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## Brief History and Basic Principles of Predictive Control

### 1.1 Generation and Development of Predictive Control

Predictive control, later also called Model Predictive Control (MPC), is a kind of control algorithm originally rising from industrial processes in the 1970s. Unlike many other control methods driven by theoretical research, the generation of predictive control was mainly driven by the requirements of industrial practice. For quite a long time the industrial process control mainly focused on regulation using the feedback control principle. The well-known PID (Proportional–Integral–Differential) controller can be used for linear or nonlinear processes, even without model information, and has few tuning parameters and is easy to use. These features are particularly suitable for the control environment in industrial processes and make it a “universal” controller that is widely used. However, the advantage of the PID controller is mainly embodied in the loop control. When the control turns from a loop to the whole system, it is difficult to achieve good control performance by using such a single-loop controller without considering the couplings between the loops. Furthermore, a PID controller can handle input constraints but is incapable of handling various real constraints on outputs and intermediate variables. Particularly, when the control goal is promoted from regulation to optimization, this kind of feedback-based controller seems powerless because it lacks understanding of the process dynamics. With the development of industrial production from a single machine or a single loop to mass production, the optimization control for constrained multivariable complex industrial processes became a new challenging problem.

During this period, modern control theory was rapidly developed and went to mature with brilliant achievements in the aerospace, aviation, and many other fields. At the same time the progress of computer technology provided a powerful tool for real-time computing. Both were undoubtedly attractive for the industrial process control engineers when pursuing higher control quality and economic benefits. They began to explore the applications of the mature modern control theory, such as optimal control, pole placement, etc., in optimization control of the complex industrial processes. However, through practice they found that a big gap existed between the perfect theory and the real industrial processes, mainly manifested in the following:

- 1) Modern control theory is based on an accurate mathematical model of the plant, while for a high-dimensional multivariable complex industrial process, it is hard to get its accurate mathematical model. Even if such a model could be established, it

should be simplified according to practical applicability and a strictly accurate mathematical model is not available.

- 2) The structure, parameters, and environment of an industrial plant are often uncertain. Due to the existence of uncertainty, the optimal control designed based on an ideal model would never remain optimal in real applications, and would even result in serious degeneration of the control performance. In an industrial environment, the control system is more focused on robustness, i.e. the ability of keeping better performance under uncertainty, rather than pursuing ideal optimality.
- 3) The industrial control must take the economics of the control tools into account. The control algorithm should satisfy the real-time requirement. But most of the algorithms in modern control theory seem complex and are difficult to implement by economic computers.

Due to the above issues coming from practice, it is hard to directly use modern control theory for complex industrial processes. To overcome the gap between theory and practice, in addition to investigating system identification, robust control, adaptive control, etc., people began to break the constraints of traditional control methods and tried to seek new optimization control algorithms in accordance with the characteristics of the industrial processes, i.e. with a low requirement for the process model, capable of dealing with multivariables and constraints, and an acceptable computational burden for real-time control. The appearance of predictive control just fitted these needs.

The earliest predictive control algorithms arising from industrial processes include Model Predictive Heuristic Control (MPHC) [1], or similarly Model Algorithmic Control (MAC) [2], and Dynamic Matrix Control (DMC) [3]. They use the process impulse or step response as a nonparametric model, which is easy to obtain from the process data, and the controller can be designed directly based on these responses without further identification. Such predictive control algorithms absorb the idea of optimization in modern control theory, but instead of solving an infinite horizon optimization problem off-line, they introduce a rolling horizon mechanism with solving a finite horizon optimization problem on-line, meanwhile at each step making feedback according to real-time information. These features make them avoid the difficulty of identifying a minimal parametric model, reduce the real-time optimization burden, and strengthen the robustness of the control systems against uncertainty, which are particularly suitable to practical requirements of industrial process control. Therefore, after their appearance, they were successfully applied to process control systems in oil refining, chemical industry, electric power, and other industries in the USA and Europe, and attracted wide interest from the industrial control community. Thereafter, various predictive control algorithms based on impulse or step response were proposed in succession, and a number of predictive control software packages for process control of different installations were soon launched and quickly generalized and applied to process control of various industries. For more than 30 years, predictive control has been successfully run in thousands of process control systems globally, and achieved great economic benefits. It has been recognized as the most efficient advanced process control (APC) method with great application potential [4].

