1

INTRODUCTION

1.1 BACKGROUNDS

Over the past few decades, the field of antenna engineering has undergone significant progress. Many new techniques and design concepts have been developed to overcome a myriad of challenges experienced in antenna engineering. Amongst the advancements, the increasing characteristic mode (CM) theory study, focusing on its extensive implementations in many critical antenna designs, is one of the exciting breakthroughs in antenna engineering. Its promising potentiality has been constantly attracting the attention of antenna engineers. The CM theory and its applications in antenna engineering are the topics of this book.

The booming of wireless communication is an important driving force for the advancement of antenna technology. Antennas are the sensors of wireless communication systems. They find wide range of applications from terminal devices (such as mobile phones) to advanced communication systems on aircrafts, ships, and so on. Antennas transfer microwave energy from transmission system to propagating waves in free-space and vice versa. Strong demands like small physical size, low weight, low cost, wideband/multiband bandwidth, reconfigurable capabilities, or even aesthetic consideration are increasingly specified as a must in modern antenna designs. The inherent challenges in these demands thus have further propelled the advance of antenna technology.

The rapid growth of numerical electromagnetic (EM) modeling techniques plays another vital role in antenna technology advancement. The numerical EM modeling

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techniques can provide an accurate way to validate antenna performance before carrying out expensive fabrications and measurements. Consequently, numerous in-house or commercial software packages based on the method of moments (MoM) [1], the finite element method (FEM) [2, 3], and the finite difference time domain (FDTD) method [4–6] are extensively used in antenna designs. Given the antenna geometry and excitation structure, numerical techniques are able to simulate any antenna parameters.

However, from the practical point of view of antenna design problem, these numerical EM modeling techniques provide little information on the physical aspects of an antenna to be designed. The lack of physical insights brings difficulties in the further optimization of the antenna structure and feedings for achieving enhanced radiation performance. Therefore, antenna designs are heavily reliant on the designer's experience and knowledge. In the worst cases, antenna designs become a trivial task where antenna engineers blindly modify the antenna and feeding structures and simulate the antenna performance via numerical EM modeling tools iteratively.

For the sake of convenience, numerical EM modeling techniques are extensively combined with modern evolution optimization algorithms such as the genetic algorithm (GA) [7] to help mitigate the heavy workload in antenna tunings. The assumption is that the optimization algorithm will eventually arrive at the expected antenna performance after the exhaustive search in their decision space. However, this is not always true in all antenna design problems. More often than not, the automatic optimization algorithm returns to a complicated antenna structure with the satisfactory level of performances. However, the complexity of the resultant antenna structure makes it too hard to understand the underlying radiation mechanism. In this case, the design will be generally regarded as a lack of scientific knowledge. Therefore, overdependence on such brute force techniques is not a good way in antenna research. At least, it should not become antenna engineers' primary choice.

It is evident that a successful antenna design is highly dependent on previous experiences and the physical understanding of antennas. To grasp such knowledge may require many years of practical exercise. The experience, however, is hard to be imparted from one to another, as such personal experience is usually formed based on one's understanding of conventional antenna design concept introduced in textbooks. With such experience, solutions to some critical antenna design problems (e.g., the problems in Chapter 6) are usually not available.

Based on authors' personal understanding, an ideal antenna design methodology should allow one to achieve optimal antenna performance in a systematic synthesis approach with very clear physical understandings. However, such antenna design methodology does not exist until the antenna community recognizes the great potential of the CM theory in antenna engineering. In the past decade, the extensive applications of the CM theory in antenna designs have witnessed the roadmap of the development of this ideal antenna design methodology.

In the new millennium, studies on the CM theory have revealed its promising potential in a variety of antenna designs. The CM theory makes antenna design much easier than ever as antenna engineers need not depend heavily on personal experiences or brute force optimization algorithms. Meanwhile, the CM theory provides an easy way to understand the physics behind many key performances such as the bandwidth, polarization, and main beam directions. These physical understandings provide a greater degree of freedom in terms of design. As compared to traditional antennas, the antennas designed with the CM theory have more attractive electrical performances and configurations. Based on the recent advances made by the authors from the Temasek Laboratories at National University of Singapore (TL@NUS), this book discusses the CM theory and the CM-based design methodologies for a wide range of antenna designs.

1.2 AN INTRODUCTION TO CHARACTERISTIC MODE THEORY

The CM theory is a relatively new topic in the antenna community. Its great potential in antenna engineering has not been widely recognized till 2000. In the following subsections, several well-known modal methods for the analysis of particular antenna problems are being reviewed first providing preliminary knowledge about what modal analysis is about. It also illustrates how these modal methods help in practical antenna design. Next, we address some fundamental questions: why the characteristic modes were proposed and what are characteristic modes? Furthermore, the primary features of the CMs would be discussed. Finally, we briefly introduce CM variants that are distinct in terms of structures and materials to meet different radiation/scattering requirements. Detailed formulations and applications of these CM variants are discussed through Chapters 2–6.

1.2.1 Traditional Modal Analysis in Antenna Engineering

There is a long history in the development of modal analysis methods for a variety of problems in electromagnetics. The most famous one would be the modal expansion technique for infinite long waveguides [8–11]. This modal expansion technique handles the electromagnetic field inside a closed structure. It calculates the cut-off frequency and modal field pattern of each propagating mode inside the waveguides. These modes are primarily determined by the boundary conditions enforced by the cross-section of the waveguide. However, it is exceptionally difficult to implement modal analysis for open problems (radiation or scattering). Thus if the resonant behavior and radiation performance of an antenna can be interpreted in terms of the mode concept, great convenience will be brought to the analysis, understanding, and design of antennas.

In antenna engineering, there are three well-known modal analysis methods for particular antenna structures, namely, spherical mode, cavity model, and dielectric waveguide model (DWM). They will be briefly reviewed to show their attractive features and primary limitations.

1.2.1.1 Spherical Mode The first modal analysis method for antenna problems was the spherical mode method proposed in 1948 [12]. It discussed the physical limitations of the omnidirectional antennas in terms of antenna quality (Q) factor. It is now known as the Chu's Q criterion. Later in 1960, Harrington used the spherical mode analysis to discuss the fundamental limitations with respect to the gain, bandwidth, and efficiency of an antenna [13]. These pioneering works were based on the assumption where the radiating fields can be expanded using the spherical modes within a sphere that completely encloses the antenna. Therefore, the radiated power can be calculated from the propagating mode within the sphere. The limitations of an electrically small antenna in terms of the Q factor can be determined from the size of the antenna. However, as the size of the antenna increases, it gives rise to many propagating modes. In order to take into account all the propagating modes, the enclosing sphere has to be large enough. On the other hand, it is difficult to compute the modal coefficients for all the propagating modes in the large sphere. Thus, in general, the spherical mode method is limited to antennas with very small electrically size.

