

## CHAPTER 1

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# LINEAR EQUATIONS AND MATHEMATICAL CONCEPTS

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### EXERCISES 1.1

1.  $3x + 1 = 4x - 5$   
 $1 = x - 5$   
 $x = 6$  conditional equation
3.  $5(x + 1) + 2(x - 1) = 7x + 6$   
 $5x + 5 + 2x - 2 = 7x + 6$   
 $7x + 3 = 7x + 6$  contradiction
5.  $4(x + 3) = 2(2x + 5)$   
 $4x + 12 = 4x + 10$  contradiction

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7.  $5x - 3 = 17$

$$5x = 20$$

$$x = 4$$

9.  $2x = 4x - 10$

$$2x - 4x = -10$$

$$-2x = -10$$

$$x = 5$$

11.  $4x - 5 = 6x - 7$

$$-5 + 7 = 6x - 4x$$

$$2 = 2x$$

$$1 = x$$

13.  $0.6x = 30$

$$x = 30/0.60 = 50$$

15.  $2/3 = (4/5)x - (1/3)$  multiply by 15 to eliminate fractions

$$1(2/3) = 15 \{(4/5)x - (1/3)\}$$

$$10 = 12x - 5$$

$$15 = 12x$$

$$5/4 = x$$

17.  $5(x - 4) = 2x + 3(x - 7)$

$$5x - 20 = 2x + 3x - 21$$

$$5x - 20 = 5x - 21$$

No solution

19.  $3s - 4 = 2s + 6$

$$s - 4 = 6$$

$$s = 10$$

$$21. \quad 7t + 2 = 4t + 11$$

$$7t - 4t = 11 - 2$$

$$3t = 9$$

$$t = 3$$

$$23. \quad 4(x + 1) + 2(x - 3) = 7(x - 1)$$

$$4x + 4 + 2x - 6 = 7x - 7$$

$$6x - 2 = 7x - 7$$

$$6x - 7x = -7 + 2$$

$$x = -5$$

$$x = 5$$

$$25. \quad \frac{x + 8}{2x - 5} = 2 \text{ multiply by } 2x - 5 \text{ to eliminate the fraction}$$

$$(x + 8) = 2(2x - 5)$$

$$x + 8 = 4x - 10$$

$$8 + 10 = 4x - x$$

$$18 = 3x$$

$$6 = x$$

(Check the result. Multiplication by a factor such as  $2x - 5$  can introduce an extraneous solution.)

$$27. \quad 8 - \{4[x - (3x - 4) - x] + 4\} = 38 - \{4[x - (3x - 4) - x] + 4\}$$

$$= 3(x + 2)$$

$$8 - \{4[x - 3x + 4 - x] + 4\} = 3x + 6$$

$$8 - \{4[-3x + 4] + 4\} = 3x + 6$$

$$8 - \{-12x + 16 + 4\} = 3x + 6$$

$$8 - \{-12x + 20\} = 3x + 6$$

$$8 + 12x - 20 = 3x + 6$$

$$12x - 12 = 3x + 6$$

$$9x = 18$$

$$x = 2$$

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29.  $6x - 3y = 9$  for  $x$

$$6x = 3y + 9$$

$$x = \frac{3y + 9}{6} = \frac{1}{2}y + \frac{3}{2}$$

31.  $3x + 5y = 15$

$$5y = 15 - 3x$$

$$y = \frac{(15 - 3x)}{5}$$

$$y = 3 - \left(\frac{3}{5}\right)x$$

33.  $V = LWH$

$$\frac{V}{LH} = W$$

35.  $Z = \frac{(x - \mu)}{\sigma}$

$$Z\sigma = x - \mu$$

$$x = Z\sigma + \mu$$

37. Let  $x =$  monthly installment (\$).

Since Sally paid \$300 down, she owes  $\$1300 - \$300 = \$1000$ .

Therefore,  $5x = 1000$  or  $x = \$200$  is the monthly installment.

39. The consumption function is  $C(x) = mx + b$ . The slope is the "marginal propensity to consume." Therefore,  $C(x) = 0.75x + b$ . The disposable income,  $x = 2$ , when consumption is  $y = 11$  yields  $11 = (0.75)(2) + b$  and  $b = 9.5$ . The consumption function is  $C(x) = 0.75x + 9.5$ .