Besides the predictive control algorithms based on nonparametric models directly from industrial process control, another kind of predictive control algorithm also appeared in the adaptive control field, motivated by improving the robustness of adaptive

control systems. In the 1980s, in adaptive control research it was found that in order to overcome the disadvantages of minimum variance control, it is necessary to absorb the multistep prediction and optimization strategy of industrial predictive control algorithms so as to improve the robustness of control systems against time delay and model parameter uncertainty. Then a number of predictive control algorithms based on identified minimum parametric models and self-adaption also appeared, such as Extended Prediction Self-Adaptive Control (EPSAC) [5], Generalized Predictive Control (GPC) [6], etc. Among those, the GPC algorithm has particularly attracted wide attentions and got applications. The identified minimum parametric model adopted in these predictive control algorithms provides a more rigorous theoretical basis for control system analysis and design, which greatly promoted the theoretical research of predictive control.

With the appearance of the above predictive control algorithms, various methods and strategies were also developed for predictive control applications to suit specific system dynamics and different application scenarios. These studies could be roughly divided into several categories:

- appropriate strategies and algorithms for specific kinds of systems, such as the Hammerstein model [7], hybrid systems [8], etc.;
- reasonable control structures for efficiently utilizing process information or reducing control complexity, such as cascade [9], hierarchical [10], decentralized [11], distributed control schemes [12, 13], etc.;
- heuristic strategies for improving the usability of predictive control algorithms, such as the blocking technique [14], multirate control [15], etc.;
- effective predictive control algorithms and strategies particularly for nonlinear systems, such as fuzzy [16], neural network [17], multimodel [18], etc.

These studies not only strongly supported the applications of predictive control but also facilitated some new sub-branches of predictive control, such as hybrid predictive control [19], hierarchical and distributed predictive control [20, 21], economic predictive control [22], and so on.

While predictive control has found successful applications in industry, its theoretical research naturally became the hotspot of control academia. To meet the application requirements, the first theoretical issue to be considered is how the design parameters in predictive control algorithms are related to control system performance, which can guide the parameter tuning to guarantee stability and tracking performance of predictive control systems and is of great significance to practical applications. This problem was explored in the  $Z$  domain and the time domain, respectively, using different tools. In the  $Z$  domain, Garcia and Morari [23] proposed Internal Model Control (IMC) in 1982 as a general framework for analyzing predictive control systems. Through transforming the predictive control algorithms into the IMC framework, some quantitative relationships between the main design parameters and the closed-loop performances were established for nonparametric model-based predictive control algorithms such as DMC [24]. Later on more theoretical results uniformly applicable to both DMC and GPC algorithms were obtained by using coefficient mapping of the characteristic polynomials of open-loop and closed-loop systems in IMC structures [25]. In the time domain, Clarke and Mohtadi [26] transformed the GPC algorithm into an optimal control problem described in state space and analyzed the system performance as well as the deadbeat property with the

help of existing results in Linear Quadratic (LQ) optimal control. Some later studies on GPC borrowed more ideas from optimal control theory and adopted the monotonically decreasing property of the receding horizon cost and the endpoint equality constraints on the tracking error to guarantee the stability of GPC [27]. All of the above theoretical researches achieved some progress, but the results were quite limited due to the essential difficulties of quantitative analysis for predictive control systems. Firstly, quantitative analysis should be based on analytical expressions of system and control law, but in most cases, predictive control should handle various constraints and the control action can only be obtained through solving a constrained optimization problem. Without analytical expression of the control law it is impossible to make a quantitative analysis. Secondly, even if analytical forms for a predictive controller and the closed-loop system could be derived in an unconstrained case, there are no direct relationships between the design parameters and the closed-loop performances, which prevents more accurate and useful quantitative results to be obtained. Therefore, most obtained analytical results in this period are only available for single-input single-output (SISO) unconstrained predictive control systems, which is far from the need of real applications.