1.2.1.2 Cavity Model The cavity model was introduced by Lo et al. in 1979. It is a well-known modal analysis method for microstrip antennas [14], where the microstrip antenna is modeled as a lossy resonant cavity. As shown in Figure 1.1, the cavity model is set up as a region bounded by electric walls on both of the top and bottom interfaces, and magnetic walls along the perimeter of the patch antenna. In general, the substrate is assumed to be very thin in terms of thickness. The thin substrate thickness ensures the suppression of the surface wave. It also ensures that the field inside the cavity is uniform in the direction of the substrate thickness [14, 15]. Meanwhile, the cavity model assumes that the fields underneath the patch are expressed as the summation of various resonant modes. Moreover, the cavity model takes into account the fringing fields along the perimeter by extending a small offset distance out of the patch periphery. The far-fields are computed from the equivalent magnetic currents around the periphery. The cavity model also accounts for the higher order resonant modes and the feed inductance. It offers a simple yet clear physical insight into the resonant behavior of microstrip antennas. However, it is only applicable to regular patch shapes such as the rectangular, circular, elliptical, and triangular patches. The computation of resonant modes for arbitrary shaped patches using cavity model is a challenging task. In addition, the cavity model is also not suitable for the analysis of multilayered or thick substrate microstrip antennas.

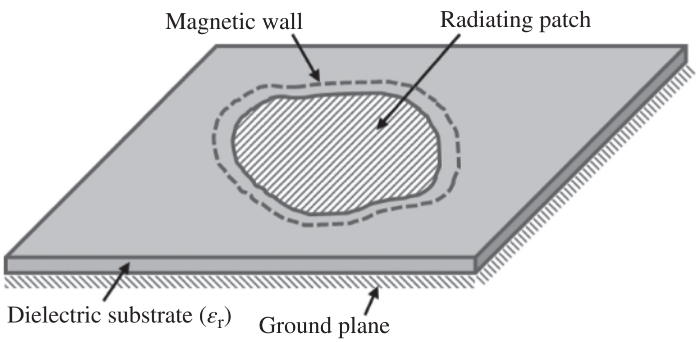

FIGURE 1.1 The cavity model of a microstrip antenna.

1.2.1.3 Dielectric Waveguide Model The DWM is another well-known modal analysis method [16], which was developed to evaluate the resonant frequencies of rectangular dielectric resonator antenna (DRA). The DWM is generally accurate enough and has been widely adopted in practical rectangular DRA designs [17–19].

In the DWM, the rectangular DRA is modeled as a truncated dielectric waveguide. Referring to the rectangular DRA sitting in the coordinate system as shown in Figure 1.2, where $a > b > d$, the first three lowest modes are TE^z_{111}, TE^y_{111}, and TE^x_{111}, respectively. As the analysis method for all of these modes is similar, only the resonant frequency determination for the TE^y_{pqr} mode is being discussed and provided below. The characteristic equations for the wavenumber k_x, k_y, and k_z by using the Marcatili's approximation technique [17] are as follows:

$$\begin{aligned} k_x a &= p\pi - 2\tan^{-1}(k_x / \varepsilon_r / k_{x0}), \quad p = 1,2,3\ldots \\ k_{x0} &= \left[(\varepsilon_r - 1)k_0^2 - k_x^2\right]^{1/2} \end{aligned} \tag{1.1}$$

$$\begin{aligned} k_y b &= q\pi - 2\tan^{-1}(k_y / k_{y0}), \quad q = 1,2,3\ldots \\ k_{y0} &= \left[(\varepsilon_r - 1)k_0^2 - k_y^2\right]^{1/2} \end{aligned} \tag{1.2}$$

$$\begin{aligned} k_z d &= r\pi - 2\tan^{-1}(k_z / \varepsilon_r / k_{z0}), \quad r = 1,2,3\ldots \\ k_{z0} &= \left[(\varepsilon_r - 1)k_0^2 - k_z^2\right]^{1/2} \end{aligned} \tag{1.3}$$

where k_{x0}, k_{y0}, and k_{z0} are the decay constants of the field outside the DRA, and k_0 is the free-space wavenumber. The wavenumbers k_x, k_y, k_z, and k_0 satisfy the following formulation:

$$k_x^2 + k_y^2 + k_z^2 = \varepsilon_r k_0^2 \tag{1.4}$$

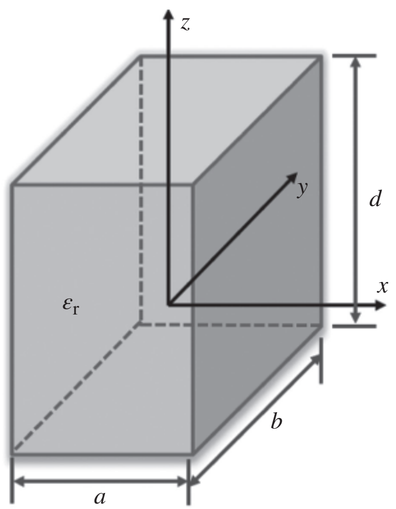

FIGURE 1.2 An isolated rectangular dielectric resonator antenna.

By solving the characteristic equations simultaneously, one can determine the resonant frequencies of the TE^{y}_{pqr} modes.

However, the DWM is only applicable to rectangular DRAs. For cylindrical DRAs and other DRAs with more complicated shapes, one would usually resort to some other engineering formulas [20] or full-wave simulations.

In summary, the modal analysis methods provide clear physical insights to the resonant behavior and modal radiation performance. It is understood that each of these modal analysis methods is theoretically developed for solving a particular kind of problems, thus they are not versatile for general antenna problems. Therefore, it is essential to develop more general modal theory for a variety of antenna problems. The CM theory was initially developed as a versatile modal analysis tool for antennas with arbitrary structures and materials. The CMs together with its metric parameters explicitly give the following useful information for antenna analysis and design:

- Resonant frequencies of dominant mode and high order modes;
- Modal radiation fields in the far-field range;
- Modal currents on the surface of the analyzed structure;
- Significances of the modes at a given frequency.

