1

Scaling of Forces

There are a number of important forces in the field of microelectromechanical systems (MEMS). However, their relative importance does not necessarily match the importance they have in the macroworld. This chapter is concerned with the scaling of these forces to small dimensions. Weight, elastic, electrostatic, capillary, piezoelectric, magnetic and dielectrophoretic forces are examined and a scaling factor identified for all of them.

1.1 Scaling of Forces Model

The integration of complex and powerful systems in silicon for a large variety of applications stems from the miniaturization of electronic devices and components. Electromechanical components that were bulky, heavy and inefficient can now be miniaturized using MEMS technology. Here, mechanical moving parts are used both for sensing devices and actuators. The main forces present in the operation of these components depend on the geometrical dimensions, and thereby, when the dimensions are scaled down, the magnitudes of these forces change, creating a different scenario compared to the macroworld.

Given a force F that depends on a number of geometrical dimensions a_i and on a number of parameters γ_j, we have

$$F = F(a_i, \gamma_j). \tag{1.1}$$

When all dimensions are scaled by the same factor α, the force changes to

$$F_\alpha = F(\alpha a_i, \gamma_j), \tag{1.2}$$

provided all the parameters γ_j do not depend on the geometrical dimensions. The ratio of the forces before and after the dimension scaling is given by

$$\frac{F_\alpha}{F} = \frac{F(\alpha a_i, \gamma_j)}{F(a_i, \gamma_j)}. \tag{1.3}$$

Understanding MEMS: Principles and Applications, First Edition. Luis Castañer.

Companion Website: www.wiley.com/go/castaner/understandingmems

Generally, when analytical models are used in simplified cases, the result of equation (1.3) provides a direct relation to a power n of the scaling factor α,

$$\frac{F_\alpha}{F} = \alpha^n, \tag{1.4}$$

meaning that when the dimensions are scaled down by a factor α, the force scales down by a factor α^n.

1.2 Weight

As our first application of the rule provided by equation (1.3) we consider the scaling of weight. Imagine that we have a body of length L, width W and thickness t. The weight of this body is given by

$$F = \rho_m g L W t, \tag{1.5}$$

where ρ_m is the material density and g the acceleration due to gravity. If all dimensions are scaled by a factor α, the length becomes αL, the width becomes αW and the thickness becomes αt, and so the scaled weight is

$$F_\alpha = \rho_m g \alpha L \alpha W \alpha t = \alpha^3 \rho_m g L W t, \tag{1.6}$$

and the ratio of forces after and before scaling is given by

$$\frac{F_\alpha}{F} = \alpha^3. \tag{1.7}$$

Equation (1.7) tells us that the weight scales down as the third power of the scaling factor, so if we reduce all dimensions by a factor of 10 ($\alpha = 0.1$), the weight is multiplied by a factor of $\alpha^3 = 0.001$).

It will become clear in the next sections that when electromechanical structures are miniaturized, the weight loses the importance it has in the macroworld and other forces become the main players.

1.2.1 Example: MEMS Accelerometer

A MEMS accelerometer has an inertial mass made up of a plate of silicon bulk material of 500 μm *side and* 500 μm *thickness. Calculate the force developed when subject to an acceleration ten times that due to gravity.*

Taking into account that the density of silicon is $2329\,\text{kg/m}^3$ and that the volume is $500 \times 500 \times 500 \times 10^{-18}\,\text{m}^3$, the force is given by

$$F = 2.85 \times 10^{-5}\,\text{N}.$$

If all dimensions are reduced by a factor of 10 ($\alpha = 0.1$) the weight reduces to $F_\alpha = 2.85 \times 10^{-8}$ N.