In view of this situation, the theoretical research for predictive control turned from quantitative analysis to qualitative synthesis in the 1990s. This study was no longer limited to existing algorithms but focused on synthesizing new algorithms to guarantee system performance. A key idea of this study is to regard predictive control as traditional optimal control with the only difference being in implementation. After formulating the system model and the rolling optimization procedure in state space, a predictive controller with guaranteed stability can be investigated with the help of mature ideas and tools in optimal control theory, particularly the ideas and methods used in Receding Horizon Control (RHC) [28] proposed in the 1970s. During this period, a great number of new predictive control algorithms with guaranteed stability were proposed by introducing novel techniques, such as terminal equality constraints [29], terminal cost function [30], terminal constraint set [31], terminal cost and constraint set [32], etc. A highlighted feature of such an investigation is to artificially impose some constraints to theoretically guarantee stability of the predictive control system. An excellent survey was comprehensively given by Mayne et al. [33] on the stability and optimality of constrained model predictive control. Promoted by stable predictive control theory and robust control theory, robust predictive control for uncertain systems was also investigated. In fact, it appeared very early [34], but has become a hotspot only since the middle of the 1990s, mainly due to introducing novel ideas and new mathematical tools. A representative work by Kothare et al. [35] in 1996 studied the robust stability of model predictive control for a large class of plant uncertainties using linear matrix inequalities (LMIs), which stimulated a lot of follow-up researches in later years.

Different from the early quantitative analysis theory, these new works concerned quite general systems and problems, including nonlinear systems and systems with constraints and various uncertainties. Taking optimal control theory as a reference, Lyapunov stability analysis as a basic method to guarantee system performance, invariant set, linear matrix inequality (LMI) as fundamental tools, and the performance analysis for rolling horizon optimization as the core of the study, they constituted rich contents and achieved fruitful results, showing academic profundity and methodological innovations. In the last two decades, hundreds of papers have appeared in main control journals and formed the mainstream of current theoretical research on predictive control [33, 36]. However, the conclusions achieved from these studies often lack clear physical

meanings and the developed algorithms still have a big gap with the requirements of industrial applications.

Throughout historical development of predictive control, roughly three climax stages appeared: the first stage is characterized by the industrial predictive control algorithms based on step or impulse response models developed in the 1970s. These algorithms are very suitable for the requirements of industrial applications, both in model selection and in control ideas, and therefore became the main algorithms used in industrial predictive control software packages. However, without theoretical guidance, application of these algorithms greatly depends on the specific knowledge and user experiences. The second stage is characterized by the adaptive predictive control algorithms developed from the adaptive control field in the 1980s. The models and control ideas of these algorithms are more familiar to the control community and thus more suitable for investigating the predictive control theory. Indeed, some quantitative analytical results for predictive control systems have been achieved. However, the essential difficulty of quantitative analysis always exists because of the lack of the analytical expression of the optimal solution for constrained optimization. The third stage is characterized by the predictive control qualitative synthesis theory developed since the 1990s. Because of the change in research ideas, this has given the predictive control theory a great leap forward and become the mainstream of current predictive control theoretical research. However, these results still have a big gap with practical applications of predictive control. Although the studies of the above three stages have had their own problems, after development of these stages, predictive control has become a diversified control branch, containing many development paths with different purposes and different characteristics. It is not only regarded as the most representative APC algorithm and favored by the majority of the industrial community, but also forms a theoretical system of stable and robust design for uncertain systems with rolling optimization characteristics. Figure 1.1 roughly gives