1.2.2 Definition of Characteristic Modes

Garbacz proposed the CM theory for the first time in 1965 [21]. At that time, the definition for CMs was not explicitly given. Instead, an assumption for the CMs was given before addressing their numerical computations:

> The basic assumption is made that the scattering or radiation pattern of any object is a linear combination of modal patterns, characterized by its shape, and excited to various degrees at the terminals (when the object acts as a radiator) or by an incident field (when the object acts as a scatterer). The current distribution over the object is assumed to be decomposed into an infinite number of modal currents, each of which radiates a characteristic modal pattern independent of all others.

Garbacz and Turpin [22] addressed the complete CM theory and the expansion method for the computation of CMs. They defined CMs as a particular set of surface currents and radiated fields that are the characteristics of the obstacle and are independent of any external source. Later, Garbacz and Pozar [23, p. 340] gave a definition for CMs:

> Characteristic modes form a useful basis set in which to expand the currents and fields scattered or radiated by a perfectly conducting obstacle under harmonic excitation. They possess orthogonality properties both over the obstacle surface and the enclosing sphere at infinity; they succinctly relate the scattering operator and impedance operator representations of the obstacle; and they exhibit interesting mathematical properties which may be interpreted physically in terms of radiated and net stored powers.

The definitions for the characteristic modes show that the characteristic modes constitute to a very special orthogonal set in the expansion of any possible induced currents on the surface of the obstacle. Moreover, the far-fields associated with these orthogonal currents possess orthogonality properties over the radiation sphere in the infinity.

1.2.3 Primary Properties of Characteristic Modes

The CM theory was initially proposed for perfectly electrically conducting (PEC) bodies. For the PEC case, there are two major features that make it attractive in antenna engineering and scattering problems [23]:

- Any PEC bodies with surface S is associated with an infinite set of real characteristic currents $\mathbf{J}_n$ on S. Each of $\mathbf{J}_n$ radiates the characteristic electric field $\mathbf{E}_n$ into the free space. The tangential component of $\mathbf{E}_n$ on S is equiphase. Associated with each CM, there is a characteristic angle $\alpha_n \in [90°, 270°]$ defining the phase lag between $\mathbf{J}_n$ and $\mathbf{E}_n^{\tan}$. The characteristic angle is an important metric parameter in the CM theory, which indicates the resonant behavior and the energy storage of each mode.
- Due to the orthogonality of $\mathbf{J}_n$ over S as well as the orthogonality of $\mathbf{E}_n$ over the radiation sphere at infinity, the CMs form a useful basis set in the expansion of any possible currents or fields associated with the perfectly conducting body.

In addition to the characteristic angle, other metric parameters were often used to describe the resonant behavior of each mode. Each of them has its unique features to illustrate the physics of each mode. These parameters and their physical interpretations will be presented in Chapter 2.

1.2.4 Variants of Characteristic Modes

There are many variants of CMs for the analysis of structures with varying configurations and materials. Among these variants, the initial work done by Garbacz has to be highlighted first [21]. Garbacz stated that arbitrary shaped PEC obstacles possessed CMs. These CMs are only dependent on the shape and size of the PEC obstacles. They are independent on any specific excitation or sources. Most importantly, these modes can be used to expand any possible currents and scattering fields due to the PEC obstacle. He further demonstrated that the CMs of a PEC sphere were identical to the spherical wave functions, and the CMs of an infinite circular PEC cylinder were identical to the associated cylindrical wave functions. This early observation showed that modal analysis for arbitrary-shaped PEC obstacles can be achieved through the implementation of CM analysis.

This initial work motivated Garbacz and Turbin to develop an approach to compute the CMs for arbitrary obstacles. In Ref. [22], they derived a generalized scattering matrix by matching the external and internal fields to the obstacle at sampling points. A characteristic equation was then formulated by diagonalizing this scattering matrix. By solving the characteristic equation, the CMs of the obstacle were obtained. Although this approach required substantial computations in getting the characteristic equation, it was the first successful attempt to compute the CMs for arbitrary PEC bodies.

In 1971, R. F. Harrington and J. R. Mautz reformulated the CM theory based on the operator in the electric field integral equation (EFIE) for PEC objects [24, 25]. This operator relates the surface current to the tangential electric field on the surface of PEC body. By diagonalizing the operator and choosing a particular weighted eigenvalue equation, they obtained the same modes as defined by Garbacz. Harrington's approach produced explicit and convenient formulas for the computation of CM currents and fields. More often than not, the CM formulation and computation implemented in recent years are primarily based on Harrington's approach.

An EM structure with many input/output ports is commonly treated as a multiport network system. Accurate analysis of the transmission and scattering properties of a multiport network system can be carried out via the port impedance matrix [Z], the port admittance matrix [Y], and the scattering matrix [S]. With these port parameter matrixes, similar to the CMs for PEC bodies [24], Harrington and Mautz developed the characteristic port modes for the eigen analysis of an N-port network [26]. The characteristic port modes were computed from the mutual impedance matrix of an N-port network system. They formed a convenient basis set for expressing the field scattered or radiated from an N-port network. It should be noted that the characteristic port modes were actually defined in a manner analogous to those for continuously loaded bodies. With the help of such kind of analogue in representing functionality, the analysis, synthesis, and optimization of the N-port systems (e.g., antenna array) become conceptually simpler. Many researchers have applied the characteristic port modes in the analysis and design of multiport antenna systems such as the multiple input multiple output (MIMO) antenna systems [27] and the wideband antenna arrays [28].

Both dielectric and magnetic materials are widely used in radiation systems. Therefore, CM theory for structures with different materials is highly demanded. In the 1970s, Harrington and his colleagues attempted to extend the CM theory to structures involving different materials [29, 30]. However, the correctness of their formulations had not been well demonstrated in solving practical problems until 2013. Several new investigations to these earlier formulations have been addressed in the recent years [31–35]. With regards to this CM variant, two new CM variants are respectively developed for the CM analysis of planar patch antennas in multilayered dielectric medium and for the CM analysis of dielectric bodies. The latter CM formulation offers useful applications in the analysis and design of DRAs. The theory and applications of these two CM variants in practical antenna designs will be discussed in depth in Chapters 3 and 4, respectively.