41. a)  $d = 4.5(2) = 9$  miles

b)  $18 = 4.5t$  and  $t = 18/4.5 = 4$  seconds

43. The tax is 6.2% or 0.062 in decimal form, so  $T = 0.062x$ , where  $x$  is  $0 \leq x \leq 87,000$ .

45. a)  $BSA = 1321 + (0.3433)(20,000) = 8187 \text{ cm}^2$

b)  $10,325 = 1321 + (0.3433)(Wt)$

$$9004 = (0.3433)(Wt)$$

$$9004/0.3433 = 26,228 \text{ g} = 26.2 \text{ kg}$$

## EXERCISES 1.2

1. Setting  $y = 0$  determines the  $x$ -intercept and setting  $x = 0$  determines the  $y$ -intercept.

- a)  $5x - 3y = 15$   $x$ -intercept 3,  $y$ -intercept  $-5$   
 b)  $y = 4x - 5$   $x$ -intercept  $5/4$ ,  $y$ -intercept  $-5$   
 c)  $2x + 3y = 24$   $x$ -intercept 12,  $y$ -intercept 8  
 d)  $9x - y = 18$   $x$ -intercept 2,  $y$ -intercept  $-18$   
 e)  $x = 4$   $x$ -intercept 4, no  $y$ -intercept (vertical line)  
 f)  $y = -2$  no  $x$ -intercept (horizontal line),  $y$ -intercept  $-2$

3. The slope is  $m = \frac{y_2 - y_1}{x_2 - x_1}$

a)  $(3, 6)$  and  $(-1, 4)$   $m = \frac{4 - 6}{-1 - 3} = \frac{-2}{-4} = \frac{1}{2}$

b)  $(1, 6)$  and  $(2, 11)$   $m = \frac{11 - 6}{2 - 1} = \frac{5}{1} = 5$

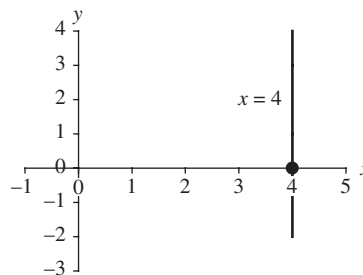
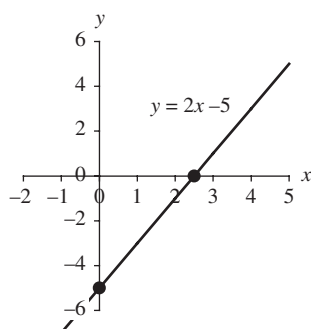
c)  $(6, 3)$  and  $(12, 7)$   $m = \frac{7 - 3}{12 - 6} = \frac{4}{6} = \frac{2}{3}$

d)  $(2, 3)$  and  $(2, 7)$   $m = \frac{7 - 3}{2 - 2} = \frac{4}{0}$  undefined

e)  $(2, 6)$  and  $(5, 6)$   $m = \frac{6 - 6}{5 - 2} = \frac{0}{3} = 0$

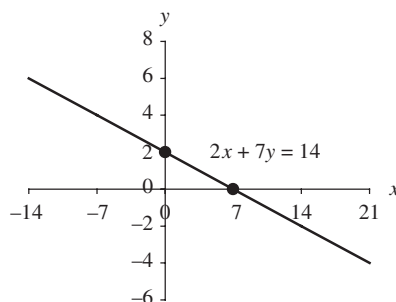
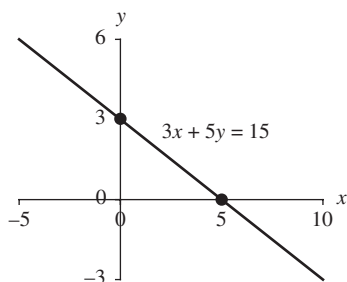
f)  $(5/3, 2/3)$  and  $(10/3, 1)$   $m = \frac{1 - 2/3}{10/3 - 5/3} = \frac{1/3}{5/3} = \frac{1}{5}$

5. a)  $x$ -intercept  $5/2$  and  $y$ -intercept  $-5$



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- c)  $x$ -intercept 5 and  $y$ -intercept 3    d)  $x$ -intercept 7 and  $y$ -intercept 2