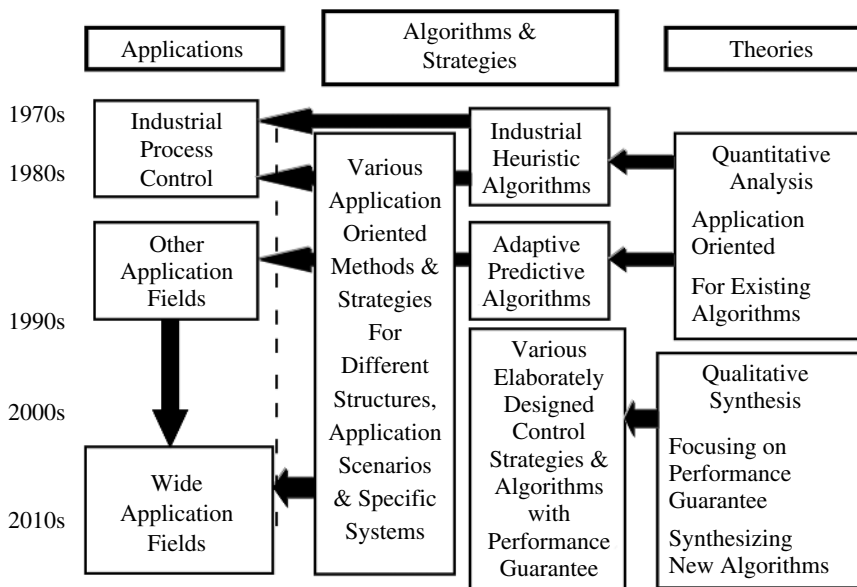


Figure 1.1 The development trajectory of predictive control.

an overall historical trajectory of predictive control where different aspects and interests are shown in parallel.

## 1.2 Basic Methodological Principles of Predictive Control

Although there exists a wide variety of predictive control algorithms with different forms, they have common characteristics in methodological principles, i.e. to predict future system dynamic behavior according to given control actions by using the system model, to control the plant in rolling style through online solving an optimization problem with required performance subject to given constraints but only implementing current control, and to correct the prediction of future system behavior by using real-time measured information at each rolling step, which can be summarized as three principles, i.e. prediction model, rolling optimization, and feedback correction [37].

### 1.2.1 Prediction Model

Predictive control is a model-based control, where the model is called a prediction model. A prediction model serves as the basis of optimization control. Its main function is to predict the future dynamic variations of system states or outputs according to historical information and assumed future system inputs. The model is particularly emphasized by its function rather than its structural form. Therefore, traditional models such as a transfer function, state space equation, etc., can be used as a prediction model. For stable linear systems, even nonparametric models such as step or impulse responses can be directly used as a prediction model without further identification. Furthermore, models for nonlinear systems, distributed parameter systems, etc., can also be used as prediction models as long as they have the above function.

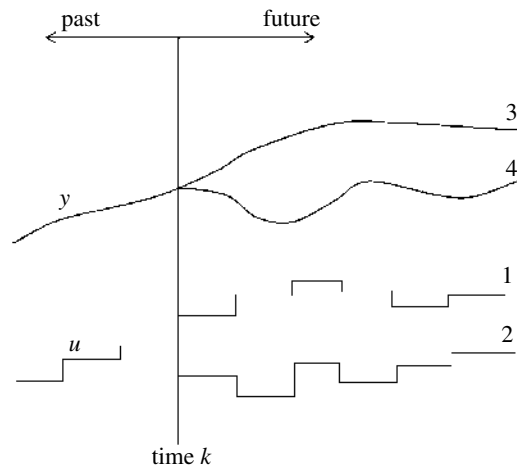
A prediction model is capable of exhibiting the future dynamic behavior of the system. Given future control policy arbitrarily, the future system states or outputs can be predicted according to the prediction model (see Figure 1.2) and, furthermore, the satisfaction of constraints can be judged and the performance index can be calculated. A prediction model thus provides the basis to compare the qualities of different control policies and is the prerequisite of optimization control.

### 1.2.2 Rolling Optimization

Predictive control is an optimization-based control. It determines future control actions through optimization with a certain performance index. This performance index involves future system dynamic behavior, for example, to minimize the output tracking errors at future sampling times or, more generally, to minimize the control energy while keeping the system output in a given range, etc. The future system dynamic behavior involved in the performance index is determined by future control actions and is based on the prediction model.

It should be pointed out that the optimization in predictive control is quite different from that in traditional discrete-time optimal control. In the predictive control

**Figure 1.2** Prediction based on model.  
 1 – control policy I; 2 – control policy II;  
 3 – output w.r.t. I; 4 – output w.r.t. II.