1.3 Elastic Force

A body is deformed when it is subject to an external force. In equilibrium, the elastic force is the restoring force that compensates the external force. If the deformation is elastic, the initial dimensions of the body are recovered after the external force disappears. In a one-dimensional geometry and according to Hooke's law [1], the elastic force, F, is proportional to the deformation length δ, collinear with the force,

$$F = k\delta, \tag{1.8}$$

where k is the stiffness constant that is not independent of the geometry as will be shown in Chapter 3; for example, for a cantilever of rectangular cross-section with length L, width W and thickness t, subject to a force applied at the tip (see Figure 1.1), the stiffness is given by

$$k = \frac{EWt^3}{4L^3}, \tag{1.9}$$

where E is Young's elasticity modulus. We now proceed as in Section 1.1 and calculate the forces F and F_α before and after scaling:

$$F = \frac{EWt^3}{4L^3}\delta, \quad F_\alpha = \frac{E\alpha W\alpha^3 t^3}{4\alpha^3 L^3}\alpha\delta = \alpha^2\frac{EWt^3}{4L^3}\delta. \tag{1.10}$$

The ratio between these two quantities is therefore

$$\frac{F_\alpha}{F} = \alpha^2. \tag{1.11}$$

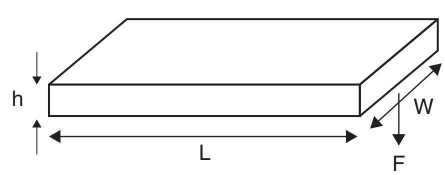

Figure 1.1 Geometry of a cantilever loaded at the tip

1.3.1 Example: AFM Cantilever

In atomic force microscopy tiny cantilevers with a very sharp tip are used to detect the force. The cantilever acts as a soft spring. Calculate the force that will deflect the cantilever by 1 μm *for* $L = 200$ μm, $W = 5$ μm *and* $h = 2$ μm.

By equation (1.9), $k = 0.081$ N/m, and by equation (1.8),

$$F = 8.1 \times 10^{-8}\text{N}.$$

Applying a dimension scaling with $\alpha = 0.1$, the force reduces to $F_\alpha = 8.1 \times 10^{-10}$ N.

1.4 Electrostatic Force

The electrostatic force between two plates is due to the electric field, **E**, that builds up when an electric potential V is applied between them.[1] This is a very common way to make mechanical parts move in today's microelectromechanical devices.

If we consider one of the two plates charged with a charge density σ, as shown in Figure 1.2(a), Gauss's law [2] allows to calculate the electric field created by the charged sheet as

$$\oint \vec{D}\vec{dS} = \int \sigma dS = Q. \tag{1.12}$$

Signs in equation (1.12) are taken as positive for an electric field directed outward from the differential volume, and $\vec{dS}$ is taken positive also directed outward from the face. As the electric field is normal to the charged surface, only integrals extending over the top and bottom surfaces of the volume are different from zero, so that

$$\int_{\text{top}} \epsilon \mathbf{E} dS + \int_{\text{bottom}} \epsilon \mathbf{E} dS = Q, \tag{1.13}$$

$$\epsilon \mathbf{E} A + \epsilon \mathbf{E} A = Q, \tag{1.14}$$

where A is the area of the surface. Then

$$\mathbf{E} = \frac{Q}{2\epsilon A}. \tag{1.15}$$

The Coulomb force that such a field exerts on the parallel plate with a charge of $-Q$ and at a distance g is

$$F = -Q\mathbf{E} = -\frac{Q^2}{2\epsilon A}. \tag{1.16}$$

[1] We denote the electric field by **E** to distinguish it from the Young's modulus E.

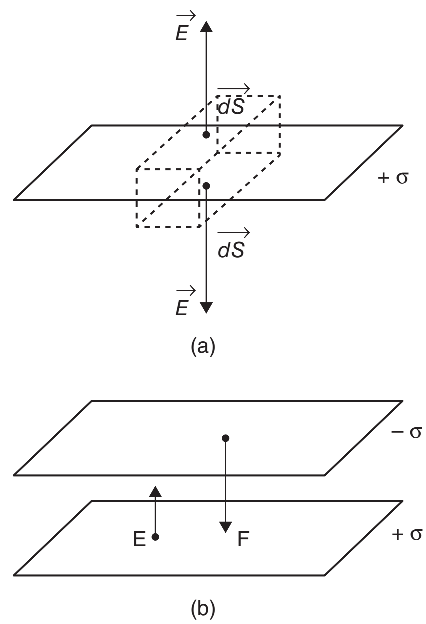

Figure 1.2 (a) Gauss's law for a sheet of charge σ, and (b) electric field and Coulomb force exerted on the upper plate

