophy. He was influenced by other thinkers of The Enlightenment.

Descartes ranks as an important and influential thinker. Some refer to him as a founder of modern philosophy as well as an outstanding mathematician. Descartes founded analytic geometry and sought simple universal laws governing all physical change. He originated Cartesian coordinates and contributed to the treatment of negative roots and the convention of exponent notation.

CHAPTER 1 SUPPLEMENTARY EXERCISES

1. Solve $9(x - 3) + 2x = 3(x + 1) - 2$.
2. Solve $\dfrac{3x + 1}{4} + \dfrac{1}{2} = x$.
3. Solve $Z = \dfrac{x - \mu}{\sigma}$ for μ.
4. Determine three consecutive odd integers whose sum is 51.
5. Graph $3x + 5y = 15$ by using the intercepts as two of the points.
6. Graph $2x - 3y = 6$.

7. Determine the equation of the line through (5, 7) and (2, 1).
8. Are the lines $5x + 2y = 7$ and $y = (2/5)\,x + 1$ parallel, perpendicular, or neither?
9. Determine the equation of the line through (2, 5) parallel to $4x - 3y = 12$.
10. Factor $6x^3y^5 - 216xy^3$ completely.
11. Factor $2x^3 - 18x^2 - 20x$ completely.
12. Solve $6x^2 - 7x - 5 = 0$ by using a) factoring b) quadratic formula.
13. Factor $2ax - 2ay - by + bx$.
14. Solve $3x^2 + 10 = 13x$ by using the quadratic formula.
15. Solve $2/3(x - 1) < x - 2$ and write the solution in interval notation.
16. Solve $2(x - 3) + 5(x + 3) \geq 2x + 19$.
17. Determine the domain of $\dfrac{2x + 5}{x^3 + 9x^2 + 8x}$.
18. Graph $y = x^2 + 4x - 12$ and make sure to determine the intercepts as well.
19. If $f(x) = x^2 + 3x + 1$ and $g(x) = x^3 - 3x^2 - 4$ find

    a) (f − g)(x) b) (f · g)(x) c) $(f \circ g)(x)$.

20. If $f(x) = 5x^3 + 7x + 4$ and $g(x) = 2x^3 - 3x^2 - 4$ find

    a) $(f + g)(x)$ b) $\left(\dfrac{f}{g}\right)(x)$ c) $(g \circ f)(x)$.

21. Simplify $\left(\dfrac{-2x^5}{3x^2}\right) \cdot \left(\dfrac{3x^{-2}}{4x^5}\right)$.
22. Simplify $\left(\dfrac{625x^{7/2}y^{13}}{x^{-9/2}y^{-3}}\right)^{1/4}$.
23. Simplify $\left(\dfrac{2x^3}{3y^{-2}z^4}\right)^{-3}$.
24. $f(x) = x^2 + 11x + 2$ find

    a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$

25. If $f(x) = x^3 + 3x + 1$ find

    a) $\dfrac{f(x+h) - f(x)}{h}$ b) $\dfrac{f(x) - f(a)}{x - a}$