7. a)  $y = (5/3)x + 2$  and  $5x - 3y = 10$ ; the slope of the first line is  $5/3$ . Solving for  $y$  in the second equation yields  $y = (5/3)x - (10/3)$ . This slope is also  $5/3$ . The slopes are both  $(5/3)$  so the lines are parallel (with different intercepts).
- b)  $6x + 2y = 4$  and  $y = (1/3)x + 1$ . The slope of the second line is easily determined (line in slope intercept form) as  $1/3$ . Again, solve for  $y$  in the first equation to determine  $y = -3x + 2$ . The slope is  $-3$ . The slopes are negative reciprocals; the lines are perpendicular.
- c)  $2x - 3y = 6$  and  $4x - 6y = 15$ . Solving for  $y$  in each equation, one determines that  $y = (2/3)x - 2$  and  $y = (2/3)x - (5/2)$ . These lines have the same slope (and different intercepts) making them parallel.
- d)  $y = 5x - 4$  and  $3x - y = 4$ . The slope of the first line is 5 and solving for  $y$  in the second equation, ( $y = 3x - 4$ ), the slope is 3. These slopes are neither the same nor negative reciprocals. They are neither parallel nor perpendicular.
- e)  $y = 5$  is a horizontal line while  $x = 3$  is a vertical line. The two lines are perpendicular.
9. A linear equation has a single  $x$ -intercept except for  $y = 0$  (the  $x$ -axis) with an infinite number of  $x$ -intercepts. Any horizontal line except  $y = 0$  has no  $x$ -intercepts. Generally, lines do not have more than one  $y$ -intercept. The exception is  $x = 0$  (the  $y$ -axis) with an infinite number of  $y$ -intercepts. Any vertical line with the exception of  $x = 0$  has no  $y$ -intercepts.

11. The ordered pairs of “time” and “machine value” are  $(0, 75,000)$  and  $(9, 21,000)$ , respectively. The slope is

$m = \frac{21,000 - 75,000}{9 - 0} = \frac{-54,000}{9} = -6000$ . The  $y$ -intercept is the purchase price, \$75,000. The equation to model the straight-line depreciation is  $V(t) = -6000t + 75,000$ , where  $V(t)$  is the machine value (\$) at time  $t$ .

13. The ordered pairs (gallons, miles) are  $(7, 245)$  and  $(12, 420)$ .

The slope is  $\frac{420 - 245}{12 - 7} = \frac{175}{5} = 35$  with  $x$  gallons and  $y$  miles.

Use either pair with the point slope-formula.

Therefore,  $y - 245 = 35(x - 7)$  or  $y = 35x$ .

15. Total cost reflects both fixed and variable costs. The fixed cost is monthly rent (\$1100). The variable cost is  $5x$ , where  $x$  is monthly production. Therefore, total cost is  $C(x) = 1100 + 5x$ .

17. a) Here, the fixed cost is \$50/day and variable cost \$0.30/mile. To rent the car for a single day costs \$50 to which the mileage cost must be added. The cost is  $C(x) = 50 + 0.30x$ .

- b) If a person has \$110 for rental, the equation to solve for the travel distance is  $110 = 50 + 0.30x$ .

Solving yields,

$$60 = 0.30x$$

$$\frac{60}{0.30} = x$$

$$200 = x$$

The person can rent the car and travel 200 miles with \$110.

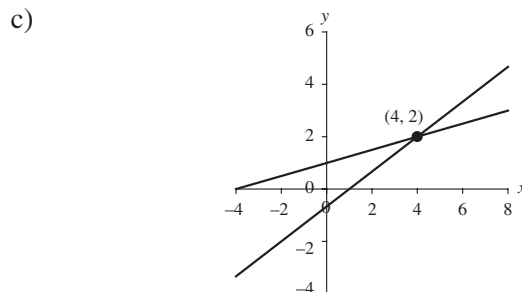
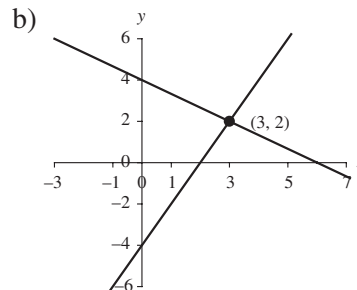
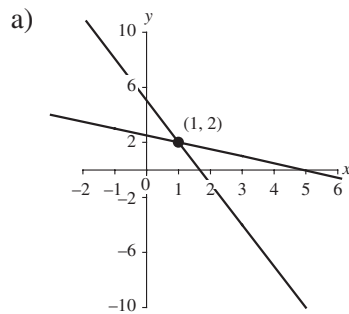
19. Since  $R$  is to be a function of  $C$ , the ordered pairs are  $(C, R)$ .

The two ordered pairs are  $(70, 84)$  and  $(40, 48)$ . The slope is  $\frac{48 - 84}{40 - 70} = \frac{36}{30} = \frac{6}{5}$ . Using either pair with the slope to yield  $R - 84 = (6/5)(C - 70)$  or  $R = (6/5)C$ .