Since $Q = CV$ and $C = \epsilon A/g$,

$$F = \frac{\epsilon A V^2}{2g^2}. \tag{1.17}$$

As can be seen the force is downwards, that is to say, it is attractive between the plates and does not depend on the sign of the applied voltage as it is squared in the force equation (1.17). When we apply the scaling method we find that

$$F_\alpha = \frac{\epsilon A \alpha^2 V^2}{2g^2 \alpha^2}. \tag{1.18}$$

In equation (1.18), A is the area of the plates which scales as α^2, and g is the value of the gap between plates. The scaling factor of the force is

$$\frac{F_\alpha}{F} = \alpha^0 = 1. \tag{1.19}$$

This is a very important result showing that the electrostatic force is independent of the scaling factor and can be very high compared to other forces in the microworld. However, it can be correctly argued that reducing the distance between plates increases the electric field and the devices may be damaged by breakdown. To prevent this situation, we can consider a different scaling scenario in which the value of the electric field is kept constant. As the electric field is $\mathbf{E} = V/g$, equation (1.18) can be written as

$$F = \frac{\epsilon A V^2}{2g^2} = \frac{\epsilon A \mathbf{E}^2 g^2}{2g^2} = \frac{\epsilon A \mathbf{E}^2}{2}, \quad F_\alpha = \frac{\epsilon A \alpha^2 \mathbf{E}^2}{2} \tag{1.20}$$

and hence

$$\frac{F_\alpha}{F} = \alpha^2. \tag{1.21}$$

Here we see that in this scenario the scaling follows an α^2 rule instead.

1.4.1 Example: MEMS RF Switch

In a MEMS RF switch two metal plates 250×250 μ*m*^{2} *are driven by a voltage of* 9 V. *Calculate the force required to close the* 5 μ*m gap between them.*

If we suppose that between the two plates there is air, the permittivity is $\epsilon = 8.85 \times 10^{-12}$ F/m, and the force can be calculated from equation (1.18):

$$F = \frac{\epsilon A V^2}{2g^2} = 8.85 \times 10^{-12} \frac{250 \times 10^{-6} \times 250 \times 10^{-6} \times 9^2}{2 \times 5^2 \times 10^{-12}} = 8.96 \times 10^{-7}\ \mathrm{N}.$$

If the dimensions are scaled by a factor of $\alpha = 0.1$ the force remains equal if equation (1.19) applies or 8.96×10^{-9} N if equation (1.21) applies.

1.5 Capillary Force

On the surface of a liquid the molecules are attracted by the other molecules inside the volume but do not have the attraction from the surroundings above the surface. This creates a situation where the molecules rearrange in order to expose the minimum surface. If an observer wants to increase the surface exposed to the ambient, he necessarily has to do some work. This work, dW, is proportional to the increase in area, dA [3]:

$$dW = \gamma dA. \tag{1.22}$$

The proportionality constant γ is the surface tension and has units of J/m^2 or, equivalently, N/m. Thus the surface tension is a measure of the surface energy per unit area.

When a liquid drop is in equilibrium, there is a pressure increase ΔP inside the drop, known as Laplace pressure, to prevent collapse. ΔP is related to the surface tension by

$$\Delta P = \gamma C, \tag{1.23}$$

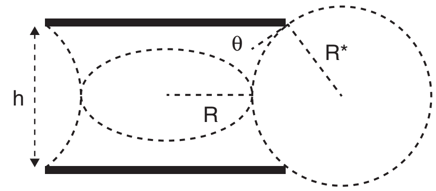

Figure 1.3 Parallel plates with droplet of liquid in between

where C is the curvature of the drop given by

$$C = \frac{1}{R} + \frac{1}{R^*}, \tag{1.24}$$

in which R and R^* are the radius of two mutually orthogonal circles drawn at a tangency point of the drop surface. R is the radius of the circle that lies inside the drop, and R^* that of the one lying outside and takes negative sign. This is shown in Figure 1.3, where the example of two parallel plates having a drop of liquid trapped inside is considered.