In many cases, the far-fields on the entire radiation sphere are often not necessary. For instance, backward radiations from a directive antenna or antenna array, backed by a large ground plane, are usually very small. It may not be necessary to consider the backward radiations in the design stage if only the radiation characteristics in the upper half-space are of interest. In such circumstance, if the far-fields derived from the CM theory can be orthogonal to each other over an interested section (say, the interested upper half-space) on the radiation sphere, such an orthogonality will bring great convenience in the analysis and design of many antennas and antenna arrays. Inagaki and Garbacz [36] developed a new CM formulation to address this issue, which is generally known as Inagaki modes. The Inagaki modes are more generalized than the conventional CMs and the orthogonal mode fields can be imposed on any section of the radiation sphere at infinity. Afterward, a modified version of the Inagaki modes was proposed by Liu [37]. This modified Inagaki modes were often referred to as the generalized CMs that provided advantages in terms of versatility and computational efficiency in the computation of the Inagaki modes.

Apertures and slots are another kind of EM structures that are widely used for radiation purpose. The aperture problems have been investigated by many researches, even back to several decades ago [38]. Small apertures in an infinite conducting plane are usually treated using the Bethe-hole theory [39]. If the small aperture is in a nonplanar surface, the Bethe-hole theory is usually used as an approximation. When the aperture increases to several wavelengths, one usually resorts to the solution of an appropriate integral equation. To solve the aperture problems with arbitrary size and shape in a conducting plane, a CM theory for various aperture problems was developed by Kabalan [40–63]. Specifically, Kabalan developed the CM theory for the following aperture problems:

- Slots in a conducting plane [40, 41].
- Rectangular aperture in a conducting plane [49].
- Slot in a conducting plane separating different media [42].
- Slots in a conducting cylinder [43, 44].
- Dielectric-filled conducting cylinder with longitudinal slot [45].
- Multiple slots in a conducting plane [46].
- Aperture-fed waveguide problem [47].
- Dielectric conducting cylinder with multiple apertures [48].
- Two-half space regions separated by multiple slot-perforated conducting planes [50].
- Parallel plate-fed slot antennas [51].

In all of these aperture CM formulations, the modes have the same attractive properties as those in the PEC objects:

- The characteristic magnetic currents are real.
- The characteristic magnetic currents are weighted orthogonal over the aperture region.
- The characteristic fields are orthogonal over the infinite radiation sphere.

1.3 CHARACTERISTIC MODES IN ANTENNA ENGINEERING

The important features of CMs have given rise to a variety of applications in antenna engineering. In this section, the CM-related antenna topics, since it was proposed in 1965 [21], are reviewed. We hope that this section will leave a deep impression on the readers where the CM theory is an efficient antenna design methodology with very clear physical insights for many challenging antenna designs.

Figure 1.3 shows the number of publications on CMs since 1965. The statistical data comes from the well-known Scopus database. It can be seen that CMs were not so popular in the first 35 years. Most of the earlier CM researches were carried out at the Syracuse University and the Ohio State University, USA. However, in the new millennium, the number of publications on CM has been gradually increasing since 2000 and a new peak was achieved in the year of 2014. This may be due to the fact that the CM approach is an attractive tool in the design of multiple antenna systems on the chassis of mobile handset devices. The rapid growth of smart phones has strongly propelled the technology advancement of multiple antenna system designs within a very crowded space. Studies have shown that the CM approach is the most promising technology to address such critical antenna designs and this trend has become more evident after the year of 2006.

Figure 1.4 shows the research topics related to CM theory and their publication percentages after the year of 1980. Thirty-six percent of publications reported about the mobile handset antenna designs using CMs. Together with Figure 1.3, it is evident that large number of CM research was conducted on the CM applications to mobile handset antenna designs since 2006. The second hottest topic after that would be the CM formulation developments for apertures and slots. As discussed in the preceding subsection, these developments were carried out by Kabalan and his colleagues at the American University of Beirut from 1987 to 2004 [40–63]. It is expected that these fruitful CM formulations will discover great applications to the analysis of transmission, reflection, coupling, and scattering in aperture problems.

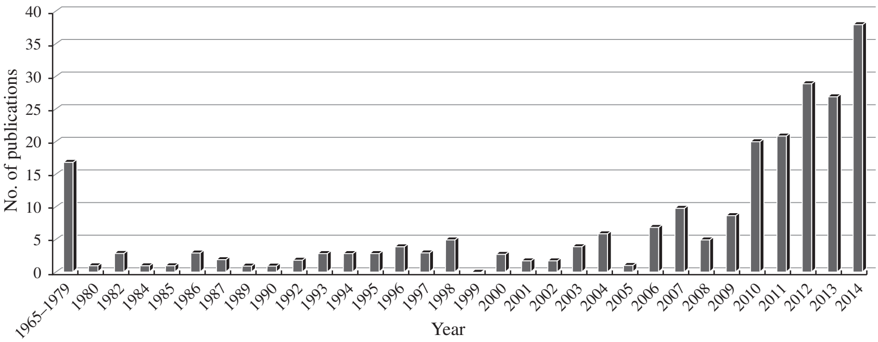

FIGURE 1.3 Yearly publications on characteristic modes. The statistical data comes from the Scopus database [64].

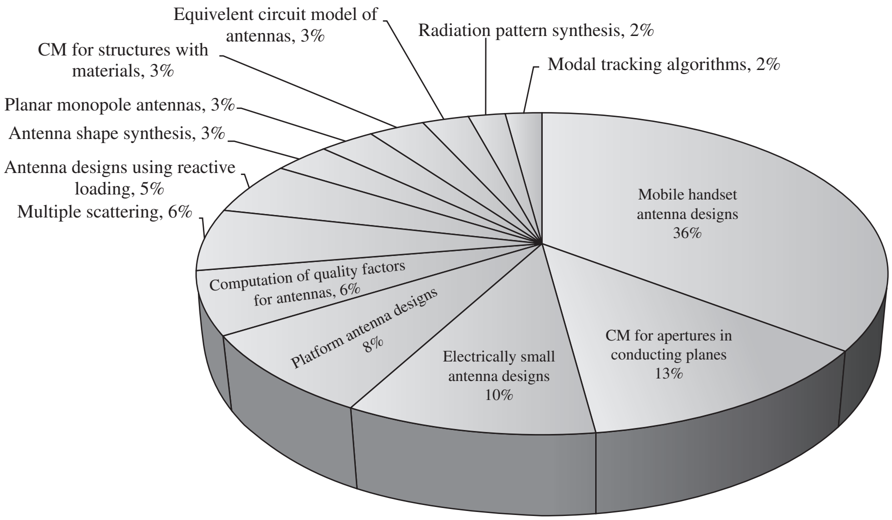

FIGURE 1.4 CM research topics and their publication percentages after the year 1980.