## EXERCISES 1.3

1. The ordered pair must satisfy each equation to be a solution to the system.
  - a)  $2(3) + 1 = 7$  is true but  $3 + 1 = 5$  is not. Therefore,  $(3, 1)$  is not a solution to the system.
  - b)  $2(2) + 3 = 7$  is true and so is  $2 + 3 = 5$ . Therefore,  $(2, 3)$  is a solution to the system.
  - c)  $2(4) + 1 = 7$  is true but  $4 + (-1) = 5$  is not. Therefore,  $(4, -1)$  is not a solution to the system.
3.
  - a)  $y = (-1/3)x + (8/3)$  and  $y = -x + 6$ ,  $m_1 = -1/3$  and  $m_2 = -1$ . Since the slopes differ, this is a consistent system.
  - b)  $y = (-1/2)x + (7/2)$  and  $y = (-1/2)x + (7/2)$ . Since both the slopes and intercepts are the same, the two equations are the same line. It is a dependent system.
  - c)  $y = (-3/2)x + (7/2)$  and  $y = (-3/2)x + 5$ . Here, the slopes are the same and the intercepts differ. The lines are parallel and the system is inconsistent.

5. The graphs and solution to each system are:



7. a) Given  $x = 3$  and substituting in the second equation yields  $3 + 3y = 9$ ,  $3y = 6$ , and  $y = 2$ . The ordered pair solution is  $(3, 2)$ .
- b) Using,  $y - 2 = 0$ ,  $y = 2$ . Substituting into the second equation yields  $x + 3(2) = 9$ ,  $x + 6 = 9$ , and  $x = 3$ . The ordered pair solution is  $(3, 2)$ .
- c) The second equation, already solved for  $y$ , substituted in the first equation yields  $x + (-x + 3) = 5$  or  $3 = 5$ . This is false, so the system is inconsistent and has no solution.
9. a) Here,  $x$  can be eliminated by simply adding the two equations as written.

$$\begin{array}{r} -x + 2y = 5 \\ x + y = 4 \\ \hline 3y = 9 \text{ or } y = 3 \end{array}$$

Next, use  $y = 3$  to determine  $x = 1$ . The ordered pair solution is  $(1, 3)$ .

- b) Here,  $y$  can be eliminated by multiplying the second equation by 3. The system is rewritten as

$$\begin{array}{r} 4x + 3y = 35 \\ 6x - 3y = 15 \end{array}$$

Adding the two equations yields  $10x = 50$  or  $x = 5$ . Using this value for  $x$  yields that  $y$  is also 5. (Check the solution in the original equation to see that  $(5, 5)$  is correct.)

- c) The second equation must first be rewritten in standard form  $ax + by = c$ , so the system to solve is

$$\begin{array}{r} x + 4y = 13 \\ 4x + 2y = 10 \end{array}$$

Multiplying the first equation by  $-4$  will allow  $x$  to be eliminated from the system.

$$\begin{array}{r} -4x - 16y = -52 \\ 4x + 2y = 10 \\ \hline -14y = -42 \text{ or } y = 3 \end{array}$$

Using  $y = 3$ , it is determined that  $x = 1$ . (Check the solution in the original system to verify the solution  $(1, 3)$ ).



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- d) Rewriting the first equation in standard form and multiplying by 2 to eliminate  $x$  yields

$$\begin{array}{r} -2x + 2y = 8 \\ 2x + 3y = 12 \\ \hline 5y = 20 \text{ or } y = 4 \end{array}$$

If  $y = 4$ , then  $x = 0$  and checking in the original system indicates that  $(0, 4)$  is the correct solution.

11. Let  $x$  represent the number of boxes of cookies and  $y$  the boxes of candy. The system to be solved is

$$x + y = 2400$$

$$4x + 5y = 10,500$$

Solving yields sales of 1500 boxes of cookies and 900 boxes of candy.

13. Let  $x =$  gallons of regular and  $y =$  gallons premium. The system of equations to be solved (by either substitution or elimination) is

$$x + y = 100$$

$$0.87x + 0.93y = 0.92 \quad (100)$$

Using elimination,

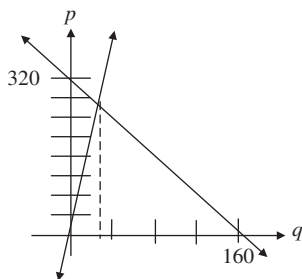
$$-0.87x - 0.87y = -87$$

$$0.87x + 0.93y = 92$$

$$\hline 0.06y = 5$$

$$y = 83 \frac{1}{3} \text{ gallons and } x = 16 \frac{2}{3} \text{ gallons}$$

15.



