1

Principles of Waveguides

1.1 Introduction

A waveguide can be defined as a structure that guides waves, such as electromagnetic or sound waves [1]. In this chapter, the basic principles of the optical waveguide will be introduced. Optical waveguides can confine and transmit light over different distances, ranging from tens or hundreds of micrometers in integrated photonics, to hundreds or thousands of kilometers in long-distance fiber-optic transmission. Additionally, optical waveguides can be used as passive and active devices such as waveguide couplers, polarization rotators, optical routers, and modulators. There are different types of optical waveguides such as slab waveguides, channel waveguides, optical fibers, and photonic crystal waveguides. The slab waveguides can confine energy to travel only in one dimension, while the light can be confined in two dimensions using optical fiber or channel waveguides. Therefore, the propagation losses will be small compared to wave propagation in open space. Optical waveguides usually consist of high index dielectric material surrounded by lower index material, hence, the optical waves are guided through the high index material by a total internal reflection mechanism. Additionally, photonic crystal waveguides can guide the light through low index defects by a photonic bandgap guiding technique. Generally, the width of a waveguide should have the same order of magnitude as the wavelength of the guided wave.

In this chapter, the basic optical waveguides are discussed including waveguides operation, Maxwell's equations, the wave equation and its solutions, boundary conditions, phase and group velocity, and the properties of modes.

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1.2 Basic Optical Waveguides

Optical waveguides can be classified according to their geometry, mode structure, refractive index distribution, materials, and the number of dimensions in which light is confined [2]. According to their geometry, they can be categorized by three basic structures: planar, rectangular channel, and cylindrical channel as shown in Figure 1.1. Common optical waveguides can also be classified based on mode structure as single mode and multiple modes. Figure 1.1a shows that the planar waveguide consists of a core that must have a refractive index higher than the refractive indices of the upper medium called the cover, and the lower medium called the substrate. The trapping of light within the core is achieved by total internal reflection. Figure 1.1b shows the channel waveguide which represents the best choice for fabricating integrated photonic devices. This waveguide consists of a rectangular channel that is sandwiched between an underlying planar substrate and the upper medium, which is usually air. To trap the light within a rectangular channel, it is necessary for the channel to have a refractive index greater than that of the substrate. Figure 1.1c shows the geometry of the cylindrical channel waveguide which consists of a central region, referred to as the core, and surrounding material called cladding. Of course, to confine the light within the core, the core must have a higher refractive index than that of the cladding.

Figure 1.2 shows the three most common types of channel waveguide structures which are called strip, rip, and buried waveguides. It is evident from the figure that the main difference between the three types is in the shape and the size of the film deposited onto the substrate. In the strip waveguide shown in Figure 1.2a, a high index film is directly deposited on the substrate with finite width. On the other hand, the rip waveguide is formed by depositing a high index film onto the substrate and performing an incomplete etching around a finite width as shown in Figure 1.2b. Alternatively, in the case of the buried waveguide shown in Figure 1.2c,

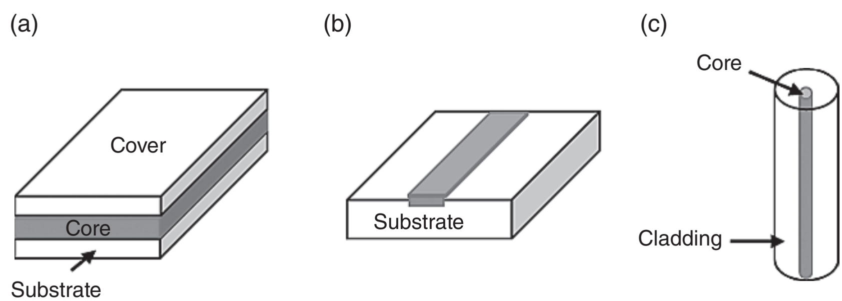

Figure 1.1 Common waveguide geometries: (a) planar, (b) rectangular, and (c) cylindrical

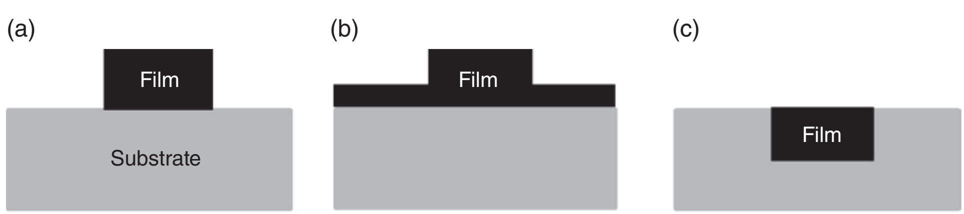

Figure 1.2 Common channel waveguides: (a) strip, (b) rip, and (c) buried

diffusion methods [2] are employed in order to increase the refractive index of a certain zone of the substrate.