Due to equilibrium of surface tensions, a liquid on a substrate has a contact angle θ shown in Figure 1.3 (see also Section 7.5). We have that

$$\frac{h}{2} = R^* \cos\theta, \tag{1.25}$$

hence

$$\Delta P = \gamma\left(\frac{1}{R} - \frac{2\cos\theta}{h}\right). \tag{1.26}$$

In many MEMS applications $R \gg h$, and then equation (1.26) simplifies to

$$\Delta P \simeq -\gamma\frac{2\cos\theta}{h}. \tag{1.27}$$

Equation (1.27) shows that if the contact angle $0 < \theta < \pi/2$, then $\Delta P < 0$ and the force is inwards with respect to the liquid, whereas for $\pi/2 < \theta < \pi$, $\Delta P > 0$ and the force is outwards. If we suppose that the plates shown in Figure 1.3 are circular with radius R, then the force developed by capillarity between the two plates is

$$F = \pi R^2 \Delta P = -\gamma\frac{2\pi\cos\theta R^2}{h}. \tag{1.28}$$

When the geometrical dimensions are scaled,

$$F_\alpha = \gamma\frac{2\pi\cos\theta\alpha^2 R^2}{\alpha g}, \tag{1.29}$$

and the ratio is given by

$$\frac{F_\alpha}{F} = \alpha^1. \tag{1.30}$$

1.5.1 Example: Wet Etching Force

We have a MEMS process involving wet etching of a sacrificial layer between two parallel circular plates of radius $R = 2500$ μm. If the gap between the plates is $g = 2$ μm, the surface tension is $\gamma = 72.9 \times 10^{-3}$ N/m and the contact angle between the liquid and the substrate is 70°, calculate the force between the plates.

As the contact angle is smaller than $\pi/2$, the force is attractive and the value can be calculated from equation (1.28):

$$F = \gamma \frac{2\pi \cos \theta R^2}{g} = 72.9 \times 10^{-3} \frac{2\pi \cos 70(2500 \times 10^{-6})^2}{2 \times 10^{-6}} = 0.48\,\mathrm{N}.$$

1.6 Piezoelectric Force

Piezoelectricity is a property of some materials that generate electric charge when mechanically stressed and undergo a deformation when biased by an electric field. This phenomenon arises from a change in the crystallization of a material when subject to a process of simultaneous application of a high electric field and a high temperature known as 'polling'. Electrical dipoles are generated during polling that remain in the material thereafter. Piezoelectric layers can be used as displacement actuators or as force generators against a restraint.

The main equation of the direct piezoelectric effect is

$$D = dT + \epsilon \mathbf{E}, \tag{1.31}$$

where D is the electrical displacement vector, d the piezoelectric coefficient, ϵ the permittivity of the material, $\mathbf{E}$ the electric field and T the mechanical stress. In equation (1.31) the piezoelectric effect is anisotropic (see Chapter 4) and d is a tensor.

It can be seen that the electrical displacement D has two components: one conventional due to the electric field applied, and the other due to the mechanical stress. Conversely, the inverse piezoelectric effect is described by the equation

$$S = sT + d^T \mathbf{E} \tag{1.32}$$

where S is the strain (or relative deformation), s the compliance and d^T the transpose of the piezoelectric coefficient tensor. When an electric field is applied, assuming that the material has a force restraint F working against deformation, equation (1.32) can be written as

$$S = -s\frac{F}{A} + d\mathbf{E}, \tag{1.33}$$

where A is the cross-section of the material. Equation (1.33) shows that in the absence of any restraint ($F = 0$), the maximum displacement, or maximum stroke, is $S_{\max} = d\mathbf{E}$ and the

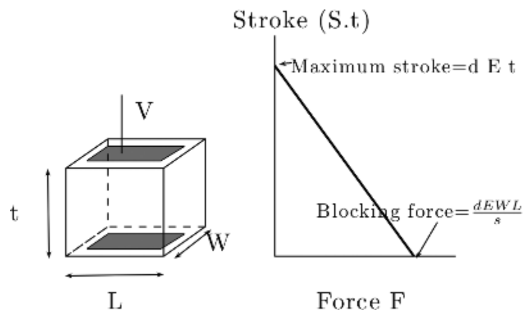

Figure 1.4 Force as a function of stroke

maximum stress $(F/A)_{\max}$ happens for zero deformation ($S = 0$) and is given, as shown in Figure 1.4. by