The Market equilibrium occurs when  $q$  is about 30 and  $y$  about 260. Using substitution

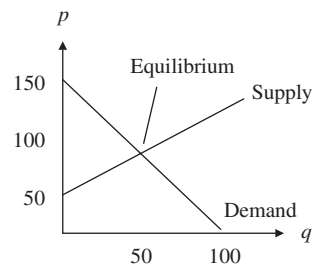
$$-2q + 320 = 8q + 20$$

$$300 = 10q$$

$$30 = q$$

$$\text{so, } p = 260$$

17.



The price is about \$75 and quantity is 50 at market equilibrium.

## EXERCISES 1.4

1. The area of a circle is  $\pi r^2$ , so,

$$\pi r^2 = \pi$$

$$r^2 = 1$$

$$r = 1$$

The radius is 1 unit, so the diameter is two units.

3. a) Pi day is March 14 (3/14)

b) Professor Yasumasa Kanada of the University of Tokyo.

c) Albert Einstein was born on March 14.

d) It is an illness thought to be attached to trying to square a circle.

e) Pi is known as die Ludolphschezahl in Germany.

5. a)  $\left(1 + \frac{1}{5}\right)^5 = 2.48832$       b)  $\left(1 + \frac{1}{10}\right)^{10} = 2.59374$

c)  $\left(1 + \frac{1}{25}\right)^{25} = 2.66584$

## EXERCISES 1.5

1. a) The expression must involve powers of 2 or 3. In this case, 8 is a power of 2, so one has  $(8)^{3x} = (2^3)^{3x} = 2^{9x}$ .

b) Rewrite as  $(27)^{2x} = (3^3)^{2x} = 3^{6x}$ .

c) Rewrite as  $(16)^{5x} = (2^4)^{5x} = 2^{20x}$ .

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3. a)  $\left(\frac{1}{8}\right)^{-4x} = \left(\frac{1}{2^3}\right)^{-4x} = (2^{-3})^{-4x} = (2)^{12x}$

b)  $\left(\frac{1}{9}\right)^{6x} = \left(\frac{1}{3^2}\right)^{6x} = (3^{-2})^{6x} = (3)^{-12x}$

c)  $\left(\frac{1}{27}\right)^{-2x} = \left(\frac{1}{3^3}\right)^{-2x} = (3^{-3})^{-2x} = (3)^{6x}$

5. a)  $\frac{10^{5x}}{5^{5x}} = (2)^{5x}$

b)  $\frac{32^{2x}}{16^{2x}} = (2)^{2x}$

c)  $\frac{4^{3x}}{12^{3x}} = \left(\frac{1}{3}\right)^{3x} = (3^{-1})^{3x} = (3)^{-3x}$

7.  $\frac{7x^3x^5y^{-2}}{x^2y^4} = \frac{7x^8}{x^2y^4y^2} = \frac{7x^6}{y^6}$

9.  $\frac{x^3}{y^{-2}} \div \frac{x}{y^5} = (x^3y^2) \cdot \left(\frac{y^5}{x}\right) = x^2y^7$

11.  $\frac{2^{5x+3}4^{x+1}}{8(2^{3x-1})} = \frac{2^{5x+3}(2^2)^{x+1}}{2^3(2^{3x-1})} = \frac{2^{5x+3}2^{2x+2}}{2^{3x+2}} = 2^{4x+3}$

13. Since the bases are the same, exponents are equal.

Therefore,  $3x = 15$  yields  $x = 5$ .

15. Equate bases as  $2^{7-x} = 2^5$  to yield  $7 - x = 5$  and  $x = 2$ .

17. Factoring yields  $5^x[(1 + x) + (3 - 2x)] = 0$

$$5^x[(4 - x)] = 0$$

Since  $5^x$  cannot be zero,  $4 - x = 0$  and  $x = 4$ .

19. Factoring yields  $7^x[(x^2 + 4x) + (x + 6)] = 0$ . Simplifying yields

$$7^x[x^2 + 5x + 6] = 0. \text{ Therefore, } [x^2 + 5x + 6] = (x + 3)(x + 2) = 0.$$

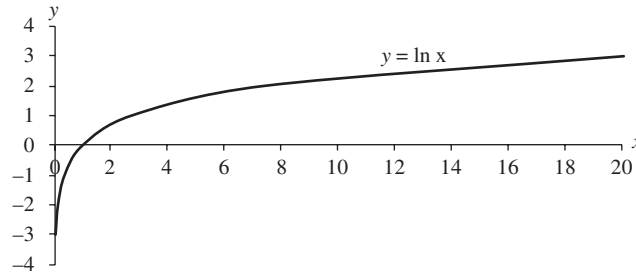
The solutions are  $x = -2$  or  $x = -3$ .