Figure 1.3 shows the classification of optical waveguides based on the number of dimensions in which the light rays are confined. In planar waveguides, the confinement of light takes place in a single direction and so the propagating light will diffract in the plane of the core. In contrast, in the case of channel waveguides, shown in Figure 1.3b, the confinement of light takes place in two directions and thus diffraction is avoided, forcing the light propagation to occur only along the main axis of the structure. There also exist structures that are often called photonic crystals that confine light in three dimensions as revealed from Figure 1.3c. Of course, the light confinement in this case is based on Bragg reflection. Photonic crystals have very interesting properties, and their use has been proposed in several devices, such as waveguide bends, drop filters, couplers, and resonators [3].

Classification of optical waveguides according to the materials and refractive index distributions results in various optical waveguide structures, such as step index fiber, graded index fiber, glass waveguide, and semiconductor waveguides. Figure 1.4a shows the simplest form of step index waveguide that is formed by a homogenous cylindrical core with constant

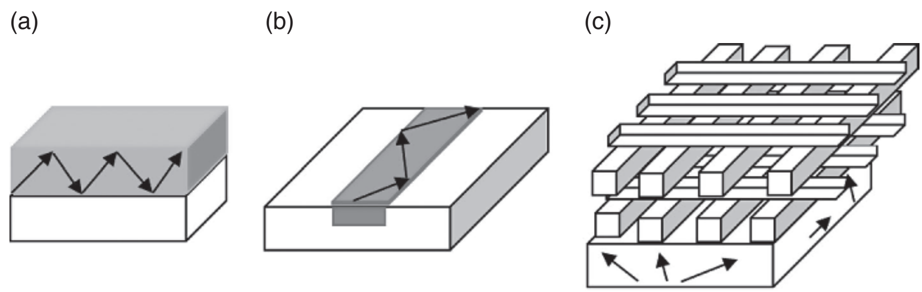

Figure 1.3 Common waveguide geometries based on light confinement: (a) planar waveguide, (b) rectangular channel waveguide, and (c) photonic crystals

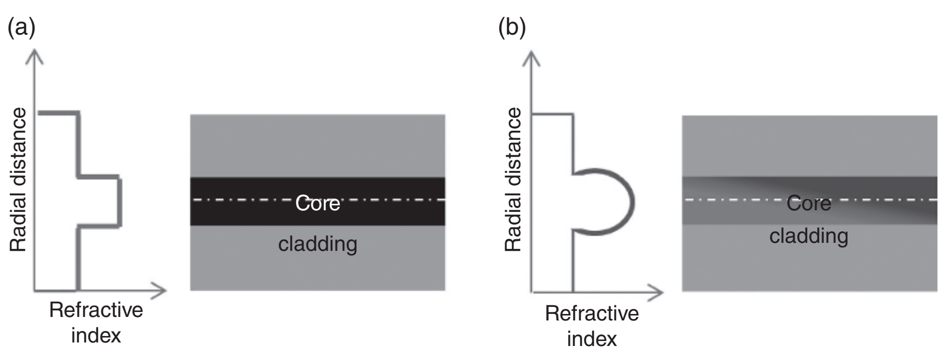

Figure 1.4 Classification of optical waveguide based on the refractive index distributions: (a) step-index optical fiber and (b) graded-index optical fiber

Table 1.1 The differential and integral forms of Maxwell's equations

Differential form	Integral form	
$\nabla \times E = \frac{-\partial B}{\partial t}$	$\oint_{\text{loop}} E \cdot dl = -\frac{\partial}{\partial t} \oint_{\text{area}} B \cdot dS$	(1.1)
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_{\text{loop}} H \cdot dl = \int_{\text{area}} J \cdot dS + \frac{\partial}{\partial t} \int_{\text{area}} D \cdot dS$	(1.2)
$\nabla \cdot B = 0$	$\int_{\text{surface}} B \cdot dS = 0$	(1.3)
$\nabla \cdot D = \rho$	$\int_{\text{surface}} D \cdot dS = Q_{\text{enclosed}}$	(1.4)

refractive index surround by cylindrical cladding of a different, lower index. Figure 1.4b shows the graded index planar waveguide where the refractive index of the core varies as a function of the radial distance [4].