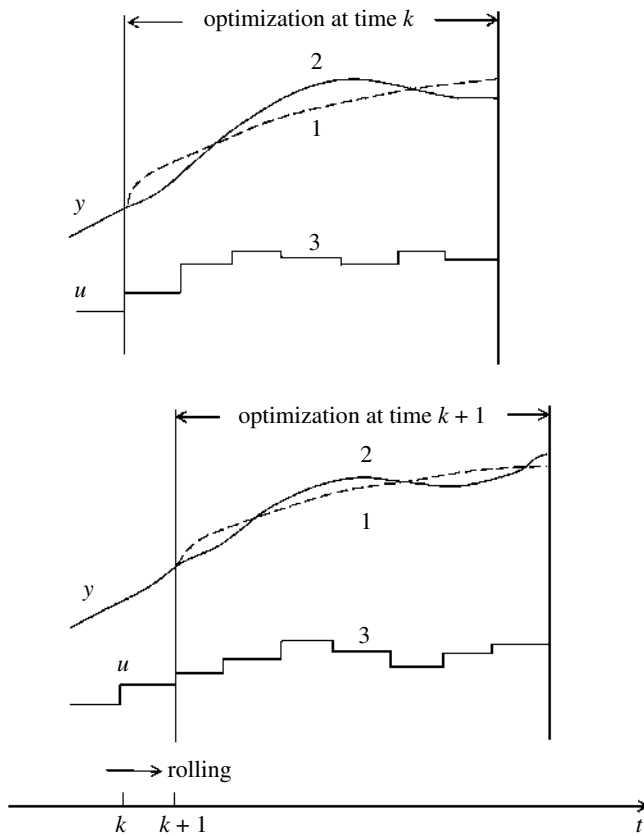


algorithms used in industrial processes, the optimization is commonly over a finite horizon and implemented in a rolling style. At each sampling time, an open-loop optimization problem with a performance index over a future finite time horizon is solved where a certain number of future control actions are taken as optimization variables. However, the solved optimal control actions do not need to be implemented one by one; instead only the current control action is implemented. At the next sampling time, the optimization horizon is moved one step forward (see Figure 1.3). Therefore, in predictive control, the optimization performance index is time dependent. The relative formulation of the performance index at each time is the same, but the concrete time interval is moved forward and is different. This indicates that the predictive control uses on-line repeatedly performed finite horizon optimization to replace the infinite horizon off-line optimization once solved in traditional optimal control, which characterizes the meaning of rolling optimization and is the specific feature of the optimization in predictive control.

### 1.2.3 Feedback Correction

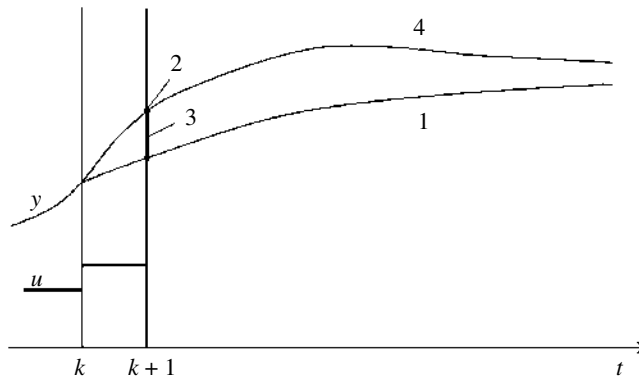
Predictive control is also a feedback-based control. The above-mentioned rolling optimization might be an open-loop optimization if it is only based on the prediction model. In practice, there unavoidably exist uncertainties such as model mismatch, unknown disturbance, etc., and the real system behavior may deviate from the ideal optimal one. For compensating the influence of various uncertainties to system behavior to a certain extent, a closed-loop mechanism is introduced into predictive control. At each sampling time, the measured real-time information for system states or outputs should be utilized to update or correct the future system behavior before solving the optimization problem. In this way, prediction and optimization at that time could be put on a basis that is closer to the real status. We call this procedure feedback correction.