$$\left.\frac{F}{A}\right|_{\max} = \frac{d}{s}\mathbf{E}. \tag{1.34}$$

If we consider, as an indicator for the scaling scenario, the maximum force value, or blocking force, and that the applied electric field is given by V/t, where t is the material thickness and V the applied voltage, the force scales as

$$F = \frac{dAV}{st}. \tag{1.35}$$

Applying the scaling model,

$$\frac{F_\alpha}{F} = \alpha^1, \tag{1.36}$$

and if the scaling is performed at constant electric field, then

$$\frac{F_\alpha}{F} = \alpha^2. \tag{1.37}$$

1.6.1 Example: Force in Film Embossing

We have a piezoelectric material 2 μ*m thick and* 200 μ*m* × 200 μ*m in area. We apply a voltage of 10 V across the film and we want to know the deformation in the direction of the electric field that is achieved. Calculate the value of the maximum force that can be put in a wall preventing the deformation of the material, such as occurs in film embossing. We know that the film is made of ZnO,* $d = 12 \times 10^{-12}$ CN^{-1} *and* $s = 7 \times 10^{-12}$.

We first calculate the maximum strain $S_{\max}$,

$$S_{\max} = d\mathbf{E} = d\frac{V}{t} = 6 \times 10^{-4}, \quad \Delta t = St = 1.2\,\text{nm},$$

and the maximum force (or blocking force) for zero deformation is given by

$$F = A\frac{V}{t}\frac{d}{s} = (200 \times 10^{-6})^2 \frac{100}{2 \times 10^{-6}} \frac{12 \times 10^{-12}}{7 \times 10^{-12}} = 0.34\,\text{N}.$$

As can be seen, the piezoelectric actuators can generate large forces but small displacements.

1.7 Magnetic Force

One important MEMS application is the measurement of magnetic field for compasses [4, 5]. One way to detect a magnetic field uses the Lorentz force that develops when a wire carrying an electric current intensity I is immersed in a magnetic flux density B. If we know the intensity value and the length of the wire L, the Lorentz force is given by

$$F = I\vec{L} \times \vec{B} \tag{1.38}$$

where × indicates the cross product of the magnetic field vector and the wire length vector. This force is orthogonal to both the magnetic field and the wire direction. If the magnetic field and the wire are orthogonal, then the magnitude of the force is simply given by

$$F = ILB. \tag{1.39}$$

When scaling equation (1.39), one has to take into account that decreasing dimensions, most of the time, require also reducing the cross-section of the wire. If the current I is constant in the scaling, then the current density will increase and the ohmic losses will also increase as the resistance of the wire increases. It is then useful to consider that the magnitude that is kept constant in the scaling is the current density $J = I/A$, where A is the cross-section of the wire. Hence, equation (1.39) can be written as

$$F = JALB, \quad F_\alpha = J\alpha^2 A\alpha LB = \alpha^3 JALB, \tag{1.40}$$

and the ratio is given by

$$\frac{F_\alpha}{F} = \alpha^3. \tag{1.41}$$

1.7.1 Example: Compass Magnetometer

A magnetometer for a compass has to detect the Earth's magnetic field within the range of 0.25×10^{-4} *T to* 0.65×10^{4} *T. Calculate the force produced by a value* $B = 0.5 \times 10^{-4}$ *T in a wire of length* 2000 μ*m conducting an electrical current of intensity* $I = 10$ *mA.*

Using equation (1.40),

$$F = 10 \times 10^{-3} \times 2000 \times 10^{-6} \times 0.5 \times 10^{-4} = 1 \times 10^{-9}\text{N}.$$

1.8 Dielectrophoretic Force

The dielectrophoretic force is the force that a non-uniform electric field exerts on a particle [6, p. 5]. The particle can be modelled as an electric dipole as shown in Figure 1.5.