1.3 Maxwell's Equations

Maxwell's equations are used to describe the electric and magnetic fields produced from varying distributions of electric charges and currents. In addition, they can explain the variation of the electric and magnetic fields with time. There are four Maxwell's equations for the electric and magnetic field formulations. Two describe the variation of the fields in space due to sources as introduced by Gauss's law and Gauss's law for magnetism, and the other two explain the circulation of the fields around their respective sources. In this regard, the magnetic field moves around electric currents and time varying electric fields as described by Ampère's law as well as Maxwell's addition. On the other hand, the electric field circulates around time varying magnetic fields as described by Faraday's law. Maxwell's equations can be represented in differential or integral form as shown in Table 1.1. The integral forms of the curl equations can be derived from the differential forms by application of Stokes' theorem.

where E is the electric field amplitude (V/m), H is the magnetic field amplitude (A/m), D is the electric flux density (C/m^2), B is the magnetic flux density (T), J is the current density (A/m^2), ρ is the charge density (C/m^3), and Q is the charge (C). It is worth noting that the flux densities, D and B, are related to the field amplitudes E and H for linear and isotropic media by the following relations:

$$B = \mu H. \tag{1.5}$$

$$D = \varepsilon E. \tag{1.6}$$

$$J = \sigma E. \tag{1.7}$$

Here, $\varepsilon = \varepsilon_o \varepsilon_r$ is the electric permittivity (F/m) of the medium, $\mu = \mu_o \mu_r$ is the magnetic permeability of the medium (H/m), σ is the electric conductivity, ε_r is the relative dielectric constant, $\varepsilon_o = 8.854 \times 10^{-12}$ F/m is the permittivity of free space, and $\mu_o = 4\pi \times 10^{-7}$ H/m is the permeability of free space.

1.4 The Wave Equation and Its Solutions

The electromagnetic wave equation can be derived from Maxwell's equations [2]. Assuming that we have a source free ($\rho=0$, $J=0$), linear (ε and μ are independent of E and H), and isotropic medium. This can be obtained at high frequencies ($f > 10^{13}$ Hz) where the electromagnetic energy does not originate from free charge and current. However, the optical energy is produced from electric or magnetic dipoles formed by atoms and molecules undergoing transitions. These sources are included in Maxwell's equations by the bulk permeability and permittivity constants. Therefore, Maxwell's equations can be rewritten in the following forms:

$$\nabla \times E = \frac{-\partial B}{\partial t} \tag{1.8}$$

$$\nabla \times H = \frac{\partial D}{\partial t} \tag{1.9}$$

$$\nabla \cdot B = 0 \tag{1.10}$$

$$\nabla \cdot D = 0 \tag{1.11}$$

The resultant four equations can completely describe the electromagnetic field in time and position. It is revealed from Eqs. (1.8) and (1.9) that Maxwell's equations are coupled with first-order differential equations. Therefore, it is difficult to apply these equations when solving boundary-value problems. This problem can be solved by decoupling the first-order equations, and hence the wave equation can be obtained. The wave equation is a second-order differential equation which is useful for solving waveguide problems. To decouple Eqs. (1.8) and (1.9), the curl of both sides of Eq. (1.8) is taken as follows:

$$\nabla \times (\nabla \times E) = \nabla \times \frac{-\partial B}{\partial t} = \nabla \times \frac{-\partial \mu H}{\partial t} \tag{1.12}$$

If $\mu(r, t)$ is independent of time and position, Eq. (1.12) becomes thus:

$$\nabla \times (\nabla \times E) = -\mu \left(\nabla \times \frac{\partial H}{\partial t} \right) \tag{1.13}$$

Since the functions are continuous, Eq. (1.13) can be rewritten as follows:

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} (\nabla \times H) \tag{1.14}$$

Substituting Eq. (1.9) into Eq. (1.14) and assuming ε is time invariant, we obtain the following relation:

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} \left(\frac{\partial D}{\partial t} \right) = -\mu\varepsilon \frac{\partial^2 E}{\partial t^2} \tag{1.15}$$