The topic of platform antenna designs using CMs ranks third among various CM topics. When CM theory found its applications in platform antenna designs in the late 1970s [65], the structural antenna concept using CM was also formed at the time of this pioneering study. In this approach, the existing platform was excited and used as the radiator, and the CMs of the platform were exploited to control the currents on the platform for designated radiation performance. Later, CM theory was extensively adopted in various platform antenna designs, especially in low-frequency bands such as the HF, VHF, and UHF.

Due to the vast number of publications on CM-based antenna topics, it is impossible to include a full collection of all the CM researches in this chapter. Therefore, with reference to Figure 1.4, the CM-related topics would be categorized into the following 11 subjects:

1. Mobile handset antenna designs
2. Electrically small antenna designs
3. Platform antenna designs
4. Computation of quality factors for antennas
5. Antenna designs using reactive loading
6. Antenna shape synthesis
7. Planar monopole antennas
8. CM for structures with dielectric materials
9. Equivalent circuit model of antennas
10. Radiation pattern synthesis and decomposition
11. Modal tracking algorithms

The following subsections will give a comprehensive introduction to each of the listed subjects. It is presented according to the reporting time of the CM research works, that is, carried out before 1990 and after 1990. Readers are encouraged to consult the references for more detailed explanation on the theory, methodology, implementation, and/or experimental results in each of the subject.

1.3.1 Pioneering CM Studies (1965–1990)

In 1972, the CM theory found its first application in the synthesis of desired radar scattering patterns [66]. In this work, the authors showed that any real current could be resonated by reactively loading the scatterers. This real current may be chosen to meet the specifications concerning required performance parameters, such as minimize the quality factor for enhanced bandwidth, increase the gain, and synthesize a desired radiation pattern. In particular, they developed a synthesis procedure to obtain the real current such that the far-field pattern approximates the desired one in a least mean-square sense. This initial concept laid the foundation for further radiation pattern or scattering pattern synthesis using the CM theory. It also pointed out that these reactive loads could be realized in the form of tuned slots and stubs on the scatterers.

In 1973, Yee and Garbacz developed simple quadratic forms for the self- and mutual-admittances of delta-gaps located along a perfectly conducting thin wire [67]. They demonstrated that once the characteristic currents and eigenvalues had been resolved, it was easy to calculate the admittances for any number of gaps located on the wires. The developed quadratic forms were useful for small array design and analysis, which formed a logical basis for decoupling the gaps for possible radiation pattern control. These quadratic forms were also useful in some antenna array optimizations. This was the first time that the CM theory was applied in the analysis of antennas.

In 1974, Harrington and Mautz developed a procedure to maximize the radar cross-section by reactively loading an N-port scatterer at the current maximum [68]. In this work, they first synthesized the real port currents that would maximize the radiation power gain. Then, reactive loads for the resonating of these real port currents were computed using the characteristic port modes. This work enlightened the applications of the characteristic port modes in antenna array designs. We will address the technical details of this topic in Chapter 5.

Later, Newman proposed a very attractive structural antenna concept for platform-integrated antenna designs in 1977 [65]. He demonstrated that the efficiency of a small antenna could be substantially increased through properly locating it on its supporting structure. CMs of the supporting structure were used to determine the optimum locations of such small antenna. He highlighted this new concept in Ref. [65], where the small antenna does not function as the primary radiator, but rather as a probe to excite the currents on the supporting structure. Since the supporting structure is not always electrically small, it can be an effective radiator. This was the first paper reported that the CMs of an antenna platform would dominate the radiation performance. The CM-based structural antenna concept was then successfully applied in the design of vehicle mounted sky-wave communication antennas [69, 70]. The vehicle platform and its installed loop antenna were treated as an entire radiating structure. CM analysis was then performed for this platform-loop integrated structure. The appropriate excitation and control of key modes was attained to realize the synthesis of the desired radiation patterns and significant performance enhancement over those from conventional whip antennas. It was illustrated that the platform CMs offered many design freedoms in terms of controlling of radiation patterns, enhancing gain and efficiency, as well as the optimal feeding placement. More technique details will be covered in Chapter 6.

To obtain a designated far-field radiation pattern from an unknown antenna structure, Garbacz and Pozar proposed an approach to synthesize the shape of the antenna such that the far fields of its dominant CM approximate the designated one [23]. With the synthesized antenna shape, one would only need to excite the dominant CM and suppress the unwanted modes. They also investigated many practical feedings to effectively excite the desired modes while ineffectively exciting high-order undesired modes. This investigation paved the way for the feeding designs for the excitation of platform CMs.

In 1985, Harrington and Mautz reported another important contribution to the CM theory [71]; the CM theory for problems consisting of two regions coupled by an aperture of arbitrary size and shape was developed. It created a new area of study for the CMs. Later, Kabalan's research group followed the work reported in Ref. [71] and reported a complete CM theory for various aperture problems [40–63]. Readers can refer to the preceding sections for more details of Kabalan's studies on aperture CMs.

The above work was carried out before 1990. Major applications of these findings include platform antenna designs, radar scattering controlling, radiation pattern synthesis, and antenna shape synthesis for designated radiation pattern. Theoretical investigations to aperture problems and electromagnetic structures with dielectric materials were also carried out in the earlier times. We shall see, in the next subsection, how the major application of the CM theory turns to the mobile handset antenna designs. The resurgence of the CM theory in antenna engineering is indeed due to the increasing demands on personal mobile communication devises.

1.3.2 Recent CM Developments (1991–2014)

This section describes the recent CM developments with regard to the 11 subjects as mentioned in the preceding sections. The challenges, CM-based techniques, and major achievements of each subject are presented in the following.

1.3.2.1 Mobile Handset Antenna Designs In the past decade, CM theory found the widest applications in the developments of mobile handset antenna designs. A large number of publications have reported on the mobile handset antenna designs using the CM theory [72–136]. Evidently, the CM theory has become a standard approach in these developments. Based on the CMs of the chassis of the mobile handset devices, the antenna community has made the following achievements:

- Bandwidth enhancement of mobile handset antennas: If many efficient radiating modes are distributed evenly within a frequency band, all of these modes can be excited at the same time to achieve wide bandwidth. However, the bandwidth of each chassis mode is usually quite narrow. Thus, many techniques were proposed to generate a set of efficient radiating CMs along a wide frequency band. Typical designs included adding a bezel above the chassis [72–74] or introducing a strip along the length of the chassis [75–77]. To achieve wideband handset multiple antenna system designs, capacitive coupling element and inductive coupling elements were studied [78–82].