The force acting on the dipole is

$$\vec{F} = Q\vec{\mathbf{E}}(\vec{r} + \vec{d}) - Q\vec{\mathbf{E}}(\vec{r}). \tag{1.42}$$

Linearizing the electric field by the first term of the Taylor expansion,

$$\vec{\mathbf{E}}(\vec{r} + \vec{d}) = \vec{\mathbf{E}} + \vec{d}\nabla\vec{\mathbf{E}}(\vec{r}), \tag{1.43}$$

and substituting in equation (1.42),

$$\vec{F} = Q\vec{d}\nabla\vec{\mathbf{E}}(\vec{r}). \tag{1.44}$$

In the limit we consider that when $r \to 0$ the dipole moment $\vec{p} = Q\vec{d}$ remains finite. According to a more detailed derivation in Chapter 6, the dipolar moment of a particle can be written as an effective dipole moment $p_{\text{eff}} \simeq Qd$, and then

$$\vec{F} = \overrightarrow{p_{\text{eff}}}\nabla\vec{\mathbf{E}}. \tag{1.45}$$

The effective dipole moment is shown to be (see Chapter 7)

$$p_{\text{eff}} = 4\pi\epsilon_1 R^3 K\vec{\mathbf{E}}, \tag{1.46}$$

where R is the particle radius, ϵ_1 is the medium permittivity and K is the Clausius–Mossotti factor

$$K = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}, \tag{1.47}$$

in which ϵ_2 is the particle permittivity.

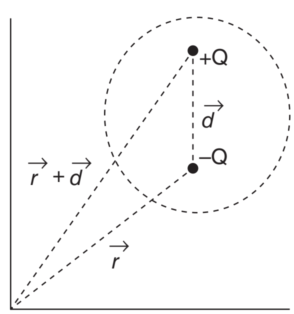

Figure 1.5 Dielectrophoretic force

The force on the particle is

$$\vec{F} = 4\pi\epsilon_1 R^3 K\vec{\mathbf{E}}\nabla\vec{\mathbf{E}}. \tag{1.48}$$

As the electric field is irrotational,

$$\vec{E}\nabla\vec{\mathbf{E}} = \frac{1}{2}\nabla(\vec{\mathbf{E}}\vec{\mathbf{E}}) = \frac{1}{2}\nabla|\mathbf{E}|^2. \tag{1.49}$$

The dielectrophoretic force is

$$\vec{F} = 2\pi\epsilon_1 R^3 K\nabla|\mathbf{E}|^2. \tag{1.50}$$

Equation (1.50) indicates that the force has the same direction as the gradient of the square of the modulus of the electric field and depends on the third power of the particle radius. If we suppose, as an example, that the electric field is created by a charge $Q*$ located at the origin of our system coordinates, the electric field has spherical symmetry and is $\mathbf{E} = Q^*/r^2$.

The dielectrophoretic force on a sphere of radius R is then given by

$$\vec{F} = \frac{8\pi\epsilon_1 R^3 K Q^{*2}}{r^5}. \tag{1.51}$$

If the scaling scenario considers that the radius of the particle and the distance from the particle to the centre are equally scaled, then the scaling factor is

$$\frac{F_\alpha}{F} = \alpha^{-2}. \tag{1.52}$$

If the distance is not scaled, then

$$\frac{F_\alpha}{F} = \alpha^{3}. \tag{1.53}$$

1.8.1 Example: Nanoparticle in a Spherical Symmetry Electric Field

Consider a polystyrene nanoparticle of radius 300 nm ($\epsilon_r = 2.5$) and assume that $\nabla|\mathbf{E}|^2 = 3.3 \times 10^{23}$ V^2/m^3. Calculate the force at a distance 10 times the diameter of the sphere.

Taking into account that the Clausius–Mossotti factor is 0.333, the force is $F = 6.59 \times 10^{-9}$ N.

1.9 Summary

The different forces involved in microelectromechanical devices scale differently when the dimensions are scaled down. Looking at the α^n scaling law, the larger the value of n the more significantly the forces are reduced when the dimensions are reduced. As far as the forces examined in this chapter are concerned, the weight is the force that will become less and less important in the microworld. On the other hand, in the examples shown in this chapter we can also see that the forces present in common examples of today's MEMS devices vary quite widely in magnitude.

Table 1.1 shows a comparison of the scaling laws and a summary of the results of the examples worked in this chapter. It can be seen that the capillary and piezoelectric forces are quite significant (of the order of tenths of newtons), whereas magnetic and elastic forces, for the examples selected, are quite small.