The resultant equation is a second-order differential equation with $(\nabla \times \nabla \times)$ operator and with only the electric field E as one variable. By applying the vector identity,

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E, \tag{1.16}$$

where the ∇^2 operator in Eq. (1.16) is the vector Laplacian operator that acts on the E vector [2]. The vector Laplacian can be written in terms of the scalar Laplacian for a rectangular coordinate system, as given by

$$\nabla^2 E = \nabla^2 E_x \,\hat{x} + \nabla^2 E_y \,\hat{y} + \nabla^2 E_z \,\hat{z}, \tag{1.17}$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors along the three axes. Additionally, the scalar ∇^2's on the right-hand side of Eq. (1.17) can be expressed in Cartesian coordinates:

$$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}. \tag{1.18}$$

In order to obtain $\nabla \cdot E$, Eq. (1.11) can be used as follows:

$$\nabla \cdot \varepsilon E = \nabla \varepsilon \cdot E + \varepsilon \nabla \cdot E = 0 \tag{1.19}$$

As a result, $\nabla \cdot E$ can be obtained as follows:

$$\nabla \cdot E = -E \cdot \frac{\nabla \varepsilon}{\varepsilon} \tag{1.20}$$

Substituting Eqs. (1.16) and (1.20) into Eq. (1.15) results in

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = -\nabla\left(E \cdot \frac{\nabla \varepsilon}{\varepsilon}\right) \tag{1.21}$$

If there is no gradient in the permittivity of the medium, the right-hand side of Eq. (1.21) will be zero. Actually, for most waveguides, this term is very small and can be neglected, simplifying Eq. (1.21) to

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \tag{1.22}$$

Equation (1.22) is the time-dependent vector Helmholtz equation or simply the wave equation. A similar wave equation can be obtained as a function of the magnetic field by starting from Eq. (1.9).

$$\nabla^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0 \tag{1.23}$$

Equations (1.22) and (1.23) are the equations of propagation of electromagnetic waves through the medium with velocity u:

$$u = \frac{1}{\sqrt{\mu\varepsilon}} \tag{1.24}$$

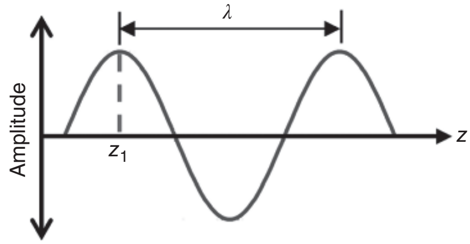

Figure 1.5 A traveling wave along the *z*-axis

It is worth noting that each of the electric and magnetic field vectors in Eqs. (1.23) and (1.24) has three scalar components. Consequently, six scalar equations for E_x, E_y, E_z, H_x, H_y, and H_z can be obtained. Therefore, the scalar wave equation can be rewritten as follows:

$$\nabla^2\Psi - \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{1.25}$$

Here, Ψ is one of the orthogonal components of the wave equations. The separation of variables technique can be used to have a valid solution [2].

$$\Psi(r,t) = \Psi(r)\phi(t) = \Psi_o \exp(jk\cdot r)\exp(j\omega t) \tag{1.26}$$

Here, Ψ_o is the amplitude, k is the separation constant which is well known as the wave vector (rad/m), and ω is the angular frequency of the wave (rad/s). The wave vector k will be used as a primary variable in most waveguide calculations. The magnitude of the wave vector that points in the propagation direction of a plane wave can be expressed as follows:

$$k = \omega\sqrt{\mu\varepsilon} \tag{1.27}$$

If the wave propagates along the z-axis, the propagation direction can be in the forward direction along the $+z$-axis with exponential term exp $(j\omega t - jkz)$ [2]. However, the propagation will be in the backward direction with $\exp(j\omega t + jkz)$. Figure 1.5 shows the real part of the spatial component of a plane wave traveling in the z direction, $\Psi(z) = \Psi_o \exp(jkz)$. The amplitude of the wave at the first peak and the adjacent peak separated by a wavelength are equal, such that

$$e^{jkz_1} = e^{jk(z_1+\lambda)} = e^{jkz_1}e^{jk\lambda} \tag{1.28}$$

Therefore, $e^{jk\lambda} = 1$, and hence $k\lambda = 2\pi$ which results in

$$k = \frac{2\pi}{\lambda} \tag{1.29}$$

1.5 Boundary Conditions

The waveguide in which the light is propagated is usually characterized by its conductivity σ, permittivity ε, and permeability μ. If these parameters are independent of direction, the material will be isotropic; otherwise it will be anisotropic. Additionally, the material is