- Port decoupling in handset MIMO antenna designs: Due to the crowding space in handset MIMO antennas, there are usually strong mutual coupling among multiple ports. The orthogonality of CMs provides an efficient way to decouple the couplings, even if the ports are physically closed with each other [83].
- Reconfigurable handset antenna designs: As the characteristic fields of each mode are orthogonal with each other, different modes feature different characteristic far-field patterns. The diversity of these characteristic fields could be exploited for reconfigurable handset antennas designs [84–86].
- Radiation efficiency enhancement: By efficiently exciting the dominant CM of the chassis, the chassis becomes the radiator. On the other hand, the size of the chassis of a handset device is usually comparable with the wavelength. Due to the large size of the chassis, handset antenna designs making use of the chassis mode usually have high radiation efficiency [87].

1.3.2.2 Electrically Small Antenna Designs Based on the clear physical investigations of electrically small antenna structures, the CM theory has produced plenty of sophisticated antenna designs. These electrically small antennas include the following:

- Radio-frequency identification (RFID) tag antenna in ultra high frequency (UHF) band [137, 138]
- Universal serial bus (USB) dongle antenna for wireless local area network (WLAN) applications [139]
- Helical spherical antennas [140–142]
- Microstrip antennas [143–146]
- Loop antenna [147]

The investigations into the CMs of these antenna structures result in a clear understanding of the natural resonance frequency, the radiating current behavior, the pattern and polarization behavior of the radiating fields in the near- and far-field zone. All these valuable investigations serve as principal guidelines in feeding designs. These CM-based antenna designs feature one or more of the following enhanced performances:

- Improved radiation efficiency and gain
- Enhanced bandwidth or multi-band performance
- Improved polarization purity
- Reduced antenna volume
- Simplified feeding designs

1.3.2.3 Platform Antenna Designs Communication antennas play a vital role in a variety of moving platforms including aircrafts [148], unmanned aerial vehicles (UAV) [149–154], ships [155], and land vehicles [69, 70, 156]. Because of the moving property, long distance transmission is generally required. In order to achieve long distance transmission, the antennas have to work at relatively low-frequency bands, such as the HF, VHF, and UHF bands. The long wavelength of the electromagnetic waves results in a large antenna size if conventional antenna design concepts are to be adopted. In consideration of the aerodynamic property and the aesthetic requirement, antennas with low profile, compact sizes or even be conformal with the platform are in demand, although this is a challenging task. The CM theory contributes a novel way to address the challenges in such kind of platform-integrated antenna design problems. The basic idea in the CM approach is to exploit the CMs of the platform and invent some miniature elements to excite one or more CMs for designated radiation pattern or wide bandwidth. The CM-based platform antennas feature with low profile or even platform conformal property.

For example, an antenna conformal to the V-shaped tail of the UAV is designed using the CMs of the UAV [152]. It radiates a vertically polarized omnidirectional pattern in the frequency range from 50 to 90 MHz. The vertical dimension of the tail is only $\lambda/17$ at 50 MHz. Evidently, if the CMs of the UAV body was not excited for radiation purpose, it is impossible for an antenna with a size of only $\lambda/17$ to radiate energy to the far-field zone. Moreover, the CM analysis of the UAV shows the optimal location for maximum energy coupling from the feeding structures.

Owing to the diversity of the CMs of platforms, the CM theory provides many design freedoms to achieve various design objectives. In the following, we list some typical CM-based platform antenna design cases:

- UAV antennas with reconfigurable radiation patterns [149, 150]
- Two-port MIMO antennas on UAV [154]
- Shipboard antenna with directional radiation pattern [155]
- Land vehicle–mounted antenna with directional radiation pattern [69, 70, 156]

It should be noted that all of these platform antennas fully make use of the CMs of the platform. As compared to traditional antenna design concept, they possess enhanced radiation efficiency and low-profile property, if conformal is not impossible.

1.3.2.4 Computation of Quality Factors for Antennas The antenna quality factor Q has been extensively studied for a long time. It is useful to measure the maximum possible antenna bandwidth and radiation efficiency, especially in the analysis of electrically small antennas. There are many approaches to formulate the Q factor of an antenna [12, 13, 157–160]. Recently, the CM theory was used to expand the total Q in terms of the modal quality factor Q_n [161–167]. Two approaches are introduced to compute Q_n from the CMs of an antenna:

- Following the Q factor formulation based on the input admittance [168], Elghannai and Rojas proposed to expand the total input admittance of an antenna in terms of the individual admittance of each CM [161]. The modal quality factor

Q_n was then computed in the form based on the Q formulation in Ref. [168]. The total Q factor can then be obtained as a summation of the modal quality factor Q_n.

- The second approach to compute the Q_n factor was based on the energy and power associated with each CM [162]. The calculation of the modal energies and the Q_n factor allows studying the effect of the radiating shape independent of the feedings. The total Q can be further formulated as a superposition of each modal quality factor Q_n.

Both the approaches are accurate for the total Q computation. Moreover, the modal quality factor, Q_n, gives more physical insights into the radiation capability of each mode. The modal quality factor, Q_n, has found wide applications in the computations of bandwidth and radiation efficiency for mobile handset antennas [163], MIMO antennas [164], Franklin antenna [165], dipole antenna [166], and microstrip antennas [167].

1.3.2.5 Antenna Designs Using Reactive Loading In most cases, the CMs of an unloaded antenna structure may not meet the expected radiation performance. Therefore, the radiating structure has to be modified to ensure the dominant CMs hold the desired radiation performance in terms of the radiation pattern, current distribution, bandwidth, and so on. Reactive loading is one of the most popular approaches to modify the CMs of an existing radiating structure. The current distribution is modified by adding lump elements such as capacitors and inductors, while the geometry of the original radiating structure does not need further modification.

There are many typical antenna designs produced by reactively loading the antennas according to the characteristic currents over the existing antenna structures:

- Pattern reconfigurable antenna array designs: In Ref. [169], pattern reconfigurable circular arrays with low sidelobe levels are synthesized by reactively loading the arrays. In this approach, the CM analysis for the antenna array was performed first to identify its different radiating modes. Based on these CMs, the differential evolution optimizer [170] was then employed to seek the reactive loading values according to the specific requirements on the radiation patterns. The benefits brought by the CM theory were observed in terms of the quality of the optimal design, the convergence rate, and the computational complexity.
- Yagi-Uda antenna designs: The CM theory was combined with the differential evolution optimizer for the optimal design of reactively loaded Yagi-Uda antenna [171]. Based on the CM analysis, the differential evolution algorithm was applied to find the optimal values of the reactance loads for the individual elements in a Yagi-Uda antenna. With the reactive loading, optimal Yagi-Uda antenna designs in terms of the antenna gain, input impedance, and sidelobe level were realized.
- Broadband antenna designs: In Refs. [172–174], a CM-based approach was proposed to systematically design antennas with broadband impedance and pattern characteristics using reactive loadings. Antennas of arbitrary geometry

have their own bandwidths. It was shown that the ideally desired current distribution of an antenna over a wide frequency range could be achieved by using a finite number of loadings. Enhanced bandwidth of a typical narrow band dipole antenna was observed through using this CM-based approach.