The ways of feedback correction are diverse. In industrial predictive control algorithms, each time after the current control action has been calculated and implemented,



**Figure 1.3** Rolling optimization. 1 – reference trajectory; 2 – predicted optimal output; 3 – optimal control action.

future outputs under this control action can be predicted by using the prediction model. Next time, the prediction error constructed by measured real output and predicted output can then be utilized to heuristically correct the prediction of future outputs (see Figure 1.4). By using predictive control algorithms developed from adaptive control, feedback correction can be implied in the adaption mechanism of adaptive control, i.e. by identifying the model using real-time input/output information and online updating the prediction model and the control law. For predictive control algorithms based on the state space model, feedback correction may be taken as different forms according to the available information. If the system states are real-time measurable, they can be directly used as a new basis for prediction and optimization each time. However, if the states are unmeasurable, it is necessary to correct the predicted states through constructing an observer according to the real-time measured system outputs. No matter which form of feedback correction is adopted, each time predictive control always tries to put the optimization on a basis that is closer to the real status by feedback of measured real-time information. Therefore, although each time predictive control performs an



**Figure 1.4** Error correction. 1 – predicted output trajectory at  $k$ ; 2 – real output at  $k + 1$ ; 3 – predictive error; 4 – predicted output trajectory after correction at  $k + 1$ .

open-loop optimization, the rolling implementation combined with feedback correction makes the whole process become a closed-loop optimization.

According to this brief introduction of the general principles of predictive control, it is not difficult to understand why predictive control is so favored in complex industrial environments. Firstly, for a complex industrial plant, the big cost of identifying its minimum parametric model always brings difficulty for control algorithms based on transfer function or state space models. However, the model used in predictive control is more convenient to obtain because it emphasizes its prediction function rather than its structural form. In many cases, the prediction model can be directly obtained by measuring the system step or impulse response without further identifying its transfer function or state space model, which is undoubtedly attractive for industrial users. More important is the fact that predictive control absorbs the idea of optimization control, but performs rolling horizon optimization combined with feedback correction instead of solving global optimization. In this way, not only is the big computational burden for solving global optimization avoided but also the uncertainty caused by the inevitable model mismatch or disturbance in industrial environment can be constantly taken into account and their influence can be timely corrected. Thus predictive control can achieve stronger robustness as compared with the traditional optimal control only based on the model. In this sense, predictive control may be regarded as a type of new control algorithm that can amend the shortcomings of traditional optimal control applying to industrial processes.

The basic principles of predictive control actually reflect the general thinking of humans when handling problems where optimization is sought but with uncertainty. For example, when a person goes across a road, there is no need to see whether there are vehicles far away; only the vehicle status a few tens of meters away and the estimation of their velocities (model) are important. However, it is wise to look into the distance while walking in order to avoid accidents caused by new vehicles entering (disturbance) or unpredicted acceleration of vehicles (model mismatch). This repeated decision process indeed includes optimization based on the model and feedback based on real-time information. Actually, this kind of rolling optimization method in an uncertain environment has already appeared in economics and management fields. Both predictive

economics and rolling planning in business management adopt the same thinking. In predictive control the methodological principle within was absorbed and combined with control algorithms, so it can be effectively applied to controlling complex industrial processes.

### 1.3 Contents of this Book

For over 30 years, predictive control has been developed into a diversified research field, including algorithms, strategies, theories, and applications. The contents of predictive control are extremely rich and relevant results and literatures are numerous. This book will comprehensively introduce the fundamentals and new developments of predictive control from a broader perspective, including basic principles and algorithms, system analysis and design, algorithm development, and practical applications, aiming to reflect the development trajectory and core contents of predictive control. The detailed contents of each chapter in the book are as follows.

Chapter 1 gives a brief introduction of the development trajectory of predictive control and its methodological principles.

In Chapter 2, three typical predictive control algorithms, based on the step response model, stochastic linear difference equation model, and state space model, respectively, are introduced for unconstrained SISO systems, focusing on illustrating how the basic principles are embodied in the algorithms when different models are adopted.