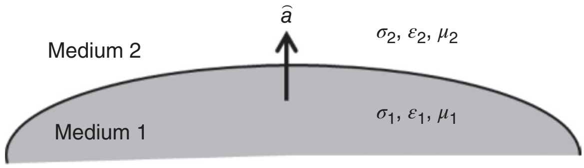

Figure 1.6 Interface between two mediums

homogeneous if σ, ε, and μ are not functions of space variables; otherwise, it is inhomogeneous. Further, the waveguide is linear if σ, ε, and μ are not affected by the electric and magnetic fields; otherwise, it is nonlinear. The electromagnetic wave usually propagates through the high index material surrounded by the lower index one. Therefore, the boundary conditions between the two media should be taken into consideration. Figure 1.6 shows the interface between two different materials 1 and 2 with the corresponding characteristics (σ_1, ε_1, μ_1) and (σ_2, ε_2, μ_2), respectively. The following boundary conditions at the interface [3] can be obtained from the integral form of Maxwell's equations with no sources, (ρ, $J=0$):

$$\hat{a}\times\left(E_{2t}-E_{1t}\right)=0 \tag{1.30}$$

$$\hat{a}\times\left(H_{2t}-H_{1t}\right)=0 \tag{1.31}$$

$$\hat{a}\cdot\left(B_{2n}-B_{1n}\right)=0 \tag{1.32}$$

$$\hat{a}\cdot\left(D_{2n}-D_{1n}\right)=0 \tag{1.33}$$

Here, $\hat{a}$ is a unit vector normal to the interface between medium 1 and medium 2, and subscripts t and n refer to tangent and normal components of the fields. It is revealed from Eqs. (1.30) and (1.31) that the tangential components of E and H are continuous across the boundary. In addition, the normal components of B and D are continuous through the interface, as shown in Eqs. (1.32) and (1.33), respectively.

1.6 Phase and Group Velocity

1.6.1 Phase Velocity

The propagation velocity of the electromagnetic waves is characterized by the phase velocity and the group velocity. Consider a traveling sinusoidal electromagnetic wave in the z direction. A point on one crest of the wave with specific phase must move at specific velocity to stay on the crest such that [5]

$$e^{-j(kz-\omega t)}=\text{constant} \tag{1.34}$$

This can be obtained if $(kz-\omega t)=$constant, and hence $z(t)$ must satisfy the following:

$$z(t)=\frac{\omega t}{k}+\text{constant} \tag{1.35}$$

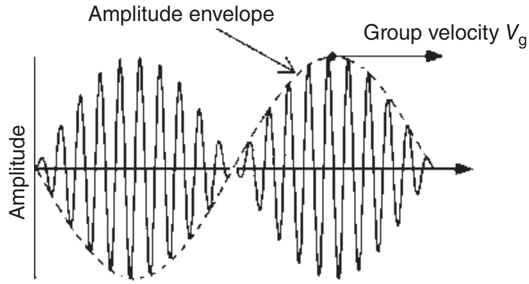

Figure 1.7 The superposition of two waves of different frequencies

The phase velocity, $v(t)=v_p$ can be obtained by differentiating $z(t)$ with respect to time as follows:

$$\frac{dz}{dt}=\frac{\omega}{k}=v_p \tag{1.36}$$

Therefore, the phase velocity v_p is a function of the angular frequency $\left(\omega = k/\sqrt{\mu\varepsilon}\right)$ and the magnitude of the wave vector. Then, the phase velocity can be rewritten in the following form:

$$v_p=\frac{1}{\sqrt{\mu\varepsilon}} \tag{1.37}$$

1.6.2 Group Velocity

The group velocity v_g [5] is used to describe the propagation speed of a light pulse. The group velocity can be expressed by studying the superposition of two waves of equal amplitude E_o but with different frequencies $\omega_1=\omega+\Delta\omega$, and $\omega_2=\omega-\Delta\omega$. Additionally, the corresponding wave vectors will be $k_1=k+\Delta k$, $k_2=k-\Delta k$, respectively. The superposition between the two waves can be expressed as follows:

$$E_t=E_1+E_2=E_o\left(\cos\left[(\omega+\Delta\omega)t-(k+\Delta k)z\right]+\cos\left[(\omega-\Delta\omega)t-(k-\Delta k)z\right]\right) \tag{1.38}$$