21.  $2^{3+h} = 2^h(2^3)$

23.  $7^{x+5} - 7^{2x} = 7^{2x}(7^{x+5} - 1)$

25. This is a difference of cubes  $(7^h)^3 - 8 = (7^h - 2)[7^{2h} + 2(7^h) + 4]$

27. Rewrite as  $e^y = x$  to determine ordered pairs and graph as



29.  $\log_3 243 = x$  so  $3^x = 3^5$ , which yields  $x = 5$ .

31.  $\log_5 125 = x$  so  $5^x = 125$ , which yields  $x = 3$ .

33.  $\log_3 \frac{1}{81} = x$  so  $3^x = 3^{-4}$ , which yields  $x = -4$

35. Given  $\ln e^x = x$ , then  $\ln e^7 = 7$ .

37. Given  $\ln e^x = x$ , then  $\ln e^{-3.4} = -3.4$ .

39.  $e^{\ln 1} = e^0 = 1$

41.  $\log_{25} 125 = x$  so  $25^x = 125$ , which yields  $5^{2x} = 5^3$ .  
Therefore,  $2x = 3$  or  $x = 3/2$ .

43.  $\log_5 625 = x$  so  $5^x = 625 = 5^4$ . Therefore,  $x = 4$ .

45.  $\log_4(1/64) = x$  so  $4^x = 4^{-3}$ . Therefore,  $x = -3$ .

47. Rewriting  $\log_x 64 = 3$  in exponential form yields  $x^3 = 64$  and  $x = 4$ .

49. Rewriting  $\log_2(x^2 + 7x) = 3$  yields  $2^3 = x^2 + 7x$ . Solving  
 $x^2 + 7x - 8 = 0$  yields  $(x + 8)(x - 1) = 0$  and  $x = 1$  or  $x = -8$ .

51. Rewrite the equation as  $e^{5x} = 2$ . Taking natural logarithms of both  
sides yields  $5x = \ln 2$  or  $x = (1/5) \ln 2$ .

53. In exponential form  $e^{1/3} = 7 - x$ . Solving for  $x$  yields  $x = 7 - e^{1/3}$ .

55.  $\ln \frac{(x-1)^4}{(2x+3)^5(x-4)^2} = \ln(x-1)^4 - \ln(2x+3)^5 - \ln(x-4)^2$   
 $= 4 \ln(x-1) - 5 \ln(2x+3) - 2 \ln(x-4)$

57. Using the rules for logarithms yields  $\ln \frac{2(7)}{3} = \ln \frac{14}{3}$ .

59. Using logarithms,  $10^{3x-1} = 100,100$  is written as  
 $\log_{10} 10^{3x-1} = \log_{10} 100,100$ . This yields  $3x - 1 = \log_{10} 100,100$ .  
 Therefore,  $x = \frac{1 + \log_{10} 100,100}{3}$ .
61. Taking natural logarithms on both sides of the equation yields  
 $\ln e^{3x+1} = \ln 22$ . Therefore,  $3x + 1 = \ln 22$  and  $x = \frac{-1 + \ln 22}{3}$ .

**EXERCISES 1.6**

1. a) Direct variation. The greater the speed, the farther the distance traveled.  
 b) Inverse variation. The greater the speed, the less time to complete the race.  
 c) Direct variation. The larger the income, the higher the tax rate.  
 d) Direct variation. The larger the area of a circle, the larger its diameter.  
 e) Direct variation. The larger the distance on the blue print, the larger the actual distance.
3. Here,  $y = kr^3$ . Use this to determine  $k$ .  
 Therefore,  $32 = k(4)^3$  yields  $1/2$  for  $k$ . When  $r = 6$ ,  
 $y = (1/2)(6)^3 = (1/2)(216) = 108$ .
5. Here,  $y = kx$ . Use this to determine  $k$ .  
 Therefore,  $15 = k(10)$  yields  $3/2$  for  $k$ . When  $y = 22.5$ ,  
 $x = (22.5)/(3/2) = (45/2)(2/3) = 15$ .
7. Since  $x$  varies directly as  $y$ , let  $x = ky$ . Values appear in the table.

$x$	2	4	8
$y$	12.5	25	50

Therefore,  $2 = k(12.5)$  yields  $k = 0.16$ . The direct relationship is  $x = (0.16)y$ . As a check,  $4 = 0.16(25)$  and  $8 = (0.16)50$  are true.