- Frequency reconfigurable antenna designs: As a further extension to the work in Ref. [172], the CM theory was applied in frequency reconfigurable antenna designs [175]. It was demonstrated that the reactive loadings could be determined systematically to resonate the antenna at many frequency points over a wide frequency range. A dipole antenna with a wide tuning range of 1:4 bandwidth is achieved by simply tuning it at four loading ports. This approach was also implemented in a frequency reconfigurable planar inverted-F antenna design.

1.3.2.6 Antenna Shape Synthesis The performance of an antenna is dependent on the antenna geometry. At the University of Ottawa, Canada, research on the antenna shape synthesis has been going for many years [176–178]. Owing to the source independent solution of the CM analysis, the proposed antenna shape synthesis allows shaping of the antenna geometry prior to any specific feeding structures. It reduces the constraints placed on the optimization process and leads to new designs. The CM-based shape synthesis approach intrinsically ensures the optimized structures having efficient radiating modes for best impedance matching at a given frequency. At the end of the antenna shape optimization, an optimal feed point can be easily determined from the modal currents. Reported examples demonstrate that the quality factor Q of the resulting shaped antenna closely approaches the fundamental Q bounds.

The CM research group at the University of Ottawa has also developed a sub-structure CM concept [179]. It extended the applicability of the antenna shape-synthesis technique where only parts of the entire antenna structure need shape synthesis.

1.3.2.7 Planar Monopole Antennas With regard to many of its attractive features, like wide frequency band, omnidirectional pattern, compact size, and low cost, the planar monopole antennas are very popular in many wireless communication systems and UWB systems. The CM analysis provides interesting physical insights into the radiation phenomena taking place in the planar monopole antennas. It revealed that the wideband performance could be characterized by the modal voltage-standing-wave ratio, modal current distribution, and modal significance [180]. The modal analysis also showed that the dominant mode controls the antenna's behavior in the lower band, and the higher order modes control the behavior in the upper band [181–183].

Modal analysis of band-notched planar monopoles illustrates that the resonant modes are due to the embedded narrowband slot structure [184]. By electronically controlling the excitation of the slot mode, planar monopoles with switchable band-notched behavior can be obtained. Similarly, a tunable band-notched UWB antenna can be realized by controlling the resonance of this slot mode. As mode excitation is proportional to current amplitude at the feed point, the bandwidth can be improved by properly combining and exciting more than one mode through multiple excitation sources. These in-depth understandings pave the way for the proposal of novel designs of planar monopole antennas through the control of the excitation and resonance of specific modes [185].

1.3.2.8 CM for Structures with Dielectric Materials CM analysis for structures with dielectric materials is far more complicated than PEC problems. Specifically, the volume integral equation raises large number MoM unknowns and the CM analysis is time-consuming, even for an electrically small dielectric body [186]. As the surface integral equation is invoked for CM analysis, it involves the surface electric currents and magnetic currents. These two types of currents are dependent on each other. This may lead to some unphysical modes if the MoM matrix is directly used for the CM analysis of structures with dielectric materials [31, 33, 187, 188]. The CM analysis of structures with dielectric materials will be further discussed in Chapter 4.

1.3.2.9 Equivalent Circuit Model of Antennas Physics based on equivalent circuit models are helpful in the design and optimization of antennas. As opposed to the mathematically fitted model, the equivalent circuit model derived from the CM theory provides physical insights and allows the exploration of the limits of attainable performance. The representative research on the developments of equivalent circuit model for antennas using the CM theory includes:

- Equivalent circuit model for conducting chassis with capacitive and inductive coupling elements was developed [189]. In this model, most of the elements can be directly obtained without numerical optimization. The equivalent circuit model developed found its applications in the design of mobile handset antennas.
- The series and parallel resonances of an antenna's input impedance are explained from the point view of CMs [190]. It showed that the parallel resonance corresponds to the interaction of at least two nearby CMs. The eigenvalues of the two modes have opposite signs. On the other hand, the series resonance is mainly contributed by the resonance of a single CM. This finding is supported by the CM analysis for a simple wire dipole antenna, an edge-fed patch antenna, and a loop antenna.
- An approach for modeling antenna impedances and radiation fields using fundamental eigenmodes was developed in Refs. [191, 192]. Higher order modes can be more accurately modeled with added circuit complexity. Owing to the physical behavior of the CMs, the developed model accurately links the circuit models with radiation patterns and other field behavior, and the far-field patterns and antenna gain of a dipole could be accurately extrapolated over a 10:1 bandwidth.
- Very recently, Adams showed that the frequency response of each CM could be approximated by a template function related to the spherical mode [193]. By choosing the appropriate template function, the frequency response of the CMs can be modeled over a broadband. Based on this observation, the acceleration in the interpolation of antenna's impedance could be achieved over a wide bandwidth.

1.3.2.10 Radiation Pattern Synthesis and Decomposition CMs consist of a complete set of orthogonal modes. The far-fields of each mode are orthogonal with each other. Any radiating far-fields can be expressed as a superposition of the CMs. This property is known as the modal solution of CMs. The synthesis and decomposition of radiation patterns is a direct application of the modal solution and orthogonal property of the CMs.

In radiation pattern synthesis problem, the objective is to find the optimal weightings for each of the CMs, such that these weightings results in a radiation pattern with satisfactory specifications. Recently, a CM-based radiation pattern synthesis procedure was developed by using a multiobjective optimizer [149, 194]. The weightings for each of the CMs also provided the corresponding radiating currents.

In radiation pattern decomposition problem, however, the objective is to express a given radiation pattern into a set of CMs. In Refs. [195, 196], this far-field decomposition problem was also called reconstruction of the CMs from radiated far fields. The far-field decomposition showed that the radiation mechanism of a complicated antenna structure could be approximated by a simplified structure. The weighting coefficients of all significant modes could be reconstructed with good accuracy even for complex real structures such as mobile phones [195]. As a consequence, the far-field decomposition provides a new way to understand the radiation mechanism of antennas with very complicated geometries.