The resultant electric field can be rewritten as follows:

$$E_t=2E_o\cos(\omega t-kz)\cos(\Delta\omega t-\Delta kz). \tag{1.39}$$

Therefore, a temporal beat at frequency $\Delta\omega$ and a spatial beat with period Δk is obtained, as shown in Figure 1.7. The envelope of the amplitude [2, 5] can be described by the $\cos(\Delta\omega t-\Delta kz)$ term and has a velocity equal to group the velocity v_g.

The group velocity can be obtained using the same procedure used in the calculation of the phase velocity. A point attached to the crest of the envelope, should move with a given speed to stay on the crest of the envelope, and hence the phase $(\Delta\omega t-\Delta kz)$ is constant.

Therefore, $z(t)$ can be expressed as follows:

$$z(t)=\frac{\Delta\omega t}{\Delta k}+\text{constant} \tag{1.40}$$

By applying the derivative of $z(t)$, the group velocity can be obtained as [5]:

$$v_g = \frac{dz(t)}{dt} = \frac{\Delta\omega}{\Delta k} \Rightarrow v_g = \lim_{\Delta\omega \to 0} \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}. \tag{1.41}$$

This means that the group velocity, v_g, is based on the first derivative of the angular frequency ω with respect to the wave vector k. Since $k = \omega n/c$, where n is the frequency-dependent material refractive index, and c is the speed of the light in a vacuum, $dk/d\omega$ can be expressed as follows:

$$\frac{dk}{d\omega} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega} = \frac{n + \omega(dn/d\omega)}{c} \tag{1.42}$$

Therefore, the group velocity can be obtained as [5]:

$$v_g = \frac{d\omega}{dk} = \left[\frac{dk}{d\omega}\right]^{-1} = \frac{c}{n - \lambda(dn/d\lambda)} \tag{1.43}$$

This can be rewritten in terms of the group index of a medium N_g as follows:

$$v_g = \frac{c}{N_g} \tag{1.44}$$

Here,

$$N_g = n - \lambda \frac{dn}{d\lambda} \tag{1.45}$$

The refractive index of the dispersive medium and hence the group index are wavelength dependent. Therefore, the phase velocity and group velocity are wavelength dependent.

1.7 Modes in Planar Optical Waveguide

In this section, we will discuss the light behavior inside the planar waveguide, shown previously in Figure 1.1a in order to estimate the propagated modes. We assume that the refractive index of the sandwiched film n_f is greater than those of the upper cover n_c and lower substrate n_s, and also that the refractive index of the substrate is greater than that of the cover. Now, we can define the critical angles for the cover–film interface θ_{1c} and for the film–substrate interface θ_{2c} as follows:

$$\theta_{1c} = \sin^{-1}\left(\frac{n_c}{n_f}\right). \tag{1.46}$$

$$\theta_{2c} = \sin^{-1}\left(\frac{n_s}{n_f}\right). \tag{1.47}$$

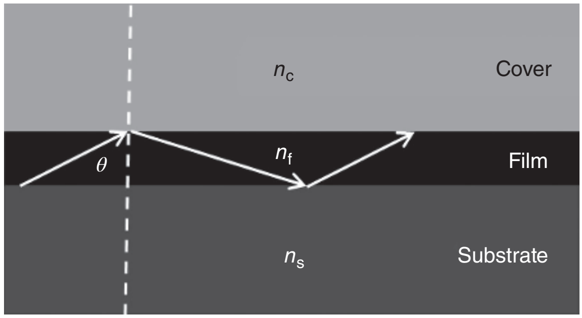

Figure 1.8 Zigzag trajectory of a confinement ray inside the film of a planar waveguide

Based on the assumption that $n_f > n_s > n_c$, we can say that $\theta_{2c} > \theta_{1c}$. Further, based on values of the propagating angle θ of the light inside the film shown in Figure 1.8, we can classify two types of modes: radiation and confinement.