9. The area of a circle with diameter  $d$  is  $A = kd^2$ . Here,  
 $36\pi = k(12)^2 = 144k$  and  $k = \pi/4$ . The area of a circle is  $A = \pi d^2 / 4$ .

11. Here,  $V = \frac{1}{3}\pi r^2 h$ . Using the data  

$$V = \frac{1}{3}\pi(3)^2(10) = 30\pi \text{ cm}^3.$$
13. Here, Hooke's Law  $y = kx$ ,  $400 = k(10)$  and  $k = 40$ . When the force is 250 Newtons,  $250 = (40)x$  and  $x = 250/40 = 6.25$  cm.
15. Let  $x$  represent the length of the shorter leg and  $y$  the length of the hypotenuse. To determine  $x$ , use  $5/x = 12/24$  and to determine  $y$ , use  $13/y = 12/24$ . Solving yields

$$\begin{array}{rcl} \frac{5}{x} = \frac{12}{24} & & \frac{13}{y} = \frac{12}{24} \\ 12x = 120 & & 12y = 312 \\ x = 10 & & y = 26 \end{array}$$

17. Here, the proportionality yields the equation

$$\begin{array}{rcl} \frac{x}{x+5} = \frac{20}{60} & & \\ \frac{x}{x+5} = \frac{1}{3} & & \\ 3x = x + 5 & & \\ 2x = 5 & & \\ x = 2.5 \quad \text{and} \quad x + 5 = 7.5 & & \end{array}$$

19. Here,  $A = kbh$  so,  $50 = k(20)(5)$  to yield  $k = 1/2$ . The area of the triangle of interest  $A = (1/2)(16)(8) = 64 \text{ in.}^2$ .
21. NASA CONNECT is an annual series of math, science, and technical instructional distance programs for students in grades 6–8. In proportionality modeling of the future in March 2000 students learned why scaling and proportion are important in the design of aircraft transportation systems.
23. Answers vary.

**EXERCISES 1.7**

$$1. 5 \text{ miles} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ m}}{39.37 \text{ in.}} = 8046.74 \text{ m}$$

3. Here, use the conversion factor from Exercise 1.4.1 (1 m = 39.37 in.) to solve.

$$\frac{100 \text{ km}}{1 \text{ h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{39.37 \text{ in.}}{1 \text{ m}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = \frac{62.14 \text{ miles}}{\text{h}}$$

$$5. \frac{50 \text{ miles}}{1 \text{ h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ m}}{39.37 \text{ in.}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \\ \approx 2235.2 \text{ cm/sec}$$

$$7. 4.3 \text{ pounds} \cdot \frac{453.6 \text{ g}}{1 \text{ pound}} = 1950.48 \text{ g}$$

$$9. \frac{320.85 \text{ g}}{45 \text{ ml}} = 7.13 \text{ g/ml}$$

11. The mass is the density times the volume (one liter here)

$$\frac{13.55 \text{ g}}{\text{cm}^3} \times 1000 \text{ ml} = 13,550 \text{ g. Next, grams are converted into}$$

pounds

$$13,550 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 29.872 \text{ lb}$$

$$13. 30 \text{ g C} \cdot \frac{1 \text{ mol C}}{12 \text{ g C}} \cdot \frac{6.02 \times 10^{23} \text{ atoms C}}{1 \text{ mol C}} = 1.505 \times 10^{24} \text{ atoms C.}$$

15. First, convert 5 tons to 10,000 pounds. Next, since volume is mass/density,  $\frac{10,000 \text{ pounds}}{5.83 \text{ pounds/gallon}} = 1715.266 \text{ gallons.}$

17. Assuming 100 g of nitrous oxide, there is 63.6 g N and 36.4 g O. Converting these to moles, there is approximately 4.543 moles N and 2.275 moles of O implying that the formula is two parts N to one part O or N<sub>2</sub>O. This formula weight is 44 g, which is the correct weight, so the subscripts are correct and nitrous oxide's chemical formula is N<sub>2</sub>O.

19. Acres are measured by square feet. An acre can have various dimensions with the area approximately 43,560 square feet.

**SUPPLEMENTARY EXERCISES CHAPTER 1**

1. To determine an average, the five scores are added and divided by five. If  $x$  represents the score on the final exam, then

$$\frac{76 + 89 + 70 + x + x}{5} = 82.6$$

$$235 + 2x = 413$$

$$2x = 178$$

$$x = 89$$

The student will need to score an 89 to receive a B in the course.