1.3.2.11 Modal Tracking Algorithms Practical wideband CM analysis requires the modes at one frequency to be associated with the modes at another frequency. The association relationships of eigenvectors at two different frequencies are generally determined from the correlation coefficients among the eigenvectors. The concept of modal tracking is introduced in the CM theory to sort the modes in the correct order at each frequency. Many tracking algorithms were developed to address this problem [197–200]. The sorted CMs clearly showed the evolution of the modes from non-resonant frequency to resonant frequency and vice versa. These modal tracking algorithms provide a new perspective to investigate the resonant behaviors of CMs. The details of the modal tracking algorithms will be discussed in Chapter 2.

1.4 CHARACTERISTIC MODES IN SCATTERING COMPUTATION

In addition to the wide applications of CM theory in antenna engineering, the CM theory is also applicable to scattering computations, especially in the multiple scattering problems. For simple targets, in most cases, a few low-order CMs are sufficient to reproduce a good approximation to the induced current on scatterers. For a multiple scattering problem consists of large number of small objects, the CMs on each object can be used as the entire domain basis function. Therefore, the MoM using these entire domain CM basis functions involves a significantly reduced number of unknowns. For this reason, it allows the analysis of large multiple scattering problems using direct methods.

At Università della Calabria, Italy, Massa and his colleagues carried out many interesting works to solve multiple scattering problems using CMs [201–211]. Their research began with the multiple scattering by arbitrary-shaped conducting cylinders [201–203]. The use of CMs as entire domain basis functions in the MoM was proved to be very effective in the analysis of the scattering from collections of arbitrarily shaped cylinders. As an extension to their previous study, CMs of elliptic cylinder were used in the computation of the TM scattering from a collection of elliptic cylinders [204]. Both computational efficiency and accuracy were demonstrated through the multiple scattering computations.

Following the same idea, the scattering from large microstrip antenna arrays was simulated by using the CMs of each antenna element as entire domain basis functions. In addition to the scattering analysis, mutual coupling [207] and radiation performance [208] of large microstrip antenna arrays were also addressed by using the CMs as entire domain basis functions in the MoM.

1.5 OUTLINE OF THIS BOOK

With the great support from Temasek Laboratories at National University of Singapore, the authors have devoted great effort in the theoretical developments as well as CM-based methodology development for a variety of critical antenna designs. Our studies on CM theory along with those from other research groups are scattered throughout many technical reports, journal, and conference papers. The intention of this book is to compile and organize these advanced research achievements made in the area of CM studies. It is the first comprehensive book on the CM theory and its applications. Thus, it will be an invaluable book for antenna researchers, engineers, and students who are seeking for antenna design methodology and concept with clear underlying physics in their antenna research and development.

Characteristic Modes: Theory and Applications in Antenna Engineering is organized into six chapters. Chapter 1 gives an introduction and review of the history developments as well as the recent advances of the CM theory and its applications in antenna engineering.

In Chapter 2, the CM theory for PEC bodies is first formulated using the EFIE. CM formulations based on the magnetic field integral equation, approximate magnetic field integral equation, and combined field integral equation (CFIE) are then discussed. This chapter also addresses the numerical implementation of the CM theory. Numerical techniques for solving generalized eigenvaule equation and tracking CMs across a wide frequency band are discussed. The physics of the characteristic modes and its relationship with the spherical modes are illustrated through numerical examples. Numerical aspects of CM computation are also discussed. Finally, the CMs of a planar inverted-F antenna are given to provide a first glance at how to apply the CM theory in practical antenna designs.

Chapter 3 discusses the CM theory for structures embedded in multilayered medium. The underlying physics of the CM theory is revealed through the comparison with the cavity modal solutions of a rectangular patch antenna printed on a dielectric substrate. The CM analysis for a triangular microstrip antenna and a concentric ring antenna is further revealed to show how the presented CM theory can be beneficial to circular polarized and pattern reconfigurable multimode microstrip antenna designs. In addition, circularly polarized microstrip antennas with corner cutting, U-slot, and E-shaped patch are described. These examples illustrate how CM-based designs can enhance the axial ratio and cross-polarization performances. It also shows that the presented CM theory allows engineers to determine the axial ratio bandwidth prior to the feeding structure design. Therefore, one can focus their effort on the feeding design once the CM analysis indicates that the antenna structure has the potential to radiate CP waves in a particular frequency band.

Chapter 4 presents the recent advances in the CM theory for dielectric bodies and its promising applications to the design of dielectric resonant antennas. Two generalized eigenvalue equations based on the PMCHWT surface integral equation formulation are introduced to predict the resonant frequencies of DRAs. Brief discussions on the CM formulation for dielectric bodies proposed in the 1970s are provided to show the essences of our newly developed CM formulations. CM analysis results for cylindrical, spherical, rectangular, notched rectangular and triangular DRAs are presented and discussed. These typical examples obtained clearly demonstrate that the presented CM theory is very promising in the understanding and designing of feeding probes for obtaining certain radiating mode. Guidelines for the excitation of a particular mode using either the coaxial probe feeding or aperture coupling feeding are presented to show how to apply the CM theory in practical DRA designs.

Chapter 5 is devoted to the CM theory for *N*-port networks and its applications in the optimization of antenna arrays. The completeness and orthogonality properties of the CMs are taken into account in the antenna array optimizations. In comparison to those array optimizations involving full-wave analysis, the efficiency of the CM-based optimization method is improved up to three orders. The mutual coupling is also considered in the CM-based method, and thus the accuracy is kept the same as that in the full-wave analysis. Concentric ring antenna arrays for main beam steering and sidelobe suppression are presented. CM-based optimal design of Yagi-Uda antennas for high directivity and low sidelobe is also presented to show the flexibility of the method. Finally, to improve the bandwidth of tightly coupled antenna arrays, the synthesis of termination conditions for edge elements using the present CM theory is presented.

Chapter 6 discusses the design of platform-integrated antenna systems using CMs. Recent advancements in antenna system designs on aircraft and ship are introduced. Using the CMs of the platforms, the current distributions on the platforms can be efficiently synthesized for any desired radiation patterns. The feedings for the excitation of these currents can either have a very low profile or be conformal to the surface of the platforms. This speciality makes the CM-based method very attractive in many practical applications, especially those operating at the HF, VHF, and UHF frequency bands. Details of the design procedure and the experimental results are presented to show how to make use of the CM theory in these designs.

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