Chapter 3 adopts the IMC structure to deduce the analytical expression of the unconstrained SISO DMC algorithm in the  $Z$  domain. After analyzing the controller and the filter in the IMC structure, some trending relationships of stability/robustness with respect to design parameters are provided, based on which the parameter tuning method is suggested with illustration examples.

Chapter 4 presents the main results of quantitative analysis theory of predictive control. The quantitative relationships between the design parameters and the closed-loop performances are achieved with the help of the Kleinman controller in the time domain and by establishing the coefficient mapping of the open-loop and the closed-loop characteristic polynomials in the  $Z$  domain, respectively.

In Chapter 5, multivariable constrained predictive control algorithms are presented with DMC as an illustration example, focusing on the description of online optimization and the solving algorithms. The decomposition concept to reduce the computational complexity is introduced and implemented by hierarchical, decentralized, and distributed predictive control algorithms.

Chapter 6 illustrates the basic ideas of the qualitative synthesis theory of predictive control. Fundamental approaches of synthesizing stable predictive control systems are presented. General conditions for a stable predictive controller are given and suboptimality is analyzed.

Fundamental materials of synthesizing robust model predictive control (RMPC) are given in Chapter 7, including basic philosophy and main developments of RMPC for polytopic uncertain systems, and typical RMPC algorithms for systems with disturbances. The main difficulties of RMPC synthesis are pointed out, for which some efficient strategies and improved algorithms are introduced.

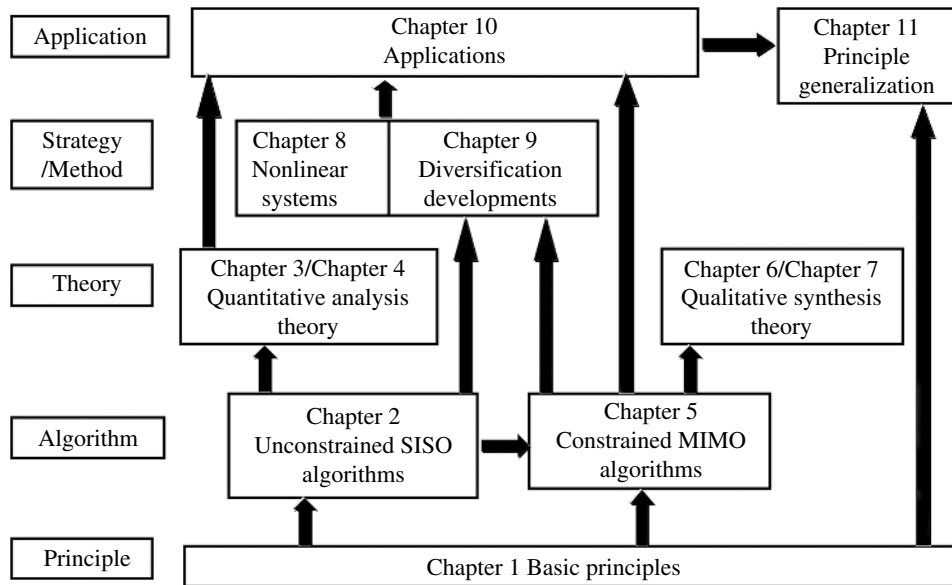


Figure 1.5 The relationship of the chapters in this book.

Chapter 8 focuses on predictive control for nonlinear systems. The general description of predictive control for nonlinear systems is given. Some commonly used strategies or algorithms are introduced, including multilayer, multimodel, neural network methods, and a specific strategy for Hammerstein models.

Chapter 9 briefly introduces the diversified development of algorithms and strategies for predictive control applications, including some effective control structures, different optimization concepts and goals, high efficiency control strategies, etc.

Chapter 10 describes the general status of industrial applications of predictive control with illustration examples. The applications of predictive control for large-scale networked systems, embedded systems, and its extension to other fields are also introduced with examples.

In Chapter 11, the basic principles of predictive control are interpreted from the viewpoints of cybernetics and information theory. These principles are then generalized to other optimization-based dynamic decision problems with illustration examples of production scheduling and robot path planning.

The relationships between the above chapters with the predictive control principles, algorithms, strategies, theories, and applications are roughly shown in Figure 1.5.

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