Table 1.1 Summary of scaling laws and examples of the magnitude of forces

Force	Scaling law	Magnitude (N)	Example
Weight	α^3	2.85×10^{-5}	1.2.1
Elastic	α^2	8.1×10^{-8}	1.3.1
Electrostatic	α^0, α^2	8.96×10^{-7}	1.4.1
Capillary	α^1	4.8×10^{-1}	1.5.1
Piezoelectric	α^1, α^2	3.4×10^{-1}	1.6.1
Magnetic	α^1, α^3	1×10^{-9}	1.7.1
Dielctrophoresis	α^{-2}, α^3	6.59×10^{-9}	1.8.1

Problems

1.1 Calculate and plot the elastic restoring force of a cantilever having width $W = 10\ \mu m$ and thickness $h = 3\ \mu m$ and for lengths from 20 μm to 2000 μm, when the deflection at the tip is 10% of the length. Take $E = 164 \times 10^9$ Pa.

1.2 For a silicon cantilever such as the one depicted in Figure 1.1, what is the most effective way to reduce the elastic constant k by a factor of 10 by changing only one of the dimensions? Similarly, what is the most effective way to increase k by a factor of 10?

1.3 We have an accelerometer based on an inertial mass from a cubic volume of silicon of 500 μm side. Find the density of silicon and calculate the force that creates such mass when accelerated at 60 times gravity. If the inertial mass is supported by a flexure having an elastic constant of 100 N/m, find the mass displacement when the two forces reach equilibrium. If the edge of the cubic volume is at 5 μm distance of a fixed electrode, find the capacitance value before the acceleration is applied to the mass and after. Assume that there is air in between the plates.

1.4 We have two plates of silver of area 250 μm × 250 μm and 50 μm thick. The upper plate is fixed and the bottom plate can move vertically. Calculate the minimum voltage that should be applied between the plates in order to start lifting the bottom plate.

1.5 We have two parallel electrodes at a distance of 4 μm in air, with a voltage $V_{CC} =$ 10V applied between them. The permittivity ϵ_1 of air is equal to the permittivity of the vacuum ϵ_0. One of the electrodes is covered by a dielectric 2 μm thick having a permittivity of $\epsilon_2 = \epsilon_0 \epsilon_r$ with $\epsilon_r = 3.9$. Calculate and plot the electric field in the air and inside the dielectric.

1.6 A thin cylindrical capillary of 5 mm diameter is immersed in water. The surface tension is $\gamma = 72.8 \times 10^{-3}$ N/m, the liquid density is $\rho_m = 10^3$ kg/m^3 and the acceleration due to gravity is $g = 9.8$ m/s^2. Calculate the height of the water inside.

1.7 Compare the surface energy of a drop of liquid, assumed spherical in shape with radius r, with its volume.

1.8 We have a spherical drop of 2 mm radius. If the surface tension is $\gamma = 72.8 \times 10^{-3}$ N/m, calculate the surface energy change if the drop radius is stretched by Δr and find the change in the internal pressure in equilibrium.

1.9 A piezoelectric actuator has to produce a displacement in the bottom plate of a reservoir to eject droplets of ink to produce 600 dots per inch. Assume that the ink dot thickness is 1 μm and that there is just one drop per dot. Calculate the diameter of the dot, the volume of the drop and the radius of the drop (assumed to be equal to the radius of the ejecting nozzle). Calculate the vertical expansion required for a piezoelectric actuator acting on a cylindrical ink reservoir of 2 mm diameter. The thickness of the piezoelectric material is 10 μm.

1.10 We have a flexure made of gold and an electrical current of 10 mA circulates through it. If we immerse the flexure in a magnetic field normal to the plane of the flexure, calculate the force and indicate the direction of the movement. Take $L = 2000$ μm, $B = 0.25 \times 10^{-4}$ T and the elastic constant of the flexure $k_x = 0.01$ N/m.

1.11 We have a 300 nm diameter polystyrene nanoparticle immersed in air, and an electric field created by a sphere carrying a total charge of Q. The distance between the particle and the sphere is 10 times the polystyrene sphere diameter. Calculate the force and the direction. Repeat the calculation if the medium is changed to ethylene glycol. Ethylene glycol has a relative permittivity of 37 and polystyrene 2.5.