3. a) A single filer with \$250,000 net taxable income has a tax of 10% on the first \$9075, 15% for next \$27,825 and so on until the \$250,000 is met.  $(9075)(0.10) + 27,825(0.15) + 52,450(0.25) + 97,000(0.28) + 63,650(0.33) = \$66,358$  tax.
- b) A married couple filing jointly with a \$250,000 taxable income has a tax of  $18,150(0.10) + 55,650(0.15) + 75,050(0.25) + 78,000(0.28) + 23,150(0.33) = \$58,405$ .
- c) A married person filing separately with \$200,000 taxable income would pay \$53,952 and the spouse making \$50,000 would pay \$8356 for a total tax of \$62,308.

It would not be advantageous for the couple to file separately in this case.

5. Two points on the line are  $(0, 8)$  and  $(5, 0)$ . The slope is  $-8/5$  since

$$m = \frac{0 - 8}{5 - 0} = -\frac{8}{5}$$

The intercept  $b$  is 8, so  $y = (-8/5)x + 8$  is the equation for the line shown.

7. a) Perpendicular lines have negative reciprocal slopes. The slope of the line through  $(2, 0)$  and  $(0, 5)$  is  $m = \frac{5 - 0}{0 - 2} = -5/2$  and the slope of a perpendicular line is  $m = 2/5$ . The equation of a line with slope  $2/5$  that passes through  $(3, 5)$  using the point-slope form, is  $y - 5 = (2/5)(x - 3)$ .
- b) A horizontal line has the form  $y = b$ . Here, the  $y$  value must be 4, since  $(1, 4)$  is to be on the line. The equation is  $y = 4$ .



18 LINEAR EQUATIONS AND MATHEMATICAL CONCEPTS

- c) A line perpendicular to the  $x$ -axis is a vertical line. Since  $(2, 7)$  is on the line, the equation is  $x = 2$ .
9. The line has a negative slope, since it “falls”. It also has a negative  $y$ -intercept. The line is d)  $-2x - 3$ .
11. Let  $x =$  number of gallons of water used (in thousands), then the bill is  $40 + 2.25x$ . Here, use  $x = 12$  to yield  $40 + 2.25(12) = \$67$ .
13. Let  $x =$  amount of water and  $y =$  amount of 100% sulfuric acid. The system of equations to solve is

$$x + y = 50$$

$$0x + 100y = 20 \quad (50)$$

The second equation yields  $y = 10$  so  $x = 40$ .  
The mix must be 40 ml of water and 10 ml of pure acid.

15. Let  $x =$  # \$50 items and  $y =$  # \$100 items. The system to solve is

$$x + y = 25$$

$$50x + 100y = 1650$$

Using elimination,

$$-50x - 50y = -1250$$

$$\underline{50x + 100y = 1650}$$

$$50y = 400 \text{ so } y = 8 \text{ and } x = 17.$$

Nine more \$50 items were sold than \$100 items. (Take care to answer the question.)

17. a)  $R(x) = 6x$    b)  $C(x) = 2000 + 2x$   
 c) Setting revenue = cost for the breakeven point,  $6x = 200 + 2x$ , so  $x = 500$ .  
 d)  $P(x) = 6x - (2000 + 2x) = 4x - 2000$  and  $P(1000) = 2000$  profit
19. a) The information is on the Internet .  
 b) There does not seem to be a digit that appears more frequently.
21.  $\frac{125^{2x-3}25^{x+1}}{5^{4x-5}} = \frac{(5^3)^{2x-3}(5^2)^{x+1}}{5^{4x-5}} = \frac{5^{6x-9}5^{2x+2}}{5^{4x-5}} = 5^{4x-2}$ .
23. Rewrite as  $\log_4 128 = x$  so  $4^x = 128$  or  $2^{2x} = 2^7$ .  
 Therefore,  $2x = 7$  or  $x = 7/2$ .

25. First, simplify as  $e^{x-1} = 5$ . Next, logarithms of both sides yield  $\ln(e^{x-1}) = \ln 5$ . Therefore,  $x - 1 = \ln 5$  or  $x = 1 + \ln 5$ .
27. Here,  $xy = \text{constant}$  and the first pair in the table indicates that the constant is  $(2)(24) = 48$ .

$x$	2	4	6	12	1
$y$	24	12	8	4	48

29.  $54 \text{ g C} \cdot \frac{1 \text{ mole C}}{12 \text{ g C}} \cdot \frac{6.02 \times 10^{23} \text{ atoms C}}{1 \text{ mole C}} = 2.709 \times 10^{24} \text{ atoms C}.$