1.7.1 Radiation Modes

These can be obtained in the following two cases:

1. If the propagating angle is less than the critical angle for the cover–film interface ($\theta < \theta_{1c}$), thus the propagating angle will also be less than the critical angle for the film–substrate interface ($\theta < \theta_{2c}$), and hence the radiation will travel in the three zones generating radiation modes.
2. If the propagating angle is greater than the critical angle for the cover–film interface ($\theta > \theta_{1c}$), and less than the critical angle for the film–substrate interface ($\theta < \theta_{2c}$), then, the light cannot penetrate the cover but can penetrate the substrate zone generating also substrate radiation modes.

1.7.2 Confinement Modes

These types of modes can be obtained if the propagating angle is greater than the critical angle for the film–substrate interface ($\theta > \theta_{2c}$), and less than $\pi/2$. In this way, the light cannot penetrate either the substrate or the cover and is totally confined in the film zone generating confinement modes where the light propagates inside the film along a zigzag path.

1.8 Dispersion in Planar Waveguide

The slab waveguide can support number of modes at which each mode can propagate with a different propagation constant. This will occur if the slab waveguide is illuminated by monochromatic radiation. It was thought that the axial ray will arrive more quickly than higher-mode rays with longer zigzag paths. However, the group velocity v_g, at which the energy or information is transported, should be taken into account [6]. Additionally, the higher-order modes penetrate more into the cladding, where the refractive index is smaller and the waves travel faster.

The group velocity v_g depends on the frequency ω and the mode propagation constant β. Therefore, the group velocity of a given mode is a function of the light frequency and the waveguide optical properties. It is worth noting that the group velocity v_g of the guided modes is frequency dependent even if the refractive indices of the composing materials are nearly constant. The dependence of the propagation constant and hence group velocity on the frequency can be called **dispersion**.

1.8.1 Intermodal Dispersion

The signal distortion that occurs in multimode fibers and other waveguides is called modal or intermodal dispersion [6]. In this case, the signal is spread in time due to the different propagation velocities for all modes of the optical signal. As a result, a number of allowed propagated modes will be excited through the waveguide when a short-duration light pulse signal is incident on the waveguide. Each mode has its own group velocity. Consequently, a broadened signal will be obtained at the receiver due to the combination of the different modes. To avoid the modal dispersion, single-mode waveguide that allows only one propagated mode can be used.

1.8.2 Intramodal Dispersion

It is worth noting that there is no ideal monochromatic light wave. Therefore, the single mode operation will also consist of various frequencies that constitute the finite spectrum of the source excitation. Each frequency propagates with specific velocity and then arrives the receiver at a given time which results in **waveguide dispersion**. Figure 1.9 shows two different wavelengths (λ_1 and λ_2) that propagate in the core region where ($\lambda_1 < \lambda_2$). As the wavelength increases, the confinement of the mode through the core region will be decreased. Therefore, the penetration of the mode through the cladding region will be increased with higher phase velocity as shown in Figure 1.9.

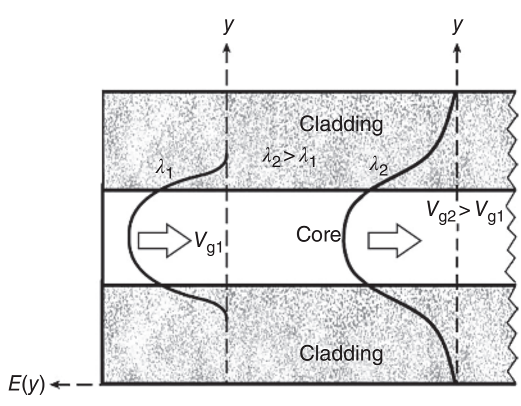

Figure 1.9 Electric field of a fundamental mode through a slab waveguide at different wavelengths (λ_1 and λ_2) where ($\lambda_1 < \lambda_2$)

The refractive index of the waveguide material is wavelength dependent which affects the propagation constant and hence the group velocity. As a result, the propagating light pulse will be broadened which is called **material dispersion** [6]. The combined effect of the waveguide and material dispersion is called **intramodal dispersion**.

1.9 Summary

In this chapter, the definition and different types of optical waveguides are first introduced. Then, Maxwell's equations and the derivation of the wave equation and its solution are presented. Additionally, the basic boundary conditions between two different mediums are discussed. Finally, the phase and group velocity, and types of propagated modes are introduced. Moreover, the dispersion in the planar waveguide is discussed.

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